Thermal effects on pure and hybrid inflation

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This paper discusses models of inflation based on global supersymmetry. It is shown that there are parameter ranges, consistent with observational constraints, for which warm inflation occurs and supergravity effects can be neglected. There is no need for any fine tuning of parameters. The thermal corrections to the inflaton potential are calculated and it is shown that they do not alter the warm inflationary evolution.

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I. INTRODUCTION

Attempts to build inflationary models based on supersymmetric Grand Unified Theories run into difficulties caused by the size of the supergravity corrections to the inflaton potential. Inflation requires severe flatness conditions on the potential, but these conflict with the F-term supergravity corrections. The solutions to this problem have meant considering models with special cancellations or models where a different supergravity correction, the D-term, dominates (see [1] for a review).

A totally different solution to the problem of supergravity corrections has recently been put forward, which is based on the realization that the dissipation associated with warm inflation relaxes the constraints on the flatness of the potential [2]. Warm inflation can exist in a parameter regime where the supergravity corrections to the potential can be safely ignored.

In warm inflation, particle production during inflation provides a damping effect on the inflaton. This idea has been around for a long time [3–6], but the general features of this scenario were described in [7]. The nonequlibrium dynamics has been expensively developed in subsequent work [2,8–15] and several phenomenological warm inflation models have been discussed in the literature [16–20].

Models in which warm inflation appears to occur spontaneously have the inflaton decaying by two stages, the first stage into a heavy particle and the second into a light particle [2,21]. An example is provided by the interaction Lagrangian density,

$$\mathcal{L}_I = -g^2\phi^2\chi^2 - \frac{1}{\sqrt{2}}h\chi\bar{\psi}_y\psi_y, \tag{1}$$

where χ is a heavy boson and $\tilde{y}$ (field ψ_y) is a light fermion. The dissipation is associated with $\phi \rightarrow \chi \rightarrow \tilde{y}\,\tilde{y}$.

The ultimate destination of the vacuum energy from the inflationary phase is into excitations of the light sector fields. In warm inflation, one has to consider the possibility that these excitations enhance the loop corrections to the inflaton potential and violate the flatness conditions which inflation requires. Because of this concern, we have calculated the loop corrections to the potential under the assumption that the light fields thermalize.

The light sector will typically have coupling terms representing self-interactions, or interactions with other light fields, in addition to the couplings given in Eq. (1). The relaxation time of the radiation should therefore be independent of the damping mechanism which is affecting the inflaton. Whether the radiation thermalizes during inflation is therefore rather arbitrary. We assume thermalisation, but some consequences of nonthermalization are mentioned in the conclusion.

We shall consider the simplest inflationary models which include global supersymmetry. These models divide naturally into two classes. In the first class, which we call pure, the vacuum energy is associated only with the inflaton field. We find that normalizing the density perturbation amplitude to the cosmic microwave background implies a mass scale of up to 10^{11} GeV and coupling constants g and h around 0.1.

In the second class of models, part of the vacuum energy can be linked to a false vacuum of the χ field. These are the supersymmetric hybrid models of inflation [22,23]. We find that normalising the density perturbation amplitude to the cosmic microwave background implies a mass scale of up to 10^{14} GeV for the false vacuum energy and coupling constants again around 0.1. Consequently, F-term supersymmetric inflation with parameters in this range is of the warm inflationary type.

While the present work was nearing completion, we learned that an independent study of hybrid models of warm inflation was also underway, being conducted by Arjun Berera and Mar Bastero-Gil [24].

II. SUPERSYMMETRIC MODELS

A. Potential and interaction terms

We have just described how the warm inflationary scenario arises when there is a two stage reheating process involving a heavy boson. Global SUSY models can easily be constructed which provide the required interactions

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[2,21]. Consider the superpotential

$$W = -g\Phi X^2, \tag{2}$$

where the scalar field components of the chiral superfields Φ and X are φ and χ respectively. The scalar interaction terms in the theory are unpacked from the superpotential using

$$\mathcal{L}_S = -|\partial_\Phi W|^2 - |\partial_X W|^2. \tag{3}$$

We identify the inflaton with $\phi = \sqrt{2}\mathrm{Re}\varphi$, and then

$$\mathcal{L}_S = -g^2|\chi|^4 - 2g^2\phi^2|\chi|^2. \tag{4}$$

Supersymmetry breaking now plays an important role in determining the shape of the inflaton potential along the flat direction $\chi = 0$ [1]. A "new inflation" type of model [25–27] results from introducing a soft SUSY breaking mass M_s for the χ field. The inflaton potential $V(\phi)$ is determined by the one loop correction [28],

$$V(\phi) = \frac{1}{2}g^2M_s^2\left(\phi^2\log\frac{\phi^2}{\phi_0^2} + \phi_0^2 - \phi^2\right). \tag{5}$$

Supercooled inflation requires $\phi_0 > m_p$, but in this parameter range the inclusion of supergravity F-term corrections would typically prevent the inflation from occurring.

