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Gravitational constant G measured with a superconducting gravimeter

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We report in this paper the results of a measurement of the gravitational constant G obtained in a laboratory at distances of about 1 m, using a superconducting gravimeter. The instrument measured the gravitational effect due to an annular mass of about 280 kg moving up and down around the gravimeter. The experiment yielded for the gravitational constant the value $G = (6.675 \pm 0.007)10^{-11}\mathrm{Nm}^2/\mathrm{kg}^2$ which agrees, within its uncertainty, with the last CODATA value.

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I. INTRODUCTION

In spite of its important role as coupling constant in a fundamental interaction, the gravitational constant G is still known with a relatively large uncertainty. This is mainly due to the weakness of this interaction, which becomes relevant only if a large mass is involved. The most recent accepted value is $(6.673 \pm 0.010)10^{-11}\mathrm{Nm}^2/\mathrm{kg}^2$ [1].

Almost all the existing measurements were performed in the laboratory, where the gravitational law was commonly tested by means of torsion balances and torsion pendula. Some experiments of the last decade have very small uncertainties. Unfortunately, as shown in Fig. 1, where the values obtained in the last decades [2–41] are reported, in some cases their central values disagree. This situation is typical in physics when the measurement is affected by the occurrence of systematic errors. In such cases it is particularly important to measure the same quantity in different ways.

For these reasons, we set up an experiment aimed at measuring G by using a superconducting gravimeter (SCG). We accomplished the task using a SCG already employed a few years ago [39] to test Newton's inverse square law for distances of some tens of meters. Such an instrument is the most precise existing device to measure time variations of the acceleration due to gravity g. We now employed it to measure g variations induced by moving up and down along the vertical axis of an annular mass of (273.40 ± 0.01) kg placed around the instrument. The G value was obtained on comparing the measured effect with the results of a simulation of the gravitational acceleration induced on the SCG sensor by the mass distribution of the moving body.

II. THE EXPERIMENT WITH THE SCG

The gravity probe used to obtain the G value is a GWR instrument [42]. The SCG operation [43] is based on the levitation of a small superconducting Niobium sphere floating in a magnetic field due to a couple of superconducting coils. The position of the sphere is the result of the equilibrium between the gravitational force pulling down and the magnetic field pulling up. As g changes, the sphere position would change, but this is avoided by the variation of the current flowing in a third nonsuperconducting coil. The variation of the potential difference at the coil ends

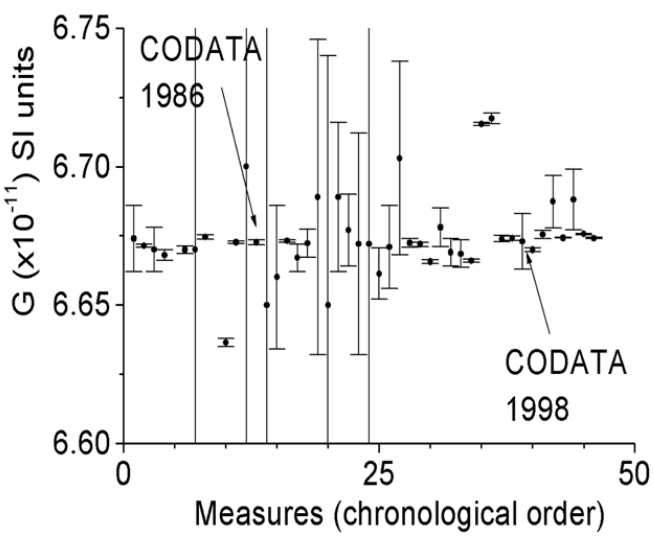

FIG. 1. Experimental values of G obtained in the last decades. The data are in chronological order; for references see the text.

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turns out to be proportional to the g variations. Therefore, to convert the instrument output from volts into an acceleration, expressed in nms^{-2}, a calibration is needed. We have operated such a SCG (type D60/T015) for a long time in recent years. The instrument is located in the Brasimone research center of ENEA, in the Apennines mountains between Bologna and Florence, Italy. In 2002 the instrument was upgraded, now GWR type T042, mainly in order to eliminate a drift in the output signal, and was put into operation again in December 2002. Other important upgrade operations were a new gravity card, a system to know exactly when the instrument is in operation (cooled down at 4.2 K) from outside the horizontal sphere position, the placement on the instrument axis of two tiltmeters that control the SCG verticality.

