New fat Higgs: Increasing the MSSM Higgs mass with natural gauge unification

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In this paper we increase the minimal supersymmetric standard model tree level Higgs mass bound to a value that is naturally larger than the LEP-II search constraint by adding to the superpotential a $\lambda S H_u H_d$ term, as in the next to minimal supersymmetric standard model, and UV completing with new strong dynamics *before* λ becomes nonperturbative. Unlike other models of this type, the Higgs fields remain elementary, alleviating the supersymmetric fine-tuning problem while maintaining unification in a natural way.

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I. INTRODUCTION

Finding a satisfactory explanation for the large difference between the weak scale and the Planck scale, known as the hierarchy problem, is an issue that has concerned particle physicists for more than two decades, and is the reason why the standard model Higgs sector is widely held to be incomplete. Supersymmetry (SUSY) provides arguably the most attractive solution for this hierarchy, since it comes with gauge coupling unification as an automatic consequence. However, its simplest implementation, the minimal supersymmetric standard model (MSSM), is looking increasingly fine-tuned as recent results from LEP-II have pushed it to regions of parameter space where a light Higgs seems unnatural.[1] This is problematic for the MSSM since SUSY relates the quartic coupling of the Higgs to the electroweak gauge couplings, which at tree level bounds the mass of the lightest Higgs to be less than that of the Z. Radiative corrections can help increase this bound, with the largest contribution coming from the top yukawa, giving

$$m_{h^0}^2 \approx m_Z^2 + \frac{3}{8\pi^2} h_t^4 v^2 \log\frac{m_{\tilde{t}}^2}{m_t^2} \tag{1}$$

for large $\tan\beta$ [3]. Since this effect is only logarithmic in the stop mass, however, consistency with the LEP-II mass bound requires the stops to be pushed up to at least 500 GeV. At the same time radiative corrections to $m_{H_u}^2$ are quadratic in the stop mass

$$\delta m_{H_u}^2 \approx -\frac{3}{4\pi^2} m_{\tilde{t}}^2 \log\frac{\Lambda}{m_{\tilde{t}}}. \tag{2}$$

There is therefore a conflict between our expectation that the stop is heavy enough to significantly increase the Higgs mass through radiative corrections and yet light enough to cut off the quadratic divergence in a natural way.[2] Requiring consistency with LEP-II results therefore forces us to live with a fine-tuning of a few percent.

One way to resolve this issue is to generate a larger tree level quartic coupling for the Higgses. This can be accomplished through new F-terms as in the next to minimal supersymmetric standard model (NMSSM) [5,6]; new D-terms by charging the Higgs under a new gauge symmetry [7]; or by using "hard" SUSY breaking at low scales [8]. We will choose to focus on the NMSSM, where the addition of a gauge singlet S allows for the following term in the superpotential,

$$W = \lambda S H_u H_d, \tag{3}$$

and results in an additional quartic coupling for the Higgses of the form $|\lambda|^2 |H_u H_d|^2$. Unfortunately, the requirement of perturbativity up to the GUT scale limits the size of λ at the electroweak scale [9] giving a maximum Higgs mass bound of about 150 GeV. This constraint was recently evaded in the fat Higgs model [10] by allowing the coupling to become nonperturbative at an energy lower than the GUT scale, where S, H_u, and H_d were seen to be composites of new strong dynamics. All couplings were asymptotically free above this point and the Higgs mass bound could be pushed up to 500 GeV. On the other hand, the composite nature of the Higgs doublets gave rise to a different problem—gauge coupling unification was not manifest and some *ad hoc* matter content had to be added to the theory to preserve it. In addition, elementary Higgs fields needed to be reintroduced in order to generate the usual standard model yukawas at low energies.

In this paper, we will argue that UV completion of the NMSSM does not require us to sacrifice the desirable properties of weak-scale SUSY. We will keep the Higgs fields elementary, making unification manifest while permitting the usual standard model yukawas to be written down. Like the fat Higgs, we use a composite S but instead we replace the λ coupling above the compositeness scale by asymptotically free yukawas. Since we will no longer have to run λ, which grows in the UV, all the way to the GUT scale, we can afford to start at a larger value at the electroweak scale. Unfortunately, our scheme will require

[1] See Refs. [1,2] for further discussion.

[2] A recent paper [4] attempted to resolve this conflict by suppressing the size of radiative corrections to $m_{H_u}^2$ from the stop.

us to compromise slightly on how heavy we can make the Higgs, but this seems a small price to pay for natural gauge coupling unification.

The outline of the rest of the paper is as follows: In Sec. II, we discuss the philosophy of this mechanism and detail a specific model; in Sec. III we discuss the bounds on the λ coupling, and the issues of fine-tuning, gauge coupling unification, and the model's phenomenology. We conclude in Sec. IV.

