Neutrino electromagnetic form factor and oscillation effects on neutrino interaction with dense matter

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The mean free path of neutrino-free electron gas interaction has been calculated by taking into account the neutrino electromagnetic form factors and the possibility of neutrino oscillation. It is shown that the form factor effect becomes significant for a neutrino magnetic moment $\mu_\nu \geq 10^{-10}\mu_B$ and for a neutrino radius $R \geq 10^{-6}$ MeV^{-1}. The mean free path is found to be sensitive to the $\nu_e - \nu_\mu$ and $\nu_e - \nu_e^R$ transition probabilities.

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Neutrino interaction with dense matter plays an important role in astrophysics, e.g., in the formation of supernova and the cooling of young neutron stars [1–8]. Earlier calculation on neutrino interactions with electrons gas, dense and hot matter, based on the standard model has been performed by Horowitz and Wehrberger [2,3]. Some relativistic calculations of neutrino mean free path in hot and dense matter have been also done in Refs. [4–7]. Recently, due to a demand on a more realistic neutrino mean free path for supernova simulations, a mean free path calculation by taking into account the weak magnetism of nucleons has been also performed [8].

However, certain phenomena such as solar neutrinos, atmospheric neutrinos problems, and some astrophysics and cosmology arguments need explanations beyond the standard model assumption of neutrino's properties such as neutrino oscillation [9,10], the helicity flipping of neutrinos [11–14] and neutrino electromagnetic form factors. We note that the upper bound of the neutrino magnetic moment extracted from the Super-Kamiokande solar data [15,16] falls in the range of $(1.1\text{–}1.5) \times 10^{-10}\mu_B$, where $\mu_B = e/2m_e$ stands for the Bohr magneton. Other experimental limits [17,18] give $\mu_\nu < 1.0 \times 10^{-10}\mu_B$, whereas signals from Supernova 1987A (SN1987A) require that $\mu_\nu \leq 1.0 \times 10^{-12}\mu_B$. These bounds have been derived by considering the helicity flipping neutrino scattering in a supernova core [19]. In the case of random magnetic fields inside the sun, one can obtain a direct constraint on the neutrino magnetic moment of $\mu_\nu \leq 1.0 \times 10^{-12}\mu_B$, similar to the bounds obtained from the star cooling [20]. In addition, data from muon neutrino- and anti neutrino-electron scatterings [21,22] and a close examination to the data over the years from Kamiokande II and Homestake according to Mourão *et al.* [23], similarly give a neutrino average squared radius $R^2 \sim 25 \times 10^{-12}$ MeV^{-2} with $R^2 = \langle R_V^2 \rangle + \langle R_A^2 \rangle$. The definitions of $\langle R_V^2 \rangle$ and $\langle R_A^2 \rangle$ will be explained later.

Therefore, in connection with the demand on realistic neutrino mean free path in dense and hot matter, an extension of the previous study [4–8] which takes into account the electromagnetic form factors of neutrinos and neutrino oscillations is inevitable. As a first step before that, in this report we calculate the mean free path of neutrino-free electrons gas where those effects are included. Here we assume that neutrinos are massless and the RPA correlations can be neglected. Furthermore, we use zero temperature approximation in this calculation.

In the standard model, where the momentum transfer is much less than the W mass, direct Z^0 and $W^\pm$ contributions to the matrix element $\mathcal{M}$ can be written as an effective four-point coupling [3,22]

$$\mathcal{M}_W = \frac{G_F}{\sqrt{2}}[\bar{U}(k')\gamma^\mu(1+\gamma^5)U(k)][\bar{U}(p')J_\mu U(p)], \quad (1)$$

where G_F is the coupling constant of weak interaction, $U(k)$ and $U(p)$ are neutrino and electron spinors, respectively, and the current J_μ is defined by

$$J_\mu = \gamma_\mu(C_V + C_A\gamma^5). \quad (2)$$

The vector and axial vector couplings C_V and C_A can be written in terms of Weinberg angle θ_W (where $\sin^2\theta_W \approx 0.223$ [3,4]) as $C_V = 2\sin^2\theta_W \pm 1/2$ and $C_A = \pm 1/2$ (the upper sign is for ν_e, the lower sign is for ν_μ and ν_τ).

