TeV-scale string resonances at hadron colliders

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We construct tree-level four-particle open-string amplitudes relevant to dilepton and diphoton production at hadron colliders. We expand the amplitudes into string resonance (SR) contributions and compare the total cross-section through the first SR with the *Z'* search at the Tevatron. We establish a current lower bound based on the CDF Run I results on the string scale to be about $1.1 - 2.1$ TeV, and it can be improved to about $1.5 - 3$ TeV with 2 fb⁻¹. At the LHC, we investigate the properties of signals induced by string resonances in dilepton and diphoton processes. We demonstrate the unique aspects of SRinduced signals distinguishable from other new physics, such as the angular distributions and forwardbackward asymmetry. A 95% C.L. lower bound can be reached at the LHC for $M_s > 8.2 - 10$ TeV with an integrated luminosity of 300 fb^{-1}. We emphasize the generic features and profound implications of the amplitude construction.

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I. INTRODUCTION

String theory [1] remains to be the leading candidate to incorporate gravity into a unified quantum framework of the elementary particle interactions. The string scale (M_S) is naturally close to the quantum gravity scale $M_{\text{Pl}} \approx 10^{19}$ GeV, or to a grand unification (GUT) scale $M_{GUT} \approx 10^{17}$ GeV [2]. It has been argued recently that the fundamental string scale can be much lower [3]. With the existence of large effective volume of extra dimensions beyond four, the fundamental quantum gravity scale may be as low as a TeV. This is thought to have provided an alternative approach to the hierarchy problem [4,5], namely, the large gap between the electroweak scale $O(100 \text{ GeV})$ and the Planck scale of M_{Pl} . What is extremely interesting is that these scenarios would lead to very rich phenomenology at low energies in particle physics [6–8] and astroparticle physics [9–11] that may be observable in the next generation of experiments.

One generic feature of string models is the appearance of string resonances (SR) in scattering of particles in the energy region above the string scale. The scattering amplitudes are of the form of the Veneziano amplitudes [1,8,12], which may develop simple poles. In the *s*-channel, the poles occur at $\sqrt{s} = \sqrt{n} M_S(n = 1, 2, ...)$ with degeneracy for different angular momentum states. It has been argued [6,8] that the scattering involving gravitons (closed strings) is perturbatively suppressed by higher power of string coupling with respect to the open-string scatterings which therefore are the dominant phenomena at energies near and above the string scale.

In this paper, we consider the possibility of producing the string resonances of a TeV-scale mass and studying their properties at colliders. We adopt the simplest openstring model in the D-brane scenario [8,12]. It is assumed that all standard model (SM) particles are identified as open strings confined to a D3-brane universe, while a graviton is a closed string propagating freely in the bulk. For a given string realization of the SM, one should be able to calculate the open-string scattering amplitudes, in particular the Chan-Paton factors [13] that are determined by the group structure of the particle representations and their interactions. Unfortunately, there is no fully satisfactory construction of the SM from string theory and we are thus led to parameterize our ignorance. We demand that our stringy amplitudes reproduce the SM amplitudes at low energies. The zero-modes of the scattering amplitudes are all identified as the massless SM particles and no new exotic states of the zero-modes are present. By taking Chan-Paton factors to be free parameters, a nontrivial stringy extension of the SM amplitudes to a higher energy region is accomplished by a unique matching between stringy amplitudes and those of the SM at low energies.

In fact, this scheme has been exploited in some earlier works. These include possible low energy effects from the string amplitudes on four-fermion interactions [14], and searching for signals in cosmic neutrino interactions [10,11]. In this paper, we explore the search and detailed study of their properties for these string resonances at hadron colliders such as the Fermilab Tevatron and the CERN Large Hadron Collider (LHC). In the string models, we expect a series of resonances with a predicted mass we expect a series of resonances with a predicted mass
relation $\sqrt{n}M_S(n = 1, 2, ...)$. Moreover, the angular distributions of the SR signals in parton-parton c.m. frame present distinctive shapes in dileptonic and diphotonic channels due to the angular momentum decomposition. Rather small forward-backward asymmetry is another fea-

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ture of the model. These are all very unique and remarkably specific in contrast to signals from other sources of new-physics. It is found that the LHC experiments may be sensitive to a string scale of $M_S \sim 8$ TeV.

The rest of the paper is organized as follows. We first construct tree-level open-string scattering amplitudes for the dileptonic and diphotonic production processes in Sec. II, which reproduce the SM amplitudes at low energies and extend to include string resonances. In Sec. III, string resonance approximation is discussed and each string resonance is expanded into partial waves to see their angular momentum states. Using the $Z¹$ constraints at the Tevatron, lower bounds on the string scale are obtained in Sec. IV. The analysis at the LHC is carried out in Sec. V. We summarize in Sec. VI our results and emphasize the generic features and profound implications of the amplitude construction. The complete expressions for the scattering amplitudes and the decay widths are given in two appendices.

II. CONSTRUCTION OF OPEN-STRING AMPLITUDES

The 4-point tree-level open-string amplitudes can be expressed generically [1,8,12]

$$
A_{\text{string}} = S(s, t)A_{1234}T_{1234} + S(t, u)A_{1324}T_{1324}
$$

$$
+ S(u, s)A_{1243}T_{1243}, \qquad (1)
$$

where $(1, 2, 3, 4)$ represents external massless particles with incoming momenta. A_{ijkl} are kinematic parts for $SU(N)$ amplitudes [15], which are given in Appendix A. The Mandelstam variables at parton-level are denoted by *s, t* and *u*. For physical process $(12 \rightarrow 34)$, the *s, t* and *u*-channels are labeled by $(1, 2)$, $(1, 4)$ and $(1, 3)$, respectively. T_{ijkl} are the Chan-Paton factors and in the usual construction,

$$
T_{1234} = tr(\lambda_1 \lambda_2 \lambda_3 \lambda_4) + tr(\lambda_4 \lambda_3 \lambda_2 \lambda_1). \tag{2}
$$

Following Ref. [15], we adopt the normalization of $tr(\lambda_a \lambda_b) = \delta_{ab}$. Since a complete string model construction for the electroweak interaction of the standard model is unavailable, we will assume that these Chan-Paton factors are free parameters and T_{ijkl} is typically in range of -4 to 4. $S(s, t)$ is essentially the Veneziano amplitude

$$
S(s,t) = \frac{\Gamma(1 - \alpha's)\Gamma(1 - \alpha't)}{\Gamma(1 - \alpha's - \alpha't)},
$$
\n(3)

where the Regge slope $\alpha' = M_S^{-2}$, and the amplitude approaches unity as either s/M_S^2 or $t/M_S^2 \rightarrow 0$.