II. CONSTRUCTING A MODEL

In SUSY models gauge contributions to anomalous dimensions are negative, tending to make yukawa couplings asymptotically free. The yukawas themselves, on the other hand, contribute positive anomalous dimensions. These competing effects, which are evident in the renormalization group equation (RGE) for the NMSSM λ coupling,

$$\frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2}\left[4\lambda^2 + 3h_t^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] + \cdots, \qquad (4)$$

result in an asymptotically free λ only when the gauge couplings involved are larger than λ itself. Even when they do not dominate the running, maximizing the negative contributions from the gauge sector by adding as many SU(5) $\mathbf{5} + \bar{\mathbf{5}}$ multiplets as are allowed by perturbative unification gives an upper bound on the low energy λ coupling [9]. The benefit is small here, however, since the electroweak gauge couplings remain quite weak for the majority of the running and g_3 only affects h_t at one loop. This makes it difficult to significantly increase the low energy value of λ.

One way to improve the situation is to introduce new gauge dynamics through the following superpotential:

$$W_\lambda = \lambda_1 \phi X H_u + \lambda_2 \phi^c X^c H_d + M_X X X^c + M_{\tilde{X}} \tilde{X}\tilde{X}^c. \qquad (5)$$

We have added the fields $\phi, \phi^c, X, X^c, \tilde{X}, \tilde{X}^c$, which are charged under a new strong gauge symmetry, with the X's also charged under the standard model as seen in Table I. We choose SU(n) to be our strong group as this permits our scheme to be most easily implemented. Since the strong gauge coupling (g_s) can now dominate the running, the $\lambda_{1,2}$ yukawas can be asymptotically free for larger initial values and the resulting gain in λ will be more substantial. The two X fields have been given a supersymmetric mass, M_X, and are completed into $(\mathbf{5}, \mathbf{n}) + (\bar{\mathbf{5}}, \bar{\mathbf{n}})$ multiplets of $SU(5) \times SU(n)_s$ by the $\tilde{X}$'s and thus maintain gauge coupling unification. Note that this does not require any MSSM particles to be gauged under $SU(n)_s$. The fields that are gauged under both the standard model and the new group have large supersymmetric mass terms and thus decouple from low energy physics.

TABLE I. Preliminary charge assignments for the new particles.

	$SU(3) \times SU(2)_L \times U(1)_Y$	$SU(n)_s$
ϕ	(1, 1, 0)	$\mathbf{n}$
ϕ^c	(1, 1, 0)	$\bar{\mathbf{n}}$
X	$(1, 2, -\frac{1}{2})$	$\bar{\mathbf{n}}$
X^c	$(1, 2, \frac{1}{2})$	$\mathbf{n}$
$\tilde{X}$	$(\bar{\mathbf{3}}, 1, \frac{1}{3})$	$\bar{\mathbf{n}}$
$\tilde{X}^c$	$(\mathbf{3}, 1, -\frac{1}{3})$	$\mathbf{n}$

Below the scale M_X and $M_{\tilde{X}}$, integrating out the X's and $\tilde{X}$'s generates the nonrenormalizable operator:

$$W_{\rm eff} = -\frac{\lambda_1\lambda_2}{M_X}\phi\phi^c H_u H_d. \qquad (6)$$

There are two ways in which the NMSSM λ coupling can be obtained from this operator. One is to break $SU(n)_s$ by giving a vacuum expectation value (vev) to ϕ; as long as this breaking takes place close to the M_X scale, λ can be satisfactorily large. A simpler approach, which we adopt in this paper, is to use the fact that below $M_X, M_{\tilde{X}}$ there are five fewer flavors of the strong group, making the gauge coupling get strong at low energies, forcing the ϕ fields to confine into an NMSSM singlet which we will call S.

Building a realistic theory from this philosophy is simply a matter of deciding what n will be. We use the fact that there is a restriction on the number of SU(5) flavors that can be added to the standard model for gauge couplings to perturbatively unify given that the added SU(5) fundamentals do not decouple until the TeV scale.[3] This requires four flavors or less and, hence, $n \leq 4$. Another important constraint is on the number of flavors of $SU(n)_s$ that remain after the five flavors in X and $\tilde{X}$ have been integrated out. We want to avoid $n_f < n$ where there is an Affleck-Dine-Seiberg vacuum instability [12] and will ignore the potentially interesting case $n_f = n$, where the quantum modified moduli space constraint might shed some light on the μ problem. Instead we will choose to start with $n + 6$ flavors of $SU(n)_s$, where integrating out the five flavors gives $n_f = n + 1$, making the theory s-confine. Now combining the requirement for asymptotic freedom ($n + 6 < 3n$) with the perturbative unitarity constraint ($n \leq 4$) discussed earlier uniquely fixes $n = 4$.[4]

[3]The possibility of a model with accelerated unification [11] and a lowered unification scale will not be considered here.

