Light quark mass ratio from the Dalitz plot of $\eta \to \pi^+\pi^-\pi^0$ decay

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High statistics Dalitz-plot distribution of $\eta \to \pi^+\pi^-\pi^0$ decay obtained recently by KLOE collaboration is fitted to the results of corresponding theoretical calculations in Chiral Perturbation Theory (ChPT) with unitarity corrections taken into account. The quark mass ratio $Q = \sqrt{[m_s^2 - (m_d + m_u)^2/4]/(m_d^2 - m_u^2)}$ can be obtained from this analysis. We get $Q = 22.8 \pm 0.4$ which differs from the value $Q_{DT} = 24.2$ that follows from Dashen's theorem and agrees with recently calculated electromagnetic kaon mass difference.

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The possibility to extract light quark mass difference from $\eta \to \pi^+\pi^-\pi^0$ decay is known for a long time [1]. In Chiral Perturbation Theory (ChPT) the decay width Γ depends on quark mass ratios and theoretically calculable factor $\bar{\Gamma}$ [2]:

$$\Gamma = \left(\frac{Q_{DT}}{Q}\right)^4 \bar{\Gamma}, \tag{1}$$

where

$$Q^{-2} = \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}, \quad \hat{m} = \frac{m_d + m_u}{2}, \tag{2}$$

m_u, m_d, m_s are up, down and strange quark masses,

$$Q_{DT}^{-2} = \frac{[(m_{K^0}^2 - m_{K^+}^2) - (m_{\pi^0}^2 - m_{\pi^+}^2)]m_{\pi^0}^2}{(m_K^2 - m_{\pi^0}^2)m_K^2} = (24.2)^{-2}, \tag{3}$$

with $m_K^2 = (m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2)/2$.

Note that $Q_{DT} = Q$ if electromagnetic mass differences of kaons and pions are equal to each other as Dashen's theorem states [3]. Experimental (Particle Data Group) value $\Gamma = 291 \pm 21$ eV [4] is far from one-loop ChPT value $\bar{\Gamma} = 167 \pm 50$ eV [2] and from the values $\bar{\Gamma} = 209 \pm 20$ eV [5] and $\bar{\Gamma} = 219 \pm 22$eV [6] obtained with higher order corrections taken into account by dispersion method. In [5] the subtraction polynomial was taken from the decomposition of one-loop order amplitude and had therefore uncertainties connected to higher orders ChPT corrections. These uncertainties were further fixed [7] by fitting the experimental data [8] on Dalitz-plot distribution in the decay considered: $\bar{\Gamma} = 213^{+3}_{-12}$ eV. There was a conjecture in [7] that new experimental data on Dalitz-plot distribution will give slightly different value of $\bar{\Gamma}$. Now these new experimental data (contradicting to the old ones [8]) are available [9]. In what follows we will use them to get new value of $\bar{\Gamma}$ and new value of quark mass ratio Q, as a consequence.

We use the method of work [7] and remind it here for completeness. In order to simulate the experimental Dalitz-plot distribution we take it in a form

$$1 + ay + by^2 + cy^3 + dx^2 \tag{4}$$

with $a = -1.075 \pm 0.008$, $b = 0.118 \pm 0.009$, $c = 0.13 \pm 0.02$, $d = 0.049 \pm 0.008$ [9] and y, x defined in a standard way

$$y = \frac{3T_0}{Q} - 1,$$

$$x = \frac{\sqrt{3}}{Q}(T_+ - T_-), Q = T_+ + T_- + T_0,$$

T_+, T_-, T_0 are the kinetic energies of pions in the rest frame $\eta \to \pi^+\pi^-\pi^0$ decay. We divide the Dalitz-plot in 10×10 bins ($x \times y$) that have equal number of events for the distribution considered. Then the number of events in each bin (n) is simulated by Gaussian distribution with variance equal to n. We used $n = 10\,000$ to get the full statistics $N = 100n = 1\,000\,000$ like in the experiment [9]. From theoretical point of view the amplitude of $\eta \to \pi^+\pi^-\pi^0$ decay have an approximate solution from Eq. (5.28) of [5]. It contains the subtraction polynomial

$$P(s) = \alpha + \beta s_a + \gamma s_a^2 + \delta(s_b - s_c)^2, \tag{5}$$

where s_a, s_b and s_c are invariant masses squared of $\pi^+\pi^-$, $\pi^+\pi^0$ and $\pi^-\pi^0$ pairs, respectively. For the values of parameters α, β, γ and δ within the regions

$$\begin{aligned} &\alpha = -1.28 \pm 0.14, \quad \beta = 21.81 \pm 1.52 \text{ GeV}^{-2} \\ &\gamma = 4.09 \pm 3.18 \text{ GeV}^{-4}, \quad \delta = 4.19 \pm 1.08 \text{ GeV}^{-4} \end{aligned} \tag{6}$$

(the case of zero subtraction points [5]) the "Minuit" fit of above simulated experimental Dalitz-plot distribution has terminated on the values

$$\begin{aligned} &\alpha_0 = -1.14, \quad \beta_0 = 23.33 \text{ GeV}^{-2} \\ &\gamma_0 = 1.03q \text{ GeV}^{-4}, \quad \delta_0 = 5.27 \text{ GeV}^{-4} \end{aligned} \tag{7}$$

with $\chi^2/Nd.o.f. = 152/(100 - 4)$.

Three from four parameters are at the boundary of allowed region (6). This probably means that the guess [5] of the size of allowed region should be changed.

Equally possible is the fit with the scaled values of parameters α, β, γ and δ because the normalization factor of the amplitude is not defined by the Dalitz-plot distribution. In our case the scaling of parameters α, β, γ and δ puts them outside the allowed region (6) and no freedom in the scaling (no error in $\bar{\Gamma}$) is possible. This way we get $\bar{\Gamma} =$ 229eV what corresponds according to Eqs. (1)–(3) to the light quark mass ratio $Q = 22.8 \pm 0.4$. The errors here are due to the errors in the experimental value of the width Γ. So, we conclude, high statistics Dalitz-plot distribution gives the value of light quark mass ratio Q slightly lower than that from the assumption of equality of kaon and pion electromagnetic mass differences ($Q_{DT} = 24.2$). This is in agreement with calculations of electromagnetic mass differences for pions and kaons [10,11] which find large violations to Dashen's theorem ($Q = 22.0 \pm 0.6$). Our result agrees also very well with that of works [5,6], where the values $Q = 22.4 \pm 0.9$ and $Q = 22.7 \pm 0.8$ were obtained, correspondingly.

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