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Conformal symmetry of brane world effective actions

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A simple derivation of the low-energy effective action for brane worlds is given, highlighting the role of conformal invariance. We show how to improve the effective action for a positive- and negative-tension brane pair using the AdS/CFT correspondence.

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One of the most striking ideas to emerge from string theory is that the universe we inhabit may be a brane embedded in, or bounding, a higher-dimensional spacetime. The brane construction naturally removes the extra dimensions from view, and gives a different perspective on the nature of the gravitational force. It also leads to important restrictions on the form of the low-energy four-dimensional effective action.

In this article, we show in particular how the brane construction automatically implies conformal invariance of the four-dimensional effective theory. This explains the detailed form of the low-energy effective action, previously found using other methods. The AdS/CFT correspondence may then be used to improve the effective description, and we show how this works in detail for a positive- and negative-tension brane pair.

We start by considering a pair of four-dimensional positive- and negative-tension Z_2-branes bounding a five-dimensional bulk with a negative cosmological constant [1]. This is the simplest setting incorporating branes with a nontrivial warp factor in the bulk. As is well-known, the model possesses a one-parameter family of static solutions representing flat branes located at arbitrary Y in a static AdS bulk $\mathrm{d}Y^2 + e^{2Y/L}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$, where L is the AdS radius, x^μ, $\mu = 0, 1, 2, 3$, parametrize the four dimensions tangent to the branes and Y parameterizes the dimension normal to the branes. The locations of the branes, $Y^\pm$, are moduli.

For the general, nonstatic solution to the same model it is convenient to choose coordinates in which the bulk metric takes the form

$$\mathrm{d}s^2 = \mathrm{d}Y^2 + g_{\mu\nu}(x, Y)\mathrm{d}x^\mu \mathrm{d}x^\nu. \tag{1}$$

The brane loci are now $Y^\pm(x)$ and the metric induced on each brane is

$$g^\pm_{\mu\nu}(x) = \partial_\mu Y^\pm(x)\partial_\nu Y^\pm(x) + g_{\mu\nu}[x, Y^\pm(x)]. \tag{2}$$

At low energies we expect the configuration to be completely determined by the metric on one brane and the normal distance to the other brane, $Y^+ - Y^-$. That is, we are looking for a four-dimensional effective theory consisting of gravity plus one physical scalar degree of freedom. What we will now show is that this theory may be determined on symmetry grounds alone. (See also [2] for related ideas).

The full five-dimensional theory is diffeomorphism invariant. This invariance includes the special set of transformations

$$Y' = Y + \xi^5(x), \quad x'^\mu = x^\mu + \xi^\mu(x, Y), \tag{3}$$

with $\xi^\mu(x, Y)$ satisfying

$$\partial_Y \xi^\mu(x, Y) = -g^{\mu\nu}(x, Y)\partial_\nu \xi^5(x), \tag{4}$$

which preserve the form (1) of the metric. Equation (4) may be integrated to give $\xi^\mu(x, Y) = \xi^\mu[x, Y^-(x)] - \int_{Y^-(x)}^{Y} dY g^{\mu\nu}(x, Y)\partial_\nu \xi^5(x)$, where $\xi^\mu[x, Y^-(x)]$ are the parameters of a four-dimensional diffeomorphism on the minus brane. The transformation (3) displaces the $Y^\pm(x)$ coordinates of the branes, $Y^\pm(x) \to Y^\pm(x) + \xi^5 - \xi^\sigma \partial_\sigma Y$, and alters $g^{\mu\nu}(x, Y)$ via the usual Lie derivative. Using (4), one finds that the combined effect on each brane metric (2) is the four-dimensional diffeomorphism $x'^\mu = x^\mu + \xi^\mu[x, Y^\pm(x)]$. In fact, by departing from the gauge (1) away from the branes, we can construct a five-dimensional diffeomorphism for which ξ^μ vanishes on the branes. To see this, we can set $\xi^\mu(x, Y) = \xi^\mu_0(x, Y) - f(Y)\xi^\mu_0(x, Y^+)$, where $\xi^\mu_0(x, Y)$ is the solution to (4) which vanishes on the minus brane and $f(Y)$ is a function chosen to satisfy $f(Y^-) = 0$, $f(Y^+) = 1$, and $f'(Y^-) = f'(Y^+) = 0$ for all x.