Of special interests for this article are the $2 \rightarrow 2$ processes that may lead to clear experimental signatures at the Tevatron and LHC. We thus concentrate on two clean channels: the Drell-Yan (DY) dilepton production $(\ell \bar{\ell})$ and the diphoton production ($\gamma\gamma$), from $q\bar{q}$ annihilation and possibly gluon-gluon fusion. In this section, we explicitly construct the string amplitudes for these production processes.

A. Dilepton Production

At hadron colliders, the $2 \rightarrow 2$ dilepton production processes are $q\bar{q}$, $gg \to \ell \bar{\ell}$. The tree-level process for $gg \to$ $\ell \bar{\ell}$ is absent in the SM. In the massless limit of the fermions, we label their helicities by the chirality α , β = *L*, *R*. For the process with initial state $q\bar{q}$, we have two cases depending on the helicity combination of the final state leptons. The nonvanishing amplitudes are those for $\alpha \neq \beta$. The external particle ordering is (12 \rightarrow 34).

l. $q\bar{q}$ *annihilation* $q_{\alpha}\bar{q}_{\beta} \rightarrow \ell_{\alpha}\bar{\ell}_{\beta}$ *:*

With the notation as in Appendix A, this process belongs to a type of $f^{\pm}f^{\mp}f^{\mp}f^{\pm}$, with \pm denoting the helicity of the particle with respect to incoming momentum. Our construction thus leads to the physical amplitude

$$
A_{\text{string}}(q_{\alpha}\bar{q}_{\beta} \rightarrow \ell_{\alpha}\bar{\ell}_{\beta}) = ig^{2} \bigg[T_{1234}S(s, t) \frac{t}{s} + T_{1324}S(t, u) \frac{t}{u} + T_{1243}S(u, s) \frac{t^{2}}{us} \bigg].
$$
 (4)

The corresponding standard model amplitude is via the electroweak interaction,

$$
A_{SM} = ig_L^2 \frac{t}{s} F_{\alpha \alpha}, \tag{5}
$$

where the photon and *Z* contributions are given by

$$
F_{\alpha\beta} = 2Q_{\ell}Q_{q}x_{w} + \frac{s}{s - m_{Z}^{2}} \frac{2g_{\alpha}^{\ell}g_{\beta}^{q}}{1 - x_{w}}.
$$
 (6)

Here $x_w = \sin^2 \theta_W$ and the $SU(2)_L$ coupling $g_L =$ $e/\sin\theta_W$. The neutral current couplings are $g_L^f = T_{3f}$ - $Q_f x_w$, $g_R^f = -Q_f x_w$.

The crucial assumption for our approach is to demand the string expression Eq. (4) to reproduce the standard model amplitude in the low-energy limit when $s/M_S^2 \rightarrow$ 0. This can be achieved by identifying the string coupling with the gauge coupling $g = g_L$, and matching the Chan-Paton factors *Tijkl* as

$$
T_{1243} = T_{1324} \equiv T; \quad T_{1234} = T + F_{\alpha\alpha}.\tag{7}
$$

We then obtain the full result

$$
A_{\text{string}}(q_{\alpha}\bar{q}_{\beta} \to \ell_{\alpha}\bar{\ell}_{\beta}) = ig_{L}^{2}S(s, t)\frac{t}{s}F_{\alpha\alpha} + ig_{L}^{2}T\frac{t}{us}f(s, t, u),
$$
 (8)

$$
f(s, t, u) = uS(s, t) + sS(t, u) + tS(u, s).
$$
 (9)

For simplicity, we will take the Chan-Paton parameter *T* to be positive and $0 \le T \le 4$. Taking *T* to be negative will not change our numerical results appreciably.

A few interesting features are worthwhile commenting. First, we see that the string amplitude Eq. (8) consists of two terms: one proportional to the SM result multiplied by a Veneziano amplitude $S(s, t)$; the other purely with string origin proportional to an unknown Chan-Paton parameter *T*. In the low-energy limit $s \ll M_S^2$, $f(s, t, u) \to s + t +$ $u = 0$, reproducing the SM result regardless of *T*. This implies that *T* cannot be determined unless one specifies the detailed embedding of the SM to some more generalized group structure in a string setup. The seemingly disturbing fact is that one of the Chan-Paton factors T_{1234} must be made dependent upon the *Z*-pole, rather than pure gauge couplings. This reflects our ignorance of treating the electroweak symmetry breaking in our approach.

As for the other helicity combination $q_{\alpha} \bar{q}_{\beta} \rightarrow \ell_{\beta} \bar{\ell}_{\alpha}$, it belongs to the class of $f^{\pm} f^{\mp} f^{\pm} f^{\mp}$. We apply the same methods as stated above and find the crossing relation $t \leftrightarrow$ *u* and an index interchange in the *F* factor,

$$
A_{\text{string}}(q_{\alpha}\bar{q}_{\beta} \to \ell_{\beta}\bar{\ell}_{\alpha}) = ig_{L}^{2}S(s, u) \frac{u}{s}F_{\beta\alpha} + ig_{L}^{2}T\frac{u}{t_{S}}f(s, t, u). \quad (10)
$$

with $T \equiv T_{1234} = T_{1324}$.

2. Gluon fusion $g_{\alpha}g_{\beta} \to \ell_{\alpha} \bar{\ell}_{\beta}$:

In our open-string model, there is the possibility of dilepton production via two initial state gluons. This amplitude vanishes at tree-level in the standard model, but could be nonzero in the open-string model if the gluons and leptons belong to some larger gauge group in which the Chan-Paton trace is nonvanishing. The amplitude belongs to a type of $g^{\pm}g^{\mp}f^{\mp}f^{\pm}$ according to Appendix A. With $T \equiv T_{1234} = T_{1324} = T_{1243}$, the result reads

$$
A_{\text{string}}(g_{\alpha}g_{\beta} \to \ell_{\alpha}\bar{\ell}_{\beta}) = ig_L^2 T \frac{1}{s} \sqrt{\frac{t}{u}} f(s, t, u), \qquad (11)
$$

where *T* may be different for each helicity combination of external particles. In fact, there exists an intrinsic ambiguity for the string coupling identification since there are both strong interaction and electroweak interaction involved simultaneously. Coupling identification for this subprocess would not be determined without an explicit string model construction. This problem is beyond the scope of this article. To be conservative, we have identified the string coupling with the weak coupling g_L .

