Towards the solution to the giant graviton puzzle

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In this paper we present several ideas toward the solution to the giant graviton puzzle—the apparent multiplicity of supergravity states dual to field theory chiral primary operators. We use the fact that, for certain ranges of the angular momentum, giant gravitons can be mapped into vacua of a dual theory to argue that the sphere and AdS giant gravitons have very different boundary descriptions and that an unpolarized Kaluza Klein graviton is unphysical in the regime where giant gravitons exist. We also show that a generic boundary state can correspond to different giant graviton configurations, which have nonoverlapping ranges of validity.

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I. REVIEW OF THE PUZZLE

In the $\mathrm{AdS}_5 \times S^5$ supergravity dual of $\mathcal{N} = 4$ super Yang Mills (SYM) we have three distinct types of configurations which correspond to a boundary state with large $\mathcal{R}$ charge. One is a graviton circling the "equator" of the S^5, the second is a graviton polarized [1] into D3 branes wrapping an $S^3 \subset S^5$ (the giant graviton or sphere giant graviton) [2], and the third is a graviton polarized into D3 branes wrapping an $S^3 \subset \mathrm{AdS}_5$ (the dual giant graviton or AdS giant graviton.) [3–5]

The field theory interpretation of these states is an interesting issue. The original puzzle was that single-trace chiral primary operators in the field theory with $\mathcal{R}$-charge L should be dual to single particle states with angular momentum L on the S^5. The natural candidate for these states is the graviton. However, at finite N, there is a cutoff in the field theory since there are no independent single-trace operators with $L > N$, yet there is no obvious upper bound on the angular momentum of the graviton. The predicted upper bound was thought to be due to stringy effects and dubbed the "stringy exclusion principle" [2]. Instead, the resolution turned out to be an IR effect where the gravity dual was identified as the giant graviton. The size of the S^3 which the brane wraps grows with the angular momentum until the upper bound of $L = N$ is reached where the radius of the S^3 reaches the radius of the S^5.

Unfortunately there are two problems with the above picture. The first is essentially a technical point—for L of order $N^{2/3}$ the single-trace operators are no longer orthogonal (even at large N). However, this does not affect the argument since the correct operators are subdeterminant operators [6] which are also cut off at $L = N$. The second point, which we address in this paper, is that there is not only the question of whether the extended giant gravitons should be preferred over the pointlike gravitons but that the extended AdS giant gravitons also carry the same quantum numbers, appear to have similar properties to the giant gravitons, but crucially have no upper limit on L since the sphere they wrap can be arbitrarily large within the AdS spacetime. So, clearly the giant graviton is the one which should correspond to the field theory state, but how do we rule out the other two states?

The presence of these two extra states has long been a puzzle, and different arguments have been made about their fate. One possible explanation is that they correspond to two short multiplets which combine to form a long multiplet, whose dimension is no longer protected [3]. However, as we will see, this is not what seems to happen.

The existence of many bulk states with polarized branes dual to only one boundary state is highly reminiscent of a similar problem in gauge-gravity dualities. When one discusses the supergravity dual of the $N = 1^*$ theory [7], one also encounters three bulk vacua dual to one gauge theory vacuum. As we will review in the first chapter, a generic field theory vacuum can naively have three supergravity duals. One candidate dual bulk contains D3 branes polarized into NS5 branes, another one contains D3 branes polarized into D5 branes, and the third one has a singularity and does not contain any polarized branes.

Fortunately the solution to this puzzle is known [7]. The first piece of the solution is that the two candidate duals which contain polarized branes have nonoverlapping ranges of validity. The second piece of the solution is that the vacuum where the D3 branes are not polarized is unphysical. Indeed, in [7] *all* the $\mathcal{N} = 1^*$ vacua (found in the field theory analysis of [8,9]) were mapped to supergravity brane configurations, and there is no field theory vacuum which is dual to the bulk vacuum with no polarization. Therefore, that vacuum has too big an action compared to the polarized configurations and does not contribute to the AdS-CFT duality [10].