We conclude that the four-dimensional theory, in which $Y^\pm(x)$ are represented as scalar fields, must possess a local symmetry $\xi^5(x)$ acting nontrivially on those fields. The dimensionless exponentials $\psi^\pm(x) \equiv e^{Y^\pm(x)/L}$ transform as conformal scalars: $\psi^\pm(x) \to e^{\xi^5/L}\psi^\pm(x)$, while the induced brane metrics $g^\pm_{\mu\nu}$ remain invariant. The only local, polynomial, two-derivative action possessing such a symmetry involves gravity with two conformally-coupled scalar fields. After diagonalizing and rescaling the fields, this may be expressed as

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$$m^2 \int d^4x\sqrt{-g}(c_+\psi^+\Delta\psi^+ + c_-\psi^-\Delta\psi^-), \qquad (5)$$

where $\Delta \equiv \Box - \frac{1}{6}R$, $c_\pm = \pm 1$, and m is a constant with dimensions of mass. It should be stressed that the metric $g_{\mu\nu}$ appearing in this expression is that of the effective theory, which is in general different to the induced metric on the branes $g^\pm_{\mu\nu}$. Potential terms are excluded by the fact that flat branes, with arbitrary constant $\psi^\pm$, are solutions of the five-dimensional theory, i.e., the $\psi^\pm$ are moduli.

By construction, the theory possesses local conformal invariance under

$$\psi^\pm \to \Omega(x)^{-1}\psi^\pm, \quad g_{\mu\nu} \to \Omega(x)^2 g_{\mu\nu}. \qquad (6)$$

For $c_+ = -c_-$, without loss of generality we can set $c_+ = -1$. Provided $(\psi^+)^2 - (\psi^-)^2 > 0$, we obtain the usual sign for the Einstein term, so there are no ghosts in the gravitational sector. We can then set $\psi^+ = A\cosh\phi/\sqrt{6}$ and $\psi^- = -A\sinh\phi/\sqrt{6}$. The field A has the wrong sign kinetic term, but it can be set equal to a constant by a choice of conformal gauge. Therefore, in this case there are no physical propagating ghost fields. In contrast, a similar analysis reveals that when $c_+ = c_-$ the theory possesses physical ghosts either in the gravitational wave sector (wrong sign of R) or in the scalar sector, no matter how the conformal gauge is fixed. We conclude that the low-energy effective action must be

$$m^2 \int d^4x\sqrt{-g}(-\psi^+\Delta\psi^+ + \psi^-\Delta\psi^-). \qquad (7)$$

We know from the above argument that the brane metrics are conformally-invariant: from this and from general covariance they must equal $g_{\mu\nu}$ times homogeneous functions of order two in ψ^+ and ψ^-. But in the model under consideration, we have static solutions $g^\pm_{\mu\nu} = e^{2Y^\pm/L}\eta_{\mu\nu}$ for all $Y^+ > Y^-$. The only choice consistent with this and with $(\psi^+)^2 - (\psi^-)^2 > 0$ is

$$g^\pm_{\mu\nu} = \frac{(\psi^\pm)^2}{6} g_{\mu\nu}, \qquad (8)$$

which is a conformally-invariant equation. We have introduced the numerical factor for later convenience.

It is instructive to fix the conformal gauge in several ways. First, set $\psi^+ = \sqrt{6}$, so that $g_{\mu\nu} = g^+_{\mu\nu}$ and the metric appearing in (7) is actually the metric on the plus (positive-tension) brane. The action (7) then consists of Einstein gravity (with Planck mass m) plus a conformally-invariant scalar field ψ^- which has to be smaller than $\sqrt{6}$:

$$m^2 \int d^4x\sqrt{-g^+}\Big[\Big(1 - \frac{1}{6}(\psi^-)^2\Big)R^+ - (\partial\psi^-)^2\Big]. \qquad (9)$$

Changing variables to $\chi = 1 - (\psi^-)^2/6$ produces the alternative form [3]

$$m^2 \int d^4x\sqrt{-g^+}\Big[\chi R^+ - \frac{3}{2(1-\chi)}(\partial\chi)^2\Big]. \qquad (10)$$

Conversely, if we set $\psi^- = \sqrt{6}$, then $g_{\mu\nu}$ is the metric on the minus (negative-tension) brane and ψ^+, which has to be larger than $\sqrt{6}$, is a conformally-coupled scalar field. (However, the relative sign between the gravitational and kinetic terms in the action is now wrong, and so this gauge possesses ghosts). If we add matter coupling to the metric on the plus and minus branes, we find that matter on the minus brane couples in a conformally-invariant manner to the plus brane metric and the field ψ^-, and conversely for matter on the plus brane. Note that we are not implying conformal invariance of the matter itself: it is simply that matter coupled to the brane metrics will be trivially invariant under the transformation (6) as the brane metrics are themselves invariant.