For $g_{\alpha}g_{\beta} \to \ell_{\beta} \bar{\ell}_{\alpha}$, we have $t \leftrightarrow u$ of the above expression.

B. Diphoton Production

Another clean signal in addition to dilepton production at hadron colliders is the diphoton final state. We therefore construct the string amplitudes for diphoton processes in this section. We again label the helicities by α , β , and as in the dileptonic processses, the nonvanishing amplitudes are those with $\alpha \neq \beta$.

1. $q\bar{q}$ annihilation $q_{\alpha}\bar{q}_{\beta} \rightarrow \gamma_{\alpha}\gamma_{\beta}$:

Using the kinematic amplitudes for fermions and gauge bosons $f^{\mp} f^{\pm} g^{\pm} g^{\mp}$ as given in Appendix A and the matching techniques between the string and SM amplitudes described in the previous section, we obtain the following open-string amplitudes for $T \equiv T_{1234} = T_{1243}$,

$$
A_{\text{string}}(q_{\alpha}\bar{q}_{\beta} \to \gamma_{\alpha}\gamma_{\beta}) = 2ie^{2}Q_{q}^{2}\sqrt{\frac{t}{u}}S(t, u)
$$

$$
+ ie^{2}T\frac{1}{s}\sqrt{\frac{t}{u}}f(s, t, u), \quad (12)
$$

which correctly reproduce SM amplitudes at low energies, given by the first term. For the other helicity combination $\gamma_{\beta} \gamma_{\alpha}$, the amplitude can be obtained by $t \leftrightarrow u$.

2. Gluon fusion $g_{\alpha}g_{\beta} \rightarrow \gamma_{\alpha}\gamma_{\beta}$: Identifying this process with $g^{\pm}g^{\mp}g^{\mp}g^{\pm}$, one has

$$
A_{\text{string}}(g_{\alpha}g_{\beta} \to \gamma_{\alpha} \gamma_{\beta}) = ie^2 T \frac{t}{us} f(s, t, u), \qquad (13)
$$

with $T \equiv T_{1234} = T_{1324} = T_{1243}$. Note that this amplitude is of purely stringy origin. There exists the same ambiguity for the string coupling identification as in $gg \to \ell \bar{\ell}$. To be conservative, we have matched the string coupling with the electromagnetic interactions.

For the other helicity combination $\gamma_{\beta} \gamma_{\alpha}$, the amplitude can be obtained by $t \leftrightarrow u$.

III. STRING RESONANCES AND PARTIAL WAVES EXPANSION

The factor $\Gamma(1 - s/M_S^2)$ in the Veneziano amplitude develops simple poles at $s = nM_S^2(n = 1, 2, 3...)$, implying develops simple poles at $s = h m_S (h - 1, 2, 3...),$ implying
resonant states with masses $\sqrt{n} M_S$. At energies near the string scale, string resonances thus become dominating. One can perform a resonant expansion,

$$
S(s,t) \approx \sum_{n=1}^{\infty} \frac{t(\frac{t}{M_S^2} + 1) \dots (\frac{t}{M_S^2} + n - 1)}{(n-1)!(s - nM_S^2)}.
$$
 (14)

Thus, by neglecting $S(t, u)$ which does not contain *s*-channel poles,

$$
f(s, t, u) = uS(s, t) + sS(t, u) + tS(u, s)
$$

\n
$$
\approx 2 \sum_{n=\text{odd}}^{\infty} \frac{ut(\frac{t}{M_S^2} + 1)...(\frac{t}{M_S^2} + n - 1)}{(n - 1)!(s - nM_S^2)}.
$$
 (15)

It is a remarkable result that this purely stringy function $f(s, t, u)$ has only odd-*n* SRs due to the crossing symmetry between *t* and *u*. It represents the stringy effects of spinexcitations along the string worldsheet, which are suppressed at low-energy. These are the generic features of stringy effects we wish to explore at the high energy experiments.

A. String Resonances in Dileptonic and Diphotonic Amplitudes

The open-string amplitude construction for Drell-Yan processes predicts the existence of exotic intermediate states such as leptoquarks in the *u*-channel and higher spin bosonic excitations in the *s*-channel as string resonances. Because of the limited c.m. energy accessible at collider experiments, we need to keep only the first few resonances. Applying the general results of Eqs. (14) and (15) to the dilepton string amplitudes, we obtain the amplitude formula for the first two resonances, with θ defined as angle between initial quark and final antilepton in the parton c.m. frame,

$$
A_{SR}(q_{\alpha}\bar{q}_{\beta}) \approx \begin{cases} i g_L^2 \frac{(1-\cos\theta)^2}{4} \left[\frac{s}{s-M_S^2} (F_{\alpha\alpha} + 2T) + \frac{s}{s-M_S^2} F_{\alpha\alpha} \cos\theta \right] & \text{for } \ell_{\alpha}\bar{\ell}_{\beta} \\ i g_L^2 \frac{(1+\cos\theta)^2}{4} \left[\frac{s}{s-M_S^2} (F_{\beta\alpha} + 2T) - \frac{s}{s-M_S^2} F_{\beta\alpha} \cos\theta \right] & \text{for } \ell_{\beta}\bar{\ell}_{\alpha}. \end{cases}
$$

The full amplitude then will appear as a sum

$$
A \approx A_{SM} + A_{SR}.\tag{16}
$$

A few remarks on the amplitudes are in order. Firstly, even if the free Chan-Paton parameter *T* is set to zero, there are still contributions from string resonances. This can be seen from the Veneziano factor multiplying to the SM term in the string formula. Significant differences from the standard model cross-sections can be expected if the string scale is accessible at future colliders. Second, the amplitude for the first (odd-*n*) string resonance depends on the Chan-Paton parameter *T*, while the second (even-*n*) resonance does not. The even resonances are completely determined by the gauge factors *F* in the standard model.