[4]The case of SU(3) with nine flavors might also be useful for our purpose. This model has been argued to have a linear family of conformal fixed points in (λ_i, g) space [13] and would therefore be convenient when we discuss the possibility of having a new superconformal fixed point in Sec. II B. Alternatively, if the $\tilde{X}$'s required for unification were not also charged under the strong group, satisfying the resulting constraints would be easier since we would have more room to maneuver. However, this theory is not naturally unified, and so will not be pursued here.

A. Details of the model

We now summarize the content and interactions of the model. There is a strong $SU(4)_s$ gauge group, with the particle content shown in Table II. The superpotential contains

$$W = W_\lambda + W_S + W_d, \quad (7)$$

where

$$W_S = m\phi\phi^c \quad (8)$$

$$W_d = y(T^i\phi\psi_i^c + T^{ci}\psi_i\phi^c + T^{ij}\psi_i\psi_j^c) + \frac{y'}{M_{GUT}^2} \times (\epsilon^{ijkl}T_i^B\phi\psi_j\psi_k\psi_l + \epsilon^{ijkl}T_i^{B^c}\phi^c\psi_j^c\psi_k^c\psi_l^c), \quad (9)$$

where we have introduced some singlets denoted by T. After confinement, W_S gives a linear term in S as in the fat Higgs [10] while W_d decouples the extra mesons by giving them mass terms with the singlets T^i, T^{ci}, T^{ij}. Note that in the second line of W_d there is a nonrenormalizable mass term for the baryons with the T^B's which is suppressed by the GUT scale M_{GUT} and thus gives rise to light baryon states. The constraints imposed by these light states will be discussed in Sec. III C. Note that there is a nonanomalous $U(1)_R$ symmetry [under which ψ_i, ψ_i^c are neutral and all other $SU(4)_s$ flavors have charge 1] that makes the given superpotential natural.

B. Conformality and confinement

At high energies the strong group has ten flavors and is within the conformal window ($\frac{3}{2}n < 10 < 3n$) implying, in the absence of $\lambda_{1,2}$, that the theory flows to an interacting fixed point in the IR [12]. As discussed previously, the strong gauge coupling gives large negative contributions to the beta functions of the $\lambda_{1,2}$ couplings making them asymptotically free for $g_s \gg \lambda_{1,2}$. Ignoring electroweak couplings and the top yukawa, near Seiberg's fixed point we have the RGE:

$$\frac{d\lambda_{1,2}}{dt} = \frac{7\lambda_{1,2}^3}{16\pi^2} + \gamma_*\lambda_{1,2} + \cdots. \quad (10)$$

The first term is the usual one loop term due to the yukawa couplings while the second term contains contributions from all orders in the fixed point gauge coupling g_*. If the theory is at the fixed point, then we have very precise information on the value of γ_* in the weak limit, since this is related to the $U(1)_R$ charges of the fields by the superconformal algebra. Within the conformal window, for example, $-1 < \gamma_* < 0$ which indicates that the $\lambda_{1,2}$ couplings are relevant; they grow in the IR. Our limited understanding of strongly coupled theories prevents us from proceeding in full generality, so from now on we will restrict ourselves to two plausible types of behavior.

The first possibility is the emergence of a new superconformal phase where both the new yukawas and gauge couplings hit fixed points in the IR; in this case it is hard to be quantitative about possible values of the NMSSM λ coupling. At best, we can specify a range of fixed point values of $\lambda_{1,2}$ which give interesting λ couplings, without being able to justify if those values can be obtained. Still, the insensitivity of this scenario to UV initial conditions is very attractive.

In the second possibility, the yukawa couplings get strong and disrupt the conformality, pushing the theory away from the fixed point. In this case a reasonable bound on the sizes of $\lambda_{1,2}$ can be given using their apparent fixed point values from Eq. (10). We will refer to this as the weak limit bound. It is nontrivial that this bound on λ will be large enough to be of interest to us. In fact, the naive estimate will be in the right range but, as we will see in Sec. III A, there are many unknown order one factors that can change its size. An undesirable aspect of this case is that the UV boundary conditions for $\lambda_{1,2}$ have to be tuned to small values in order for these couplings to be just below their one loop fixed points at low energies which saturates the weak limit bound. This tuning could be improved somewhat if the gauge coupling hits its fixed point at some intermediate scale. It is also worth noting that this weak limit bound could give us a rough estimate of the fixed point values of $\lambda_{1,2}$ in the first scenario.

