Measuring the fermionic couplings of the Higgs boson at future colliders as a probe of a nonminimal flavor structure

J. L. Díaz-Cruz,[1,2] R. Noriega-Papaqui,[3] and A. Rosado[1,3]

[1] *Cuerpo Académico de Partículas, Campos y Relatividad de la Benemérita Universidad Autónoma de Puebla, Apartado Postal 1364, C. P. 72000 Puebla, Puebla, Mexico*

[2] *Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla, Apartado Postal 1364, C. P. 72000 Puebla, Puebla, Mexico*

[3] *Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, C. P. 72570 Puebla, Puebla, Mexico*

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We study the fermionic couplings of the neutral Higgs bosons in the two Higgs doublet model, assuming a four-texture structure for the Yukawa matrices. We then derive the low-energy constraints on the model, focusing in b-quark and lepton physics, and apply them to study Higgs boson detection at future colliders. We show that the bound on the flavor-violating parameter χ_{sb} obtained from the contribution due to the $b\bar{s}h^0$ coupling to the decay $b \rightarrow s + \gamma$ (roughly of the order 10^{-1}–10^{-2}) is approximately a factor 10 more restrictive than that obtained from the current bound on $\Gamma(B_s^0 \rightarrow \mu^- \mu^+)$ (which gives a bound on χ_{sb} of the order 10^0–10^{-1}). On the other hand, the analysis of the LFV decay $\mu \rightarrow e\gamma$ leads to the constraint $\chi_{\mu\tau} \lesssim 10^{-2}$. The observation of a Higgs signal at a future muon collider would confirm the results of these constraints for $\tan\beta \lesssim 15$, while for larger values of $\tan\beta$ such colliders will not improve the bounds derived from low-energy data. At a hadron collider it is further possible to study the Higgs boson coupling $h^0 b\bar{b}$ by searching for the associated production of the Higgs boson with $b\bar{b}$ pairs.

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I. INTRODUCTION

Despite the success of the standard model (SM) in the gauge and fermion sectors, the Higgs sector remains the least tested aspect of the model, and the mechanism of electroweak symmetry breaking (EWSB) is still a puzzle. However, the analysis of radiative corrections within the SM [1] points towards the existence of a Higgs boson with mass of the order of the EW scale, which could be detected in the early stages of CERN LHC [2]. On the other hand, the SM is often considered as an effective theory, valid up to an energy scale of O(TeV), and eventually it will be replaced by a more fundamental theory, which will explain, among other things, the physics behind EWSB and perhaps even the origin of flavor. Several examples of candidate theories, which range from supersymmetry [3] to deconstruction [4], include a Higgs sector with two scalar doublets, which has a rich structure and predicts interesting phenomenology [5]. The general two-Higgs doublet model (THDM) has a potential problem with flavor-changing neutral currents (FCNC) mediated by the Higgs bosons, which arise when each quark type (u and d) is allowed to couple to both Higgs doublets, and FCNC could be induced at large rates that may jeopardize the model. The possible solutions to this problem of the THDM involve an assumption about the Yukawa structure of the model. To discuss them it is convenient to refer to the Yukawa Lagrangian, which is written for the quarks fields as follows:

$$\mathcal{L}_Y = Y_1^u \overline{Q}_L \Phi_1 u_R + Y_2^u \overline{Q}_L \Phi_2 u_R + Y_1^d \overline{Q}_L \Phi_1 d_R + Y_2^d \overline{Q}_L \Phi_2 d_R \qquad (1)$$

where $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)^T$ denote the Higgs doublets. The specific choices for the Yukawa matrices $Y_{1,2}^q$ ($q = u, d$) define the versions of the THDM known as I, II, or III, which involve the following mechanisms that are aimed either to eliminate the otherwise unbearable FCNC problem or at least to keep it under control, namely:

(1) *DISCRETE SYMMETRIES.* A discrete symmetry can be invoked to allow a given fermion type (u or d quarks for instance) to couple to a single Higgs doublet, and in such a case FCNC are absent at tree level. In particular, when a single Higgs field gives masses to both types of quarks (either $Y_1^u = Y_1^d = 0$ or $Y_2^u = Y_2^d = 0$), the resulting model is referred to as THDM-I. On the other hand, when each type of quark couples to a different Higgs doublet (either $Y_1^u = Y_2^d = 0$ or $Y_2^u = Y_1^d = 0$), the model is known as the THDM-II. This THDM-II pattern is highly motivated because it arises at tree level in the minimal supersymmetry (SUSY) extension for the SM (MSSM) [5].

(2) *RADIATIVE SUPPRESSION.* When each fermion type couples to both Higgs doublets, FCNC could be kept under control if there exists a hierarchy between $Y_1^{u,d}$ and $Y_2^{u,d}$. Namely, a given set of Yukawa matrices is present at tree level, but the other ones arise only as a radiative effect. This occurs for instance in the MSSM, where the type-II THDM structure is not protected by any symmetry, and is transformed into a type-III THDM (see below) through the loop effects of sfermions and gauginos. Namely, the Yukawa couplings that are already

present at tree level in the MSSM (Y_1^d, Y_2^u) receive radiative corrections, while the terms (Y_2^d, Y_1^u) are induced at one-loop level.

In particular, when the ''seesaw'' mechanism [6] is implemented in the MSSM to explain the observed neutrino masses [7,8], lepton flavor violation (LFV) appears naturally in the right-handed neutrino sector, which is then communicated to the sleptons and from there to the charged leptons and Higgs sector. These corrections allow the neutral Higgs bosons to mediate LFV; in particular, it was found that the (Higgs-mediated) tau decay $\tau \to 3\mu$ [9] as well as the (real) Higgs boson decay $H \to \tau\mu$ [10] can enter into possible detection domain. Similar effects are known to arise in the quark sector; for instance, $B \to \mu\mu$ can reach branching fractions at large $\tan\beta$ that can be probed at Run II of the Tevatron [11,12].

(3) *FLAVOR SYMMETRIES.* Suppression for FCNC can also be achieved when a certain form of the Yukawa matrices that reproduce the observed fermion masses and mixing angles is implemented in the model, which is then named as THDM-III. This could be done either by implementing the Frogart-Nielsen mechanism to generate the fermion mass hierarchies [13], or by studying a certain ansatz for the fermion mass matrices [14]. The first proposal for the Higgs boson couplings along these lines was posed in [15,16]; it was based on the six-texture form of the mass matrices, namely,

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & 0 & B_q \\ 0 & B_q^* & A_q \end{pmatrix}.$$

Then, by assuming that each Yukawa matrix $Y_{1,2}^q$ has the same hierarchy, one finds $A_q \simeq m_{q_3}$, $B_q \simeq \sqrt{m_{q_2} m_{q_3}}$, and $C_q \simeq \sqrt{m_{q_1} m_{q_2}}$. Then, the fermion-fermion′-Higgs boson ($ff'\phi^0$) couplings obey the following pattern: $Hf_if_j \sim \sqrt{m_{f_i} m_{f_j}}/m_W$, which is known as the Cheng-Sher ansatz. This brings under control the FCNC problem, and it has been extensively studied in the literature to search for flavor-violating signals in the Higgs sector [17].

In this paper we are interested in studying the THDM-III. However, the six-texture ansatz seems disfavored by current data on the Cabibbo-Kobayashi-Maskawa mixing angles. More recently, mass matrices with four-texture ansatz have been considered, and are found in better agreement with the observed data [18,19]. It is interesting then to investigate how the Cheng-Sher form of the $ff'\phi^0$ couplings get modified when one replaces the six-texture matrices by the four-texture ansatz. This paper is aimed precisely to study this question; we want to derive the form of the $ff'\phi^0$ couplings and to discuss how and when the resulting predictions could be tested, both in rare quark and lepton decays and in the phenomenology of the Higgs bosons [10]. Unlike previous studies, we keep in our analysis the effect of the complex phases, which modify the FCNC Higgs boson couplings.

The organization of the paper goes as follows: In Sec. II, we discuss the Lagrangian for the THDM with the four-texture form for the mass matrices and present the results for the $ff'\phi^0$ vertices in the quark sector. Then, in Sec. III we study the constraints imposed on the parameters of the model from low-energy flavor-violating processes. Section IV includes the predictions of the model for both flavor-conserving (FC) and flavor-violating (FV) Higgs boson decays. While in Sec. V, we discuss the capabilities of future $\mu^+\mu^-$ and hadron colliders to detect such decays. Finally, Sec. VI contains our conclusions.

II. THE QUARK SECTOR OF THE THDM-III WITH FOUR-TEXTURE MASS MATRICES

The Yukawa Lagrangian of the THDM-III is written for the quark fields as follows:

$$\mathcal{L}_Y^q = Y_1^u \overline{Q}_L \Phi_1 u_R + Y_2^u \overline{Q}_L \Phi_2 u_R + Y_1^d \overline{Q}_L \Phi_1 d_R + Y_2^d \overline{Q}_L \Phi_2 d_R \tag{2}$$

where $\Phi_{1,2} = (\phi_{1,2}^+, \phi_{1,2}^0)^T$ denote the Higgs doublets. The specific choices for the Yukawa matrices $Y_{1,2}^q$ ($q = u, d$) define the versions of the THDM known as I, II, or III.

After spontaneous symmetry breaking the quark mass matrix is given by

$$M_q = \frac{1}{\sqrt{2}}(v_1 Y_1^q + v_2 Y_2^q). \tag{3}$$

We will assume that both Yukawa matrices Y_1^q and Y_2^q have the four-texture form and are Hermitic; following the conventions of [18], the quark mass matrix is then written as

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & \tilde{B}_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix};$$

when $\tilde{B}_q \to 0$ one recovers the six-texture form. We also consider the hierarchy: $|A_q| \gg |\tilde{B}_q|, |B_q|, |C_q|$, which is supported by the observed fermion masses in the SM.

Because of the Hermiticity condition, both $\tilde{B}_q$ and A_q are real parameters, while the phases of C_q and B_q, Φ_{B_q,C_q}, can be removed from the mass matrix M_q by defining $M_q = P_q^\dagger \tilde{M}_q P_q$, where $P_q = \mathrm{diag}[1, e^{i\Phi_{C_q}}, e^{i(\Phi_{B_q}+\Phi_{C_q})}]$, and the mass matrix $\tilde{M}_q$ includes only the real parts of M_q. The diagonalization of $\tilde{M}_q$ is then obtained by an orthogonal matrix O_q, such that the diagonal mass matrix is $\overline{M}_q = O_q^T \tilde{M}_q O_q$.