Hybrid inflationary models can be constructed if we change the superpotential slightly [22,23],

$$W = g\Phi\Lambda^2 - g\Phi X^2 + g\Phi X'^2. \tag{6}$$

where Λ is a constant and X and X' are a pair of superfields. The interaction terms are now

$$\mathcal{L}_S = -g^2|\chi^2 - \chi'^2 - \Lambda^2|^2 - 2g^2\phi^2(|\chi|^2 + |\chi'|^2). \tag{7}$$

In hybrid models, the χ field is stable at $\chi = 0$ during inflation and the potential is dominated by the constant term $g^2\Lambda^4$. The χ field becomes unstable at when ϕ falls below the critical value $\phi_c = \Lambda$.

The supersymmetry is broken by the inlaton field resulting in a nonvanishing one loop contribution to the inflaton potential. The presence of the second superfield helps produce a potential which is suitable for inflation by reducing the size of the quantum corrections. For $\phi \gg \Lambda$,

$$V(\phi) = g^2\Lambda^4 + \frac{g^4}{4\pi^2}\Lambda^4\ln\left(\frac{2g^2\phi^2}{\Lambda^2}\right). \tag{8}$$

The heavy sector plays a double role in contributing to the vacuum energy and damping the inflaton field.[1]

For an efficient two stage reheating process, we introduce an additional light sector Y, which can be coupled through a superpotential

$$W = -g\Phi X^2 - hXY^2. \tag{9}$$

The Yukawa interaction terms are recovered from

$$\mathcal{L}_Y = -\frac{1}{2}\frac{\partial^2 W}{\partial\phi_n\partial\phi_m}\bar{\psi}_n P_L\psi_m - \frac{1}{2}\frac{\partial^2 W^*}{\partial\phi_n^*\partial\phi_m^*}\bar{\psi}_n P_R\psi_m, \tag{10}$$

where ϕ_m is a superfield and $P_L = 1 - P_R = (1 + \gamma_5)/2$. The interactions contain terms such as those in Eq. (1), and lead to a friction term $\propto \dot{\phi}$ in the inflaton field equation [2]. They also have an effect on the vacuum polarization of the χ field, which in turn can affect the inflaton potential. The full set of interaction terms and the vacuum polarization are discussed in Sec. III.

B. Inflationary dynamics

In an expanding, homogeneous universe, the inflaton equation of motion is given by

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{T,\phi} = 0, \tag{11}$$

where $V_T(\phi, T)$ is the thermodynamic potential and $\Gamma(\phi, T)$ is the damping term due to interactions between the inflaton ϕ and surrounding fields. For supercooled inflation, this damping term is negligible compared to the Hubble damping term. The interesting regime of warm inflation is characterized by large damping terms. To distinguish between the two inflationary scenarios, a dimensionless parameter, r, is introduced to denote the relative strength between the damping terms

$$r = \frac{\Gamma}{3H}.$$

We shall take warm inflation in the limit $r \gg 1$.

As the inflaton evolves, energy dissipates into radiation and entropy is produced. Simple thermodynamic relations lead to a definition of entropy density,

$$s(\phi, T) = -V_{T,T} = \frac{4\pi^2}{90}g_*T^3 + \ldots, \tag{12}$$

where g_* is the effective particle number and the dots denote contributions from the thermal correction to the potential. In the warm inflationary scenario, inflation is characterized by three slow-roll equations [29]

$$\dot{\phi} = -\frac{V_{T,\phi}}{3H(1 + r)}, \qquad Ts = r\dot{\phi}^2, \qquad 3H^2 = 8\pi G V_T. \tag{13}$$

The second equation denotes conservation of energy, while the third is the usual Friedmann equation. Slow-roll automatically implies inflation, $\ddot{a} > 0$, and the consistency of slow-roll is governed by a set of slow-roll parameters:

[1]For comparison with Dvali *et al.* [23], their $\kappa = 2g$ and $\mu = g^{1/2}\Lambda$.

$$\epsilon = \frac{m_p^2}{16\pi}\left(\frac{V_{T,\phi}}{V_T}\right)^2, \quad \eta = \frac{m_p^2}{8\pi}\left(\frac{V_{T,\phi\phi}}{V_T}\right),$$
$$\beta = \frac{m_p^2}{8\pi}\left(\frac{\Gamma_{,\phi} V_{T,\phi}}{\Gamma V_T}\right), \quad \delta = \frac{T V_{T,\phi T}}{V_{T,\phi}}, \tag{14}$$

where m_p^{-2} is Newton's constant. The slow-roll approximation is consistent when the above parameters are less than r. Supergravity F-term corrections, without special cancellations, lead to η of order unity [22]. We shall therefore concentrate on the range,

$$1 < \eta < r. \tag{15}$$

The thermal corrections to the potential will be calculated in the next section. For the remainder of this section we shall examine the situation where the corrections are small, $V_T = V$ and the slow-roll parameter $\delta = 0$.