At energies around the mass of the X's and their colored partners $\tilde{X}$, these five flavors are integrated out of the theory. The terms in the RGEs for the supersymmetric masses typically give an ordering $m < M_X < M_{\tilde{X}}$. Thus, the colored partners are integrated out first which leaves seven flavors of $SU(4)_s$; this is still within the conformal window and, in the electric description, has a stronger fixed point than the UV theory. This would take $|\gamma_*|$ from $1/5$ to $5/7$ and also increase the weak limit bound on $\lambda_{1,2}$ at the scale M_X. For the coupling to approach this fixed point, the ratio $M_X/M_{\tilde{X}}$ must be small. As discussed in Sec. III B, there are no constraints on the size of this parameter from unification as long as $M_X = M_{\tilde{X}}$ at the GUT scale.

Below M_X, X and X_c are integrated out and the theory becomes a $SU(4)_s$ gauge theory with five flavors $\Psi_I = (\phi, \psi_i)$, $\Psi_I^c = (\phi^c, \psi_i^c)$ for $I = 0, \ldots, 4$. There is a dynamically generated superpotential,

TABLE II. Final charge assignments for new particles.

	$SU(3) \times SU(2)_L \times U(1)_Y$	$SU(4)_s$
ϕ	$(1, 1, 0)$	$\mathbf{4}$
ϕ^c	$(1, 1, 0)$	$\bar{\mathbf{4}}$
ψ_i for $i = 1, \ldots, 4$	$(1, 1, 0)$	$\mathbf{4}$
ψ_i^c for $i = 1, \ldots, 4$	$(1, 1, 0)$	$\bar{\mathbf{4}}$
X	$(1, 2, -\frac{1}{2})$	$\bar{\mathbf{4}}$
X^c	$(1, 2, \frac{1}{2})$	$\mathbf{4}$
$\tilde{X}$	$(\bar{\mathbf{3}}, 1, \frac{1}{3})$	$\bar{\mathbf{4}}$
$\tilde{X}^c$	$(\mathbf{3}, 1, -\frac{1}{3})$	$\mathbf{4}$

$$W_{\rm dyn} = \frac{1}{\Lambda^7}[M_{IJ}B^I B^{cJ} - \det M], \tag{11}$$

written in terms of gauge invariant mesons ($M_{IJ} \sim \Psi_I \Psi_J^c$) and baryons ($B^I \sim \epsilon^{IJKLM}\Psi_J \cdots \Psi_M$). At the scale $\Lambda \lesssim M_X$, this theory confines and the superpotential should be written in terms of the canonically normalized meson and baryon fields. Since the gauge coupling is strong, the sizes of the interactions after matching are in principle unknown. However, estimating their sizes by naive dimensional analysis (NDA) [14] gives

$$W = W_{\rm eff} + W_S + W_{\rm d} + W_{\rm dyn}, \tag{12}$$

where

$$W_{\rm eff} \to \left[\sqrt{n}\frac{\lambda_1\lambda_2}{4\pi}\frac{\Lambda}{M_X}\right] S H_u H_d, \tag{13}$$

$$W_S \to \frac{m\Lambda}{4\pi} S, \tag{14}$$

$$W_{\rm d} \to \frac{y\Lambda}{4\pi}(T^i M_{0i} + T^{ci}M_{i0} + T^{ij}M_{ij}) + \frac{y'\Lambda^3}{4\pi M_{\rm GUT}^2}(T_i^B B^i + T_i^{B^c} B^{ci}), \tag{15}$$

$$W_{\rm dyn} \to \left[(4\pi)M_{IJ}B^I B^{cJ} - \frac{(4\pi)^3}{\Lambda^2}\det M\right], \tag{16}$$

and we have defined M_{00} to be S. The first two terms give us an NMSSM-like model at energy scales below Λ. In the $W_{\rm eff}$ term we have done not only the normal NDA analysis, but also the large n counting—notice that this partly compensates for the 4π NDA suppression. Up to an unknown O(1) constant, this results in a value for λ at the confinement scale of

$$\lambda = \sqrt{n}\frac{\lambda_1\lambda_2}{4\pi}\frac{\Lambda}{M_X}. \tag{17}$$

W_S contains a term linear in S that favors electroweak symmetry breaking and explicitly breaks the Peccei-Quinn symmetry that would give rise to an undesirable light axion. $W_{\rm d}$ marries up the superfluous baryons and mesons with singlet T partners as desired. In addition, integrating out the heavy mesons will decouple their interactions in $W_{\rm dyn}$. It is also possible to add interactions that will give rise to the standard NMSSM S^3 coupling to eliminate the new μ problem arising from the supersymmetric parameter m but we will not address this or the μ problems of M_X and $M_{\tilde{X}}$ here.