The Lagrangian (2) can be expanded in terms of the mass eigenstates for the neutral (h^0, H^0, A^0) and charged

Higgs bosons ($H^\pm$). The interactions of the neutral Higgs bosons with the d-type and u-type are given by ($u, u' = u, c, t$ and $d, d' = d, s, b$),

$$\begin{aligned}
\mathcal{L}_Y^q &= \frac{g}{2}\left(\frac{m_d}{m_W}\right)\bar{d}\left[\frac{\cos\alpha}{\cos\beta}\delta_{dd'} + \frac{\sqrt{2}\sin(\alpha-\beta)}{g\cos\beta}\left(\frac{m_W}{m_d}\right)(\tilde{Y}_2^d)_{dd'}\right]d'H^0 \\
&+ \frac{g}{2}\left(\frac{m_d}{m_W}\right)\bar{d}\left[-\frac{\sin\alpha}{\cos\beta}\delta_{dd'} + \frac{\sqrt{2}\cos(\alpha-\beta)}{g\cos\beta}\left(\frac{m_W}{m_d}\right)(\tilde{Y}_2^d)_{dd'}\right]d'h^0 \\
&+ \frac{ig}{2}\left(\frac{m_d}{m_W}\right)\bar{d}\left[-\tan\beta\delta_{dd'} + \frac{\sqrt{2}}{g\cos\beta}\left(\frac{m_W}{m_d}\right)(\tilde{Y}_2^d)_{dd'}\right]\gamma^5 d'A^0 \\
&+ \frac{g}{2}\left(\frac{m_u}{m_W}\right)\bar{u}\left[\frac{\sin\alpha}{\sin\beta}\delta_{uu'} - \frac{\sqrt{2}\sin(\alpha-\beta)}{g\sin\beta}\left(\frac{m_W}{m_u}\right)(\tilde{Y}_2^u)_{uu'}\right]u'H^0 \\
&+ \frac{g}{2}\left(\frac{m_u}{m_W}\right)\bar{u}\left[\frac{\cos\alpha}{\sin\beta}\delta_{uu'} - \frac{\sqrt{2}\cos(\alpha-\beta)}{g\sin\beta}\left(\frac{m_W}{m_u}\right)(\tilde{Y}_2^u)_{uu'}\right]u'h^0 \\
&+ \frac{ig}{2}\left(\frac{m_u}{m_W}\right)\bar{u}\left[-\cot\beta\delta_{uu'} + \frac{\sqrt{2}}{g\sin\beta}\left(\frac{m_W}{m_u}\right)(\tilde{Y}_2^u)_{uu'}\right]\gamma^5 u'A^0. \qquad (4)
\end{aligned}$$

The first term, proportional to $\delta_{qq'}$, corresponds to the modification of the THDM-II over the SM result, while the term proportional to $\tilde{Y}_2^q$ denotes the new contribution from THDM-III. Thus, the $ff'\phi^0$ couplings respect CP invariance, despite the fact that the Yukawa matrices include complex phases; this follows because of the Hermiticity conditions imposed on both Y_1^q and Y_2^q.

The corrections to the quark FC and FV couplings depend on the rotated matrix $\tilde{Y}_2^q = O_q^T P_q Y_2^q P_q^\dagger O_q$. We will evaluate $\tilde{Y}_2^q$ assuming that Y_2^q has a four-texture form, namely,

$$Y_2^q = \begin{pmatrix} 0 & C_2^q & 0 \\ C_2^{q*} & \tilde{B}_2^q & B_2^q \\ 0 & B_2^{q*} & A_2^q \end{pmatrix}, \qquad |A_2^q| \gg |\tilde{B}_2^q|, |B_2^q|, |C_2^q|. \qquad (5)$$

The matrix that diagonalizes the real matrix $\tilde{M}_q$ with the four-texture form is given by

$$O_q = \begin{pmatrix} \sqrt{\frac{\lambda_2^q\lambda_3^q(A_q-\lambda_1^q)}{A_q(\lambda_2^q-\lambda_1^q)(\lambda_3^q-\lambda_1^q)}} & \eta_q\sqrt{\frac{\lambda_1^q\lambda_3^q(\lambda_2^q-A_q)}{A_q(\lambda_2^q-\lambda_1^q)(\lambda_3^q-\lambda_2^q)}} & \sqrt{\frac{\lambda_1^q\lambda_2^q(A_q-\lambda_3^q)}{A_q(\lambda_3^q-\lambda_1^q)(\lambda_3^q-\lambda_2^q)}} \\ -\eta_q\sqrt{\frac{\lambda_1^q(\lambda_1^q-A_q)}{(\lambda_2^q-\lambda_1^q)(\lambda_3^q-\lambda_1^q)}} & \sqrt{\frac{\lambda_2^q(A_q-\lambda_2^q)}{(\lambda_2^q-\lambda_1^q)(\lambda_3^q-\lambda_2^q)}} & \sqrt{\frac{\lambda_3^q(\lambda_3^q-A_q)}{(\lambda_3^q-\lambda_1^q)(\lambda_3^q-\lambda_2^q)}} \\ \eta_q\sqrt{\frac{\lambda_1^q(A_q-\lambda_2^q)(A_q-\lambda_3^q)}{A_q(\lambda_2^q-\lambda_1^q)(\lambda_3^q-\lambda_1^q)}} & -\sqrt{\frac{\lambda_2^q(A_q-\lambda_1^q)(\lambda_3^q-A_q)}{A_q(\lambda_2^q-\lambda_1^q)(\lambda_3^q-\lambda_2^q)}} & \sqrt{\frac{\lambda_3^q(A_q-\lambda_1^q)(A_q-\lambda_2^q)}{A_q(\lambda_3^q-\lambda_1^q)(\lambda_3^q-\lambda_2^q)}} \end{pmatrix},$$

where $m_1^q = |\lambda_1^q|$, $m_2^q = |\lambda_2^q|$, $m_3^q = |\lambda_3^q|$, and $\eta_q = \lambda_2^q/m_2^q$ ($q = u, d$), with $m_u = m_1^u$, $m_c = m_2^u$, and $m_t = m_3^u$; $m_d = m_1^d$, $m_s = m_2^d$, and $m_b = m_3^d$.

Then the rotated form $\tilde{Y}_2^q$ has the general form

$$\tilde{Y}_2^q = O_q^T P_q Y_2^q P_q^\dagger O_q = \begin{pmatrix} (\tilde{Y}_2^q)_{11} & (\tilde{Y}_2^q)_{12} & (\tilde{Y}_2^q)_{13} \\ (\tilde{Y}_2^q)_{21} & (\tilde{Y}_2^q)_{22} & (\tilde{Y}_2^q)_{23} \\ (\tilde{Y}_2^q)_{31} & (\tilde{Y}_2^q)_{32} & (\tilde{Y}_2^q)_{33} \end{pmatrix}. \qquad (6)$$

However, the full expressions for the resulting elements have a complicated form, as it can be appreciated, for instance, by looking at the element $(\tilde{Y}_2^q)_{22}$, which is displayed here:

$$\begin{aligned}
(\tilde{Y}_2^q)_{22} &= \eta_q[C_2^{q*}e^{i\Phi_{C_q}} + C_2^q e^{-i\Phi_{C_q}}]\frac{(A_q-\lambda_2^q)}{m_3^q-\lambda_2^q}\sqrt{\frac{m_1^q m_3^q}{A_q m_2^q}} \\
&+ \tilde{B}_2^q\frac{A_q-\lambda_2^q}{m_3^q-\lambda_2^q} + A_2^q\frac{A_q-\lambda_2^q}{m_3^q-\lambda_2^q} \\
&- [B_2^{q*}e^{i\Phi_{B_q}} + B_2^q e^{-i\Phi_{B_q}}]\sqrt{\frac{(A_q-\lambda_2^q)(m_3^q-A_q)}{m_3^q-\lambda_2^q}} \qquad (7)
\end{aligned}$$

where we have taken the limits $|A_q|, m_3^q, m_2^q \gg m_1^q$. The free parameters are $\tilde{B}_2^q, B_2^q, A_2^q, A_q$.

To derive a better suited approximation, we will consider the elements of the Yukawa matrix Y_2^l as having the same hierarchy as the full mass matrix, namely,

$$C_2^q = c_2^q \sqrt{\frac{m_1^q m_2^q m_3^q}{A_q}}, \tag{8}$$

$$B_2^q = b_2^q \sqrt{(A_q - \lambda_2^q)(m_3^q - A_q)}, \tag{9}$$

$$\tilde{B}_2^q = \tilde{b}_2^q (m_3^q - A_q + \lambda_2^q), \tag{10}$$

$$A_2^q = a_2^q A_q. \tag{11}$$

Then, in order to keep the same hierarchy for the elements of the mass matrix, we find that A_q must fall within the interval $(m_3^q - m_2^q) \leq A_q \leq m_3^q$. Thus, we propose the following relation for A_q:

$$A_q = m_3^q (1 - \beta_q z_q), \tag{12}$$

where $z_q = m_2^q / m_3^q \ll 1$ and $0 \leq \beta_q \leq 1$.

Then, we introduce the matrix $\tilde{\chi}^q$ as follows:

$$(\tilde{Y}_2^q)_{ij} = \frac{\sqrt{m_i^q m_j^q}}{v} \tilde{\chi}_{ij}^q = \frac{\sqrt{m_i^q m_j^q}}{v} \chi_{ij}^q e^{i\vartheta_{ij}^q}, \tag{13}$$

which differs from the usual Cheng-Sher ansatz not only because of the appearance of the complex phases, but also in the form of the real parts $\chi_{ij}^q = |\tilde{\chi}_{ij}^q|$.

Expanding in powers of z_q, one finds that the elements of the matrix $\tilde{\chi}^q$ have the following general expressions:

$$\begin{aligned}
\tilde{\chi}_{11}^q &= [\tilde{b}_2^q - (c_2^{q*} e^{i\Phi_{C_q}} + c_2^q e^{-i\Phi_{C_q}})]\eta_q + [a_2^q + \tilde{b}_2^q - (b_2^{q*} e^{i\Phi_{B_q}} + b_2^q e^{-i\Phi_{B_q}})]\beta_q, \\
\tilde{\chi}_{12}^q &= (c_2^q e^{-i\Phi_{C_q}} - \tilde{b}_2^q) - \eta_q [a_2^q + \tilde{b}_2^q - (b_2^{q*} e^{i\Phi_{B_q}} + b_2^q e^{-i\Phi_{B_q}})]\beta_q, \\
\tilde{\chi}_{13}^q &= (a_2^q - b_2^q e^{-i\Phi_{B_q}})\eta_q \sqrt{\beta_q}, \\
\tilde{\chi}_{22}^q &= \tilde{b}_2^q \eta_q + [a_2^q + \tilde{b}_2^q - (b_2^{q*} e^{i\Phi_{B_q}} + b_2^q e^{-i\Phi_{B_q}})]\beta_q, \\
\tilde{\chi}_{23}^q &= (b_2^q e^{-i\Phi_{B_q}} - a_2^q)\sqrt{\beta_q}, \\
\tilde{\chi}_{33}^q &= a_2^q.
\end{aligned} \tag{14}$$

While the diagonal elements $\tilde{\chi}_{ii}^q$ are real, we notice [Eqs. (14)] the appearance of the phases in the off-diagonal elements, which are essentially unconstrained by present low-energy phenomena. As we will see next, these phases modify the pattern of flavor violation in the Higgs sector. For instance, while the Cheng-Sher ansatz predicts that the LFV couplings $(\tilde{Y}_2^q)_{13}$ and $(\tilde{Y}_2^q)_{23}$ vanish when $a_2^q = b_2^q$, in our case this is no longer valid for $\cos\Phi_{B_q} \neq 1$. Furthermore the LFV couplings satisfy several relations, such as $|\tilde{\chi}_{23}^q| = |\tilde{\chi}_{13}^q|$, which simplifies the parameter analysis.