An important observational constraint on the model is set by the density perturbation amplitude. In our case, where we have assumed that the radiation has thermalized, the thermal fluctuations induce scalar density fluctuations. The amplitude Δ can be obtained analytically [29,30], and for $r \gg 1$,

$$V_h^{1/4} = \alpha r_h^{-3/4} \epsilon_h^{1/4} \Delta^{2/3} m_p, \tag{16}$$

where $\alpha \approx 0.68 g_*^{-1/12}$ and the parameters are evaluated at the time t_h that the perturbation scale crossed the horizon. The value of Δ inferred from cosmic microwave observations is around $\Delta \approx 5.4 \times 10^{-5}$ on the 500 MPc scale [31].

Limits on the mass parameters can be found by combining the slow-roll limits (15) with the constraint from the density perturbations (16). The pure inflation model with potential (5) has two mass parameters gM_s and ϕ_0. Order of magnitude estimates can be obtained by taking $\phi_h \sim \phi_0$ for the value of ϕ at horizon crossing (which is consistent with numerical solutions [28]) and $\epsilon \approx \eta$. The normalization condition (16) gives

$$gM_s \approx 3.3 \times 10^{-6} \eta_h r_h^{-3/2} m_p. \tag{17}$$

The upper limit for gM_s set by (15) is of the order 10^{13} GeV.

For hybrid inflation, a similar approximation can be made when the vacuum energy $g^2\Lambda^4$ dominates the potential and $\phi \gg \Lambda$. In this case,

$$\epsilon \approx \frac{g^2}{4\pi^2}\eta \approx \frac{g^4 m_p^2}{64\pi^4 \phi^2}. \tag{18}$$

The normalization condition (16) gives

$$\Lambda \approx 3.88 \times 10^{-4} \eta_h^{1/4} r_h^{-3/4} m_p. \tag{19}$$

When combined with the conditions $r_h > \eta_h > 1$, the upper limit on Λ is of order 10^{15} GeV.

More detailed limits can be placed on the parameters when we know the form of the friction term in the inflaton equation. For the interactions in Eq. (1), the friction term has been calculated in the zero-temperature limit [21] and is given to leading order in h by

$$\Gamma \approx \gamma\phi. \tag{20}$$

The value of γ depends on the decay process. For $\chi \to 2\tilde{y}$ and $\chi \to 2y$, the leading order contributions to γ are

$$\gamma(\chi \to 2\tilde{y}) = \gamma(\chi \to 2y) = \frac{\sqrt{2}g^3h^2}{128\pi^2}. \tag{21}$$

For the fermionic channel $\tilde{\chi} \to y\tilde{y}$,

$$\gamma(\tilde{\chi} \to y\tilde{y}) = \frac{3\sqrt{2}g^3h^2}{64\pi^2}. \tag{22}$$

Hence the total

$$\gamma = \frac{\sqrt{2}g^3h^2}{16\pi^2}. \tag{23}$$

To be in the perturbative regime, with $g < 1$ and $h < 1$, sets a requirement $\gamma < 8.9 \times 10^{-3}$.

We can relate r to γ using the slow-roll Eqs. (14),

$$r = \frac{\Gamma}{3H} = \frac{\gamma\phi m_p}{(24\pi V)^{1/2}}. \tag{24}$$

The nonhybrid models can be regarded as having three parameters gM_s, ϕ_0 and γ. The normalization from the density fluctuation amplitude provides one constraint which can be used to eliminate one parameter, let us say ϕ_0. The warm inflationary regime can then be displayed as bounds on the remaining two parameters.

The horizon crossing timescale depends on the number N of e-folds of the scale factor before the end of inflation. As a rough guide, we can take $r_h = N\eta_h$. The normalization condition (16) gives

$$\phi_0 = 8.2 \times 10^{-6} \gamma^{-1} N^{-1/2} m_p. \tag{25}$$

Note that $\phi_0 < m_p$ is needed for η to remain larger than 1 throughout the inflationary era, which is a necessary requirement for the neglect of supergravity corrections. After eliminating ϕ_0, there is a consistency requirement

$$\gamma g M_s < 4.8 \times 10^{-11} N^{-2} m_p \tag{26}$$

for warm inflation.

The hybrid models can be taken to have parameters $M = g^{1/2}\Lambda$, g and γ. The horizon crossing timescale is given approximately by the relation $r_h = 3N\eta_h$. The normalization condition (16) allows us to express g as

$$g \approx 1.5 \times 10^{21} \gamma^2 M^2 N^{5/2} m_p^{-2}. \tag{27}$$

The condition $\eta > 1$ becomes

$$\gamma > 1.3 \times 10^{-6} N^{-1/2}. \tag{28}$$

Taken together with the expression for γ given by Eq. (23), this gives lower limits on g and h. For example, if $g \approx h$, then $g > 0.1$. Another condition follows from $\phi > \Lambda$, which implies a consistency condition

$$\gamma M > 3.8 \times 10^{-12} N^{-13/8} m_p \tag{29}$$

for the warm inflationary regime.