A third conformal gauge maps the theory to Einstein gravity with a minimally-coupled scalar field ϕ, taking the values $-\infty < \phi < 0$. Starting from (7), we can set $\psi^+ = A\cosh\phi/\sqrt{6}$ and $\psi^- = -A\sinh\phi/\sqrt{6}$, as noted earlier, to obtain the action

$$-m^2 \int d^4x\sqrt{-g}\Big[A\Delta A + \frac{A^2}{6}(\partial\phi)^2\Big]. \qquad (11)$$

Now choosing the conformal gauge $A = \sqrt{6}$ we find

$$m^2 \int d^4x\sqrt{-g}(R + \phi\Box\phi), \qquad (12)$$

i.e., gravity plus a minimally-coupled massless scalar. In this gauge Eqs. (8) read:

$$g^+_{\mu\nu} = \cosh^2\Big(\frac{\phi}{\sqrt{6}}\Big)g_{\mu\nu}, \quad g^-_{\mu\nu} = \sinh^2\Big(\frac{\phi}{\sqrt{6}}\Big)g_{\mu\nu}, \qquad (13)$$

in agreement with explicit calculations in the moduli space approach [4].

The present treatment also goes some way towards explaining the moduli space results. For example, the fact that the moduli space metric is flat is seen to be a consequence of conformal invariance. Specifically, for solutions with cosmological symmetry one can pick a conformal gauge in which the metric is static. The scale factors on the two branes are determined by $\psi^\pm$. From (7), the moduli space metric is just two-dimensional Minkowski space.

A couple of results for conformal gravity follow from the above discussion. First, in the $\psi^+ = \sqrt{6}$ gauge, we have $\psi^- = -\sqrt{6}\tanh(\phi/\sqrt{6})$. Any solution for a minimally-coupled scalar ϕ, with metric $g_{\mu\nu}$, thus yields a corresponding solution for a conformally-coupled scalar ψ^-, with $|\psi^-| < \sqrt{6}$ and metric $g^+_{\mu\nu}$ as in (13), and vice versa. Second, in the $\psi^- = \sqrt{6}$ gauge, we have $\psi^+ = -\sqrt{6}\coth(\phi/\sqrt{6})$. Hence we may also obtain a solution for a conformally-coupled scalar ψ^+, with $|\psi^+| > \sqrt{6}$ and metric $g^-_{\mu\nu}$ given in (13). Thus solutions to conformal scalar gravity come in pairs: if $g_{\mu\nu}$ and ψ are a solution, then $(\psi^2/6)g_{\mu\nu}$ and $\tilde{\psi} = 6/\psi$ is another solution. In terms of branes, this merely states that if $g^+_{\mu\nu}$ and ψ^- are known in the gauge $\psi^+ = \sqrt{6}$, then it is possible to reconstruct $g^-_{\mu\nu}$ and ψ^+ in the gauge $\psi^- = \sqrt{6}$.

The argument given above establishing the conformal symmetry of the effective action is of a very general nature: the only step at which we specialized to the Randall-Sundrum model was in the identification of the brane metrics in terms of the effective theory variables (8). This required only the knowledge of a one-parameter family of solutions.

To derive the effective theory for other brane models, it is only necessary to generalize this last step. For example, in the case of tensionless branes compactified on an S_1/Z_2, the bulk warp is absent and so we know that a family of static solutions is given by the ground state of Kaluza-Klein theory (in which all fields are independent of the extra dimension, and so the additional Z_2 orbifolding present in the tensionless brane case is irrelevant). Ignoring the gauge fields, the Kaluza-Klein ansatz for the five-dimensional metric is

$$\mathrm{d}s^2 = e^{2\sqrt{2/3}\phi(x)}\mathrm{d}y^2 + e^{-\sqrt{2/3}\phi(x)}g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu, \quad (14)$$

where ϕ and $g_{\mu\nu}$ extremize an action identical to (12). For branes located at constant y, the induced metrics are $e^{-\sqrt{2/3}\phi}g_{\mu\nu}$, independent of y.

Using the effective action in the form (11), conformal invariance of the induced brane metrics dictates that

$$g^{\pm}_{\mu\nu} = A^2 f^{\pm}(\phi)g_{\mu\nu}, \quad (15)$$

for some unknown functions $f^{\pm}$. Upon fixing the conformal gauge to $A = \sqrt{6}$ one recovers the action (12), which is just the standard Kaluza-Klein low-energy effective action. The functions $f^{\pm}$ are thus both equal to $\frac{1}{6}e^{-\sqrt{2/3}\phi}$ and we have

$$g^{\pm}_{\mu\nu} = e^{-\sqrt{2/3}\phi}g_{\mu\nu} = \frac{1}{6}(\psi^+ + \psi^-)^2 g_{\mu\nu}. \quad (16)$$

Note that this is consistent with the $\phi \to -\infty$ limit of the Randall-Sundrum theory (13): as the brane separation goes to zero, the warping of the bulk becomes negligible and the Randall-Sundrum theory tends to the Kaluza-Klein limit [5].