The purpose of this paper is to extend this analysis to giant gravitons and to show that the giant graviton puzzle is solved in an essentially identical way. To do this one needs to make a conceptual jump, from regarding the giant gravitons as *states* in the four-dimensional boundary theory to regarding them as *vacua* of an auxiliary theory, which lives on L coincident gravitons in this background. This

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theory has been discussed in [11] and was successfully used in [12] to give a microscopic description to some of the giant gravitons.[1] When the giant gravitons sit in the ''near-graviton'' region, they are indeed dual (by the BDHM extension of the AdS-CFT correspondence [13]) to vacua of this theory.

Introducing this auxiliary theory into the puzzle makes the dictionary between the giant gravitons and the CFT chiral primaries two-step. One first relates the states of the CFT to the vacua of the auxiliary theory, and then relates these vacua to the giant gravitons. Fortunately, each step is conceptually rather simple. The dictionary between the gauge theory states and the vacua of the auxiliary theory is one to one and not hard to guess. As we will see in Sec. III, this dictionary relates vacua and states of two theories which are both strongly coupled when supergravity is weakly coupled, and vice versa. Thus the fact that it is one to one is quite natural.

The three to one degeneracy we had in the original dictionary is now mapped into a three to one degeneracy in the map between a vacuum of the auxiliary theory and its three candidate dual giant graviton configurations. Fortunately, this problem is almost identical to the one solved in [7].

The physics behind the two maps is conceptually rather simple. Like in the $N = 1^*$ case, the unpolarized configuration [the Kaluza Klein (KK) graviton] is unphysical, and has no field theory dual.[2] Moreover, a given chiral primary can have two giant graviton duals, which have nonoverlapping ranges of validity. The sphere and AdS giant gravitons have therefore completely different origins. Intuitively, the sphere giant comes from one ''single particle'' state with angular momentum/$\mathcal{R}$-charge L (roughly speaking a single-trace operator of L Φ's), while the AdS giant comes from L single particle states with angular momentum/$\mathcal{R}$-charge one (an operator with a product of L traces of Φ.).

We also should note that the map we propose matches very well with, and extends the proposal of Corley, Jevicki, and Ramgoolam (CJR) [14] for the chiral primaries dual to the AdS and sphere giant gravitons. Although this proposal gives a better understanding of the mapping between extended objects and field theory operators, it does not solve the giant graviton problem, since an operator represented by a rectangular Young tableau of size $L \times H$ can correspond to either L sphere giants of angular momentum H, or to H AdS giants of angular momentum L. As we will see, our proposal maps this degeneracy to the one encountered in $N = 1^*$ theory, and thus resolves this puzzle rather nicely.

[1] In this paper we analyze this auxiliary theory only implicitly, by relating it via dualities to much better understood theories.

[2] This was also argued in the original giant graviton paper [2], by estimating the action of the KK graviton and finding it divergent.

In Sec. II we dualize the geometries sourced by KK gravitons in the AdS $\times$ S spacetimes to geometries where the bulk-boundary duality is very well understood. We then argue that the solution corresponding to the unpolarized graviton is nonphysical, by using the dual field theory. The pointlike graviton with angular momentum of order N can also be directly ruled out as a sensible classical solution [2] since it receives large quantum corrections because of its very large energy density. Our T-duality arguments extend this to all pure graviton states in the regime where giant gravitons exist.

In Sec. III we review the $N = 1^*$ duality (which is the prototypical duality between field theories and bulks with many polarized branes) as well as its trivial extension to the theory on a large number of D0 branes. We also present the one to one map between CFT chiral primaries and vacua of the auxiliary theory, and thus complete our proposal. In the Appendix A we explore the ranges of validity of our construction.

Before proceeding we should remark that in the case of the $AdS_5 \times S^5$ giant gravitons, the brane configurations corresponding to the two gravitons are very similar. Therefore, we will initially concentrate on the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ cases, where the distinction between sphere and AdS giant gravitons is more clear and then argue that the same picture extends to the $AdS_5 \times S^5$ case.[3]

II. DUALIZING THE GIANT GRAVITONS

Perhaps the easiest way to understand the three (giant) graviton states of angular momentum L is to do a T-duality along the momentum of the graviton. The resulting static configuration consists locally of L F1 strings, in some transverse fields. These F1 strings can appear in three incarnations—by themselves and polarized into a D4 brane of geometry $R^{1,1} \times S^3$ where the S^3 can sit in either group of four (picked out by the background flux) of the eight transverse directions.