In order to perform our phenomenological study we find it convenient to rewrite the Lagrangian given in Eq. (4) in terms of the $\tilde{\chi}_{qq'} = \tilde{\chi}_{ij}^q$ as follows:

$$\begin{aligned}
\mathcal{L}_Y^q = &\frac{g}{2}\bar{d}\left[\left(\frac{m_d}{m_W}\right)\frac{\cos\alpha}{\cos\beta}\delta_{dd'} + \frac{\sin(\alpha-\beta)}{\sqrt{2}\cos\beta}\left(\frac{\sqrt{m_d m_{d'}}}{m_W}\right)\tilde{\chi}_{dd'}\right]d' H^0 \\
&+ \frac{g}{2}\bar{d}\left[-\left(\frac{m_d}{m_W}\right)\frac{\sin\alpha}{\cos\beta}\delta_{dd'} + \frac{\cos(\alpha-\beta)}{\sqrt{2}\cos\beta}\left(\frac{\sqrt{m_d m_{d'}}}{m_W}\right)\tilde{\chi}_{dd'}\right]d' h^0 \\
&+ \frac{ig}{2}\bar{d}\left[-\left(\frac{m_d}{m_W}\right)\tan\beta\,\delta_{dd'} + \frac{1}{\sqrt{2}\cos\beta}\left(\frac{\sqrt{m_d m_{d'}}}{m_W}\right)\tilde{\chi}_{dd'}\right]\gamma^5 d' A^0 \\
&\times \frac{g}{2}\bar{u}\left[\left(\frac{m_u}{m_W}\right)\frac{\sin\alpha}{\sin\beta}\delta_{uu'} - \frac{\sin(\alpha-\beta)}{\sqrt{2}\sin\beta}\left(\frac{\sqrt{m_u m_{u'}}}{m_W}\right)\tilde{\chi}_{uu'}\right]u' H^0 \\
&+ \frac{g}{2}\bar{u}\left[\left(\frac{m_u}{m_W}\right)\frac{\cos\alpha}{\sin\beta}\delta_{uu'} - \frac{\cos(\alpha-\beta)}{\sqrt{2}\sin\beta}\left(\frac{\sqrt{m_u m_{u'}}}{m_W}\right)\tilde{\chi}_{uu'}\right]u' h^0 \\
&+ \frac{ig}{2}\bar{u}\left[-\left(\frac{m_u}{m_W}\right)\cot\beta\,\delta_{uu'} + \frac{1}{\sqrt{2}\sin\beta}\left(\frac{\sqrt{m_u m_{u'}}}{m_W}\right)\tilde{\chi}_{uu'}\right]\gamma^5 u' A^0,
\end{aligned} \tag{15}$$

where $u, u' = u, c, t$ and $d, d' = d, s, b$, and unlike the Cheng-Sher ansatz, $\tilde{\chi}_{qq'}$ $(q \neq q')$ are complex.

Finally, for completeness we display here the corresponding Lagrangian for the charged lepton sector, which has already been reported in our previous work [20], namely,

$$\mathcal{L}_Y^l = \frac{g}{2}\bar{l}\bigg[\left(\frac{m_l}{m_W}\right)\frac{\cos\alpha}{\cos\beta}\delta_{ll'} + \frac{\sin(\alpha-\beta)}{\sqrt{2}\cos\beta} \times \left(\frac{\sqrt{m_l m_{l'}}}{m_W}\right)\tilde{\chi}_{ll'}\bigg]l'H^0 + \frac{g}{2}\bar{l}\bigg[-\left(\frac{m_l}{m_W}\right)\frac{\sin\alpha}{\cos\beta}\delta_{ll'} + \frac{\cos(\alpha-\beta)}{\sqrt{2}\cos\beta}\left(\frac{\sqrt{m_l m_{l'}}}{m_W}\right)\tilde{\chi}_{ll'}\bigg]l'h^0 + \frac{ig}{2}\bar{l}\bigg[-\left(\frac{m_l}{m_W}\right)\tan\beta\,\delta_{ll'} + \frac{1}{\sqrt{2}\cos\beta}\left(\frac{\sqrt{m_l m_{l'}}}{m_W}\right)\tilde{\chi}_{ll'}\bigg]\gamma^5 l'A^0 \quad (16)$$

where $l, l' = e, \mu, \tau$.

On the other hand, one can also relate our results with the SUSY-induced THDM-III, for instance, by considering the effective Lagrangian for the couplings of the charged leptons to the neutral Higgs fields, namely,

$$-\mathcal{L} = \bar{L}_L Y_l l_R \phi_1^0 + \bar{L}_L Y_l(\epsilon_1 \mathbf{1} + \epsilon_2 Y_\nu^\dagger Y_\nu) l_R \phi_2^{0*} + \text{h.c.} \quad (17)$$

In this language, LFV results from our inability to simultaneously diagonalize the term Y_l and the nonholomorphic loop corrections $\epsilon_2 Y_l Y_\nu^\dagger Y_\nu$. Thus, since the charged lepton masses cannot be diagonalized in the same basis as their Higgs boson couplings, this will allow neutral Higgs bosons to mediate LFV processes with rates proportional to ϵ_2^2. In terms of our previous notation we have $\tilde{Y}_2 = \epsilon_2 Y_l Y_\nu^\dagger Y_\nu$. Thus, our result will cover (for some specific choices of parameters) the general expectations for the corrections arising in the MSSM.

III. BOUNDS ON THE FLAVOR-VIOLATING HIGGS PARAMETERS

Constrains on the FV-Higgs interaction can be obtained by studying FV transitions. In this section we consider the radiative decay $b \to s\gamma$ and the decay $B_s^0 \to \mu^-\mu^+$, which together with LFV bounds derived in [20], constrain the parameter space of THDM-III, and determine possible Higgs boson signals that may be detected at future colliders.

A. Radiative decay $b \to s\gamma$

We will make an estimation of the contribution due to the flavor-violating $ff'\phi^0$ couplings to the standard model branching ratio of $b \to s\gamma$ as follows:

$$\Delta\text{Br}(b \to s\gamma) = \Delta\Gamma(b \to s\gamma) \times \bigg[\sum_{l=e,\mu,\tau}\Gamma(b \to cl\bar{\nu}_l)\bigg]^{-1}. \quad (18)$$

Such contribution to the branching ratio of $b \to s\gamma$ at one-loop level is then given by [21]

$$\Delta\text{Br}(b \to s\gamma) = \frac{\alpha_{em} m_s m_b^3 \cos^2(\alpha-\beta)}{16\pi m_{h^0}^4 |V_{cb}|^2 \cos^4\beta}\chi_{sb}^2 \bigg| -\sin\alpha + \frac{\cos(\alpha-\beta)}{\sqrt{2}}\tilde{\chi}_{bb}\bigg|^2 \bigg|\ln\frac{m_b^2}{m_{h^0}^2} + \frac{3}{2}\bigg|^2. \quad (19)$$

From Eqs. (14) we have $\chi_{sb} = \chi_{db} = |(a_2^d - b_2^d e^{-i\Phi_{B_d}})|\sqrt{\beta_d}$. We will make use of the good agreement between the current experimental value for $\text{Br}(b \to s\gamma) = (3.3 \pm 0.4) \times 10^{-4}$ and the theoretical value obtained for $\text{Br}(b \to s\gamma) = (3.29 \pm 0.33) \times 10^{-4}$ in the context of the standard model [22] to constrain any new contribution to $\text{Br}(b \to s\gamma)$, namely $\Delta\text{Br}(b \to s\gamma) \lesssim 10^{-5}$, and hence to bound $\chi_{sb}(= \chi_{db})$ as a function of m_{h^0}, $\tilde{\chi}_{bb}$, α, and $\tan\beta$.

(a) Assuming $m_{h^0} = 120$ GeV and $\chi_{bb} = 0$, we depict in Fig. 1(a) the values of the upper bound on χ_{sb} ($(\chi_{sb})_{u.b.}^{b\to s\gamma}$) as a function of $\tan\beta$, for $\alpha = \beta, \beta - \pi/4, \beta - \pi/3$.

(b) Taking $\alpha = \beta - \pi/4$ and $\chi_{bb} = 0$, we plot in Fig. 1(b) the results for $(\chi_{sb})_{u.b.}^{b\to s\gamma}$ as a function of $\tan\beta$, for $m_{h^0} = 80$ GeV, 120 GeV, 160 GeV.

(c) We show in Fig. 1(c), taking $m_{h^0} = 120$ GeV and $\alpha = \beta - \pi/4$, our numerical results for $(\chi_{sb})_{u.b.}^{b\to s\gamma}$ as a function of real values[1] of $\tilde{\chi}_{bb}$ for $\tan\beta = 5, 25, 50$.

From Fig. 1, we conclude that the upper bound on the LFV parameter χ_{sb}, from the radiative decay $b \to s + \gamma$ measurements, is much more restrictive for large values of $\tan\beta$, $\tilde{\chi}_{bb} \sim -1$, $m_{h^0} \approx 80$ GeV, and $\alpha \approx \beta$. However, one can still say that at the present time the coupling χ_{sb} is not highly constrained when $\tan\beta \sim 5$–10, or even for larger values of $\tan\beta$ provided that $\tilde{\chi}_{bb} \to +1$ or $\alpha \to \beta - \pi/2$, thus $\tilde{\chi}_{sb}$ could induce interesting direct LFV Higgs boson signals at future colliders.

B. $B_s^0 \to \mu^-\mu^+$ decay

The formula to calculate the width of the decay $B_s^0 \to \mu^-\mu^+$ at the tree level is given as follows [11]:

$$\Gamma(B_s^0 \to \mu^-\mu^+) = \frac{G_F^2 \eta_{\text{QCD}}^2 m_B^3 f_B^2 m_s m_b m_\mu^2 \cos^2(\alpha-\beta)}{128\pi m_{h^0}^4 \cos^4\beta}\chi_{sb}^2 \times \bigg| -\sin\alpha + \frac{\cos(\alpha-\beta)}{\sqrt{2}}\tilde{\chi}_{\mu\mu}\bigg|^2 \quad (20)$$

where $G_F = 1.16639^{-5}$ GeV^{-2}, $\eta_{\text{QCD}} \approx 1.5$, $m_B \simeq 5$ GeV, and $f_B = 180$ MeV.

[1] We will study the dependency on the phases ϑ_{ij}^f ($\tilde{\chi}_{ij}^f = \chi_{ij}^f e^{\vartheta_{ij}^f}$) of the Higgs phenomenology in a forthcoming paper.