The electromagnetic properties of Dirac neutrinos are described in terms of four form factors, i.e., $f_{1\nu}$, $g_{1\nu}$, $f_{2\nu}$, and $g_{2\nu}$, which stand for the Dirac, anapole, magnetic, and electric form factors, respectively. The matrix element for the neutrino-electron interaction which contains electromagnetic form factors reads [22]

$$\mathcal{M}_{EM} = \frac{4\pi\alpha}{q^2}[\bar{U}(p')\gamma_\mu U(p)]\Big\{\bar{U}(k')\Big[f_{m\nu}\gamma^\mu + g_{1\nu}\gamma^\mu\gamma^5 - (f_{2\nu} + ig_{2\nu}\gamma^5)\frac{P^\mu}{2m_e}\Big]U(k)\Big\}, \quad (3)$$

where $f_{m\nu} = f_{1\nu} + (m_\nu/m_e)f_{2\nu}$, $P^\mu = k^\mu + k^{\mu\prime}$, m_ν, and m_e are neutrino and electron masses, respectively. In the static limit, the reduced Dirac form factor $f_{1\nu}$ and the neutrino anapole form factor $g_{1\nu}$ are related to the vector and axial vector charge radii $\langle R_V^2 \rangle$ and $\langle R_A^2 \rangle$ through [22]

$$f_{1\nu}(q^2) = \frac{1}{6}\langle R_V^2\rangle q^2 \quad \text{and} \quad g_{1\nu}(q^2) = \frac{1}{6}\langle R_A^2\rangle q^2. \tag{4}$$

In the limit of $q^2 \to 0$, $f_{2\nu}$ and $g_{2\nu}$ define, respectively, the neutrino magnetic moment $\mu_\nu^m = f_{2\nu}(0)\mu_B$ and the (CP violating) electric dipole moment $\mu_\nu^e = g_{2\nu}(0)\mu_B$ [22,24]. Here we use $\mu_\nu^2 = \mu_\nu^{m\,2} + \mu_\nu^{e\,2}$.

Next, we can obtain the differential cross section per volume V for scattering of neutrinos with the initial energy E_ν and final energy E'_ν on the electrons gas. It consists of the contributions from weak (W) interaction, electromagnetic (EM) interaction, as well as their interference (INT) term, i.e.,

$$\left(\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu}\right)_{\nu_e} = -\frac{1}{16\pi^2}\frac{E'_\nu}{E_\nu}\left[\left(\frac{G_F}{\sqrt{2}}\right)^2 L_\nu^{\mu\nu}\Pi_{\mu\nu}^{\rm Im(W)} + \left(\frac{4\pi\alpha}{q^2}\right)^2 L_\nu^{\mu\nu}\Pi_{\mu\nu}^{\rm Im(EM)} + \frac{8G_F\pi\alpha}{q^2\sqrt{2}} L_\nu^{\mu\nu}\Pi_{\mu\nu}^{\rm Im(INT)}\right]. \tag{5}$$

For each contribution, the neutrino tensors are given by

$$L_\nu^{\mu\nu(\rm W)} = 8[2k^\mu k^\nu - (k^\mu q^\nu + k^\nu q^\mu) + g^{\mu\nu}(k\cdot q) - i\epsilon^{\alpha\mu\beta\nu}k_\alpha k'_\beta], \tag{6}$$

$$L_\nu^{\mu\nu(\rm EM)} = 4(f_{m\nu}^2 + g_{1\nu}^2)[2k^\mu k^\nu - (k^\mu q^\nu + k^\nu q^\mu) + g^{\mu\nu}(k\cdot q)] - 8if_{m\nu}g_{1\nu}\epsilon^{\alpha\mu\beta\nu}(k_\alpha k'_\beta) - \frac{f_{2\nu}^2 + g_{2\nu}^2}{m_e^2}(k\cdot q)[4k^\mu k^\nu - 2(k^\mu q^\nu + q^\mu k^\nu) + q^\mu q^\nu], \tag{7}$$