The parameter ranges are shown in Fig. 1. The number of e-folds of inflation has been taken to be $N = 60$. It is clear from the figure, that the warm inflation occurs for a broad range of parameters. In the case of hybrid inflation, range includes $0.03 < g < 1$ which means that many F-term inflationary models should be have warm rather than cold inflation [23].

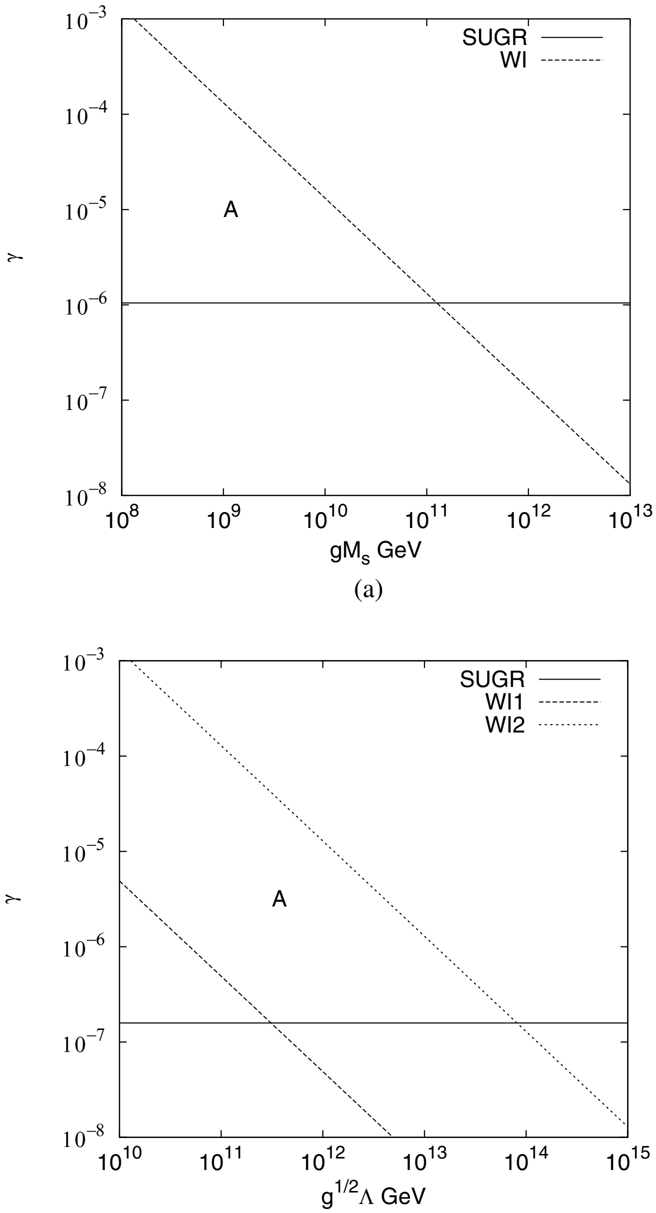

FIG. 1. The figures show, approximately, the allowed values (region A) of the parameters γ, gM_s and $g^{1/2}\Lambda$ for (a) pure and (b) hybrid inflation. The line SUGR shows the limit for which SUGR corrections can be neglected. In case (a), the limit of consistent warm inflation is shown as WI. In case (b), the limits for consistent warm inflation are $\phi_h = \Lambda$ (WI1) and $g = 1$ (WI2).

Finally, we can use the slow-roll equations to determine what the temperature is during the inflationary phase. The slow-roll Eqs. (13) give

$$Ts = \frac{2}{3} r^{-1} \epsilon V. \tag{30}$$

Once again making use of the normalization provided by the density fluctuation amplitude, and the slow-roll relation

$$V_{,\phi\phi} = \frac{8\pi\eta V}{m_p^2}, \tag{31}$$

we can compare T_h to the slope of the potential,

$$T_h \approx 230 r_h^{1/2} \eta_h^{-1/2} (V_{,\phi\phi})^{1/2}. \tag{32}$$

Note that T_h is always larger than the mass scale responsible for the slope of the potential. In this situation, we should be concerned that thermal corrections to the potential may make the models untenable. The calculation of these thermal corrections to the potential is therefore necessary.

In the case of pure inflation, $V_{,\phi\phi} \sim 2g^2 M_s^2$ and the temperature during inflation is approximately

$$T_h \sim 320 N^{1/2} g M_s. \tag{33}$$

In the case of hybrid inflation, using Eq. (30) and Eq. (18),

$$T_h \approx \left(\frac{45}{16\pi^5}\right)^{1/4} g^{1/2} N^{-1/4} M. \tag{34}$$

Note that the vacuum energy of the hybrid model after inflation, before the second field decays, is approximately M^4. The values of the temperature are sufficiently high such that, if we have local supersymmetry, then we are in danger of violating constraints set by the thermal production of gravitinos [32–37]. These constraints can be satisfied by taking small values of the mass parameters, corresponding to large values of r [38]. The allowed parameter ranges permit this, but so far this appears to be an unnatural feature of the models under discussion.