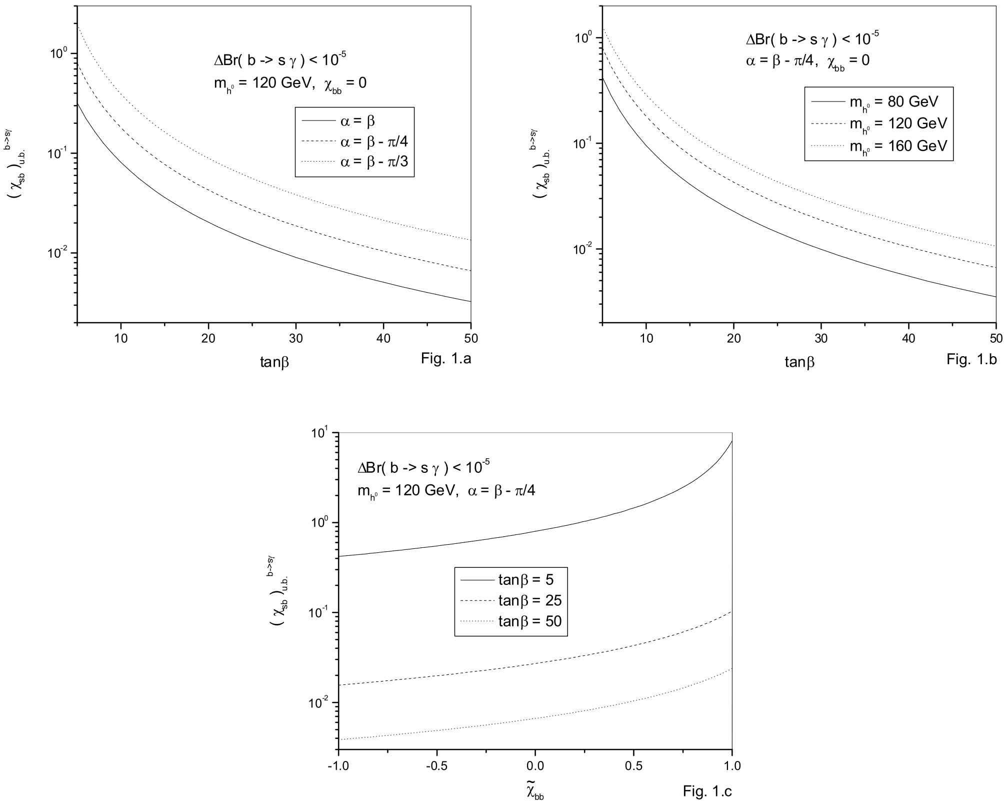

FIG. 1. The upper bound $(\chi_{sb})^{b\to s\gamma}_{u.b.}$, with $\Delta\mathrm{Br}(b\to s\gamma) < 10^{-5}$, as a function of (a) $\tan\beta$, for $\alpha=\beta$, $\alpha=\beta-\pi/4$, $\alpha=\beta-\pi/3$, taking $m_{h^0}=120$ GeV and $\chi_{bb}=0$; (b) $\tan\beta$, for $m_{h^0}=80$ GeV, 120 GeV, 160 GeV, taking $\alpha=\beta-\pi/4$ and $\chi_{bb}=0$; (c) $\tilde\chi_{bb}$, for $\tan\beta=5$, 25, 50, taking $m_{h^0}=120$ GeV and $\alpha=\beta-\pi/4$.

We will make use of the current experimental limit for $\Gamma(B^0_s\to\mu^-\mu^+) < 8.7\times10^{-19}$ GeV [11,23] to constrain the LFV parameter $\chi_{sb}(=\chi_{db})$ and the resulting upper bound will be shown as a function of m_{h^0}, $\tilde\chi_{\mu\mu}$, α, and $\tan\beta$.

(a) Assuming $m_{h^0}=120$ GeV and $\chi_{\mu\mu}=0$, we depict in Fig. 2(a) the values of the upper bound on χ_{sb} ($(\chi_{sb})^{B^0_s\to\mu\mu}_{u.b.}$) as a function of $\tan\beta$, for $\alpha=\beta, \beta-\pi/4, \beta-\pi/3$.

(b) Taking $\alpha=\beta-\pi/4$ and $\chi_{\mu\mu}=0$, we plot in Fig. 2(b) the results for $(\chi_{sb})^{B^0_s\to\mu\mu}_{u.b.}$ as a function of $\tan\beta$, for $m_{h^0}=80$ GeV, 120 GeV, 160 GeV.

(c) We show in Fig. 2(c), taking $m_{h^0}=120$ GeV and $\alpha=\beta-\pi/4$, our numerical results for $(\chi_{sb})^{B^0_s\to\mu\mu}_{u.b.}$ as a function of real values (see footnote 1 of $\tilde\chi_{\mu\mu}$ for $\tan\beta=$ 5, 25, 50.

From Fig. 2, we conclude that the upper bound on the LFV parameter χ_{sb}, obtained from the experimental bound for the width of the decay $B^0_s\to\mu^-\mu^+$, is more restrictive for large values of $\tan\beta$, $\tilde\chi_{\mu\mu}\sim-1$, $m_{h^0}\approx 80$ GeV, and $\alpha\approx\beta$. However, one can still say again that at the present time the coupling χ_{sb} is not highly constrained for $\tan\beta\sim$ 5–10, or even larger values of $\tan\beta$ provided that $\tilde\chi_{\mu\mu}\to +1$ or $\alpha\to\beta-\pi/2$.

From Eqs. (18) and (19), we obtain the following relation:

$$\Gamma(B^0_s\to\mu^-\mu^+) = \frac{1.22\times10^{-14}\ \mathrm{GeV}}{|\ln\frac{m_b^2}{m_{h^0}^2}+\frac{3}{2}|^2} \times\frac{|-\sin\alpha+\frac{\cos(\alpha-\beta)}{\sqrt2}\tilde\chi_{\mu\mu}|^2}{|-\sin\alpha+\frac{\cos(\alpha-\beta)}{\sqrt2}\tilde\chi_{bb}|^2}\Delta Br(b\to s\gamma). \quad (21)$$

Assuming that $\tilde\chi_{\mu\mu}=\tilde\chi_{bb}$ (or $\chi_{\mu\mu}\lesssim10^{-2}$ and $\chi_{bb}\lesssim 10^{-2}$) and taking $\Delta Br(b\to s\gamma) < 10^{-5}$, which is a conservative bound [22], we get

$$\Gamma(B^0_s\to\mu^-\mu^+) < \begin{cases} 6.7\times10^{-21}\ \mathrm{GeV} & \text{for } m_{h^0}=80\ \mathrm{GeV}\\ 4.8\times10^{-21} GeV & \text{for } m_{h^0}=120\ \mathrm{GeV}\\ 3.8\times10^{-21} GeV & \text{for } m_{h^0}=160\ \mathrm{GeV}.\end{cases} \quad (22)$$

Thus, we conclude from (22) that the bound on the parameter χ_{sb} obtained from the constraint on the contribution due to the $b\bar s h^0$ coupling to the theoretical branching ratio of the radiative decay $b\to s+\gamma$ is approximately a factor 10 more restrictive than the one

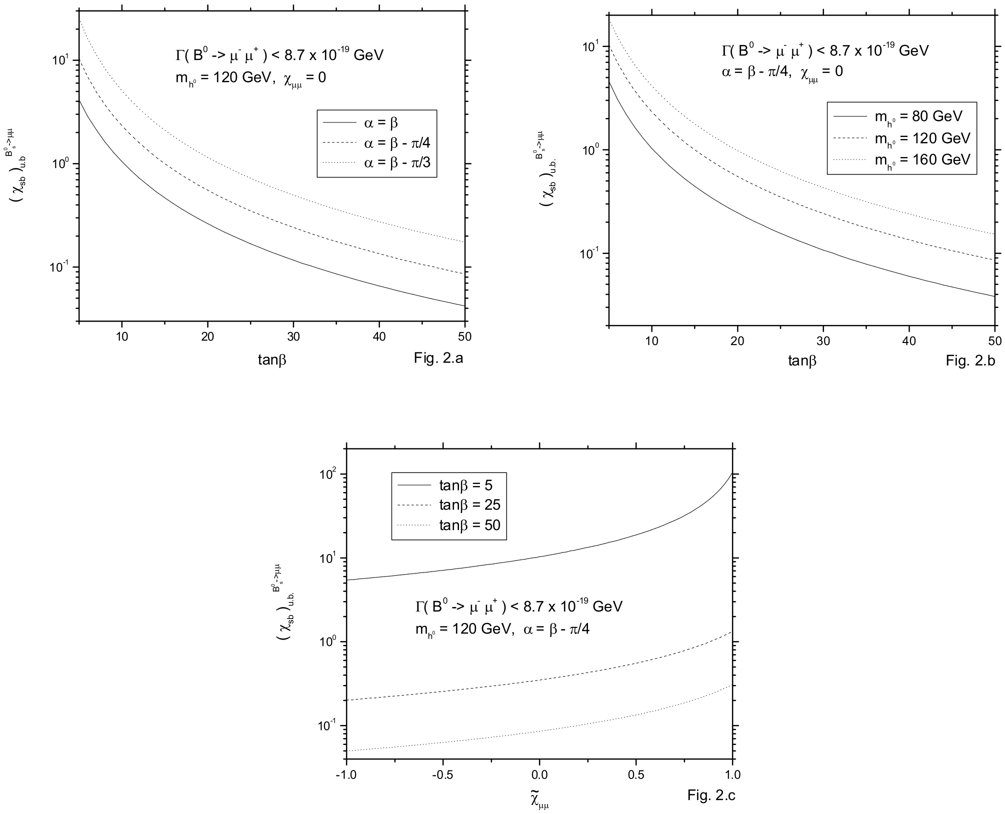

FIG. 2. The upper bound $(\chi_{sb})_{u.b.}^{B_s^0\to\mu\mu}$, with $\Gamma(B_s^0 \to \mu^-\mu^+) < 8.7 \times 10^{-19}$ GeV, as a function of (a) $\tan\beta$, for $\alpha = \beta$, $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, taking $m_{h^0} = 120$ GeV and $\chi_{\mu\mu} = 0$; (b) $\tan\beta$, for $m_{h^0} = 80$ GeV, 120 GeV, 160 GeV, taking $\alpha = \beta - \pi/4$ and $\chi_{\mu\mu} = 0$; (c) $\tilde{\chi}_{\mu\mu}$, for $\tan\beta = 5$, 25, 50, taking $m_{h^0} = 120$ GeV and $\alpha = \beta - \pi/4$.

obtained from the current experimental bound for $\Gamma(B_s^0 \to \mu^-\mu^+)$ already mentioned [11,23].

IV. HIGGS BOSON DECAYS IN THE THDM-III

One of the distinctive characteristic of the SM Higgs boson is the fact that its coupling to another particle is proportional to the mass of that particle, which in turn determines the search strategies proposed so far to detect it at future colliders. In particular, the decay pattern of the Higgs boson is dominated by the heaviest particle allowed to appear in its decay products. When one considers extensions of the SM it is important to study possible deviations from the SM decay pattern as it could provide a method to discriminate among the different models [24].

Within the context of the THDM-III, which we have been studying, not only are modifications of the Higgs boson couplings predicted, but also the appearance of new channels with flavor violation, both in the quark and leptonic sectors [10,25].

To explore the characteristics of Higgs boson decays in the THDM-III, we will focus on the lightest CP-even state (h^0), which could be detected first at LHC. The light Higgs boson-fermion couplings are given by Eqs. (15) and (16), where we have separated the SM from the corrections that appear in a THDM-III. In fact, we have also separated the factors that arise in the THDM-III too. We notice that the correction to the SM result depends on $\tan\beta$, α (the mixing angle in the neutral CP-even Higgs sector) and the factors $\tilde{\chi}_{ij}$ that induce FCNC transitions (for $i \neq j$) and further corrections to the SM vertex.