III. DISCUSSION

A. λ and the Higgs mass bound

So far we have shown how our model approximately reduces to the NMSSM below the confinement scale. Before analyzing this further, it is important to determine what range of λ will be most useful for our purposes. The value of the Higgs quartic can be found by running the λ coupling from the compositeness scale down to the electroweak scale (μ). We can solve for λ in Eq. (4) by ignoring all except the λ^3 term to obtain

$$\lambda(\mu)^2 = \left(\frac{1}{\lambda(\Lambda)^2} + \frac{1}{2\pi^2}\ln\frac{\Lambda}{\mu}\right)^{-1}. \tag{18}$$

We summarize the resulting running in Fig. 1, in which the low energy value $\lambda(\mu)$ is plotted as a function of the initial value $\lambda(\Lambda)$, for Λ/μ of different orders of magnitude. Notice that the value of λ at low energies is largely insensitive to its value at the confinement scale for $\lambda(\Lambda) \gtrsim 3$; it is this crucial feature that allows this model to compare favorably with the fat Higgs. Unlike the fat Higgs, however, we do not have to start in the limit of strong coupling to get $\lambda(\mu)$ parametrically higher than the NMSSM bound of 0.8 [9]. In the analysis that follows, we will arbitrarily choose as our region of interest $\lambda(\mu) \gtrsim 1.5$, which translates to $\lambda(\Lambda) \gtrsim (1.8, 2.2, 3.3)$ for running over one, two, and three decades, respectively.

Returning to the first scenario in which there is a new superconformal fixed point, we can now relate the above values of λ to the fixed point values of $\lambda_{1,2}$. Using Eq. (17) and assuming comparable fixed points for the two yukawas, we see that we need $\lambda_{1,2} \gtrsim (3.4, 3.7, 4.5)$ at M_X. Unfortunately, we cannot say whether the actual fixed points satisfy this condition, although these values are at least feasible since the flatness of the RGE running of $\lambda(\mu)$ means that λ does not have to equal 4π at the confinement scale. It would be interesting to do a more detailed study to determine whether this occurs.

It is possible to be more quantitative than this in the second case by relying on our knowledge of the model in the weak limit. Using Eq. (10) we see that

$$\lambda(\Lambda) \sim \sqrt{4}\frac{\lambda_1\lambda_2}{4\pi}\frac{\Lambda}{M_X} \lesssim -\frac{\Lambda}{2\pi M_X}\frac{16\pi^2}{7}\gamma_* \sim \frac{8\pi}{7}\gamma_* \sim 3.6\gamma_*. \tag{19}$$

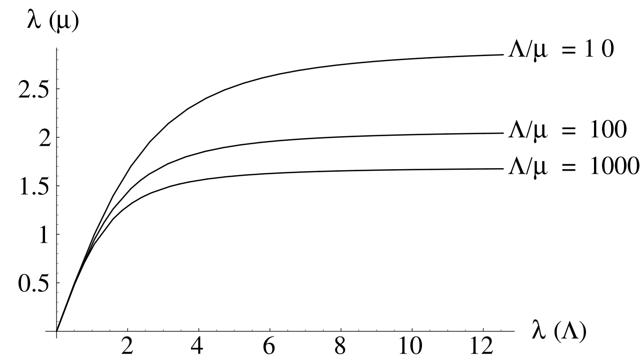

FIG. 1. The low energy values of the λ coupling after running from the compositeness scale Λ down to the scale μ.

If we start with all ten flavors of $SU(4)_s$, we have $\gamma_* = -1/5$ and $\lambda(\Lambda) \lesssim 0.7$ which is too low to be of interest. However, integrating out the $\tilde{X}$'s leaves us with seven flavors, which at the fixed point gives $\gamma_* = -5/7$ and $\lambda(\Lambda) \lesssim 2.6$. We saw that this gives rise to a λ that is in the interesting range for almost three decades of running between the confinement scale and the electroweak scale, suggesting that there are regions of parameter space where the low energy λ coupling is large enough to be of interest.

We can calculate the tree level bound on the Higgs mass by assuming that we are somewhere in the region $1.5 \lesssim \lambda(\mu) \lesssim 2$ and using the NMSSM equation,

$$m_h^2 \leq m_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta/2, \tag{20}$$

to obtain

$$m_h \lesssim 260\text{–}350 \text{ GeV}, \tag{21}$$

which is a substantial improvement over the MSSM bound of 90 GeV. Taking the largest $\lambda(\mu)$ in Fig. 1 pushes this bound up to 490 GeV, but this is probably less generic in the parameter space. Radiative corrections from the top sector can increase this further although these are no longer necessary to satisfy the LEP-II bound.

We emphasize that it is rather surprising to obtain interesting results in the weak limit bound in spite of the NDA suppression factor of 4π. This is a direct consequence of λ not having to start off at 4π; moderately large coupling is sufficient. However, the robustness of our conclusions in the weak limit depends on a number of O(1) unknowns which we ignored in the above analysis. These are listed below and discussed in turn.