III. THERMAL CORRECTIONS

The full set of interaction terms obtained from the superpotential (9) are

$$\mathcal{L}_S = g^2(\Lambda^2 - |\chi|^2)^2 + 4g^2|\varphi|^2|\chi|^2 + 4h^2|y|^2|\chi|^2 + h^2|y|^4 + 2gh(y^2\varphi^\dagger\chi^\dagger + y^{\dagger 2}\varphi\chi) \tag{35}$$

$$\mathcal{L}_Y = g(\varphi\overline{\psi}_\chi P_L\psi_\chi + \varphi^\dagger\overline{\psi}_\chi P_R\psi_\chi) + h(\chi\overline{\psi}_y P_L\psi_y + \chi^\dagger\overline{\psi}_y P_R\psi_y) + 2g(\chi\overline{\psi}_\chi P_L\psi_\varphi + \chi^\dagger\overline{\psi}_\chi P_R\psi_\varphi) + 2h(y\overline{\psi}_y P_L\psi_\chi + y^\dagger\overline{\psi}_y P_R\psi_\chi). \tag{36}$$

Thermalization conditions of the light fermion, $\tilde{y}$ depend directly on the mass of the fermion and its self-interaction. These properties are inherent to quadratic and cubic terms in the superpotential ($\mu_\chi Y^2$, λY^3), which we have not specified. We assume that the interactions are such that the light fermions thermalize and we calculate the corresponding thermal effects. The ψ_y interactions to fields other than the inflaton, ϕ or χ fields, will have no effect on the thermal corrections to the inflaton effective potential. We can therefore disregard the exact nature of these interactions.

For the model considered, if $\tilde{y}$ thermalizes but χ and ψ_χ do not, then important simplifications can be made. The thermal corrections in the action appear as a result of the self-energies of the χ and $\tilde{\chi}$ fields. Inside the self-energy loops, y and $\tilde{y}$ are taken to be very light, so the Hard Thermal Loop (HTL) approximation can be made, i.e., $T \gg m_y, m_{\tilde{y}}$. Outside the loop, however, the χ fields are heavy, and $T \ll m_\chi, m_{\tilde{\chi}}$.

We use the imaginary time formalism and adopt the notation that 4-momenta are written in upper case and 3-momenta as written in lower case, so that $P^\mu = (\omega, \mathbf{p})$. The boson and fermion propagators of the χ fields are G and S respectively. The contribution to the effective potential of the inflaton field from the χ fields is given by

$$V_\chi = \int \frac{d^4P}{(2\pi)^4} \ln\det(G^{-1}) - \int \frac{d^4P}{(2\pi)^4} \ln\det(S^{-1}S^{*-1})^{1/2} \tag{37}$$

after regularization has been applied. The detailed calculation of the thermal corrections to the fermionic and bosonic masses m_χ and $m_{\tilde{\chi}}$ will be given in the following sections.

A. Fermion Contribution

If we set $y = (y_1 + iy_2)/\sqrt{2}$, then the $y_i\overline{\psi}_y\psi_\chi$ terms in Eq. (36) lead to two similar fermion self-energy diagrams (Fig. 2) for $\tilde{\chi}$. The vertex factors are $i\sqrt{2}h$ and $\sqrt{2}h\gamma_5$ respectively. Using the properties of γ_5, contributions from both diagrams are found to be identical. Thermal feynman rules applied to the diagram in Fig. 2 lead to an expression for the fermion self-energy, Σ,

$$\Sigma(P) = -4h^2T\sum_n \int \frac{d^3k}{(2\pi)^3}(\not{K} - \not{P})\Delta(K)\tilde{\Delta}(P-K), \tag{38}$$

where $\Delta(K) \approx K^{-2}$, $k^0 = 2n\pi T$ for bosons and $k^0 = (2n+1)\pi T$ for fermions (denoted by a tilde).

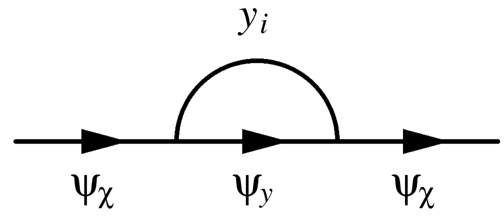

FIG. 2. Diagram for the fermionic self-energy

Many similar diagrams appear in the literature, for example [39–42], specifically with regards to the HTL loop corrections to electron propagators in QCD. The calculation given here follows [41] with only slight changes, due to having a scalar field rather than a vector field. Therefore we only need quote the result,

$$\Sigma(P) = \frac{m_f^2}{2p}\gamma_0 Q_0\left(\frac{i\omega}{p}\right) + \frac{m_f^2}{2p}\gamma\cdot\hat{p}\left[1 - \frac{i\omega}{p}Q_0\left(\frac{i\omega}{p}\right)\right]. \tag{39}$$