In what follows, we will include the decay widths for all the modes that are allowed kinematically for a Higgs boson with a mass in the range $80\ \text{GeV} < m_{h^0} < 160$ GeV. Namely, we study the branching ratios for the decays $h^0 \to b\bar{b}, c\bar{c}, \tau\bar{\tau}, \mu\bar{\mu}$ and the flavor-violating $h^0 \to b\bar{s}(s\bar{b}), \tau\bar{\mu}(\mu\bar{\tau})$, as well as the decays into pairs of gauge bosons with one real and the other one virtual, i.e., $h^0 \to WW^*, ZZ^*$.

Making use of Eqs. (15) and (16) we obtain

$$\Gamma(h^0 \to d\bar{d}) = 3\frac{g^2 m_{h^0} m_d^2}{32\pi m_W^2}\left| -\frac{\sin\alpha}{\cos\beta} + \frac{\cos(\alpha-\beta)}{\sqrt{2}\cos\beta}\tilde{\chi}_{dd}\right|^2 \left(\frac{\lambda(m_d^2, m_d^2, m_{h^0}^2)}{m_{h^0}^4}\right)^{3/2}, \tag{23}$$

$$\Gamma(h^0 \to d\bar{d}') = 3\frac{g^2 m_{h^0} m_d m_{d'}}{64\pi m_W^2}\frac{\cos^2(\alpha-\beta)}{\cos^2\beta} \times \left(\frac{\lambda(m_d^2, m_{d'}^2, m_{h^0}^2)}{m_{h^0}^4}\right)^{3/2}\chi_{dd'}^2, \tag{24}$$

$$\Gamma(h^0 \to u\bar{u}) = 3\frac{g^2 m_{h^0} m_u^2}{32\pi m_W^2}\left|\frac{\cos\alpha}{\sin\beta} - \frac{\cos(\alpha-\beta)}{\sqrt{2}\sin\beta}\tilde{\chi}_{uu}\right|^2\left(\frac{\lambda(m_u^2, m_u^2, m_{h^0}^2)}{m_{h^0}^4}\right)^{3/2}, \tag{25}$$

$$\Gamma(h^0 \to u\bar{u}') = 3\frac{g^2 m_{h^0} m_u m_{u'}}{64\pi m_W^2}\frac{\cos^2(\alpha-\beta)}{\sin^2\beta} \times \left(\frac{\lambda(m_u^2, m_{u'}^2, m_{h^0}^2)}{m_{h^0}^4}\right)^{3/2}\chi_{uu'}^2, \tag{26}$$

$$\Gamma(h^0 \to l\bar{l}) = \frac{g^2 m_{h^0} m_l^2}{32\pi m_W^2}\left|-\frac{\sin\alpha}{\cos\beta} + \frac{\cos(\alpha-\beta)}{\sqrt{2}\cos\beta}\tilde{\chi}_{ll}\right|^2\left(\frac{\lambda(m_l^2, m_l^2, m_{h^0}^2)}{m_{h^0}^4}\right)^{3/2}, \tag{27}$$

$$\Gamma(h^0 \to l\bar{l}') = \frac{g^2 m_{h^0} m_l m_{l'}}{64\pi m_W^2}\frac{\cos^2(\alpha-\beta)}{\cos^2\beta} \times \left(\frac{\lambda(m_l^2, m_{l'}^2, m_{h^0}^2)}{m_{h^0}^4}\right)^{3/2}\chi_{ll'}^2, \tag{28}$$

where $\lambda(x, y, z) = (x-y-z)^2 - 4yz$; $u, u' = u, c, t$; $d, d' = d, s, b$; and $l, l' = e^-, \mu^-, \tau^-$. For the decays $h^0 \to WW^*, ZZ^*$ we use the corresponding expressions given in Ref. [2].

We calculate the branching ratios for all the relevant decay modes that are allowed kinematically in the range 80 GeV $< m_{h^0} <$ 160 GeV, taking $\alpha = \beta - 3\pi/8$ and assuming $\tilde{\chi}_{ij} = 0.1$ for $i = j$ and $i \neq j$. We consider the following cases: $\tan\beta = 2$, 2.61, 5, and 15. Our results

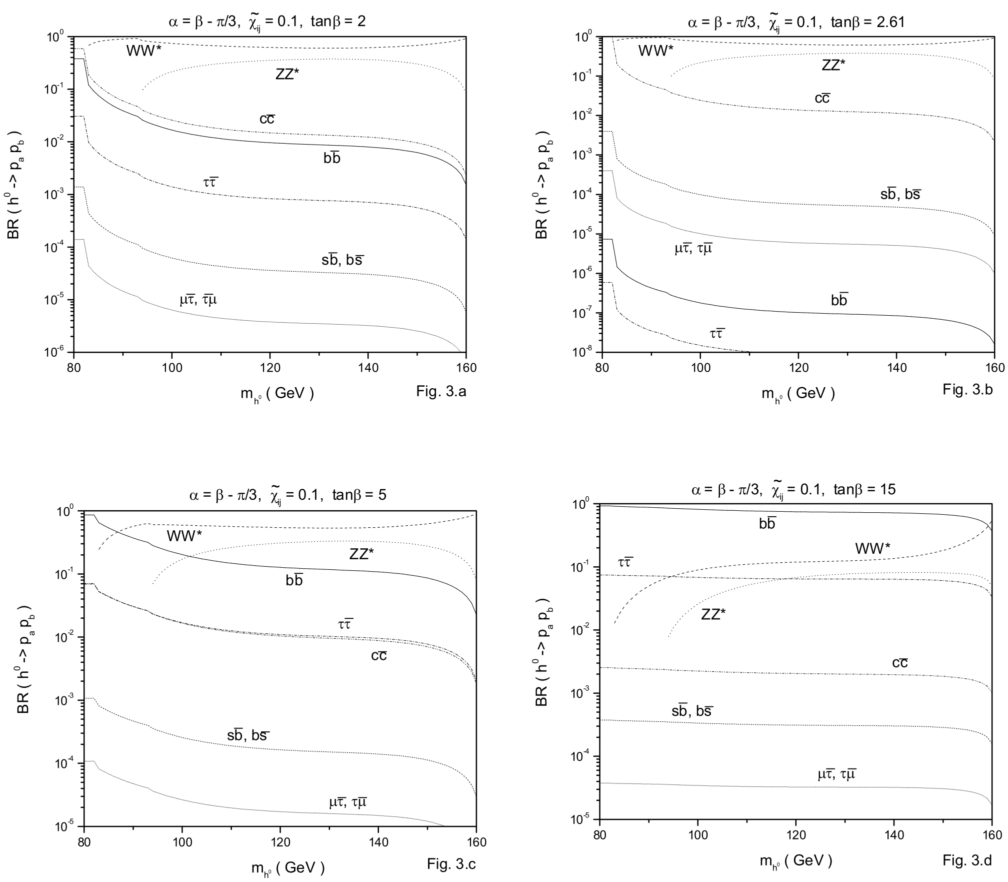

FIG. 3. Branching ratios for all the relevant decay modes that are allowed kinematically for 80 GeV $< m_{h^0} <$ 160 GeV, taking $\alpha = \beta - 3\pi/8$ and assuming $\tilde{\chi}_{ij} = 0.1$ for $i = j$ and $i \neq j$—for (a) $\tan\beta = 2$; (b) $\tan\beta = 2.61$; (c) $\tan\beta = 5$; (d) $\tan\beta = 15$.

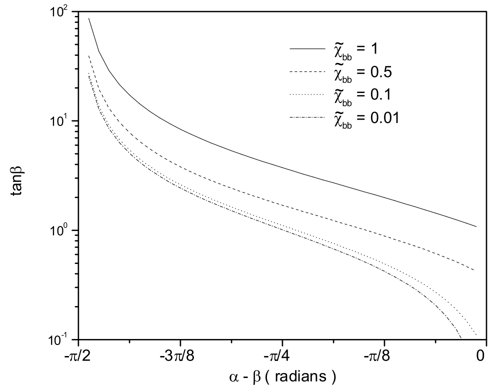

FIG. 4. Curves in the plane $(\alpha - \beta)$- $\tan\beta$ in which the coupling $b\bar{b}h^0$ vanishes, for $\tilde{\chi}_{bb} = 0.01$, 0.1, 0.5, and 1.

are displayed in Fig. 3, where we notice the important effect that the factor $\tilde{\chi}_{bb}$ has on the mode $h^0 \to b\bar{b}$, which could be dominant for a certain range of parameters, but it could be suppressed for other choices. Figure 4 clarifies what is going on; it shows the region in the plane $(\alpha - \beta)$- $\tan\beta$, where the coupling $h^0 b\bar{b}$ vanishes, and one can notice that this happens even for small values of the parameter $\tilde{\chi}_{bb}$ (≈ 0.01).

We also notice in Fig. 3 that the branching ratio for the FCNC mode $h^0 \to b\bar{s}(\bar{b}s)$ reaches values above 10^{-4} and the LFV mode $h^0 \to \tau\bar{\mu}(\bar{\tau}\mu)$ reaches values above 10^{-5} for $5 \lesssim \tan\beta \lesssim 50$ and $80\ \mathrm{GeV} \lesssim m_{h^0} \lesssim 155\ \mathrm{GeV}$. Further, in the mass range when $Br(h^0 \to b\bar{b})$ is not dominant, we find that the modes $h^0 \to WW^*, ZZ^*$ become the dominant ones.

Overall, our results show that the usual search strategies to look for the SM Higgs boson in this mass range may need to be modified in order to cover the full parameter space of the THDM-III.

In the following sections we will discuss how the Higgs boson signals could be searched at a future $\mu^+\mu^-$ collider. We will also study the reach in parameter space that could be obtained through the Higgs boson production in association with a pair of b quarks at LHC, which was found to be relevant in the large $\tan\beta$ limit for the MSSM [26].

V. PROBING THE FERMIONIC HIGGS BOSON COUPLINGS AT FUTURE COLLIDERS

In order to probe the Higgs vertices we will consider first the search for the LFV Higgs boson decays at future muon colliders, which was proposed some time ago [27]; namely,