A gravity signal is made of several components. The amplitude A_i of every wave component is usually expressed by means of a gravimetric factor δ_i (>1), so that $A_i = \delta_i A_{\mathrm{theor},i}$, where $A_{\mathrm{theor},i}$ depends on Moon and Sun positions. Among these factors, as computed theoretically [44] and verified experimentally [45] in the last years as a consequence of data accumulation from several SCG around the world, the gravimetric factor δO_1 of the diurnal lunar wave O_1 is usually adopted as a reference wave because its amplitude is large, the ocean load is well known and the atmospheric influence is weak. Its value turns out to be practically constant over Europe. Its average value over 20 European stations, before correcting for the oceanic loading, is $\delta O_1 = (1.1483 \pm 0.0008)$ [46,47]. Normally the SCG calibration is accomplished by comparison with an absolute gravimeter. Since the absolute instrument is not as sensible as the SCG, a series of periodic comparisons are required to achieve a more precise calibration factor. The Brasimone instrument was calibrated in a different way, that is by checking that the δO_1 value obtained from the wave component analysis agrees with the European average. This procedure provided a more precise calibration factor then that resulting from the usual calibration method.

When in operation, the SCG measures g variations due to the tide and to geological noise. In addition to that, we measured the effect of the moving annular mass, which has an inner diameter of 80 cm, outer diameter of 91 cm and $11 \times 11\ \mathrm{cm}^2$ cross section. The total moving mass, including four supporting clamps, was (281.58 ± 0.01) kg. In order to move it, we build around the instrument an elevator, taking particular care to avoid the influence of the mechanical vibrations of the device on the SCG. The vertical position of the mass was measured by means of an optical lever having a 0.01 mm resolution. Figure 2 is a picture of the experimental set up at the Brasimone site.

The horizontal position of the sphere, relative to the annular mass, was measured when the instrument was in operation by using a vertical rod inserted in a receptacle, put inside the instrument for this purpose. The measure must be performed when the instrument is cold because of the deformations accompanying the cooling down. A microphotogrammetric method was then used to locate within ± 1 mm the rod position with respect to the annular mass. We found a (13 ± 1) mm shift of the sphere position with respect to the symmetry axis of the mass.

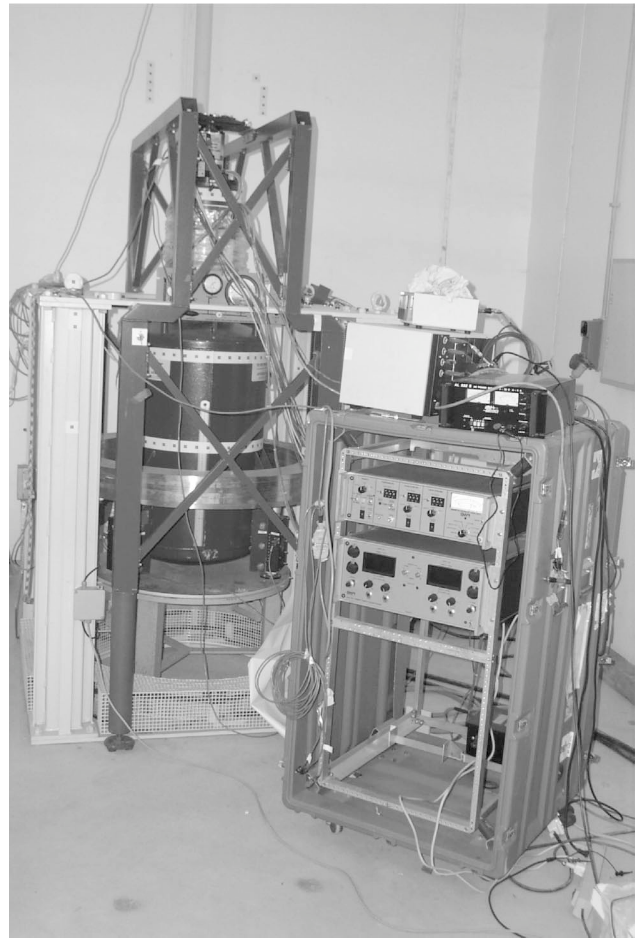

FIG. 2. The experimental apparatus. On the left the annular ring and the dewar containing the gravity sensor are visible. On the right is the electronics controlling the system.