Note that even though the geometry resulting after the T-duality is singular at the poles of the sphere, we know that string theory on that background makes sense. The singularity of the T-dual supergravity solution comes from the fact that winding modes near the origin can shrink to zero size, and thus it is an artifact, signaling the breakdown of the supergravity approximation to string theory. However, as we will see in the Appendix A, the auxiliary theory describes the giant gravitons only in a region near the equator. Thus, the breakdown of the supergravity approximation happens in a region away from the F1 strings we are analyzing.

Near the F1 strings, one cannot consider them any more as a perturbation on the geometry. They become a *source* for the geometry. The geometry near the strings is the near-

[3] The $AdS_3 \times S^3$ case appears to be very different from the other cases [15].

horizon F1-string geometry perturbed with some transverse fluxes (coming from the 5-form field strength of the $AdS_5 \times S^5$ geometry), which can cause the strings to be polarized into D4 branes. The M-theory lift of this geometry is very reminiscent of the geometry dual to the massive flow of the M2 brane worldvolume theory [16]. In fact, it is quite easy to see (using the fact that both geometries allow brane polarization and that both are supersymmetric) that this M-theory lift is the massive $AdS_4 \times S^7$ flow geometry in which M2 branes in transverse fields polarize into M5 branes.

One can also examine the M-theory giant gravitons and see that by dimensionally reducing them along their momentum they correspond locally to D0 branes polarized into D2 branes and NS5 branes, respectively. This geometry is also the gravity dual of the field theory on the D0 branes perturbed by a chiral multiplet mass. Like in the previous case, the IIA geometry obtained by reducing the M-theory giant gravitons along their momentum is singular at the poles. However, this only signifies the breakdown of the IIA supergravity approximation, and the background makes perfect sense if one considers the full M-theory. As before, the physics we are interested in happens in a region away from where IIA supergravity breaks down.

The theory on the D0 branes is just supersymmetric matrix quantum mechanics. The background fluxes in which these D0 branes sit are a transverse Neveu-Schwarz-Neveu-Schwarz 3-form field strength and a transverse Ramond-Ramond 6-form field strength. It is not hard to see (either by T-dualizing 3 times the Polchinski-Strassler setup, or by analyzing the non-Abelian Born Infeld action of the D0 branes) that the effect of these fluxes on the D0 branes is to induce a mass for three of the chiral multiplets of the supersymmetric matrix quantum mechanics. The supergravity dual of the perturbed theory has now many vacua, which contain polarized branes. The structure of these vacua is identical for any Dp branes put in a transverse H_3 and F_{6-p}. The D3 brane case has been analyzed in [7] and the D2 brane case has been analyzed in [17].

In the next section we review the example of D3 branes in transverse H_3 and F_3. This is the best understood case where one boundary vacuum naively corresponds to three bulk configurations. In that case, the dual field theory intuition helps us understand very well how this discrepancy is resolved. We then use the fact that the D0 setup is related to the D3 setup case by T-duality to argue that this resolution extends to the D0 brane case and consequently to the giant gravitons.

A. The map between giant gravitons and vacua of the D0 brane theory

As we explained in the introduction, in the string theory dual of the $\mathcal{N} = 1^*$ theory one generically has vacua containing D3 branes polarized into D5 branes and NS5 branes. Brane polarization happens because the D3 branes are placed in transverse Ramond-Ramond and Neveu-Schwarz-Neveu-Schwarz 3-form fluxes.

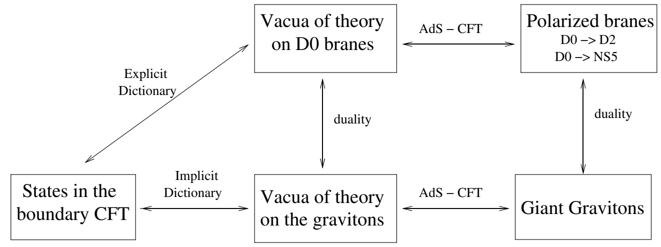

FIG. 1. The dualities behind the proposed solution.