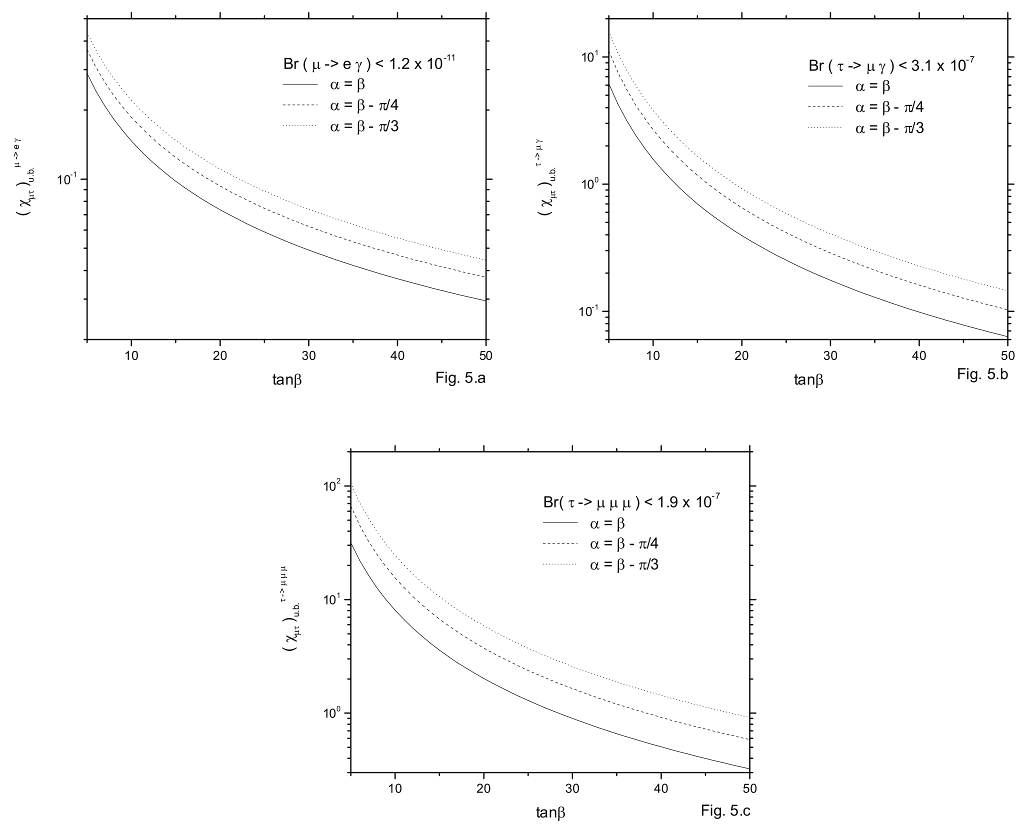

FIG. 5. The upper bound on $\chi_{\mu\mu}$ as a function of $\tan\beta$, for $\alpha = \beta$, $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, taking $m_{h^0} = 120$ GeV, $m_{H^0} = 300$ GeV, $m_{A^0} = 300$ GeV, and $\chi_{\mu\mu} = 0$, $\chi_{\tau\tau} = 0$. (a) From the decay $\mu \to e\gamma$, with $\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$; (b) From the decay $\tau \to \mu\gamma$, with $\mathrm{Br}(\tau \to \mu\gamma) < 3.1 \times 10^{-7}$; (c) From the decay $\tau^- \to \mu^-\mu^+\mu^-$, with $\mathrm{Br}(\tau^- \to \mu^-\mu^+\mu^-) < 1.9 \times 10^{-7}$.

we will evaluate the reaction $\mu^-\mu^+ \to h^0 \to f'f''$. Then we will consider the production of Higgs bosons at the LHC, to probe both LFV and $h^0 b\bar{b}$ couplings.

A. Tests of LFV/FCNC Higgs boson couplings at $\mu^-\mu^+$ colliders

An option to search for LFV $ff'\phi^0$ couplings could be provided by the reaction $\mu^-(p_a) + \mu^+(p_b) \to \phi^0 \to f'(p_c) + \bar{f}''(p_d)$. The s-channel Higgs boson cross section (on resonance) is given by

$$\sigma_{\phi^0}(\mu^-\mu^+ \to f'\bar{f}'') = 4\pi \frac{\Gamma(\phi^0 \to \mu^+\mu^-)\Gamma(\phi^0 \to f'\bar{f}'')}{(s - m_{\phi^0}^2)^2 + m_{\phi^0}^2(\Gamma_{\rm tot}^{\phi^0})^2} \quad (29)$$

where ϕ^0 denotes a neutral Higgs boson which decays to a final state $f'\bar{f}''$. The effective cross section $\bar{\sigma}_{\phi^0}$ is obtained by convoluting with the Gaussian distribution in $\sqrt{s}$ [28]:

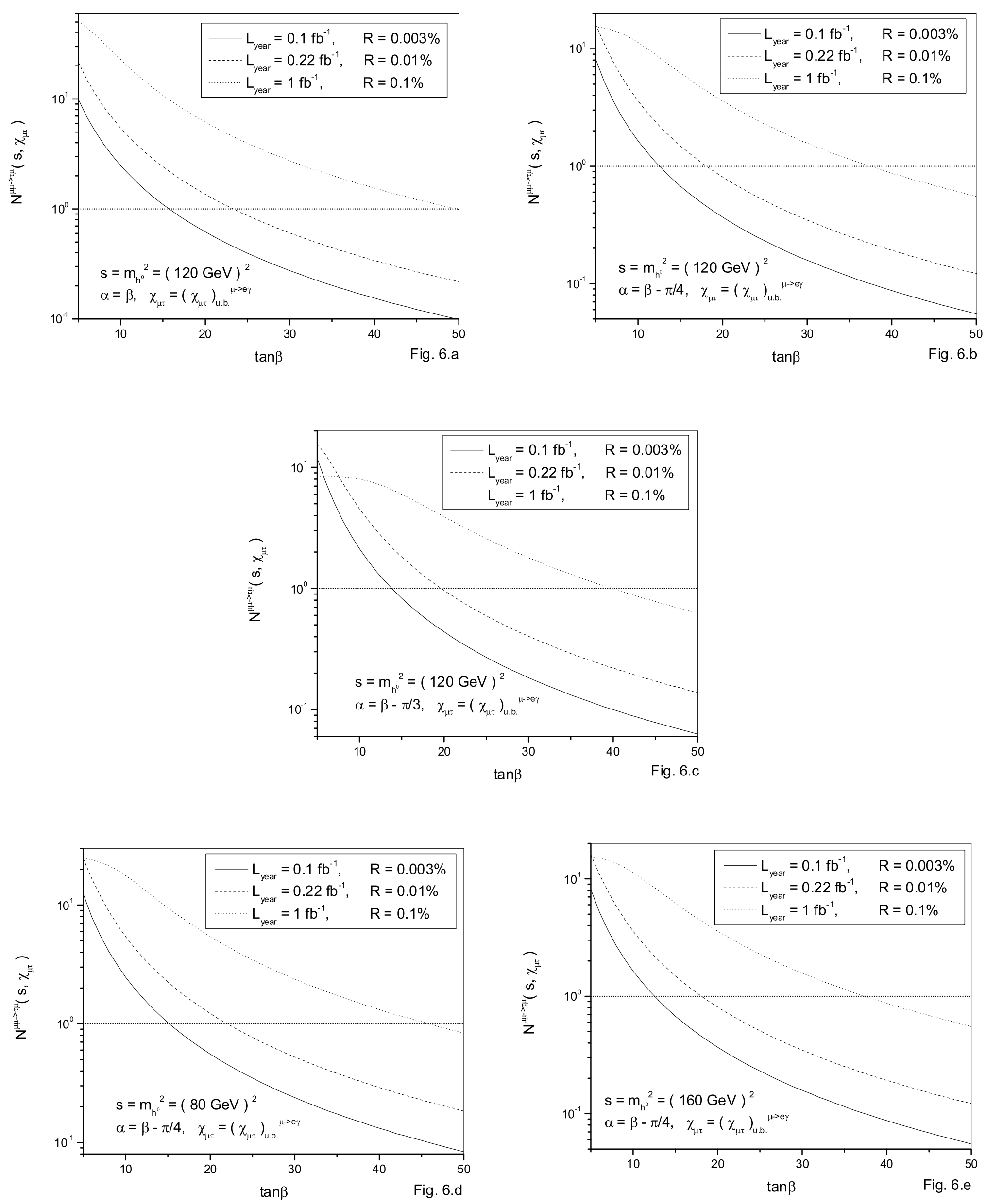

FIG. 6. Number of events $N^{\mu\mu\to\tau\mu}$ as a function of $\tan\beta$, taking $\chi_{\mu\tau} = (\chi_{\mu\tau})_{u.b.}^{\mu\to e\gamma}$ with ${\rm Br}(\mu^+ \to e^+\gamma) < 1.2 \times 10^{-11}$, for (a) $s = m_{h^0}^2 = (120\ {\rm GeV})^2$, $\alpha = \beta$; (b) $s = m_{h^0}^2 = (120\ {\rm GeV})^2$, $\alpha = \beta - \pi/4$; (c) $s = m_{h^0}^2 = (120\ {\rm GeV})^2$, $\alpha = \beta - \pi/3$; (d) $s = m_{h^0}^2 = (80\ {\rm GeV})^2$, $\alpha = \beta - \pi/4$; (e) $s = m_{h^0}^2 = (160\ {\rm GeV})^2$, $\alpha = \beta - \pi/4$. We consider yearly integrated luminosities $L_{\rm year} = 0.1, 0.22, 1\ {\rm fb}^{-1}$ and beam energy resolutions of $R = 0.003\%$, 0.01%, 0.1%, respectively.

$$\overline{\sigma}_{\phi^0}(\mu^-\mu^+ \to f'\overline{f}'') \simeq \frac{4\pi}{m_{\phi^0}^2}\frac{\mathrm{Br}(\phi^0 \to \mu^+\mu^-)\mathrm{Br}(\phi^0 \to f'\overline{f}'')}{[1+\frac{8}{\pi}(\frac{\sigma_{\sqrt{s}}}{\Gamma_{\mathrm{tot}}^{\phi^0}})^2]^{1/2}}. \tag{30}$$

$\sigma_{\sqrt{s}}$ can be expressed in terms of the root-mean-square (rms) Gaussian spread of the energy of an individual beam, R, as follows:

$$\sigma_{\sqrt{s}} = (2\ \mathrm{MeV})\left(\frac{R}{0.003\%}\right)\left(\frac{\sqrt{s}}{100\ \mathrm{GeV}}\right). \tag{31}$$

In this work, we will restrict our numerical analysis to the case of the light neutral scalar, i.e., $\phi^0 = h^0$, and for the most relevant cases $f'f'' = \tau^-\mu^+(\tau^+\mu^-)$, $b\overline{s}(\overline{b}s)$.

The calculation of $\overline{\sigma}_{h^0}$ requires the evaluation of the following quantities: $\Gamma(h^0 \to \tau^-\mu^+)$, $\Gamma(h^0 \to b\overline{s})$,