III. RESULTS AND DISCUSSION

In the first part of the experiment the SCG acquired tidal data, which were used to compute the calibration factor, as mentioned in the previous section. A simultaneous barometric measure allowed also to obtain the local relation between changes of pressure and gravity and to correct the data for that effect. Afterwards, the annular mass was moved up and down for some days. The overall mass displacement was about 0.8 m, centered around the sphere. The mass was moved at a constant speed of 0.17 cm/s, adding to the tide a 16'' periodic signal whose amplitude was about 0.108 volt or 69.7 nms^{-2}, taking into account the calibration factor. A typical tidal signal and the SCG output when the mass was moved are shown in Fig. 3 and 4.

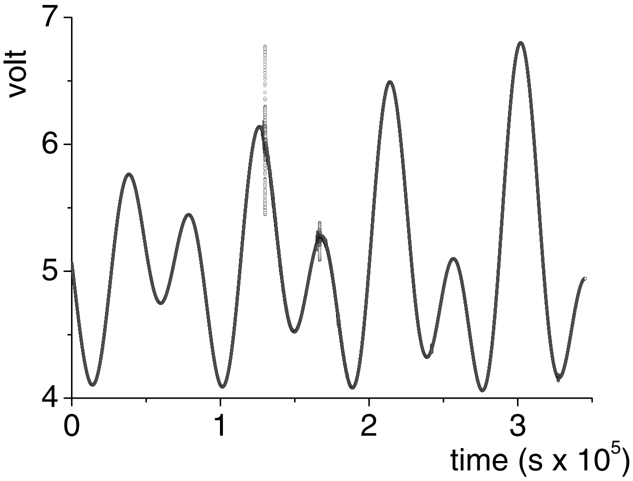

FIG. 3. Four days of tidal signal as acquired by the SCG at the Brasimone site during 2003. The overall intensity of the tide was sligtly less then 3 volts, corresponding to about 2000 nms^{-2}. The spikes in the signal are due to seismic noise.

With the aim at obtaining the calibration factor, tidal data acquired with a Keithley K2000 multimeter from the SCG at a 1 s rate for a period of several months were analyzed as follows. A data pre-elaboration was done using the Tsoft software [48] to correct for steps, spikes and gaps. With the same program we numerically filtered and decimated the data to an hourly rate. By using Eterna3 software [49], the measured tide (in volts) was compared with a tidal model of the site (in nms^{-2}) resulting from several years of data [50] obtained with the T015 instrument and comparisons with absolute instruments [51,52]. By changing the calibration factor, an iterative procedure was performed in order to minimize the difference between the two data sets. This analysis provided amplitudes, gravimetric factors and phases for the main tidal species of the Tamura 1987 catalogue [53], together with a mean real barometric pressure admittance. In particular, we obtained $\delta O_1 = (1.14825 \pm 0.00008)$, well consistent with the European average (the same agreement is also found after correcting both values for the oceanic loading). The calibration factor resulting from this analysis is (645.0 ± 0.2) nms^{-2}/volt. Its uncertainty cannot be however known better than the gravimeter factor δO_1 which is known at the 7×10^{-4} level. Because of the linearity between δO_1 and the calibration factor (which we verified in the data analysis process) the overall error on f becomes 7.7×10^{-4}, yielding $f = (645.0 \pm 0.5)$ nms^{-2}/volt.

The gravitational effect produced by the moving mass, shown in Fig. 5, was obtained by subtracting the tide from the experimental data. It was analyzed through a linear regression between simulated and experimental data. The gravitational signal, S_{exp}, induced by the moving mass is proportional at any position z of the annular mass along the vertical axis to G, so that it can be written as: $S_{exp} = \alpha(z)G$. The same is true for the simulated signal, S_{th}, except that the CODATA value ($G_{cod} = 6.673 \times 10^{-11} Nm^2/kg^2$) was used: $S_{th} = \alpha(z)G_{cod}$. This latter data set was generated using a Montecarlo algorithm computing the acceleration due to the annular mass and the supporting clamps as a function of the coordinate along an axis shifted (13 ± 1) mm from the symmetry axis. The simulated signal was generated considering the gravitational effect induced on a point moving with the same law with which the annular mass was moved. It is straightforward to see that the experimental signal is proportional to the simulated one and that the proportionality constant is G/G_{cod}. Therefore, when the two data sets, which consist of about 85 000 points, are plotted one as a function of the other, they lie along a line whose slope provides G, see