(i) the value of the factor Λ/M_X
(ii) the running of the nonrenormalizable operator in Eq. (6) due to gauge coupling contributions in the region $\Lambda \leq E \leq M_X$
(iii) the coefficient in the NDA matching that was used in Eq. (17)
(iv) loop-level corrections from g_* to the coefficient of $\lambda_{1,2}^3$ in Eq. (10)
(v) restrictions due to the large top yukawa

The first tends to suppress the value of λ at low energies. The strong dynamics after flavor decoupling suggests that this factor is close to 1, but it cannot be determined exactly since we do not have detailed information on the fixed point value and exact running of g_s below M_X. It might, however, be compensated by the effect of the second which enhances λ; hence, we might be able to make a case for neglecting them both, especially since this allows us to make a quantitative prediction. The O(1) coefficient in the third item parametrizes our ignorance of the physics of strong coupling and unfortunately cannot be eliminated. The fourth point is that we ignored higher order gauge corrections to the $\lambda_{1,2}^3$ term in Eq. (10) at the gauge coupling fixed point. If the coefficient of this term decreases, the upper bound on the λ coupling increases and vice versa. Notice, however, that higher loop $\lambda_{1,2}$ corrections to the RGE are suppressed and have been rightfully ignored since the loop suppression factor $\lambda_{1,2}^2/(16\pi^2) \lesssim -\gamma_*/7 \leq 1/7 \ll 1$. Finally, the fact that the top yukawa is not negligible at low energies places some constraints on how large we can make λ_1 without losing perturbativity for both these couplings to the GUT scale. Doing a simple one loop analysis, for $\tan\beta$ near 1 (where the gain in the tree level bound is greatest), the λ_1 fixed point is about half of the value in the above analysis which in turn halves the size of $\lambda(\Lambda)$. In general, we expect that there is some O(1) suppression from this effect, but there is no comparable suppression in λ_2 due to the smallness of the bottom yukawa. Although it is unfortunate that these factors cannot be evaluated to determine a more specific bound, that the naive answer is in the interesting range suggests that the actual value of λ can be similarly large.

Since we were motivated to explore this model by concerns of naturalness, we will now discuss how this scenario helps the fine-tuning. First of all, the Higgs mass bound has increased so it is no longer necessary for the top squarks to be made heavy to evade the LEP-II bound. In fact, it is now possible for all the MSSM scalars including the Higgs to have masses that are of the same order. Thus, from a bottom-up perspective, there are no unnatural hierarchies in these masses.[5] On the other hand, there is new fine-tuning introduced in the weak limit (the second scenario), since the UV initial conditions have to be precisely tuned to avoid breaking conformality. However, these parameters are at least technically natural and so could still have the right size. There is no such fine-tuning in the new superconformal phase since the attractive IR fixed points reduce the sensitivity to UV initial conditions. For further discussion of how a larger Higgs quartic coupling helps the fine-tuning issue, see [15] and Casas *et al.* in [8].

B. Gauge coupling unification

In both the fat Higgs and the new fat Higgs, SUSY guarantees that running the SM gauge couplings through the strong coupling regions does not give corrections larger than typical threshold effects. We will recount the argument here for completeness. Matching holomorphic couplings of a high energy theory containing a massive field with those of a low energy theory, with the field integrated out, is constrained by holomorphy. In particular, the matching depends only on the bare mass of the field and thus is not affected by strong dynamics [16]. For instance, taking

[5]It could be argued that the top-down approach is still problematic since starting with universal scalar and gaugino masses (m_0 and $m_{1/2}$) at the unification scale, for example, force the top squarks to be heavy given observational lower bounds on chargino and slepton masses. This is a property of current SUSY breaking scenarios, however, and it is possible to imagine alternatives with more random boundary conditions at the GUT scale that result in realistic particle spectra with light top squarks.

$M_X = M_{\tilde{X}} = M$ at the cutoff $M_{\rm GUT}$, the high and low energy SM gauge couplings (with and without the $X, \tilde{X}$, respectively) are matched at the bare mass M:

$$g_{\rm sm,le}(M) = g_{\rm sm,he}(M), \tag{22}$$

where the high energy gauge couplings have their unified value at $M_{\rm GUT}$. At other energies these holomorphic couplings are determined by their one loop running (with beta functions $b_{\rm i,le} = b_{\rm i,MSSM}$ and $b_{\rm i,he} = b_{\rm i,le} + 4$). However, during this running the coefficients of the matter kinetic terms (Z) can change. Thus, to reach a more "physical" coupling, one should go to canonical normalization for the matter fields. This rescaling is anomalous and relates the couplings by

$$\frac{8\pi^2}{g^2_{\rm le,phys}} = \frac{8\pi^2}{g^2_{\rm le}} - \sum_i T^i \ln Z_i, \tag{23}$$

where i only runs over the matter fields in the low energy theory and the T_i's are their Dynkin indices. All potential strong coupling effects are contained within the Z_i's of the low energy fields. As a matter of fact, there is actually no effect due to the RGE splitting $M_X < M_{\tilde{X}}$, since the matching in Eq. (22) of the low energy couplings occurs at M, giving no restriction on the ratio of these masses from unification. An order one $\ln Z_i$ gives a contribution of the order of a typical theshold correction; thus it takes exponentially large Z_i to adversely affect unification. In this model, such large Z_i can occur only for the Higgses when the $\lambda_{1,2}$ couplings are strong for an exponentially large region. Thus, the weak limit case is generically safe, whereas in the new superconformal phase the conformal region for $\lambda_{1,2}$ cannot be exponentially large without affecting unification. Note that a similar constraint applies to the conformal region in the fat Higgs model.