In the string model, there is a possible contribution from gluon fusion to lepton pairs, as seen in Eq. (11). Near the string resonance, we have

$$
A_{SR}(g_{\alpha}g_{\beta}) \approx ig_L^2 T \frac{s}{s - M_S^2} \frac{1 \mp \cos\theta}{2} \sin\theta, \qquad (17)
$$

where the sign " - " corresponds to $g_{\alpha}g_{\beta} \rightarrow \ell_{\alpha} \bar{\ell}_{\beta}$, and " + " to $\ell_{\beta} \bar{\ell}_{\alpha}$ with $\alpha \neq \beta$. There are only odd-*n* string resonances from this gluon contribution. This is generic for any processes if the standard model amplitude vanishes at tree-level. It is always proportional to the function $f(s, t, u)$ which vanishes in the low-energy limit, which only has odd-*n* resonances. As a comparison, for processes with the nonvanishing amplitudes in standard model at tree-level, their open-string amplitude will most likely contain both odd- and even-*n* SRs.

The only exception is when the stringy correction piece multiplying to the standard model amplitude is $S(t, u)$ which does not contain SR pole in the *s*-channel. This occurs naturally when the zero-mode (SM) tree-level exchange is in *t* or *u* but not in the *s* channel. We can see from the list in Appendix A that A_{1324} , to be multiplied with $S(t, u)$ in the full amplitude expression, never contain an *s*-channel pole. This is consistent with the physical picture that SR is the spin excitation of the zero-mode intermediate state. If the zero-mode (SM) intermediate state does not exist, then there will not exist SR interacting with the same gauge charges. An example of this kind of processes is $q\bar{q} \rightarrow \gamma\gamma$ which we can see from Eq. (12). For diphoton production, there are thus only odd-*n* string resonances. The first SR $(n = 1)$ for both processes are

$$
A_{SR}(q_{\alpha}\bar{q}_{\beta} \to \gamma_{\alpha}\gamma_{\beta}) = ie^2T\frac{s}{s - M_S^2}\frac{1 - \cos\theta}{2}\sin\theta,
$$
\n(18)

$$
A_{SR}(g_{\alpha}g_{\beta} \to \gamma_{\alpha}\gamma_{\beta}) = 2ie^2T\frac{s}{s - M_S^2}\frac{(1 - \cos\theta)^2}{4}.
$$
 (19)

The expressions for opposite helicity combinations ($\gamma_{\beta}\gamma_{\alpha}$) are given by $\theta \rightarrow \pi - \theta$. Observe that SR coupling is proportional to *T* which is completely undetermined. We will include these $n = 1$ resonances and ignore those of $n = 3$ in our LHC analysis for the diphoton signals.

B. Partial Waves Expansion of String Resonances

There is degeneracy of states with different angular momenta at each SR as can be seen from the dependence on different powers of *t* for each *n* in Eq. (14). Generically, any amplitude $A(s, t)$ can be expanded in terms of the Wigner functions $d_{mm'}^j(\cos\theta)$ [16] as

$$
A(s, t) = 16\pi \sum_{j=M}^{\infty} (2j + 1)a_j(s)d_{mm'}^j(\cos\theta),
$$
 (20)

where $M = \max(|m|, |m'|)$, and $a_j(s)$ are the partial wave amplitudes corresponding to a definite angular momentum state *j*.

For our purpose, we expand the SR amplitudes for each mass eigenstate of a given *n* by the Wigner functions as in Table I.

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TABLE I. Angular momentum decomposition for string resonance amplitudes in dilepton and diphoton processes.

DY dilepton pairs $A_{SR}^{n=1}(q_\alpha \bar{q}_\beta \rightarrow \ell_\alpha \bar{\ell}_\beta)$	$ig_L^2(F_{\alpha\alpha} + 2T) \sum_{j=1}^2 \frac{s \alpha_1^j d_{1,-1}^j}{s - M_{\sigma}^2 + i \Gamma_1^j M_S}$
$A_{SR}^{n=1}(q_\alpha \bar{q}_\beta \rightarrow \ell_\beta \bar{\ell}_\alpha)$	$ig_L^2(F_{\beta\alpha} + 2T) \sum_{j=1}^2 \frac{s\alpha_1^j d_{1,1}^j}{s - M_c^2 + i\Gamma_c^j M_s}$
$A_{SR}^{n=2}(q_{\alpha}\bar{q}_{\beta}\rightarrow \ell_{\alpha}\bar{\ell}_{\beta})$	$ig_L^2 F_{\alpha\alpha} \sum_{j=1}^3 \frac{s\alpha_1^j d_{1,-1}^j}{s-2M_s^2 + i\Gamma_2^j \sqrt{2}M_s}$
$A_{SR}^{n=2}(q_\alpha \bar{q}_\beta \rightarrow \ell_\beta \bar{\ell}_\alpha)$	$ig_L^2 F_{\beta\alpha} \sum_{j=1}^3 \frac{s \alpha_1^j d_{1,1}^j}{s - 2M_s^2 + i\Gamma_2^j \sqrt{2}M_s}$
$A_{SR}^{n=1}(g_{\alpha}g_{\beta} \to \ell_{\alpha} \bar{\ell}_{\beta}, \ell_{\beta} \bar{\ell}_{\alpha})$	$ig_L^2 T \frac{sd_{2,\mp1}^2}{s-M_{\sigma}^2+i\Gamma_1M_{\rm S}}$
Diphoton final state $A_{SR}^{n=1}(q_\alpha \bar{q}_\beta \rightarrow \gamma_\alpha \gamma_\beta, \gamma_\beta \gamma_\alpha)$	$ie^2T\frac{s d_{2,\mp1}^2}{s-M_{\mp}^2+i\Gamma_1M_{\gamma}}$
$A_{SR}^{n=1}(g_{\alpha}g_{\beta} \to \gamma_{\alpha} \gamma_{\beta}, \gamma_{\beta} \gamma_{\alpha})$	$2ie^2T\frac{sd_{2,\mp2}^2}{s-M_{\rm s}^2+i\Gamma_1 M_S}$

It becomes clear that the different angular momentum states will lead to very distinctive angular distributions of the final state leptons for the SR signals and may serve as important indicators in exploring the resonance properties. To regularize the poles, the decay widths have been included. The coefficients α_n^j , decay widths Γ_n^j , and the relevant Wigner functions are given in Appendix B.