We now turn to a discussion of the general cosmological solutions representing colliding branes. We choose a conformal gauge in which the metric is static, and all the dynamics are contained in $\psi^{\pm}$. For flat, open and closed spacetimes the spatial Ricci scalar $R = 6k$, where $k = 0, -1$ and $+1$ respectively. The action (7) yields the equations of motion

$$\ddot{\psi}^{\pm} = -k\psi^{\pm} \quad (17)$$

$$(\dot{\psi}^+)^2 - (\dot{\psi}^-)^2 = -k[(\psi^+)^2 - (\psi^-)^2]. \quad (18)$$

For $k = 0$ we have the solutions

$$\psi^+ = -At + B, \quad \psi^- = At + B, \quad t < 0 \quad (19)$$

representing colliding flat branes. It is natural to match ψ^+ to ψ^- across the collision, and vice versa, to obtain $\psi^{\pm} = \pm At + B$ for $t > 0$. This solution then describes two branes which collide and pass through each other, with the plus brane continuing to a minus brane and vice versa [5,6].

For $k = -1$, we have the three solutions

$$\begin{aligned} \psi^{(1)} &= A\sinh t; & A\cosh t; & \quad Ae^t, \\ \psi^{(2)} &= A\sinh(t - t_0); & A\cosh(t - t_0); & \quad Ae^{t-t_0}, \end{aligned} \quad (20)$$

where we set ψ^+ equal to the greater, and ψ^- equal to the lesser, of $\psi^{(1)}$ and $\psi^{(2)}$. For $k = +1$, we find the bouncing solutions $\psi^{(1)} = A\sin t$, $\psi^{(2)} = A\sin(t - t_0)$. In the absence of matter on the minus brane, the sin and sinh solutions are singular when the minus brane scale factor a_- vanishes. However, matter on the minus brane scaling faster than a_-^{-4}, for example, scalar kinetic matter, causes the solution for ψ^- to bounce smoothly at positive a_- because ψ^- has a positive kinetic term. This bounce is perfectly regular. However, the "big crunch-big bang" singularity, occurring when the positive- and negative- tension branes collide, is unavoidable.

The above example illustrates a general feature of the brane pair effective action. If the positive- and negative-tension brane solutions are continued through the collision without relabeling (this means that the orientation of the warp must flip) then the four-dimensional effective action changes sign. The relabeling restores the conventional sign. The same phenomenon is seen in string theories obtained by dimensionally reducing 11 dimensional supergravity, when the 11th dimension collapses and reappears. Brane world black hole solutions with intersecting branes are discussed in [7].

Recently it has been shown that the AdS/CFT correspondence [8] provides a powerful approach to the understanding of brane worlds. For a single positive-tension brane the four-dimensional effective description comprises simply Einstein gravity plus two copies of the dual CFT [9] (as the Z_2 symmetry implies there are two copies of the bulk). Notable successes of this program include reproducing the $O(1/r^3)$ corrections to Newton's law on the brane [10], and reproducing the modified Friedmann equation induced on the brane [11,12].

Consider for simplicity a single positive-tension brane containing only radiation. Taking the trace of the effective Einstein equations we find

$$-R = 2(8\pi G_4) < T_{CFT} >, \quad (21)$$

as the stress tensor of the radiation is traceless. The trace anomaly of the dual $\mathcal{N} = 4$ $SU(N)$ super-Yang Mills theory must then be evaluated. With the help of the AdS/

CFT dictionary, this quantity may be calculated for the case of cosmological symmetry as shown in [13], giving

$$-R = \frac{L^2}{4}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right). \tag{22}$$

Here, the usual R^2 counterterm has been added to the action in order to eliminate the $\Box R$ term in the trace, thus furnishing second order equations of motion.