$$L_\nu^{\mu\nu(\rm INT)} = 4(f_{m\nu} + g_{1\nu})[2k^\mu k^\nu - (k^\mu q^\nu + k^\nu q^\mu) + g^{\mu\nu}(k\cdot q) - i\epsilon^{\alpha\mu\beta\nu}k_\alpha k'_\beta], \tag{8}$$

whereas the polarizations read

$$\begin{aligned}\Pi_{\mu\nu}^{\rm Im(W)} &= C_V^2\Pi_{\mu\nu}^{{\rm Im}V} + 2C_VC_A\Pi_{\mu\nu}^{{\rm Im}(V-A)} + C_A^2\Pi_{\mu\nu}^{{\rm Im}A},\\ \Pi_{\mu\nu}^{\rm Im(EM)} &= \Pi_{\mu\nu}^{{\rm Im}V},\\ \Pi_{\mu\nu}^{\rm Im(INT)} &= C_V\Pi_{\mu\nu}^{{\rm Im}V} + C_A\Pi_{\mu\nu}^{{\rm Im}(V-A)}.\end{aligned} \tag{9}$$

Because of the current conservation and translational invariance, the vector polarization $\Pi_{\mu\nu}^{{\rm Im}V}$ consists of two independent components which we choose to be in the frame of $q^\mu \equiv (q_0, |\vec{q}|, 0, 0)$, i.e.,

$$\Pi_T^{\rm ImV} = \Pi_{22}^V = \Pi_{33}^V \quad \text{and} \quad \Pi_L^{\rm ImV} = -(q_\mu^2/|\vec{q}|^2)\Pi_{00}^V.$$

The axial vector and the mixed pieces are found to be

$$\Pi_{\mu\nu}^{\rm Im(V-A)}(q) = i\epsilon_{\alpha\mu 0\nu}q_\alpha\Pi_{VA} \tag{10}$$

and

$$\Pi_{\mu\nu}^{{\rm Im}A}(q) = \Pi_{\mu\nu}^{{\rm Im}V}(q) + g_{\mu\nu}\Pi_A. \tag{11}$$

The explicit forms of Π_{22}^V, Π_{00}^V, Π_{VA}, and Π_A are given in Ref. [3]. Thus the analytical form of Eq. (5) can be obtained from the contraction of every polarization and neutrino tensors couple ($L^{\mu\nu}\Pi_{\mu\nu}$).

If we take into account the possibility of the $\nu_e - \nu_\mu$ transition, the cross section can be written in the form of [25,26]

$$\frac{d^3\sigma}{d^2\Omega' dE'} = P_{ee}\left(\frac{d^3\sigma}{d^2\Omega' dE'}\right)_{\nu_e} + (1 - P_{ee})\left(\frac{d^3\sigma}{d^2\Omega' dE'}\right)_{\nu_\mu}. \tag{12}$$

Here $(d^3\sigma/d^2\Omega' dE')_{\nu_e}$ is the cross section of the $\nu_e - e$ scattering. If C_V and C_A are replaced with $C_V - 1$ and $C_A - 1$, respectively, then the cross section becomes $(d^3\sigma/d^2\Omega' dE')_{\nu_\mu}$, i.e., the cross section of the $\nu_\mu - e$ scattering. P_{ee} is the ν_e's flavor survival probability as a function of the neutrino energy.

Because of the assumption of massless neutrino, the ν_e helicity flip from left- to right-handed is only possible through its dipole moment. Thus, the cross section after taking into account this possibility ($\nu_e - \nu_e^R$ transition) reads [17]

$$\frac{d^3\sigma}{d^2\Omega' dE'} = (1 - P_{LL})\left(\frac{d^3\sigma}{d^2\Omega' dE'}\right)_{LR} + P_{LL}\left(\frac{d^3\sigma}{d^2\Omega' dE'}\right)_{\nu_e}, \tag{13}$$

where $(d^3\sigma/d^2\Omega' dE')_{LR}$ is the $\nu_e - e$ scattering via neutrino dipole moment and P_{LL} is the probability of ν_e to be still left handed.

Finally we can compute the mean free path from Eqs. (5), (12), and (13) by using

$$\frac{1}{\lambda(E_\nu)} = \int_{q_0}^{2E_\nu - q_0} d|\vec{q}| \int_0^{2E_\nu} dq_0 \frac{|\vec{q}|}{E'_\nu E_\nu} 2\pi \frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu}. \tag{14}$$

In this calculation we use a neutrino energy of 5 MeV.