IV. BOUNDS ON THE STRING SCALE FROM THE TEVATRON

At the Fermilab Tevatron, the clean channels of dileptons and diphotons have been actively searched for. The CDF collaboration has been searching for a $Z¹$ gauge boson in the dilepton channel and a lower bound M_{Z} ^{> 690} GeV had been set based on their Run I data [17] for a neutral gauge boson with SM-like couplings. Similar results were obtained by the D0 collaboration [18]. The nonexistence of a signal put an upper bound on the production cross-section and can thus be translated to stringent constraints on the string scale.

Using CTEQ5L parton distribution functions [19] , we estimate the total cross-sections for the string resonance signatures at various string scales with $T = 1 - 4$. Since there is degeneracy of state with different angular momenta at the same mass, we use partial wave expansion to split each SR pole. We regulate the resonance pole by including the decay width of each angular momentum state separately. The detailed treatment for the width calculation is given in Appendix B. For instance, for $M_s = 1$ TeV, $n = 1$ and $T = 1$, the widths of SR in the Drell-Yan process are 240 (48) GeV for $j = 1(2)$, while the width of SR in $gg \rightarrow$ $\ell \bar{\ell}$ is 19 GeV with the only $j = 2$ state. When we compare with Tevatron data on their Z' search, we need only the first

FIG. 1. Total cross-section for the DY process ($\ell = e, \mu$) via the SR versus its mass M_S , for different values of $T = 0 - 4$ (the solid curves). Detector acceptance cuts of Eq. (21) have been imposed. The horizontal dashed lines show the 95% C.L. upper bound on $\sigma(Z')B(Z' \to \ell \ell)$ for integrated luminosities 110 pb^{-1} , 1 fb⁻¹ and 2 fb⁻¹, respectively.

SR, the lightest state (including the angular momentum degeneracy).

In Fig. 1, we present the total cross-section for the DY process ($\ell = e, \mu$) via the SR versus its mass M_S , for different values of the Chan-Paton parameter $T = 0 - 4$ as shown by the solid curves. Both contributions from $q\bar{q}$ and *gg* are taken into account. To extract the lower bound on the string scale, we have simulated the experimental acceptance cuts on the invariant mass of the lepton pair, transverse momentum of the leptons, and their rapidity to be

$$
M(\ell\ell) > 50 \text{ GeV}, \quad p_T(\ell) > 18 \text{ GeV}, \quad |y_\ell| < 2.4.
$$
 (21)

We extrapolate CDF result [17] of 110 pb^{-1} on the Z^t mass bound at 95% C.L. through dilepton production to a higher mass scale to obtain an upper bound on the production cross-section, as shown by the horizontal dashed lines, corresponding to different integrated luminosities, 110 pb^{-1} , 1 fb⁻¹ and 2 fb⁻¹, respectively. The intersections between the top horizontal line from the extrapolated data and the curves calculated for string resonances are located at $1.1 - 2.1$ TeV for $T = 1 - 4$, and thus yield the current lower bound on M_S . This gives a stronger bound for the string scale than that based on a contact-interaction analysis [14]. A bound obtained from the diphoton final state is weaker than that from the DY process, and we will not present it here.

In the near future with an integrated luminosity of 2 fb⁻¹ at the Tevatron, one should be able to extend the search to $M_S \sim 1.5 - 3$ TeV for $T = 1 - 4$, as indicated in Fig. 1. It is interesting to note that even for $T = 0$, one still has some sensitivity at the Tevatron, reaching $M_S \sim 1$ TeV.

V. STRING RESONANCES AT THE LHC

At the LHC, operating at $E_{cm} = 14$ TeV with an expected luminosity of 300 fb^{-1} , could produce a sufficiently large number of events induced by SRs with masses of several TeV. We will first present various aspects of dilepton and diphoton SR-induced signals in comparison with the expected SM backgrounds. Then we will proceed to set the lower bound on the string scale if we do not see any SRinduced signals at the LHC. For illustration, we take a fixed string scale of $M_S = 2$ TeV and $T = 1$. All of the processes are calculated with the minimal acceptance cuts on the final state particles of leptons and photons

$$
p_T > 20 \text{ GeV}, \quad |y| < 2.4. \tag{22}
$$

To be more realistic in generating the resonant structure, we smear the particle energies according the electromagnetic calorimeter response with a Gaussian distribution

$$
\frac{\Delta E}{E} = \frac{5\%}{\sqrt{E/\text{GeV}}} \oplus 1\%.
$$
 (23)

A. The resonance signals

In Fig. 2, we present the invariant mass distributions of the DY dileptons for the SM background expectation and the string resonances, including both $q\bar{q}$ and *gg* contributions as labeled. At low energies, the stringy amplitudes reproduce SM results as expected. At higher energies, the resonant structure in the invariant mass distribution can be very pronounced. The dilepton processes have both evenvery pronounced. The different processes have both even-
and odd-*n* SRs, with masses M_S , $\sqrt{2}M_S$ for $n = 1, 2$. Recall that the second SR is independent of the Chan-Paton parameter *T*, in contrast to the first SR which is dependent on *T*. To illustrate this effect, we have also depicted the peak height for the choice of $T = 4$.

FIG. 2. Invariant mass distributions for DY dilepton production at the LHC, for the continuum SM expectation and the SR contributions with $M_S = 2$ TeV and $T = 1$: $q\bar{q} + gg$ (top curve) and *gg* only (dashed). The vertical bar at the $n = 1$ SR peak indicates the enhancement for $T = 4$.

Therefore, the number of events around the first SR (the cross-section) will determine the Chan-Paton parameter *T*, while the number of events around the second SR will be predicted essentially by the SM couplings. Moreover, the mass of the second string resonance is remarkably premass or the second string resonance is remarkably predicted to be $\sqrt{2}M_s$, fixed with respect to the first resonance. These essential aspects of SR signals allow us to distinguish this unique model from other new-physics. The scale on the right-hand side gives the number of events per bin for an integrated luminosity of 300 fb^{-1} .

The differential cross-sections for diphoton production are shown in Fig. 3 for the SM background and the string resonant contribution. The diphoton processes have only odd-*n* SRs and thus the peak is at M_S for $n = 1$. The contribution from $gg \rightarrow \gamma \gamma$ is again separately shown for comparison (dashed curve). Although it would just double the diphoton signals at the peak of SR by including the *gg* channel, we have pointed out earlier that the string coupling identification to *e* is ambiguous.