For a cosmological metric with scale factor a this becomes

$$2(\ddot{a}a + ka^2) = L^2(k + h^2)\dot{h}, \tag{23}$$

where $h \equiv \dot{a}/a$ and the dot denotes differentiation with respect to conformal time. Re-expressing the left-hand side as $h^{-1}\partial_t(\dot{a}^2 + ka^2)$ we can then integrate to obtain

$$h^2 + k = \frac{1}{a^2}\left(B - \frac{1}{4}k^2L^2\right) + \frac{1}{4}(h^2 + k)^2\frac{L^2}{a^2}, \tag{24}$$

where B is an integration constant. Now, we can expect to recover Einstein gravity on the brane in the limit when $L \rightarrow 0$, with other physical quantities fixed. We expand all terms in powers of L. At leading order we must obtain four-dimensional Einstein gravity, for which $8\pi G_4 = 8\pi G_5/L$. So we set $B \sim (8\pi G_5\rho_0/3L) + C$, where $\rho = \rho_0/a^4$ is the energy density of conventional radiation, and C is a constant independent of L as $L \rightarrow 0$. From (24) we then obtain the first correction to $h^2 + k$, namely

$$h^2 + k = \frac{8\pi G_5\rho_0}{3La^2} + \frac{C}{a^2} + \frac{(8\pi G_5\rho_0)^2}{36a^6} + O(L), \tag{25}$$

which, thanks to the CFT contribution, now includes the well-known dark energy and ρ^2 corrections [14].

It should come as no surprise that the AdS/CFT correspondence only approximates the Randall-Sundrum setup up to first nontrivial order in an expansion in L. The AdS/CFT scenario involves string theory on $AdS_5 \times S_5$. Since $\alpha' \sim \ell_s^2 \sim L^2$ at fixed 't Hooft coupling, and the masses squared of the Kaluza-Klein modes on the S_5 are of order $1/L^2$, we expect nontrivial corrections at second order in an expansion in L. Furthermore, one can show from the AdS/CFT dictionary that in order for the ρ^2 term to dominate in the modified Friedmann equation, the temperature of the conventional radiation must be greater than the Hagedorn temperature of the string. Clearly, the AdS/CFT correspondence cannot describe this situation.

We now extend the AdS/CFT approach to the case of a pair of positive- and negative- tension branes using the ideas developed earlier in this paper. The effective action for a single positive-tension brane is

$$\frac{1}{16\pi G_4}\int d^4x\sqrt{-g^+}R^+ + 2W_{CFT}[g^+] + S_m[g^+], \tag{26}$$

where $g^+_{\mu\nu}$ is the induced metric on the brane, S_m is the brane matter action, and W_{CFT} is the CFT effective action (including the appropriate R^2 counterterms). Substituting now for $g^+_{\mu\nu}$ using (8), the Einstein-Hilbert term $\sqrt{-g^+}R^+$ becomes $-\sqrt{-g}\psi^+\Delta\psi^+$. A negative-tension brane may then be incorporated as follows:

$$\frac{1}{16\pi G_4}\int d^4x\sqrt{-g}(-\psi^+\Delta\psi^+ + \psi^-\Delta\psi^-) + 2W_{CFT}[g^+] - 2W_{CFT}[g^-] + S_m[g^+] + S_m[g^-]. \tag{27}$$

The action for the positive- and negative-tension brane pair must take this form in order to correctly reproduce the Friedmann equation for each brane. To see this, consider again the conformal gauge in which the effective theory metric is static and all the dynamics are contained in $\psi^\pm$, which play the role of the brane scale factors. Variation with respect to the $\psi^\pm$ yields the scalar field equations

$$(\psi^\pm)^{-3}\Delta\psi^\pm = 2(8\pi G_4) < T^\pm_{CFT} >, \tag{28}$$

where the trace anomaly must be evaluated on the induced brane metric $g^\pm_{\mu\nu}$ but Δ is evaluated on the effective metric $g_{\mu\nu}$. The left-hand side evaluates to $-(\psi^\pm)^{-3} \times [\ddot{\psi}^\pm + k(\psi^\pm)^2]$. After identifying $\psi^\pm/\sqrt{6}$ with $a_\pm$ according to (8), and then dropping the plus or minus label, we recover Eq. (23). From the necessity of recovering the Friedmann equation on each brane we may also deduce that cross-terms in the action between ψ^+ and ψ^- are forbidden.

The signs of the gravity parts of the action are needed to achieve consistency with (7). Consequently, the relative sign between the gravity plus CFT part of the action and that of the matter is reversed for the minus brane, consistent with the modified Friedmann equations [14],

$$H^2_\pm = \pm\frac{8\pi G_5\rho_\pm}{3L} + \frac{(8\pi G_5\rho_\pm)^2}{36} - \frac{k}{a^2} + \frac{C}{a^4}, \tag{29}$$

where plus and minus label the positive- and negative-tension branes, and C is again a constant representing the dark radiation.

To summarize, we have elucidated the origin of conformal symmetry in brane world effective actions, and shown how this determines the effective action to lowest order. When combined with the AdS/CFT correspondence, our approach also recovers the first corrections to the brane Friedmann equations.

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