Aside from this potential constraint, gauge coupling unification occurs naturally in this theory since the additional matter is charged under the SM in complete SU(5) multiplets *and* because the Higgses are elementary [hence, the beta functions of the SM couplings are equivalent to those of the MSSM up to SU(5) symmetric terms as detailed earlier]. In comparison, the fat Higgs model had elementary preons which correctly reproduced the running of the Higgs doublets above the compositeness scale, but also contained additional fields which were put into both split GUT and non-GUT multiplets in order to restore unification. In that model, explaining why unification is natural requires a setup that generates the additional matter content as well as the required mass spectrum.

C. Phenomenology

Much of the phenomenology in this model is similar to the fat Higgs. In both theories the physics at the TeV scale is NMSSM-like with a linear term in S but no cubic. The low energy λ coupling is large and gets strong before the GUT scale, but some asymptotically free dynamics takes over to UV complete the theory. They both have similar Higgs spectra which are in concordance with precision electroweak constraints. Also, the analysis in [17] which concludes that UV insensitive anomaly mediation works in the fat Higgs should also apply to this model.

One notable difference between the two models is the additional baryon physics in our model. The B^0 and B^{c0} in this theory get a large supersymmetric mass from the S vev and are not problematic. However, we also have light baryon states, the four B^i's and B^{ci}'s that are married to the T_i^B's and $T_i^{B^c}$'s, with supersymmetric masses of order

$$M_B \sim \frac{\Lambda^3}{4\pi M^2_{\rm GUT}} \sim 10^{-13}\text{–}10^{-7}\ {\rm eV}, \tag{24}$$

for $\Lambda \sim 5$–500 TeV. The scalar components of these chiral superfields get TeV sized soft masses from SUSY breaking and it is possible to determine these from the masses of the elementary fields using the techniques in [18]. The fermionic components are more worrying since they remain light and thus give rise to some stringent cosmological constraints. For instance, they decouple at a $T_{\rm dec} \sim 10$ GeV, requiring $T_{\rm reheat} \lesssim T_{\rm dec}$ in order to be consistent with big bang nucleosynthesis. It is also unclear whether the lightest supersymmetric particle (LSP) in this theory is a good dark matter candidate, given that it is never produced with thermal abundance, or whether baryogenesis can be made to work given such a low reheat temperature.

This constraint on the reheat temperature can be relaxed by adding small mass terms for the fundamental fields of the form $W \supset m_{IJ}\Psi_I\Psi^c_J$ which become tadpoles for the mesons after confinement. The tadpoles induce meson vevs which give masses to the light baryonic states through $W_{\rm dyn}$.[6] These mass terms break the $U(1)_R$ symmetry mentioned in Sec. II A; however, even very small masses ($m_{IJ} \sim M_B \gtrsim 1$ MeV) ensure that big bang nucleosynthesis can proceed as normal, while the newly added masses are small enough for the symmetry breaking effects to be under control.

It is also possible to circumvent this issue by using the scenario with the quantum modified moduli space mentioned in Sec. II or by making models without baryons, for instance with an $Sp(2) \equiv SO(5)$ theory, starting with 18 fundamentals of the $Sp(2)$. Integrating out the $X, \tilde{X}$ will reduce to the s-confining case with eight fundamentals. At high energies, this has a vanishing one loop beta function but is not asymptotically free at two loops. With all 18 fundamentals and their yukawas, the analysis in [13] suggests that there is a superconformal fixed point for the yukawa and gauge couplings. Specifically, there is a linear family of fixed points which run through the free fixed point ($g = 0$, $\lambda_i = 0$) (see footnote [4]) and it needs to be

[6] We thank Manuel Drees for proposing this solution to us.

determined whether the fixed point values of $\lambda_{1,2}$ are large enough to be in the interesting range. We can also work in a limit analogous to our weak limit of the previous section, integrating out the $\tilde{X}$'s first; this leaves the group in the conformal window with 12 fundamentals. Thus, if $M_X/M_{\tilde{X}}$ is small enough, the theory can run to Seiberg's strong conformal fixed point before the X's are integrated out. In this case, the weak limit bound gives $\lambda(\Lambda) \lesssim 1.8$, so we would need M_X near the weak-scale or some help from the unknown order one contributions detailed above. However, since there are no baryons in $Sp(n)$ theories, we only have to decouple the extra mesons. From this reasoning we see that the physics associated with the baryons does not appear generic to all implementations of our mechanism and thus cannot be used to rule out all models of this type.