Note that the overall factor of $1/2$, which is different to the literature, is a convention we have adopted for the definition of m_f for later convenience. $Q_0(x)$ is the Legendre function of the second kind,

$$Q_0(x) = \frac{1}{2}\ln\frac{x+1}{x-1}. \tag{40}$$

Accounting for the contribution of two diagrams, the fermion thermal mass is

$$m_f^2 = \frac{h^2T^2}{2}. \tag{41}$$

In our conventions, the inverse propagator is $iS^{-1} = \not{P} - m_{\tilde{\chi}} - \Sigma$ and thus can be written

$$iS^{-1} = A_0\gamma_0 - A_s\gamma\cdot\hat{p} - m_{\tilde{\chi}}, \tag{42}$$

where

$$A_0 = i\omega - \frac{m_f^2}{2p}Q_0\left(\frac{i\omega}{p}\right) \tag{43}$$

$$A_s = p + \frac{m_f^2}{2p}\left[1 - \frac{i\omega}{p}Q_0\left(\frac{i\omega}{p}\right)\right]. \tag{44}$$

Hence the combination

$$(SS^*)^{-1} = -A_0^2 + A_s^2 + m_{\tilde{\chi}}^2. \tag{45}$$

The fermionic contribution to the effective potential (37) becomes

$$V_f = -2\int\frac{d^4P}{(2\pi)^4}\ln\left\{\left[\omega + \frac{im_f^2}{2p}Q_0\left(\frac{i\omega}{p}\right)\right]^2 + \left[p + \frac{m_f^2}{2p} - \frac{im_f^2}{2p^2}\omega Q_0\left(\frac{i\omega}{p}\right)\right]^2 + m_{\tilde{\chi}}^2\right\}. \tag{46}$$

We can obtain the leading terms using $m_\chi \gg m_f$,

$$V_f = -2\int \frac{d^4P}{(2\pi)^4} \ln\{\omega^2 + p^2 + m_{\tilde\chi}^2 + m_f^2\} + O(m_f^4). \tag{47}$$

The evaluation of this regularized integral is a standard exercise,

$$V_f = -\frac{1}{32\pi^2}(m_{\tilde\chi}^4 + 2m_{\tilde\chi}m_f^2)\ln\left(\frac{m_{\tilde\chi}^2 + m_f^2}{\mu^2}\right), \tag{48}$$

where μ has been introduced by the regularization.

B. Boson Contribution

Three diagrams are expected to contribute to the bosonic self-energy as shown in Fig. 3. Because of the interaction terms $\chi_i\bar\psi_y\psi_y$ in Eq. (36), there will be two diagrams similar to Fig. 3(a), with vertex factors $i\sqrt{2}h$ and $\sqrt{2}h\gamma_5$ respectively. The two diagrams result in identical expressions, each with a symmetry factor of 1/2 because the fermions are Majorana. The self-energy for χ_1 from Fig. 3(a) is given by

$$\Pi(P)_a = h^2T\sum_n\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}[\not K(\not K - \not P)]\tilde\Delta(K)\tilde\Delta(K-P). \tag{49}$$

In the HTL limit,

$$\Pi(P)_a = -4h^2T\sum_n\int\frac{d^3k}{(2\pi)^3}K^2\tilde\Delta(K)\tilde\Delta(K-P). \tag{50}$$

Using $\tilde\Delta(K) \approx K^{-2}$ the final result is a single fermion loop expression,

$$\Pi(P)_a = -4h^2T\sum_n\int\frac{d^3k}{(2\pi)^3}\tilde\Delta(K) = \frac{1}{6}h^2T^2. \tag{51}$$

The second contribution stems from the four tadpole diagrams coming from the terms $\chi_i^2y_i^2$ in Eq. (35). Each diagram is identical, and has a symmetry factor of 1/2. The self-energy is given by the expression,

$$\Pi(P)_b = 4h^2T\sum_n\int\frac{d^3k}{(2\pi)^3}\Delta(K) = \frac{1}{3}h^2T^2. \tag{52}$$

The Feynman diagram in Fig. 3(c) does not result in an $O(T^2)$ contribution to the self-energy, but instead gives a possibly important contribution of $O(m_\chi^2)$. The self-energy contribution is given by

$$\Pi(P)_c = 4g^2h^2\phi^2T\sum_n\int\frac{d^3k}{(2\pi)^3}\Delta(K)\Delta(K-P). \tag{53}$$

For the evaluation of this integral in the HTL limit see, for example, [43]. The result is

$$\Pi(P)_c = 4g^2h^2\phi^2\int\frac{dk}{(2\pi)^2}\frac{1}{p}\ln\left(\frac{k-p}{k+p}\right)\frac{1}{e^{kT}-1}. \tag{54}$$

The integral evaluates to

$$\Pi(P)_c \approx \frac{1}{2\pi^2}g^2h^2\phi^2\log\frac{T^2}{p^2} \tag{55}$$

in the HTL limit.