Figure 1 shows the total mean free path compared to the mean free path of weak interaction with various neutrino effective moments μ_ν, and neutrino charge radii R. The total mean free path is the coherent sum of the weak, electromagnetic and the interference contributions.

There are also evidences that $R^2 \approx 10^{-32}$ cm^{-2} or $R^2 \approx 25 \times 10^{-12}$ MeV^{-2} [21–23]. Therefore, in the left panel of Fig. 1 we use $R = 5 \times 10^{-6}$ MeV^{-1} and vary μ_ν between 0 and $10^{-9}\mu_B$. In the right panel, we use $\mu_\nu = 10^{-12}\mu_B$ as the strongest bound on the neutrino magnetic moment while R is varied between 0 and 5×10^{-5} MeV^{-1}.

It is evident from the left panel of Fig. 1 that for fixed R, the mean free path increases rapidly only after $\mu_\nu = 10^{-10}\mu_B$. As we can see from Fig. 2 this increment is due to the significant difference between total and weak cross sections starting from $\mu_\nu = 10^{-10}\mu_B$. The summa-

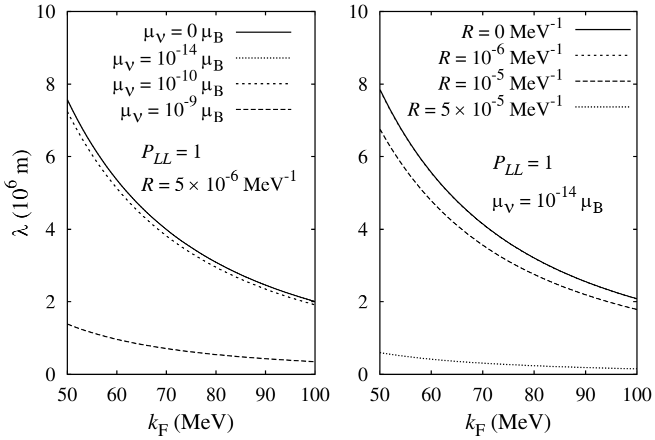

FIG. 1. Total mean free path compared to the mean free path of weak interaction with various neutrino magnetic moments μ_ν and radii R as a function of Fermi momentum k_F. In the left panel the neutrino charge radius is fixed, while the neutrino magnetic moment is varied. In the right panel, we fix the neutrino magnetic moment, but vary the neutrino radius.

tion of the longitudinal and transversal terms of the electromagnetic contribution is responsible for this. The right panel shows that for fixed μ_ν, the total mean free path and the mean free path of weak interaction show significant variance for $R \geq 10^{-6}$ MeV^{-1}. This is also due to the fact that the summation of the longitudinal and transversal terms of the electromagnetic part of the cross section increases rapidly starting at $R = 10^{-6}$ MeV^{-1}.

Figure 3 shows the effects of neutrino oscillations on the neutrino mean free path. In this case we do not calculate the transition probabilities. Instead, we only study the variation of neutrino mean free path with respect to the transition probabilities of a left handed massless neutrino-electron, ν_e, oscillates to a left-handed massless neutrino muon, ν_μ, or flips to a right-handed neutrino electron, ν_e^R.

By comparing the possibility of $\nu_e - \nu_\mu$ transition (left panel of Fig. 3) and $\nu_e - \nu_e^R$ transition (right panel), we can clearly see that these effects lengthen the neutrino mean free path, where the rate depends on their survival probabilities. For smaller P_{LL} (large flipping possibility), the path increment becomes more significant. This effect can be traced back to the value of $(d^3\sigma/d^2\Omega' dE')_{LR}$ in Eq. (13) which is smaller than that of $(d^3\sigma/d^2\Omega' dE')_{\nu_e}$. On the other hand, for small P_{ee} the possibility of $\nu_e - \nu_\mu$ oscillation does not change the neutrino mean free path dramatically. This fact arises because the difference between $(d^3\sigma/d^2\Omega' dE')_{\nu_\mu}$ and $(d^3\sigma/d^2\Omega' dE')_{\nu_e}$ in Eq. (12) is not as large as in the case of Eq. (13). Therefore different from the mean free path with flavor changing possibility, the mean free path with helicity flipping possibility depends strongly on the value of μ_ν. For example, we have also found that with decreasing P_{LL} the mean free path grows more rapidly when we use $\mu_\nu = 10^{-12}\mu_B$ rather than $\mu_\nu = 10^{-10}\mu_B$.