Let us first examine the two vacua corresponding to L D3 branes polarized into one D5 brane and one NS5 brane, respectively. The $\mathcal{N} = 1^*$ classical F-term constraints are $[X^i, X^j] = \epsilon^{ijk}X^k$. The D5 vacuum corresponds to the maximally Higgsed classical solution, where the X^i are the generators of the $L \times L$ irreducible representation of SU(2). The NS5 vacuum corresponds to the classical solution $X^i = 0 \times 11$. Thus the X^i can be thought of as the generators of the product of L trivial representations of SU(2). Quantum effects make this state acquire a nonzero $\langle X^2 \rangle$, which can be interpreted in supergravity as the D3 branes polarizing into an NS5 brane. In [7] it was shown that this polarization is caused by nonperturbative effects—indeed, the presence of NS5 branes in the bulk corresponds to confinement in the field theory, and the size of the NS5 branes gives the field theory mass gap.

As we can see, the two single-brane states come from completely different classical states. This phenomenon is generic to all polarizations of D branes. The polarization pattern is $Dp \rightarrow D(p+2)$ and $Dp \rightarrow NS5$. The $Dp \rightarrow D(p+2)$ configuration corresponds in the classical limit to the $L \times L$ irreducible representation of SU(2). The $Dp \rightarrow NS5$ configuration is made of L classical objects (corresponding to a product of N trivial representations) which via nonperturbative quantum effects create one NS5 brane.

Once we have established this correspondence, we can go ahead and analyze a more general classical configuration, and see what it corresponds to in the supergravity dual. The various duality relations are sketched in Fig. 1.

For example, a classical configuration in which the X^i are the generators of the product of L/k SU(2) irreducible representations of dimension k corresponds in the bulk to polarization into k D5 branes. However, this configuration could also correspond to polarization into L/k NS5 branes. What solves this apparent puzzle is the fact that the two configurations have nonoverlapping ranges of validity [7]. Thus, for $k^2 \gg \frac{L}{g}$ the dual bulk configuration has NS5 branes, while for $k^2 \ll \frac{L}{g}$ the dual bulk configuration has D5 branes.[4]. This gives a one to one map between classical

[4]This is explained in [7], Eqs. (83-85).

gauge theory configurations and bulk configurations with D5 and NS5 branes.

We should note also that there is no field theory vacuum corresponding to the geometry without polarized branes. It is quite likely that this configuration has a naked singularity, and thus it has too large an action to contribute in the AdS-CFT correspondence. The fact that the bulk and boundary analysis of the vacua of the $\mathcal{N} = 1^*$ theory match so precisely, and that there is no boundary vacuum dual to this geometry is a very strong argument that this state indeed does not appear in the AdS-CFT correspondence. This shows that KK graviton is probably singular when a giant graviton with the same angular momentum exists. When this giant graviton is smaller than the string scale, the KK graviton is physical.

The $N = 1^*$ classical analysis of the vacua extends trivially to the D0 brane case.[5] Thus, the maximally Higgsed irreducible representation corresponds to the D0 branes becoming one D2 brane, and the product of L trivial representations corresponds to the D0 branes polarizing into one NS5 brane. These two configurations are the reduction of the biggest M-theory sphere and AdS giant gravitons. A product representation can again be interpreted as the dual of k giants or L/k AdS giants, depending on k and the coupling constant.

Thus, we have a very clear one to one map between giant graviton configurations and vacua of the auxiliary gauge theory on the D0 branes. The map is naively three to one, but we have seen that the configuration with no polarization (dual to the KK graviton) is excluded, and the two configurations dual to the same gauge theory vacuum have nonoverlapping ranges of validity.

III. THE ONE TO ONE MAP

The auxiliary theory description presented in the previous section allows one to think about the D0 → NS5 state as corresponding classically to L particles of angular momentum one, which form a bound state because of quantum effects. In a similar way, the D0 → D2 state would correspond classically to one state of angular momentum L.