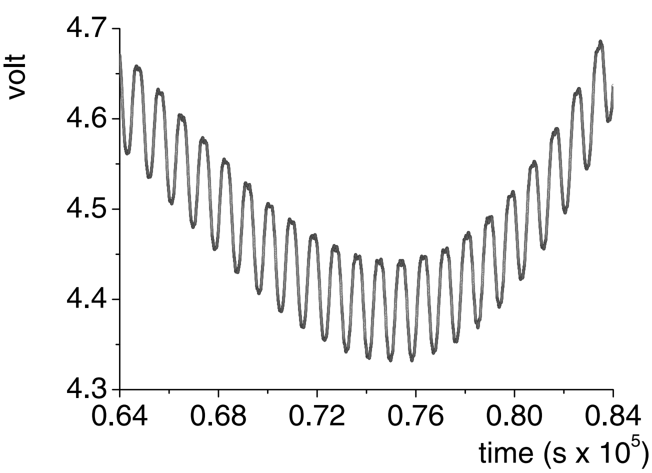

FIG. 4. SCG signal due to tide and annular mass. The 16'' oscillations due to the moving mass are superposed on the more slowly varying tide, here close to a minimum.

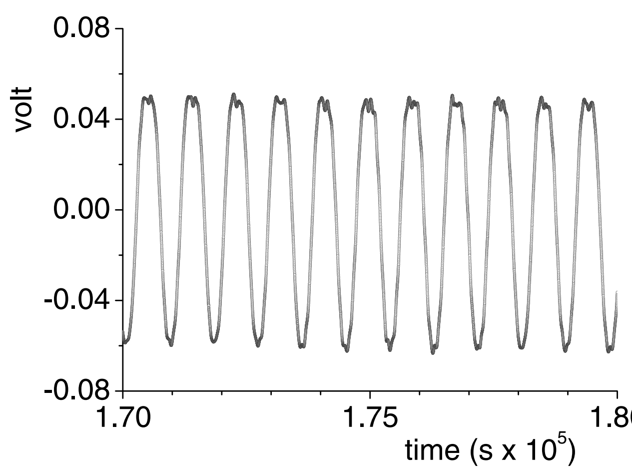

FIG. 5. After subtracting the tide what is left is the effect of the moving mass, that is a gravity signal of about 0.108 volts or 69.7 nms^{-2}.

Fig. 6. The linear regression between the two data sets yielded the value $G = (6.675 \pm 0.003)10^{-11}\mathrm{Nm}^2/\mathrm{kg}^2$.

The uncertainty quoted above for G is simply the standard error obtained from the linear regression. Its value was corrected taking into account the uncertainties induced on the experimental data by the quantities on which they depend. These are the calibration factor f and the mass, which has a relative error of 0.4×10^{-4}. The errors due to the uncertainty on the z axis position and the voltmeter accuracy turn out to be negligible. Therefore, the final value for the gravitational constant with its standard deviation are $G = (6.675 \pm 0.007)10^{-11}\mathrm{Nm}^2/\mathrm{kg}^2$.

A similar experiment was accomplished in past years using an absolute gravimeter [35] and a one-half metric ton source mass. This experiment measure a value of G with a relative uncertainty of 14×10^{-4}, indicating that a SCG is better that an absolute instrument for measuring G. In our opinion, this is due to the higher resolution and less noise provided by an SCG with respect to an absolute gravimeter. In any case a gravimetric G determination will not do better then a relative uncertainty of 7×10^{-4}, essentially arising from the present uncertainty affecting the knowledge of δO_1. The same conclusions obviously apply also to the use of an atomic gravimeter, since the above mentioned error source equally affects this type of instrument as it does all the other gravimeters [54].

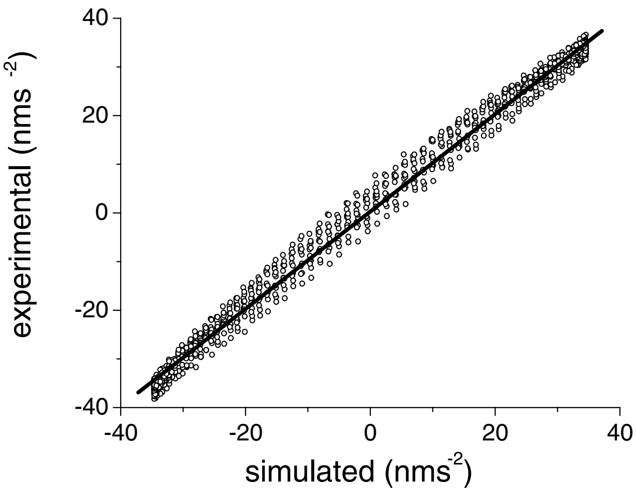

FIG. 6. Linear fit of the experimental data as a function of simulated data. The slope of the fitting line provides the ratio G/G_{cod}

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