B. Angular distributions

As already seen from Table I, there are interesting massdegeneracies with different angular momentum states. This will lead to distinctive angular distributions when the pair invariant mass is close to the string resonance. It is thus tempting to explore how this unique aspect could be studied.

We first tabulate the angular dependence for the processes with given *n; j* values in Table II. As always, the angle θ is defined in the $\ell \bar{\ell}$ or $\gamma \gamma$ rest frame with respect to the beam direction. It is indeed interesting to see the drastic differences of the angular distributions for different processes. For instance, there is a degeneracy of spin 1 and 2 at the first SR in dileptonic processes. Spin-2 contributions to dileptonic processes have two possible sources with totally

FIG. 3. Invariant mass distributions for diphoton production at the LHC, for the continuum SM expectation and the SR contributions with $M_S = 2$ TeV and $T = 1$: $q\bar{q} + gg$ (top curve) and *gg* only (dashed). The vertical bar at the $n = 1$ SR peak indicates the enhancement for $T = 4$.

different angular distributions. One is from SR of *qq* initial state and another is from SR of *gg* one as illustrated in Fig. 4 by the dashed curves. Here, the contribution of spin-2 SR from $q\bar{q}$ is one-ninth of the spin-1 contribution of the same process while the contribution from *gg* is directly proportional to the Chan-Paton parameter *T*. These two contributions of spin-2 exchange could change the angular distribution significantly from the conventional "Z'" exchange that we would encounter in many extensions of the SM [20–22]. It is obvious that this unique angular distribution is also distinguishable from new-physics models with only spin-2 exchange such as Kaluza-Klein graviton [23]. For diphoton processes, there is only spin-2 SR from both $q\bar{q}$ and *gg* initial states, as shown in Fig. 5.

In Fig. 6, the predicted angular distributions (normalized to unity) of dileptonic signals are presented with the choice of $T = 1$ for both $q\bar{q}$ and *gg* initial states, for two different mass eigenstates $n = 1, 2$. The events are selected not only by imposing the acceptance cuts of Eq. (22), but also by choosing the invariant mass around the resonance mass

FIG. 4. Normalized theoretical angular distributions of string resonances with spin 1, 2, and 3 in the DY channel $pp \rightarrow$ $\ell^+\ell^- X$.

$$
\sqrt{n}M_S - 2\Gamma_n < M < \sqrt{n}M_S + 2\Gamma_n. \tag{24}
$$

We see from the figure that the distribution for $n = 1$ is less pronounced near $\cos\theta \sim \pm 1$ than that for $n = 2$. The eventual drop is due to the acceptance cuts. One could imagine to fit the observed distributions in Fig. 6 by the combination of the functions listed in Table II to test the model prediction. Similar distribution for the $\gamma\gamma$ final state is shown in Fig. 7, where the total contribution of $q\bar{q} + gg$ (the solid curve) and that for $q\bar{q}$ only (the dashed curve) are compared at $T = 1$ for both processes.

C. The Forward-Backward asymmetry

For parton-level subprocess $q\bar{q} \rightarrow \ell \bar{\ell}$, forwardbackward asymmetry is defined as

$$
A_{FB}^{q\ell} = \frac{N_F - N_B}{N_F + N_B},\tag{25}
$$

where $N_{F(B)}$ is the number of events with final lepton moving into the forward (backward) direction. At *pp*

FIG. 5. Normalized theoretical angular distributions of string resonances with only spin-2 in $pp \rightarrow \gamma \gamma X$.

FIG. 6. Normalized angular distributions for $n = 1$ (solid) and $n = 2$ (dashed) string resonances in the DY channel $pp \rightarrow$ $\ell^+ \ell^- X$ with appropriate cuts of Eq. (22).

colliders, the annihilation process is from the valence quarks and the sea antiquarks. Therefore, the produced intermediate resonant state will most likely move along the direction of the initial valence quark due to its higher fraction of momentum [21]. With respect to one particular boost direction of the final dilepton, we can consequently extract information of the forward-backward asymmetry of the subprocess.

In our open-string model, the asymmetry is given, for $s \gg m_Z^2$, by

$$
A_{FB}^{q\ell} = \left(\frac{30}{32}\right) \frac{(G_{LL}^q)^2 + (G_{RR}^q)^2 - (G_{LR}^q)^2 - (G_{RL}^q)^2}{(G_{LL}^q)^2 + (G_{RR}^q)^2 + (G_{LR}^q)^2 + (G_{RL}^q)^2}
$$
 (26)

$$
= \begin{cases} -0.176(-0.039) & \text{for } q = u, T = 1(4) \\ 0.160(0.042) & \text{for } q = d, T = 1(4) \end{cases}
$$
 (27)

where $G_{\alpha\beta}^q = F_{\alpha\beta} + 2T$, the interaction factor of the fermions defined in Sec. II. This asymmetry is inherited from the SM part, $F_{\alpha\beta}$, in the amplitudes. The value of $A_{FB}^{q\ell}$ for SM with $s \gg m_Z^2$ is 0.61(0.69) for *u*(*d*) quark. The asymmetry is diluted by the symmetric SR contribution since typically $T > F_{\alpha\beta}$. The forward-backward asymmetry is hardly visible when $T = 4$. This also can be viewed as another feature to distinguish the SR from the other states like *Z'* which normally yields larger asymmetry [21].

D. The reach on the string scale

For the unfortunate possibility that we do not detect any signals with SR properties, the absence of signals implies certain bounds on the string scale M_S and Chan-Paton parameters *T*. We present the sensitivity reach at 95% C.L. in Fig. 8 as a function of the integrated luminosity at the LHC. The results are obtained by assuming the

FIG. 7. Normalized angular distributions for $n = 1$ string resonance in the diphoton channel $p p \rightarrow \gamma \gamma X$ with appropriate cuts of Eq. (22). The solid curve represents the total contribution of $q\bar{q} + gg$ and the dashed curve is for $q\bar{q}$ only.

Gaussian statistics and by demanding $S/\sqrt{S+B} > 3$, where the signal rate is estimated in the dilepton-mass window $[M_S - 2\Gamma_1, M_S + 2\Gamma_1]$ at the first SR. The lower bound on the string scale could reach $M_S > 8.2 - 10$ TeV for $T = 1 - 4$ at a luminosity of 300 fb⁻¹.