This picture is furthermore supported by the fact that the state with L/k AdS giants and the state with k sphere giants correspond to the same auxiliary theory configuration. Therefore, k giant gravitons of one kind correspond to k states each containing L/k particles, while k giant gravitons of the other kind correspond to L/k states each containing k particles. Hence, only one of the maximal giant gravitons found in supergravity corresponds to a gauge theory single particle state, while the other describes a "multiparticle" state.

[5]The only difference is that for D0 branes there are no oblique states, and therefore vacua do not proliferate as one goes from classical to quantum.

To summarize, the main lesson we can draw from the auxiliary theory description of the M-theory giant gravitons is that k sphere giants of angular momentum L/k correspond to the same boundary configuration as L/k dual giants of angular momentum k but in a different regime of the parameter space. This indicates that the maximal sphere and maximal AdS giants correspond to classical configurations that roughly speaking can be thought of as single particle and multiparticle states. We would like now to use this intuition to discuss the $AdS_5 \times S^5$ giant gravitons.

As we show in the Appendix , the auxiliary map of Sec. II does not extend to the case when the $\mathcal{R}$ charge is comparable to N, basically because the giant gravitons become bigger than the near-horizon region they source, and are thus not described any more by the auxiliary theory. However, we do not believe this affects the picture above. It would be rather strange if the fact that two different giant gravitons correspond to the same boundary state changed, especially in the $AdS_5 \times S^5$ case were one can go out of the near-graviton region (A7) by changing a continuous small parameter—g_s.

Through the usual correspondence, the number of particles becomes a number of traces in the field theory operator, at least for $\mathcal{R}$ charge small[6] compared to $N^{2/3}$. Thus, for small L, a maximal sphere giant should correspond to a single-trace operator, while the maximal AdS giant should correspond to a chiral primary operator with L traces. Moreover, given a total angular momentum L, k sphere giants and L/k AdS giants correspond to the same CFT state.

We can see that the picture that emerges from our description matches very well with the proposal of Corley, Jevicki, and Ramgoolam for the chiral primaries dual to the AdS and sphere giant gravitons [14]. According to this proposal, the sphere giant gravitons and AdS giant gravitons are both dual to $\mathcal{N} = 4$ SYM chiral primaries of $\mathcal{R}$-charge L. An efficient method to index the chiral primaries of this field theory is to associate to each primary a U(N) Young Tableau. For $L \ll N$ the number of traces in each operator corresponds to the number of columns of the Young tableau, and the number of fields in each trace corresponds to the number of boxes in the corresponding column. For larger L mixing becomes important, and the dictionary becomes more involved [6]. The total number of boxes in the Young tableau gives the $\mathcal{R}$-charge L of the operator, which maps to the total angular momentum of the dual supergravity state.

Now the proposal in [14] was that a single column of L boxes, i.e., the totally antisymmetric rank L representation, corresponds to the operator dual to a single sphere giant

[6]When the $\mathcal{R}$ charge becomes comparable to $N^{2/3}$, this picture starts getting corrected [6]—the single traces become subdeterminants.

graviton with angular momentum L. Clearly these states fit in with the ''stringy exclusion principle'' as they are both cut off at $L = N$. Several columns of equal length would correspond to several giant gravitons (or a single multiply wrapped giant graviton). Similarly, a single row of L boxes, i.e., the totally symmetric rank L representation, was proposed to be dual to a single AdS giant graviton with angular momentum L. For small L the corresponding field theory operator contains a product of L single traces.[7] This confirms the heuristic picture of the dual giant as a bound state of L particles with angular momentum 1, which emerged from the auxiliary theory.

By combining the auxiliary theory description of giant gravitons with the CJR proposal, one can formulate a map between field theory operators and auxiliary theory vacua, which naturally extends this proposal. Thus, a CFT operator described by a Young tableau with k columns of lengths L_i corresponds to the same giant graviton configuration as a classical vacuum of the auxiliary theory in which the scalars are in the SU(2) representation which is the product of k irreducible representations of size $L_i \times L_i$. The number of boxes of the Young tableau is the total angular momentum of the giant graviton, which in the auxiliary theory gives the rank of the gauge group and is equal to the sum of the L_i's. The completely vertical Young tableau is linked by this map to the maximally Higgsed vacuum, which corresponds to the D0–D2 polarization channel. The completely horizontal Young tableau is linked to the product of L trivial representations, which corresponds to the D0–NS5 polarization.