The inverse boson propagator is defined as $G^{-1} = P^2 + m_\chi^2 + \Pi$. The total contribution from the first two diagrams in Fig. 3 defines a contribution m_b^2 to the mass,

$$m_b^2 \equiv \Pi_a + \Pi_b = \frac{1}{2}h^2T^2. \tag{56}$$

Note that $m_\chi^2 = 4g^2\phi^2$, and the temperature dependent part of Π_c can be regarded as a contribution to the finite temperature coupling constant $g(T)$,

$$g^2(T) = g^2\left(1 + \frac{1}{8\pi^2}h^2\log\frac{T^2}{\mu^2}\right). \tag{57}$$

Since the inflationary dynamics is not sensitive to the precise value of g, we shall not distinguish between g and $g(T)$.

The bosonic contribution to the effective potential is obtained from the two fields χ_i,

$$V_b = \int\frac{d^4P}{(2\pi)^4}\ln\{P^2 + m_\chi^2 + m_b^2\} = \frac{1}{32\pi^2}(m_\chi^2 + m_b^2)^2\ln\left(\frac{m_\chi^2 + m_b^2}{\mu^2}\right). \tag{58}$$

C. Effective Potential

The total effective potential is given by the sum of both the fermionic and bosonic contributions. The largest of the $O(T^2)$ terms cancel due to the fact that $m_b = m_f$. This is, of course, due to the underlying supersymmetry, but it happens despite the fact that supersymmetry is broken at nonzero temperatures. The remaining $O(T^2)$ terms are due to other sources of SUSY breaking. For the case of soft SUSY breaking, we can consider the χ boson and $\tilde\chi$ fermion masses,

$$m_\chi^2 = 2g^2\phi^2 + M_s'^2, \tag{59}$$

$$m_{\tilde\chi}^2 = 2g^2\phi^2. \tag{60}$$

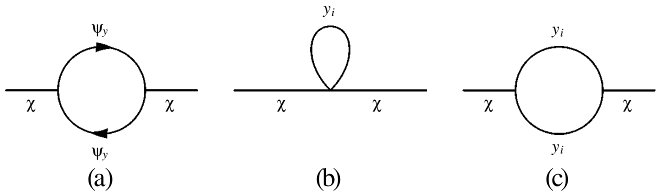

FIG. 3. Diagrams for the bosonic self-energy

The leading order terms in the potential are

$$V_\chi = \frac{1}{32\pi^2}\left\{(m_\chi^2 + m_b^2)^2 \ln\left(\frac{m_\chi^2 + m_b^2}{\mu^2}\right) - (m_{\tilde{\chi}}^2 + m_f^2)^2 \ln\left(\frac{m_{\tilde{\chi}}^2 + m_f^2}{\mu^2}\right)\right\} + \text{constant}. \quad (61)$$

For our scenario, $g\phi \gg M_s'$, which results in

$$V_\chi \approx \frac{1}{2} M_s^2\left(g^2\phi^2 + \frac{1}{4}h^2T^2\right)\left[\ln\left(\frac{g^2\phi^2 + \frac{1}{4}h^2T^2}{g^2\phi_0^2}\right) - 1\right] + \frac{1}{2}g^2 M_s^2 \phi_0^2. \quad (62)$$

We have defined $M_s^2 = M_s'^2/(8\pi^2)$, and ϕ_0 has been chosen such that both the potential and its derivative vanish at $\phi = \phi_0$ when $T = 0$.

The thermodynamic potential of the inflaton in the models under consideration is determined primarily by the χ loop contribution calculated above and the free energy of the light radiation fields,

$$V(\phi, T) = -\frac{\pi^2}{90} g_* T^4 + V_\chi(\phi, T). \quad (63)$$

Fig. 4 graphically shows the temperature dependence of the potential. The scale of the temperature corrections is set by T_0, where $hT_0 = 2g\phi_0$.

The Hybrid models (6) can be treated in a similar manner if we couple the two heavy superfields to the light superfield with equal couplings h (which is consistent with an underlying $U(1)$ symmetry). The masses of the χ and χ' fields are

$$m_\chi^2 = 2g^2\phi^2 + 2g^2\Lambda^2 \quad (64)$$

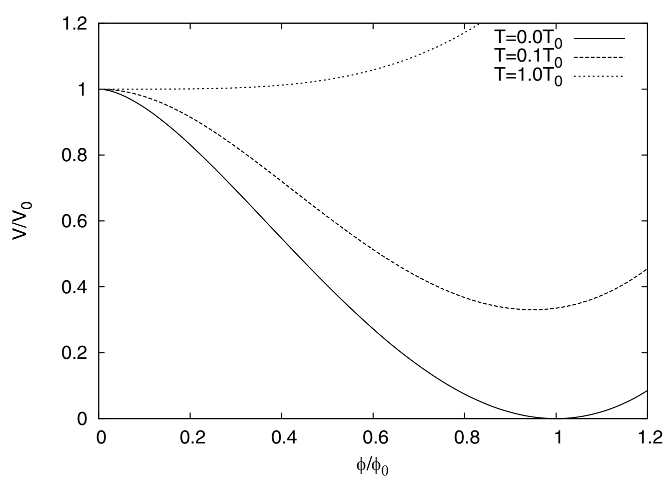

FIG. 4. The thermodynamic potential of the pure inflationary model is depicted. On the vertical axis, $V = V_T(\phi, T) - V_T(0, T)$ and $V_0 = V_T(0, 0)$. The critical temperature $hT_0 = 2g\phi_0$.