VI. SUMMARY AND CONCLUSIONS

We have constructed tree-level open-string amplitudes for dilepton and diphoton processes. The massless SM particles are identified as the stringy zero-modes. For a given $2 \rightarrow 2$ scattering process, by demanding the openstring amplitudes reproduce the SM ones at low energies, the amplitudes can be cast into a generic form

FIG. 8. Sensitivity reach at 95% C.L. of M_S at various luminosities at the LHC.

$$
A_{\text{string}} \sim A_{SM}(s, t, u) \cdot S(s, t, u) + Tf(s, t, u) \cdot g(s, t, u),
$$
\n(28)

where A_{SM} is the SM amplitude, $S(s, t, u) = S(s, t)$, $S(s, u)$ or $S(t, u)$ the Veneziano amplitudes, T the undetermined Chan-Paton parameter, $f(s, t, u)$ a kinematical function given in Eq. (9), and $g(s, t, u)$ some process-dependent kinematical function. The amplitudes have the following general features:

- (i) By construction, they reproduce the standard model amplitudes at low energies $s \ll M_S^2$, since $S(s, t) \rightarrow$ 1 and $f(s, t, u) \rightarrow 0$, and thus fixing the string couplings with respect to the SM gauge couplings.
- (ii) The Veneziano amplitude $S(s, t)$ and $f(s, t, u)$ de-The veneziano amplitude $S(s, t)$ and $J(s, t, u)$ develop stringy resonances at energies $\sqrt{s} = \sqrt{n} M (n - 1.2)$ velop stringy res
 $\sqrt{n}M_S(n = 1, 2, ...)$.
- (iii) $S(s, t)$ leads to both even- and odd-*n* resonances, while $f(s, t, u)$ yields only odd-*n* SRs. Thus, the even-*n* resonances are completely fixed by the SM interactions, independent of the unknown factor *T*.
- (iv) For the standard model processes that either vanish at tree-level (such as $gg \rightarrow \gamma\gamma$), or do not contain *s*-channel exchange (such as $q\bar{q} \rightarrow \gamma \gamma$), there will be no SRs which couple with SM charges as in the first term of Eq. (28). Yet, there can still be SR contributions from purely stringy effects, directly proportional to *T*, given in the second term of the equation.

We would like to emphasize the profound implication of our amplitude construction and the generic structure of Eq. (28). The basic assumption of this work is to take the tree-level open-string scattering amplitudes of Eq. (1) as the description of leading new physics beyond the SM near the TeV threshold. As long as one accepts this approach and demands the amplitudes to reproduce the SM counterparts at low energies, Eq. (28) would be the natural consequence. There are essentially only two unknown parameters: the string scale M_S and the Chan-Paton parameter *T*. This construction should be generic for any leading-order $2 \rightarrow 2$ processes of massless SM particle scattering, and thus be applicable for further phenomenological studies.

We have calculated numerically the total cross-section of DY through the first string resonance and compared with the CDF data for $Z¹$ production. We establish the current lower bound of the string scale at about $1.1 - 2.1$ TeV which is stronger than limits from the contact-interaction analysis [14]. The bound from Tevatron can be improved to $1.5 - 3$ TeV with an integrated luminosity of two fb⁻¹.

At the CERN LHC, with the high luminosity expected and much larger center-of-mass energy, SR-induced signals for $M_S \le 8$ TeV can be substantial and a large number of events is predicted around the SR in dilepton and diphoton processes regardless of the value of the Chan-Paton parameters *T*. The second string resonance with a Paton parameters 1. The second string resonance with a mass $\sqrt{2}M_s$ may be observed in the dilepton channel as well. Distinctive angular distributions and the forwardbackward asymmetry may serve as indicators to distinguish the SR from other new-physics. For a larger value of *MS*, SR signals become weaker and we may establish the sensitivity on the lower bound of the string scale for $T =$ $1 - 4$ to be $M_s > 8.2 - 10$ TeV at 95% C.L. with a luminosity of 300 fb^{-1} .

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APPENDIX A: KINEMATIC TABLE

Consider a tree-level scattering of four massless gauge bosons in $SU(N)$ gauge theory, with all momenta incoming. The only nonvanishing amplitudes are those with two positive and two negative helicities. There are six of them, each as a sum of three terms of independent permutations. The general formula for one permutation is given in Ref. [15] as

$$
A_{1234} = ig^2 \frac{\langle IJ \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle},\tag{A1}
$$

where *I*, *J* label the two gauge bosons with negative helicities. Obviously, the above amplitude is invariant if *I; J* are for the positive helicity gauge bosons. $\langle pq \rangle$ is the spinor product defined by

$$
\langle pq \rangle \equiv \overline{\Psi_{-}(p)} \Psi_{+}(q) \tag{A2}
$$

and $|\langle pq \rangle|^2 = 2p \cdot q$. The order of $\langle XY \rangle$ in the denominator is cyclic of 1234. For processes involving fermions, the supersymmetric relation of Eq. (4.9) in [15] can been applied. The expressions for four fermions (*ffff*) are exactly the same as those for four gauge bosons (*gggg*) for each corresponding helicity and particle permutation. The amplitudes for processes with two bosons and two fermions vanish when the two fermions (or bosons) have the same helicity. A useful list of the amplitudes relevant to our scattering amplitude construction in the text is given as follows, where the superscripts indicate the helicities with respect to the incoming momenta.

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$$
g^{\pm}g^{\mp}g^{\mp}g^{\pm}/f^{\pm}f^{\mp}f^{\pm}; \qquad A_{1234} = ig^2 \frac{\langle 14 \rangle^2}{\langle 12 \rangle^2} \qquad A_{1324} = ig^2 \frac{\langle 14 \rangle^2}{\langle 13 \rangle^2} \qquad A_{1243} = ig^2 \frac{\langle 14 \rangle^4}{\langle 12 \rangle^2 \langle 13 \rangle^2}
$$

$$
g^{\pm}g^{\mp}g^{\pm}g^{\mp}/f^{\pm}f^{\mp}f^{\pm}f^{\mp}; \qquad A_{1234} = ig^2 \frac{\langle 13 \rangle^4}{\langle 12 \rangle^2 \langle 14 \rangle^2} \qquad A_{1324} = ig^2 \frac{\langle 13 \rangle^2}{\langle 14 \rangle^2} \qquad A_{1243} = ig^2 \frac{\langle 13 \rangle^2}{\langle 12 \rangle^2}
$$

$$
g^{\pm}g^{\mp}f^{\mp}f^{\pm}/f^{\mp}f^{\pm}g^{\pm}g^{\mp}; \qquad A_{1234} = ig^2 \frac{\langle 13 \rangle^4}{\langle 12 \rangle^2} \qquad A_{1324} = ig^2 \frac{\langle 14 \rangle}{\langle 13 \rangle} \qquad A_{1243} = ig^2 \frac{\langle 14 \rangle^3}{\langle 13 \rangle \langle 12 \rangle^2}
$$