The CJR proposal, although giving a better understanding of the mapping between giant gravitons and chiral primaries, still does not solve the giant graviton problem, since an operator represented by a rectangular Young tableau of size $L \times H$ can correspond to either L sphere giants of angular momentum H or to H AdS giants of angular momentum L. Our proposed map solves this problem very easily by mapping any Young tableau to a classical vacuum of the gauge theory on the D0 branes and using the fact that the two supergravity duals of this vacuum have nonoverlapping ranges of validity (at least for rectangular Young tableaux—and we propose this is true in general). So for a given range of parameters there is always only one valid supergravity solution per chiral primary.

Although our map agrees with the CJR proposal for the maximal sphere and AdS giant, it seems to differ a bit for less symmetric cases. For example, in [14] a Young tableau with two columns of lengths L_1 and L_2, such that $0 < L_2 - L_1 \ll L_2$ was argued to correspond to two giant gravitons of momentum L_1 and to one KK graviton of momentum $L_2 - L_1$. According to our map, this Young tableau should correspond to two giant gravitons of momenta L_1 and L_2 and no free KK gravitons. As we have explained, a KK graviton of momentum $L_2 - L_1$ should be unphysical when the giant graviton of the same momentum is a valid solution. If $L_2 - L_1$ is very small, and the corresponding giant graviton does not exist, the KK graviton should correspond to a Young tableau column of length $L_2 - L_1$. It would be interesting to see if this could be independently checked.

[7]The gauge group is taken to be U(N) in [14] rather than SU(N). However, on the supergravity side, this difference may not be apparent without considering quantum corrections.

We should remark also that the duals of the sphere giant gravitons are different in the case of $AdS_7 \times S^4$ and $AdS_4 \times S^7$. In one case the sphere giant is dual to a D2 brane, and in the other to an NS5 brane. Our arguments imply that one of the giant gravitons corresponds to the single particle state and the other one to the many particle state. However, we cannot say which of the two giant gravitons is the ''single-trace'' one, essentially because the dualities used to get to the auxiliary theory description are strong-weak dualities, which can cause large mixing between single-trace and multiple-trace operators [18] (essentially in the same way in which S-duality in the $N = 1^*$ theory interchanges the D5 and the NS5 vacua). The same difficulty persists to the $AdS_5 \times S^5$ case, where the AdS and sphere giant gravitons have the same brane content. One can also see that it is difficult to distinguish which of the two giant gravitons is the dual of a single particle state because S-duality maps each brane to itself.

What seems however to be very generic is that when we have two polarization channels, only one channel corresponds to a single particle state/single-trace operator, while the other one corresponds to a many particle state/multi-trace operator. Thus the sphere giant and AdS giant have completely different field theory duals, despite their similarity in supergravity.

As we have explained in the previous section, the M-theory lift of the T-dual of the IIB near-graviton geometry is the massive flow of the M2 brane theory, which contains M2 branes polarized into M5 branes. The sphere and AdS giants correspond to the two orientations of the polarization planes.

According to our dictionary, the two duals of a gauge theory Young Tableau of size $k \times \frac{L}{k}$ would be a state with k M5 branes of M2 charge L/k each, and a state with L/k M5 branes of M2 charge k each. However, it was shown in [16] that a state with a total of L M2 branes polarized into k M5 branes only has a valid description for

$$k^2 < L. \tag{1}$$

Thus, the two duals of the gauge theory state described above have nonoverlapping ranges of validity. We should note that this is a rather nontrivial check for our conjectured dictionary, given that the bound above comes from the M5 brane action, which does not have many things in common with chiral primaries of four-dimensional $\mathcal{N} = 4$ super Yang Mills.