$$m_{\chi'}^2 = 2g^2\phi^2 - 2g^2\Lambda^2 \quad (65)$$

$$m_{\tilde{\chi}}^2 = m_{\tilde{\chi}'}^2 = 2g^2\phi^2. \quad (66)$$

The one loop correction to the inflaton potential is now

$$V_\chi = \frac{1}{4\pi^2} g^4 \Lambda^4 \ln\left(\frac{2g^2\phi^2 + \frac{1}{2}h^2T^2}{g^2\Lambda^2}\right) \quad (67)$$

for $\phi \gg \Lambda$.

D. Friction term

The zero temperature friction term calculated by Berera and Ramos [21] can be modified for our interaction terms with the decays $\phi \to \chi \to 2\tilde{y}$ and $\phi \to \chi \to 2y$,

$$\Gamma = \frac{g^4\phi^2\Gamma_\chi}{2\pi(m_\chi^2 + \Gamma_\chi^2)^{1/2}[2m_\chi(m_\chi^2 + \Gamma_\chi^2)^{1/2} + 2m_\chi^2]^{1/2}}, \quad (68)$$

where Γ_χ is the χ decay width,

$$\Gamma_\chi = \frac{h^2}{16\pi} m_\chi\left(1 - \frac{4m_y^2}{m_\chi^2}\right)^{3/2} + \frac{h^2}{16\pi} m_\chi\left(1 - \frac{4m_{\tilde{y}}^2}{m_\chi^2}\right)^{1/2}. \quad (69)$$

This reduces to the expression used earlier when $m_y = 0$ and $m_\chi^2 = 2g^2\phi^2$.

One would expect that the thermal corrections to Γ will manifest themselves as corrections to m_χ and Γ_χ. Since the latter already contains factors of h, the thermal corrections to Γ will be of order h^3T^2. The correction to Γ due to m_χ will be of order h^4T^2. The corrections to the effective potential are order h^2T^2 and are therefore taken to be the dominant effect. Corrections to the friction term can therefore be ignored.

E. Effects on the inflationary dynamics

The temperature range relevant for the warm inflationary scenario was given in Eq. (32). In the models under discussion, $h^2T^2 \ll g^2\phi^2$ and the thermal corrections make only a small change to the height of the thermodynamic potential V_T. The effect on the slope of the potential is more delicate, however, and has to be investigated separately. The slope can be quantified by the slow-roll parameter η. Consider the change in η,

$$\frac{\delta\eta}{\eta} = \frac{V_{T,\phi\phi} - V_{,\phi\phi}}{V_{,\phi\phi}}. \quad (70)$$

In the nonhybrid case,

$$\frac{\delta\eta_h}{\eta_h} \sim \frac{h^2T_h^2}{g^2\phi_0^2}. \quad (71)$$

Given Eqs. (33), (25), and (26), this correction is of order $10^{-5}h^2g^{-2}$ at most.

For hybrid inflation, using (67),

$$\frac{\delta\eta_h}{\eta_h} \approx -\frac{h^2 T_h^2}{g^2\phi_h^2}. \tag{72}$$

Proceeding as above, using Eq. (34), this correction is of order $10^{-7}h^2g^{-2}$.

IV. CONCLUSION

We have found that the inflation occurs naturally in particle models with global supersymmetry when the dissipative effects of particle production are taken into account. The warm inflationary scenario escapes the flatness problems which arise when supercooled inflation is combined with global supersymmetry. The parameter restrictions on the model are not severe, with the possible exception of a gravitino constraint, and there is a correspondence between mass parameters required for the observed density fluctuation amplitude and the parameter values of interest for supersymmetric Grand Unified Theories.

We have demonstrated that, in a two stage reheating process, the thermal corrections to the inflaton potential are small, due to fermion-boson cancellations . The assumptions used for the models have been relatively mild, consisting mainly of the following:

(i) At least one superfield has vanishing, or very weak coupling, to the inflaton and another has nonvanishing coupling. During inflation, the former will naturally become a ‘‘light’’ sector, and the latter a ‘‘heavy’’ sector.

(ii) There is either (a) soft SUSY breaking in the heavy sector, which we called the pure inflation model or (b) a false vacuum energy, and two equally coupled heavy superfields, which we identify as the hybrid model.

(iii) We have assumed that the light radiation thermalizes.

The last assumption was needed to avoid far from equilibrium calculations. However, in the absence of thermalization, the energy will still be dumped into the light sector. The important features of the light particle distribution can be described in terms of nonthermal occupation numbers $n(k)$. If the boson and fermion occupation numbers are similar, we might still expect the cancellation of correction to the effective potential which we have found here.

We have made use of the density fluctuation amplitude when setting limits on the parameters in the models. If the radiation does not thermalize, we would expect to find changes in the predicted value of the density fluctuation amplitude and this remains to be investigated further.

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