Expressions for other helicity combinations can be achieved by properly crossing two particle momenta, or by cyclic permutation under which Eq. (A1) is invariant. In doing so, some identities may be useful:

- (i) $A_{ijkl} = A_{lkji}$; $A_{ijkl} = A_{ilki}$;
- (ii) invariant under the sign change $(+ + \leftrightarrow -)$.

APPENDIX B: CALCULATION OF DECAY WIDTHS

The partial decay width of SR with a mass $m = \sqrt{n}M_S$ and angular momentum *j* to a final state $\ell \bar{\ell}$ can be written generically as

$$
\Gamma_n^j = \frac{1}{2m} \frac{1}{2j+1} \int dPS_2 |A(X_n^j \to \ell \bar{\ell})|^2.
$$
 (B1)

The two-body phase space element is $dPS_2 = d\Omega/8$, and the decay matrix element squared can be related to the scattering amplitude by

$$
|A(X_n^j \to \ell_3 \bar{\ell}_4)|^2 = (s - m^2)|A_n^j(\ell_1 \bar{\ell}_2 \to \ell_3 \bar{\ell}_4)|
$$

with $p_1 = p_3, p_2 = p_4$. (B2)

With the help of partial wave expansion in terms of the Wigner functions $d^{j}_{mm'}$ as discussed in Sec. IIIB, we have

$$
A_n^j(\ell_\alpha \bar{\ell}_\beta \to \ell_\alpha \bar{\ell}_\beta) = ig^2 G_{\alpha\alpha} \frac{s \alpha_n^j d_{1,-1}^j}{s - m^2}, \qquad (B3)
$$

where

$$
G = \begin{cases} F + 2T & \text{for odd } n, \\ F & \text{for even } n, \end{cases}
$$
 (B4)

with *F* and *T* given in text. The coefficient α_n^j satisfies normalization condition $\sum_{j=1}^{n+1} |\alpha_n^j| = 1$. The final expression for decay width of the SR is therefore

$$
\Gamma_n^j = \frac{g^2}{16\pi} \frac{\sqrt{n}M_S}{2j+1} G_{\alpha\alpha} |\alpha_n^j|
$$
 (B5)

This expression can be easily generalized to other elastic

processes. As for the case of diphoton production, the gauge coupling factor $G = T$ after absorbing the $1/2$ factor for identical particles, and the coupling $g^2/16\pi$ = $\alpha/4$, instead of $\alpha/4x_w$ as in the dilepton case. It should also be noted that even we do have a nonvanishing SM part in the $q\bar{q}\gamma\gamma$ channel, there is no corresponding contribution from an SR and consequently to the width of diphoton processes.

For completeness, in Table III we provide the expansion coefficients in Eq. (B3), and the relevant Wigner functions are

$$
d_{1,-1}^1 = \frac{1 - \cos\theta}{2}
$$
 (B6)

$$
d_{1,-1}^2 = \frac{1 - \cos\theta}{2} (2\cos\theta + 1)
$$
 (B7)

$$
d_{1,-1}^3 = \frac{1}{4} \left(\frac{1 - \cos \theta}{2} \right) (15 \cos^2 \theta + 10 \cos \theta - 1) \quad (B8)
$$

$$
d_{2,-1}^2 = -\sin\theta \left(\frac{1-\cos\theta}{2}\right) \tag{B9}
$$

$$
d_{2,-2}^2 = \left(\frac{1 - \cos\theta}{2}\right)^2 \tag{B10}
$$

with $d_{1,1}^j(x) = (-1)^{j-1} d_{1,-1}^j(-x)$ and $d_{2,m}^2(x) =$ $d_{2,-m}^2(-x)(m = 1, 2).$

TABLE III. Coefficients α_n^j of partial wave expansion in each processes. Upper (lower) sign in $q\bar{q}\ell\bar{\ell}$ corresponds to scattering of quark into lepton with like (opposite) helicity.

	\boldsymbol{n}	$j=1$	2	3
$q\bar{q}\ell\bar{\ell}$	\overline{c}	3/4 $-9/20$	$\overline{+}1/4$ $\pm 5/12$	0 $-2/15$
$q\bar{q}\gamma\gamma$	$\mathcal{D}_{\mathcal{L}}$	0 0	-1 θ	Ω Ω
$gg\ell\bar{\ell}$	2	0 0	-1	Ω Ω
88YY	2	∩		0 0

Numerically, the total widths for each processes when $T = 1$ are

$$
\Gamma_1^{1,2}(q\bar{q}\ell\bar{\ell}) = 240, \quad 48 \text{ GeV} \left(\frac{M_S}{\text{TeV}}\right), \tag{B11}
$$

$$
\Gamma_2^{1,2,3}(q\bar{q}\ell\bar{\ell}) = 46, 26, \quad 5.8 \text{ GeV} \left(\frac{M_S}{\text{TeV}}\right), \quad (B12)
$$

$$
\Gamma_1^2(gg\ell\bar{\ell}) = 19 \text{ GeV} \left(\frac{M_S}{\text{TeV}}\right),\tag{B13}
$$

$$
\Gamma_1^2(q\bar{q}\gamma\gamma) = 3.9 \text{ GeV} \left(\frac{M_S}{\text{TeV}}\right),\tag{B14}
$$

$$
\Gamma_1^2(gg\gamma\gamma) = 3.5 \text{ GeV} \left(\frac{M_S}{\text{TeV}}\right). \tag{B15}
$$

where we have included all necessary decay modes into related final states for each resonance. For instance, the width $\Gamma(q\bar{q}\ell\bar{\ell})$ includes the partial decay widths of SR into charged leptons, neutrinos, and quarks. Partial decay modes into massive bosons such as the Higgs and W^{\pm} , Z are not included.

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