In conclusion, we have studied the sensitivity of the neutrino mean free path to the neutrino electromagnetic form factors and neutrino oscillations. It is found that the electromagnetic form factor has a significant role if $\mu_\nu \geq 10^{-10}\mu_B$ and $R \geq 10^{-6}$ MeV^{-1s}. We note that these values are larger than their largest upper bounds. It would be interesting to see whether or not such a phenomenon would also appear if contributions from the neutrino-nucleon scatterings were taken into account. Future calculation

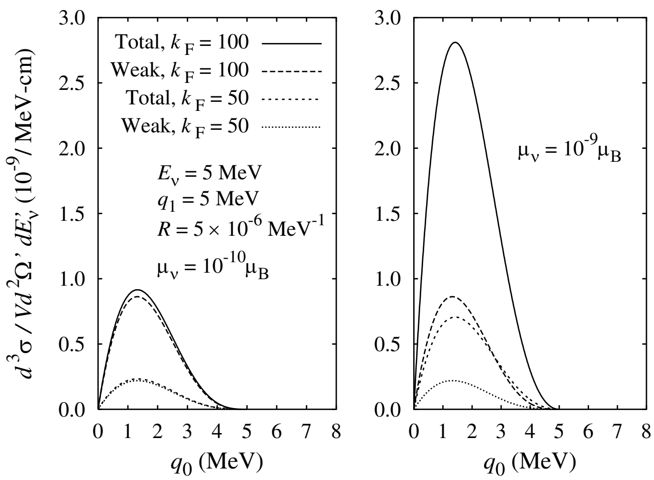

FIG. 2. Total cross section compared to the cross section of weak interaction as a function of energy transfer q_0 where momentum transfer q_1 is fixed. Here, two different neutrino magnetic moments μ_ν and Fermi momenta with a same neutrino charge radius are used. In the left panel we use $\mu_\nu = 10^{-10}\mu_B$, while in the right panel $\mu_\nu = 10^{-9}\mu_B$.

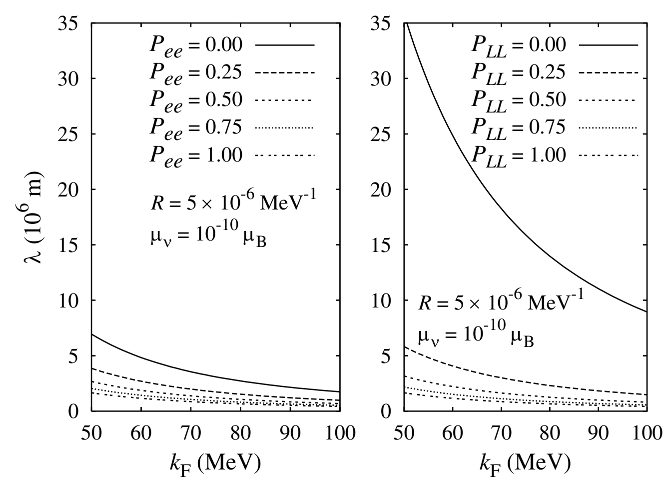

FIG. 3. Total mean free path of ν_e that allows for $\nu_e - \nu_\mu$ and $\nu_e - \nu_e^R$ transitions with various P_{LL} and P_{ee} as a function of Fermi momentum k_F. In the left panel we vary the ν_e's flavor survival probability, while in the right panel the helicity flipping probability of neutrino is varied.

should address this question. The mean free path is also found to be sensitive to the neutrino oscillations and depends on the transition probabilities of $\nu_e - \nu_\mu$ and $\nu_e - \nu_e^R$. This result clearly indicates that realistic mean free path calculations in the future should be performed with appropriate values of the $\nu_e - \nu_\mu$ and $\nu_e - \nu_e^R$ transition probabilities.

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