We should remark that the argument we give above, much like the arguments given in other ''Polchinski-Strassler''-like setups [7,16], does not ''prove'' that the vacuum degeneracy problem is solved. These papers propose a solution to the vacuum degeneracy problem in the $N = 1^*$ theory and show that the most obvious way to implement the proposal (by constructing two bulk vacua dual to the same boundary vacuum) does not work. However, the whole method of constructing these kinds of configurations is rather intricate, and therefore it is legitimate to ask whether the fact that one cannot construct two bulk vacua per boundary vacuum is strong evidence for a solution to vacuum degeneracy, or merely a reflection of the limitations of the technology in [7,16]. The only method to prove that vacuum degeneracy does not exist is to construct the full Polchinski-Strassler supergravity solutions and to observe the transition from one bulk vacuum to the other as one changes the parameters of the boundary theory. This has been done in two papers that appeared after this paper was posted on the arxiv—[19,20]—where it was found that all M2–M5 and giant graviton solutions are governed by a harmonic function and that the interpretation of a giant graviton as a wrapped brane breaks down as one goes away from the regime of parameters described in (1). This confirms the correctness of the solution to the vacuum degeneracy problem that we propose.

Another prediction of our map is that there is an upper bound on the total number of AdS giants. Indeed, the number of AdS giants and the angular momentum of the sphere giants have the same auxiliary theory interpretation, and thus both should be cut off at N. To see that this is the case we recall that the AdS giant is a spherical brane domain wall in AdS, and therefore the flux which supports the AdS $\times$ S geometry jumps across the brane. Let us now imagine having exactly N AdS giant gravitons. The flux inside this configuration is zero, and so if one tries to form another AdS giant graviton there is no flux to support it from collapsing. If there are more than N AdS giants, the flux in the region inside them changes sign, and pulls down some of the AdS giants, reducing the number below N.

IV. CONCLUSIONS

We have examined the expansion of pointlike gravitons into giant gravitons, and have argued that when giant gravitons exist, the pointlike gravitons have no field theory dual. Most probably they are not valid solutions of supergravity. Moreover, we have argued by analogy to the study of the $\mathcal{N} = 1^*$ theory by Polchinski and Strassler [7] that the two different types of giant graviton have distinct interpretations in the field theory.

In the supergravity description the giant graviton and dual giant graviton appear to be similar objects, arising from the coupling of the pointlike graviton to a background field, either electrically or magnetically. However, we have presented arguments that only one of these two expanded configurations corresponds classically to a single particle/single-trace state, while the other one corresponds classically to many particles, which form a bound state via quantum effects. This interpretation also ties in nicely with the map between giant gravitons and field theory operators via Young tableaux [14].

Moreover, we have shown that a collection of L/k AdS giants with angular momentum k and a collection of k sphere giants with angular momentum L/k correspond to the auxiliary theory vacuum, and hence to the same CFT chiral primary, but in different regimes of parameters. We have thus presented a solution to the problem that there are apparently many more configurations involving giant and dual giant gravitons than there are appropriate dual field theory operators.

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APPENDIX: RANGES OF VALIDITY

Here we explore the range of angular momenta when the giant graviton states can be described as vacua of the auxiliary theory living on the gravitons. In order for this to happen, the size r_g of a giant graviton of momentum L must be smaller than the size of the near-horizon region of the gravitons r_0. Moreover, when the size of the giant graviton becomes smaller than the string or Planck scale, it makes sense to treat it as a KK graviton.

To estimate r_0 we use the fact that in ten dimensions, the harmonic function sourced by L gravitons is of the form

$$Z - 1 \sim \frac{\frac{L}{R} g_s^2}{r^6} = \frac{r_0^6}{r^6}, \tag{A1}$$

while in eleven-dimensional supergravity the harmonic function is

$$Z - 1 \sim \frac{\frac{L}{R}}{r^7} = \frac{r_0^7}{r^7}. \tag{A2}$$

The total energy in the gravitons is $\frac{L}{R}$, and the extra factor of g^2 in (A1) comes from using string units instead of Planck units.