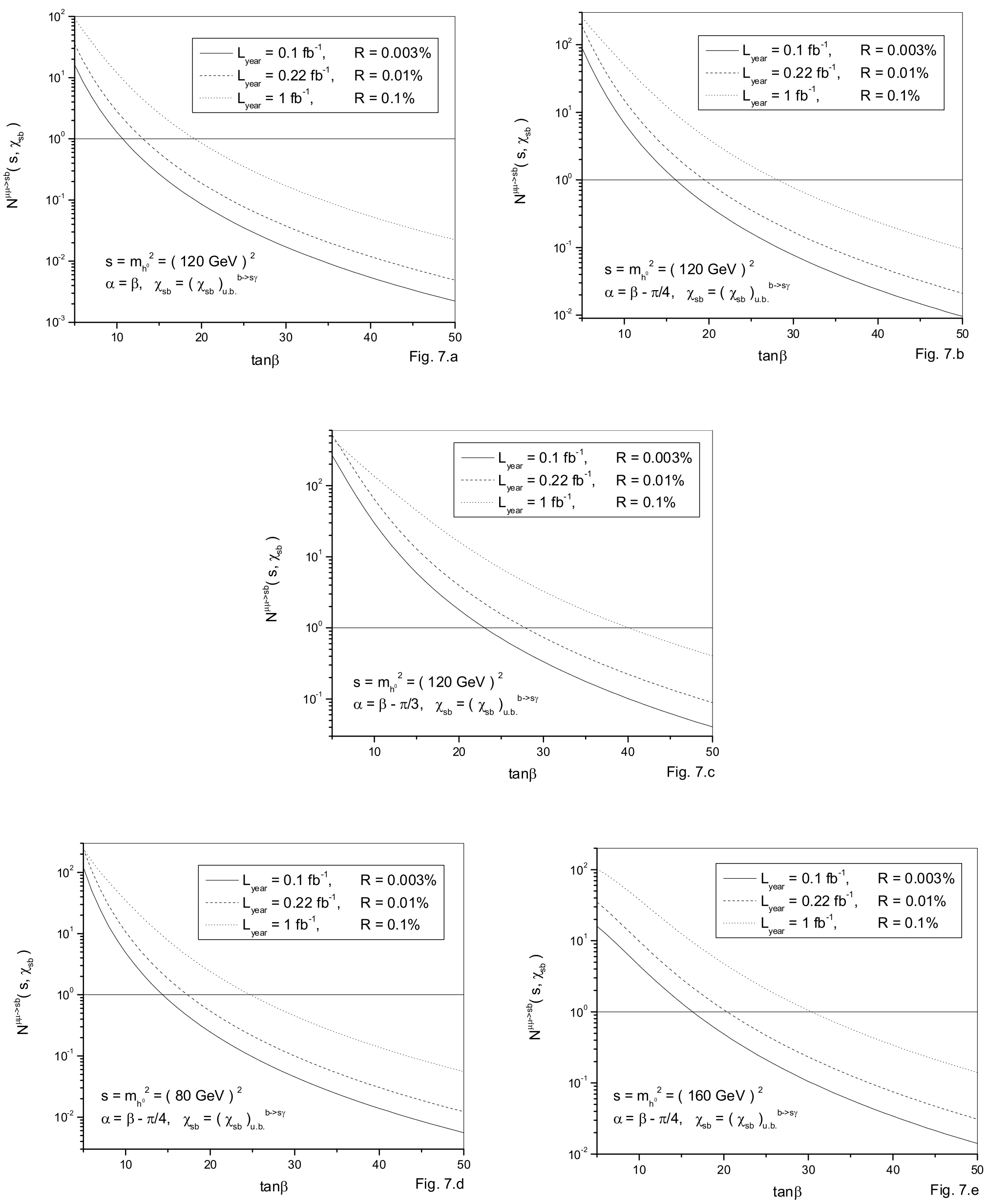

FIG. 7. Number of events $N^{\mu\mu\to bs}$ as a function of $\tan\beta$, taking $\chi_{sb} = (\chi_{sb})_{u.b.}^{b\to s\gamma}$ with $\Delta\mathrm{Br}(b\to s\gamma) < 10^{-5}$, for (a) $s = m_{h^0}^2 = (120\ \mathrm{GeV})^2$, $\alpha = \beta$; (b) $s = m_{h^0}^2 = (120\ \mathrm{GeV})^2$, $\alpha = \beta - \pi/4$; (c) $s = m_{h^0}^2 = (120\ \mathrm{GeV})^2$, $\alpha = \beta - \pi/3$; (d) $s = m_{h^0}^2 = (80\ \mathrm{GeV})^2$, $\alpha = \beta - \pi/4$; (e) $s = m_{h^0}^2 = (160\ \mathrm{GeV})^2$, $\alpha = \beta - \pi/4$. We consider yearly integrated luminosities $L_{\mathrm{year}} = 0.1, 0.22, 1\ \mathrm{fb}^{-1}$ and beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively.

$\Gamma(h^0 \to \mu^-\mu^+)$, and $\Gamma^{h^0}_{\rm tot}$, which are given in Eqs. (22)–(27).

By performing a detailed numerical analysis one can show that[2]

$$0.98 < \frac{\Gamma(h^0 \to \mu^-\mu^+)}{\Gamma(h^0 \to \mu^-\mu^+)|_{\tilde\chi_{\mu\mu}=0}} < 1.02 \qquad (32)$$

provided that 80 GeV $\leq m_{h^0} \leq$ 160 GeV, $-\pi/3 \leq \alpha - \beta \leq 0$, $5 \leq \tan\beta \leq 50$, $|\tilde\chi_{\mu\mu}| \lesssim 0.01$. Hence, under the previous conditions we have

$$\Gamma(h^0 \to \mu^-\mu^+) \simeq \Gamma(h^0 \to \mu^-\mu^+)|_{\tilde\chi_{\mu\mu}=0} = \frac{g^2 m_{h^0} m_\mu^2}{32\pi m_W^2}\frac{\sin^2\alpha}{\cos^2\beta}\left(1 - 4\frac{m_\mu^2}{m_{h^0}^2}\right)^{3/2}. \qquad (33)$$

Assuming 80 GeV $\leq m_{h^0} \leq$ 160 GeV, we can write

$$\Gamma^{h^0}_{\rm tot} = \sum_{f'}\Gamma(h^0 \to f'\bar f') + \sum_{f',f''}\Gamma(h^0 \to f'\bar f'') + \Gamma(h^0 \to WW^*) + \Gamma(h^0 \to ZZ^*), \qquad (34)$$

where $f', f'' \neq t$ quark.

It is also possible to show numerically that (see footnote 2)

$$0.98 < \frac{\Gamma^{h^0}_{\rm tot}}{\Gamma^{h^0}_{\rm tot}|_{\tilde\chi_{f'f''}=0}} < 1.06, \qquad (35)$$

provided that the following conditions are satisfied: $-\pi/3 \leq \alpha - \beta \leq 0$, $5 \leq \tan\beta \leq 50$, $|\tilde\chi_{ff}| \lesssim 0.01$, $|\tilde\chi_{f'f''}| \lesssim 1 (f' \neq f'')$. Hence, under the previous conditions we can approximate

$$\Gamma^{h^0}_{\rm tot} \simeq \Gamma^{h^0}_{\rm tot}|_{\tilde\chi_{f'f''}=0}. \qquad (36)$$

We can write the cross sections of the processes $\mu^-\mu^+ \to \tau^-\mu^+$ and $\mu^-\mu^+ \to b\bar s$ as follows:

$$\bar\sigma_{h^0}(\mu^-\mu^+ \to \tau^-\mu^+) \simeq \frac{4\pi}{m_{h^0}^2}\frac{{\rm Br}(h^0 \to \mu^+\mu^-){\rm Br}(h^0 \to \tau^-\mu^+)}{[1 + \frac{8}{\pi}(\frac{\sigma_{\sqrt s}}{\Gamma^{h^0}_{\rm tot}})^2]^{1/2}}, \qquad (37)$$

$$\bar\sigma_{h^0}(\mu^-\mu^+ \to b\bar s) \simeq \frac{4\pi}{m_{h^0}^2}\frac{{\rm Br}(h^0 \to \mu^+\mu^-){\rm Br}(h^0 \to b\bar s)}{[1 + \frac{8}{\pi}(\frac{\sigma_{\sqrt s}}{\Gamma^{h^0}_{\rm tot}})^2]^{1/2}}, \qquad (38)$$

where

$$\begin{aligned}{\rm Br}(h^0 \to \mu^-\mu^+) &\simeq \frac{\Gamma(h^0 \to \mu^-\mu^+)|_{\tilde\chi_{\mu\mu}=0}}{\Gamma^{h^0}_{\rm tot}|_{\tilde\chi_{f'f''}=0}},\\ {\rm Br}(h^0 \to \tau^-\mu^+) &\simeq \frac{\Gamma(h^0 \to \tau^-\mu^+)}{\Gamma^{h^0}_{\rm tot}|_{\tilde\chi_{f'f''}=0}},\\ {\rm Br}(h^0 \to b\bar s) &\simeq \frac{\Gamma(h^0 \to b\bar s)}{\Gamma^{h^0}_{\rm tot}|_{\tilde\chi_{f'f''}=0}}\end{aligned} \qquad (39)$$

provided that $|\tilde\chi_{f'f''}| \lesssim 10^{-2}$ for $f' = f''$, and $|\tilde\chi_{f'f''}| \lesssim 1$ for $f' \neq f''$.

In order to estimate the number of events $\tau^-\mu^+$ ($\tau^+\mu^-$) produced in a muon collider, we will take into account the most restrictive bound on $\chi_{\mu\tau}$ obtained from low-energy data. In Fig. 5, we plot the upper bound on $\chi_{\mu\tau}$ as a function of $\tan\beta$, for $\alpha = \beta$, $\alpha = \beta - \pi/4$, $\alpha = \beta - \pi/3$, taking $m_{h^0} = 120$ GeV, $m_{H^0} = 300$ GeV, and $m_{A^0} = 300$ GeV, assuming $\chi_{\mu\mu} = 0$ and $\chi_{\tau\tau} = 0$. (a) From the experimental bound for the decay $\mu \to e\gamma$, ${\rm Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [22]; (b) From the experimental bound for the decay $\tau \to \mu\gamma$, ${\rm Br}(\tau \to \mu\gamma) < 3.1 \times 10^{-7}$ [29]; (c) From the experimental bound for the decay $\tau^- \to \mu^-\mu^+\mu^-$, ${\rm Br}(\tau^- \to \mu^-\mu^+\mu^-) < 1.9 \times 10^{-7}$ [30]. We observe that the most restrictive bound on $\chi_{\mu\tau}$ is the one obtained from the experimental bound for the decay $\mu \to e\gamma$.[3]

[2]We will discuss the dependency on the parameters χ^f_{ij} and the phases ϑ^f_{ij} ($\tilde\chi^f_{ij} = \chi^f_{ij}e^{\vartheta^f_{ij}}$) of the decay widths $\Gamma(\phi^0 \to f_i\bar f_j)$ for $\phi^0 = h^0, H^0$, and A^0 in a forthcoming paper.

[3]The branching ratio of $\tau \to \mu\gamma$ at one-loop level is given by [21]

$$\begin{aligned}{\rm Br}(\tau \to \mu\gamma) = \frac{3}{5}\frac{\alpha_{em} m_\mu m_\tau^3}{16\pi\cos^4\beta}\chi^2_{\mu\tau}\Bigg\{&\frac{\sin^2\alpha\cos^2(\alpha-\beta)}{m_{h^0}^4} \times \left|\ln\frac{m_\tau^2}{m_{h^0}^2} + \frac{3}{2}\right|^2\\ &+ \frac{\cos^2\alpha\cos^2(\alpha-\beta) + \sin^2\alpha\sin^2(\alpha-\beta)}{m_{h^0}^2 m_{H^0}^2} \times \left|\ln\frac{m_\tau^2}{m_{h^0}^2} + \frac{3}{2}\right|\left|\ln\frac{m_\tau^2}{m_{H^0}^2} + \frac{3}{2}\right|\\ &+ \frac{\cos^2\alpha\sin^2(\alpha-\beta)}{m_{H^0}^4}\left|\ln\frac{m_\tau^2}{m_{H^0}^2} + \frac{3}{2}\right|^2 + \frac{\sin^2\beta}{m_{A^0}^4}\left|\ln\frac{m_\tau^2}{m_{A^0}^2} + \frac{3}{2}\right|^2\Bigg\}\end{aligned}$$

where we have assumed $|\tilde\chi_{\tau\tau}| \ll 1$. The expressions for ${\rm Br}(\mu \to e\gamma)$ and ${\rm Br}(\tau^- \to \mu^-\mu^+\mu^-)$ are given in Ref. [20].