IV. CONCLUSION

Supersymmetry does extremely well in solving the hierarchy problem but, as more precise measurements have told us, the minimal implementation of weak-scale supersymmetry (the MSSM) is becoming fine-tuned at about the percent level. Approaches that attempt to alleviate this problem have been many and varied, all of which have their own advantages and disadvantages. Led by the positive aspects of the MSSM, we analyzed a UV complete NMSSM model which justifies the presence of a large λ at low energies, resulting in a similarly large Higgs quartic coupling. We did this by splitting the λ coupling into two asymptotically free yukawa couplings, allowing the theory to be continued above the apparent strong coupling scale. The simple model pursued in this paper is similar in spirit to the fat Higgs model: We start at the electroweak scale with a large λ coupling which grows with increasing energy scale. Rather than waiting for it to hit 4π before UV completing, we do this at a lower scale, leaving a theory with a composite S only (see Fig. 2). There is no need for a dynamically generated superpotential because the induced λ coupling never becomes nonperturbative; instead moderately strong coupling is sufficient to achieve a large tree level Higgs mass bound without making the Higgs fields composite. This results in a Higgs that is not as fat as in the fat Higgs, but gauge coupling unification, arguably the best evidence for weak-scale SUSY, is naturally maintained.

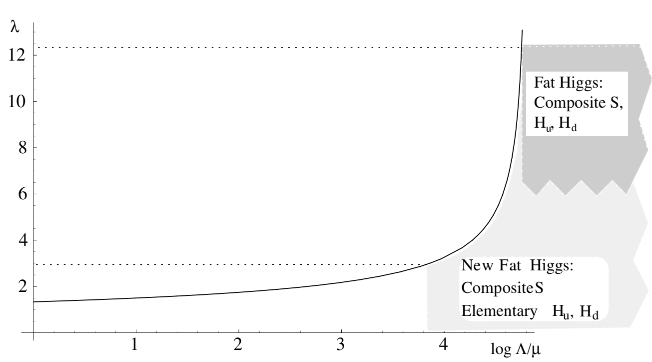

FIG. 2. A comparison of UV completion scales in the fat Higgs and the new fat Higgs.

We did not study in depth the potentially interesting scenario where the theory hit a superconformal fixed point, since it was tricky to make any definitive statements about the fixed point values of $\lambda_{1,2}$. The strong coupling dynamics also made it difficult to give exact results in the second case we considered, but we were able to set a reasonable upper bound on λ at low energies, up to some unknown order one coefficients, using the properties of Seiberg's fixed point and superconformality in the weak limit. That this bound turned out to give large enough λ is comforting, since it suggests the possibility of realizing our mechanism for a generic parameter space with similar results. However, to say any more requires a detailed understanding of both the RGE equations at strong coupling and matching at the confinement scale.

Finally, we discussed some of the implications of our model. We saw that the fine-tuning issue was indeed ameliorated, at least from a bottom-up perspective, and that unification was not affected by the strong coupling. We also discussed the equivalence of the phenomenology to that of the fat Higgs Model in that there was little difference in their Higgs spectra or compatibility with precision electroweak constraints. One notable difference was the presence of light fermionic baryons in our theory. It would be interesting to analyze the new baryon physics in more detail, especially since they give rise to an interesting cosmological constraint. As discussed, this can be relaxed by adding mass terms that weakly break the nonanomalous $U(1)_R$. Furthermore, the existence of models which do not have baryons suggests that light states are not generic to this framework. In such models we expect the dark matter abundance and baryogenesis analysis to proceed along the lines of [19].

In a few years, the LHC will start to explore the possible presence of weak-scale supersymmetry, and it is important to continue to study SUSY models so as to compare data with experiment. Using naturalness as a guideline, it already seems that the simplest SUSY models are fine-tuned, which motivates us to attempt to generalize them. With this intuition we have analyzed a theory which improves the naturalness of weak-scale SUSY in a simple way without losing the natural unification of the MSSM. However, only experiment can ultimately determine the accuracy of our guesses for what comes beyond the standard model.

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Note added.—As this paper was being finished, a paper appeared that analyzed a very similar model [20]. However, they did not notice the mechanism we have described for generating the NMSSM λ coupling and had too few flavors of $SU(4)_s$ to avoid the Affleck-Dine-Seiberg vacuum instability after integrating out the X fields. Also, their results on the suppression of the μ and m_{H_u} corrections are not as crucial when the tree level upper bound on the Higgs mass is increased.

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