For AdS$_7 \times S^4$, size of the near-graviton region is (A2)

$$r_0^7 \sim \frac{L}{R} \sim \frac{L}{N^{1/3}}, \tag{A3}$$

while the size of a giant graviton of angular momentum L_i is [2]

$$r_g = \frac{L_i R}{N} \sim \frac{L_i}{N^{2/3}}. \quad (A4)$$

Therefore, a state containing a single giant graviton is described by the auxiliary theory for $L < N^{13/18}$. Moreover, the requirement that the size of the giant graviton be bigger than the Plank length gives a lower bound on L_i: $L_i > N^{2/3}$. If we have more giant gravitons, the near-graviton region increases, while the size of the giants remains the same, so the range described by the auxiliary theory increases.

For $AdS_5 \times S^5$, the size of the near-graviton region is (A1)

$$r_0^6 \sim \frac{L g_s^2}{R}, \quad (A5)$$

where $R^4 \sim g_s N$. The size of the giant graviton of angular momentum L_i is [2]

$$r_g^2 = \frac{L_i R^2}{N}. \quad (A6)$$

Therefore, a state containing a single giant graviton is described by the auxiliary theory for

$$L^2 < NR/l_s \sim N^{5/4} g_s^{1/4}. \quad (A7)$$

The requirement that the size of the giant graviton be bigger than the string length gives the lower bound: $L^2 > N g_s^{-1}$.

For $AdS_4 \times S^7$, the horizon size is (A2)

$$r_0^7 \sim \frac{L}{R} \sim \frac{L}{N^{1/6}}, \quad (A8)$$

while the size of a giant graviton is [2]

$$r_g^4 = \frac{L_i R^4}{N} \sim \frac{L_i}{N^{1/3}}. \quad (A9)$$

Therefore, a state with one giant graviton is described by the auxiliary theory for $L < N^{5/9}$. The lower bound on L is $L > N^{1/3}$.

We should note that the window of parameters in which giant gravitons are described by the auxiliary theory grows with N. Moreover, the window can be made larger if one considers states with many giant gravitons. This window covers but a fraction of the available parameter space. However, as we have explained in Sec. III the basic physics which the auxiliary theory analysis reveals remains valid throughout the whole parameter space.

[1] R. C. Myers, J. High Energy Phys. 12 (1999) 022.
[2] J. McGreevy, L. Susskind, and N. Toumbas, J. High Energy Phys. 06 (2000) 008.
[3] M. T. Grisaru, R. C. Myers, and O. Tafjord, J. High Energy Phys. 08 (2000) 040.
[4] A. Hashimoto, S. Hirano, and N. Itzhaki, J. High Energy Phys. 08 (2000) 051.
[5] S. R. Das, A. Jevicki, and S. D. Mathur, Phys. Rev. D **63**, 044001 (2001).
[6] V. Balasubramanian, M. Berkooz, A. Naqvi, and M. J. Strassler, J. High Energy Phys. 04 (2002) 034.
[7] J. Polchinski and M. J. Strassler, hep-th/0003136.
[8] C. Vafa and E. Witten, Nucl. Phys. **B431**, 3 (1994).
[9] R. Donagi and E. Witten, Nucl. Phys. **B460**, 299 (1996).
[10] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[11] B. Janssen and Y. Lozano, Nucl. Phys. **B643**, 399 (2002).
[12] B. Janssen and Y. Lozano, Nucl. Phys. **B658**, 281 (2003); B. Janssen, Y. Lozano, and D. Rodriguez-Gomez, Nucl. Phys. **B669**, 363 (2003).
[13] T. Banks, M. R. Douglas, G. T. Horowitz, and E. Martinec, hep-th/9808016.
[14] S. Corley, A. Jevicki, and S. Ramgoolam, Adv. Theor. Math. Phys. **5**, 809 (2002).
[15] O. Lunin, S. D. Mathur, and A. Saxena, Nucl. Phys. **B655**, 185 (2003).
[16] I. Bena, Phys. Rev. D **62**, 126006 (2000).
[17] I. Bena and A. Nudelman, Phys. Rev. D **62**, 086008 (2000); I. Bena and A. Nudelman, Phys. Rev. D **62**, 126007 (2000).
[18] D. J. Gross and H. Ooguri, Phys. Rev. D **58**, 106002 (1998).
[19] I. Bena and N. P. Warner, hep-th/0406145.
[20] H. Lin, O. Lunin, and J. Maldacena, J. High Energy Phys. 10 (2004) 025.