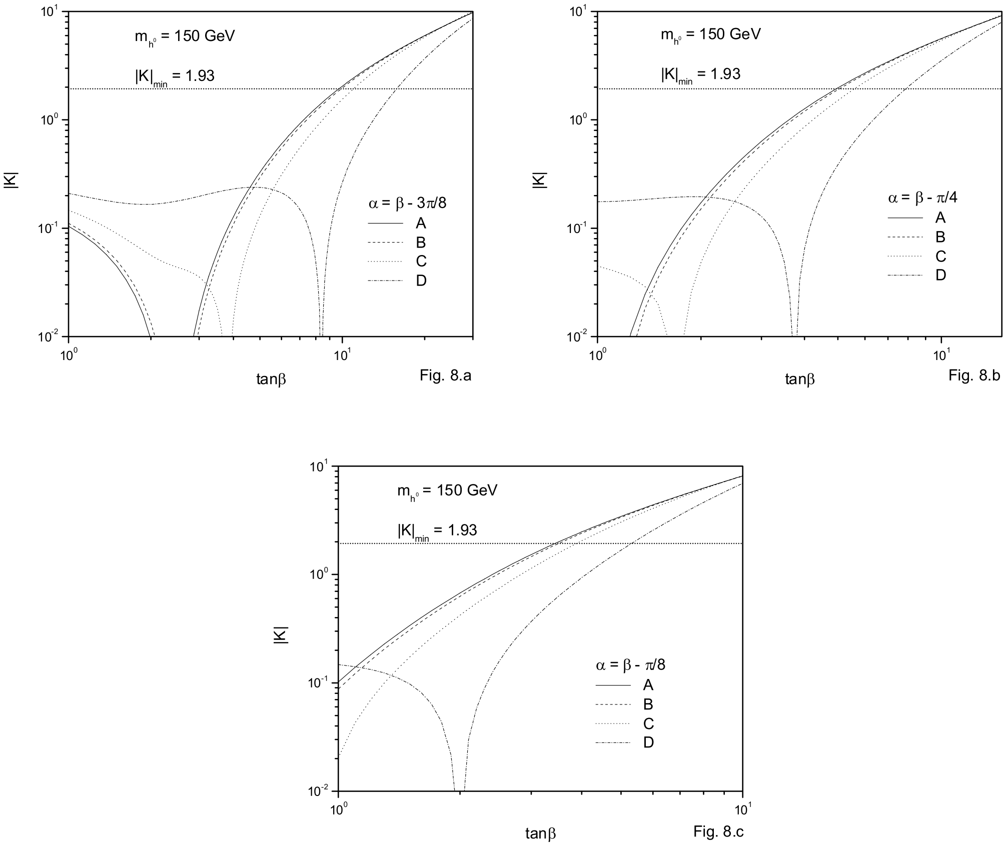

FIG. 8. $|K|$ as a function of $\tan\beta$, taking $m_{h^0} = 150$ GeV, assuming $\tilde{\chi}_{ij} = 0.1$ for $i \neq j$ and (i) $\tilde{\chi}_{ii} = 0.01$ (line A); (ii) $\tilde{\chi}_{ii} = 0.1$ (line B); (iii) $\tilde{\chi}_{ii} = 0.5$ (line C); (iv) $\tilde{\chi}_{ii} = 1$ (line D)—for (a) $\alpha = \beta - 3\pi/8$; (b) $\alpha = \beta - \pi/4$; (c) $\alpha = \beta - \pi/8$.

We will calculate the number of events $\tau^-\mu^+$ $(\tau^+\mu^-)$ produced in a $\mu^-\mu^+$ collider,

$$N^{\mu\mu\to\tau\mu} = \overline{\sigma}_{h^0}(\mu^-\mu^+ \to \tau^-\mu^+) \times L_{\text{year}}. \tag{40}$$

Then our numerical results for $N^{\mu\mu\to\tau\mu}(s = m_{h^0}^2, \chi_{\mu\tau})$ are shown in Fig. 6, as a function of $\tan\beta$. We have taken $\chi_{\mu\tau} = (\chi_{\mu\tau})_{u.b.}^{\mu\to e\gamma}$, i.e., the value of the upper bound on $\chi_{\mu\tau}$ obtained from the experimental measurement of the radiative decay $\mu^+ \to e^+\gamma$ [20]. We have used the current experimental result $\text{Br}(\mu^+ \to e^+\gamma) < 1.2 \times 10^{-11}$ [22]. We plot curves for $m_{h^0} = 80$ GeV, 120 GeV, 160 GeV, taking $\alpha = \beta$, $\beta - \pi/4$, $\beta - \pi/3$, assuming yearly integrated luminosities $L_{\text{year}} = 0.1, 0.22, 1$ fb^{-1} for beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively [27].

The production rates of $\tau^-\mu^+$ $(\tau^+\mu^-)$ pairs with a $\mu^-\mu^+$ collider are drastically reduced (we calculate the number of such events using the constraint on $\chi_{\mu\tau}$ obtained from the experimental bound on the branching ratio of the LFV process $\mu^+ \to e^+\gamma$), especially for large values of $\tan\beta$ ($\gtrsim 15$), as can be observed in Fig. 6. We can conclude that for large $\tan\beta$ the expected bounds on $\chi_{\mu\tau}$, at future muon colliders, will not improve with respect to the present bounds from low-energy data.

On the other hand, the nonobservation of at least an event $\tau^-\mu^+$ (or $\tau^+\mu^-$) in a year would imply that

$$N^{\mu\mu\to\tau\mu}(s = m_{h^0}^2, \chi_{\mu\tau}) < 1, \tag{41}$$

which would also allow us to put an upper bound on $\chi_{\mu\tau}$, namely,

$$(\chi_{\mu\tau})_{u.b.}^{\mu\mu\to\tau\mu}(s = m_{h^0}^2) = [N^{\mu\mu\to\tau\mu}(s = m_{h^0}^2, \chi_{\mu\tau} = 1)]^{-1/2}. \tag{42}$$

According to Fig. 6, the $\mu^-\mu^+$ collider measurements could improve the bound on $\chi_{\mu\tau}$ obtained from the radiative decay $\mu^+ \to e^+\gamma$, $(\chi_{\mu\tau})_{u.b.}^{\mu\to e\gamma}$, only if $\tan\beta \lesssim 15$.

Then, for the quark signals, we will calculate the number of events $b\overline{s}(\overline{b}s)$ produced in a $\mu^-\mu^+$ collider, given by

$$N^{\mu\mu\to bs} = \overline{\sigma}_{h^0}(\mu^-\mu^+ \to b\overline{s}) \times L_{\text{year}}. \tag{43}$$

We depict our numerical results for $N^{\mu\mu\to bs}(s = m_{h^0}^2, \chi_{sb})$ in Fig. 7, as a function of $\tan\beta$, by taking $\chi_{sb} = (\chi_{sb})_{u.b.}^{b\to s\gamma}$, the value for the upper bound on χ_{sb} obtained in Sec. III A from the good agreement between the experimental and theoretical value of the radiative decay $b \to s\gamma$. We plot curves for $m_{h^0} = 80$ GeV, 120 GeV, 160 GeV, taking $\alpha = \beta$, $\beta - \pi/4$, $\beta - \pi/3$, assuming yearly integrated luminosities $L_{\rm year} = 0.1, 0.22, 1\ {\rm fb}^{-1}$ for beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively [27].

The production rates of $b\bar{s}(\bar{b}s)$ pairs at a $\mu^-\mu^+$ collider (calculated by using the constraint on χ_{sb} imposed by the branching ratio of the process $b \to s\gamma$) are drastically reduced for $\tan\beta \gtrsim 15$, as can be observed in Fig. 7. Again, we can conclude that for large $\tan\beta$ the expected bounds on χ_{sb}, at future muon colliders, will not improve with respect to the present bounds from low-energy data.

Similarly, the nonobservation of at least an event of the type $b\bar{s}$ (or $\bar{b}s$) in a year, could be used to improve the bound on χ_{sb} obtained from the radiative decay $b \to s\gamma$, $(\chi_{sb})^{b\to s\gamma}_{u.b.}$, only if $\tan\beta \lesssim 15$.

B. Search for Higgs boson in associated production with *b*-quark pairs at LHC

The associated production of the Higgs boson in association with a quark pair $b\bar{b}$ has been found useful to detect the neutral Higgs bosons of the MSSM [5], especially in the large-$\tan\beta$ domain. Here we will show that this reaction can also be useful to constrain the coupling $h^0 b\bar{b}$ in the THDM-III.

As shown in Ref. [26], the reaction $pp \to h^0(\to b\bar{b}) + b\bar{b} + X$ produces a large sample of events which could be detectable provided a K factor is above a certain value, which depends on the Higgs boson mass and the coupling $h^0 b\bar{b}$ (which enter in the event rate both from the Higgs boson production and decay); this factor is defined as

$$K = \frac{(g_{h^0 b\bar{b}})_{\rm THDM-III}}{(g_{\phi^0 b\bar{b}})_{\rm SM}} \sqrt{Br(h^0 \to b\bar{b})}. \qquad (44)$$

To have a detectable signal at LHC for $m_{h^0} = 150$ GeV, the modulus of this factor has to be above $|K|_{\rm min} = 1.93$, as obtained from a detailed analysis of signal and backgrounds performed in Ref. [26], to which we refer for details of kinematical cuts, acceptances, and parton distributions.

In Fig. 8, we show the region of the plane $\tan\beta$-$(\alpha - \beta)$, where the signal for $m_{h^0} = 150$ GeV is detectable. One can notice that the effect of the parameter $\tilde{\chi}_{bb}$, even for small values, can have a dramatic impact on the extension of the region of parameters where the signal is detectable. Therefore, LHC will be able to constrain the presence of a nonminimal flavor structure (which is reflected on the parameters $\tilde{\chi}_{ij}$), and provide a decisive test of the fermionic coupling of the Higgs boson.

VI. CONCLUSIONS

We have studied in this paper the $ff'\phi^0$ couplings that arise in the THDM-III, using a Hermitic four-texture form for the fermionic Yukawa matrix. Because of this, although the $ff'\phi^0$ couplings are complex, the CP properties of h^0, H^0 (even) and A^0 (odd) remain valid.

We have derived bounds on the parameters of the model, using current experimental bounds on LFV and FCNC transitions. One can say that the present bounds on the couplings χ_{ij} still allow the possibility to study interesting direct flavor-violating Higgs boson signals at future colliders. However, we should note that for large values of $\tan\beta$ such colliders will not improve the bounds derived from low-energy data. Although the event rates for the signals grow with $\tan\beta$, one should consider that the constraints obtained from low-energy data become more restrictive too.

In particular, the LFV couplings of the neutral Higgs bosons can lead to new discovery signatures of the Higgs boson itself. For instance, the branching fraction for $h^0 \to \tau\bar{\mu}(\bar{\tau}\mu)$ can be as large as 10^{-5}, while ${\rm Br}[h \to b\bar{s}(\bar{b}s)]$ is also about 10^{-4}. These LFV Higgs modes complement the modes $B^0 \to \mu\mu$, $\tau \to 3\mu$, $\tau \to \mu\gamma$, and $\mu \to e\gamma$, as probes of flavor violation in the THDM-III, which could provide key insights into the form of the Yukawa mass matrix.

Thus, the coming generation of colliders will provide a decisive test of the Yukawa sector of the SM and its extensions, as well as other properties of the gauge-Higgs sector [31].

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