

Photon-meson transition form factors of light pseudoscalar mesons

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The photon-meson transition form factors of light pseudoscalar mesons π^0 , η , and η' are systematically calculated in a light-cone framework, which is applicable as a light-cone quark model at low Q^2 and is also physically in accordance with the light-cone perturbative quantum chromodynamics approach at large Q^2 . The calculated results agree with the available experimental data at high-energy scale. We also predict the low Q^2 behaviors of the photon-meson transition form factors of π^0 , η , and η' , which are measurable in the $e + A(\text{Nucleus}) \rightarrow e + A + M$ process via the Primakoff effect at JLab and DESY.

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I. INTRODUCTION

The meson-photon and photon-meson transition form factors contain interesting physics concerning the QCD structure of both photons and mesons. The pion-photon transition form factor provides a very simple example for the perturbative QCD (pQCD) analysis to exclusive processes, and was first analyzed by Brodsky and Lepage [1] at large Q^2 . It has been shown [2] that the applicability of pQCD can be extended to lower Q^2 around a few GeV^2 by taking into account the transverse momentum contributions in both the hard scattering amplitude and pion wave function. In our recent study [3] within the light-cone quark model, it is shown that the pion-photon transition form factor is identical to the photon-pion transition form factor when taking into account only QCD and QED contributions. Therefore the formalism that applies to the pion-photon transition form factor is also applicable to the photon-pion transition form factor. Taking the minimal quark-antiquark Fock states of both the photon and pion as their wave functions, we could calculate the photon-pion transition form factor by using the Drell-Yan-West assignment. This framework is applicable at low Q^2 as a light-cone quark model approach, and it is also physically in accordance with the light-cone pQCD approach at large Q^2 . Thus we can describe the photon-pion form factors at both low Q^2 and high Q^2 within the same framework. The purpose of this work is to apply this framework [3] for a systematic description of the photon-meson transition form factors of pseudoscalar mesons π^0 , η , and η' , at both $Q^2 \rightarrow 0$ and $Q^2 \rightarrow \infty$ limits, and to make predictions in a wide Q^2 range.

The photon-meson transition form factor $\gamma^* \gamma \rightarrow M$ can be realized in $e + e \rightarrow e + e + M$ or $e + A(\text{Nucleus}) \rightarrow$

$e + A + M$ processes. The $\gamma^* \gamma \rightarrow M$ transition form factors of π^0 , η , and η' at medium to high Q^2 have been measured at Cornell [4] and at DESY [5] through the $e^+ + e^- \rightarrow e^+ + e^- + M$ process, while the latter process $e + A(\text{Nucleus}) \rightarrow e + A + M$ is convenient to provide measurement of the photon-meson transition form factors at low Q^2 . Moreover, high precision measurements of the electromagnetic properties of these pseudoscalar mesons via the Primakoff effect are proposed by PrimEx Collaboration at the Thomas Jefferson National Accelerator Facility (JLab) [6], which would give the experimental value of transition form factors $F_{\gamma^* \gamma \rightarrow M}(Q^2)$ of π^0 , η , and η' at low Q^2 ($0.001\text{--}0.5 \text{ GeV}^2$), and lead to a clarification of the obvious disagreement between the former Primakoff experiment and collider cases in the measurements of $\Gamma(\eta \rightarrow \gamma\gamma)$ and a more precise determination of the η - η' mixing angle. Similar measurements can also be performed by HERMES Collaboration at DESY [7]. Therefore, theoretical predictions at low Q^2 are necessary and essential for comparison with future experimental measurements.

It is well known that the physical η and η' states dominantly consist of a flavor SU(3) octet η_8 and singlet η_0 in the SU(3) quark model, respectively. The usual mixing scheme reads

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}. \quad (1)$$

Using different sets of experimental data, we recalculate the value of the mixing angle θ by employing the limiting method developed by Cao-Signal [8]. Our results are also compatible with other approaches [9] for the mixing angle and scheme.

In general, people use the chiral perturbation theory [10] or some other methods [11] which deal with current quark masses in order to take the chiral symmetry and chiral

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anomaly into account, since the chiral symmetry predominates the $\pi^0(\eta, \eta')\gamma\gamma$ vertex at large Q^2 [12], and the chiral anomaly determines the $\pi^0(\eta, \eta')$ transition form factors at $Q^2 = 0$ [Eqs. (30)–(33)]. In addition, the chiral perturbation theory is also very useful and effective in discussing the η and η' mixing properties [13]. Since we are consistently using the valence quark masses in the light-cone treatment to the form factor calculation, it is not very applicable to start with the current quark mass within the chiral symmetry and investigate the chiral limits in the transition form factor computation. However, our main purpose of this paper is to employ the new light-cone $\gamma \rightarrow q\bar{q}, s\bar{s}$ wave functions [3,14,15] to compute the transition form factors of the light mesons. Moreover, we consider the chiral symmetry when we choose the η and η' mixing scheme, and take the chiral limit approximation when we try to determine and fix the parameters. Therefore our results respect the chiral symmetry and its breaking to some extent. Phenomenologically, we could give the predictions of the η and η' mixing angle within the light-cone formalism, as well as the photon-meson transition form factor which is applicable at both low- and high-energy scales.

This paper is organized as follows. In Sec. II we present the formalism for the photon-meson transition form factor using the minimal quark-antiquark Fock states of the photon and pion as wave functions. In Sec. III, we will introduce the η - η' mixing scheme used in our calculation. In Sec. IV, we calculate systematically the photon-meson transition form factors of π^0 , η , and η' , and show that the calculated results agree with the available experimental data at a medium to large Q^2 scale. We also predict the low Q^2 behaviors of the photon-meson transition form factors of π^0 , η , and η' , which are measurable in the $e + A(\text{Nucleus}) \rightarrow e + A + M$ process via the Primakoff effect at JLab and DESY. In Sec. V, we present a brief summary.

II. FORMALISM OF PHOTON-MESON TRANSITION FORM FACTOR

We work in the light-cone formalism [16], which provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and for the application of perturbative QCD to exclusive processes. The transition form factor $F_{\gamma^*\gamma \rightarrow M}$ ($M = \pi^0, \eta$, and η'), in which an on-shell photon is struck by one off-shell photon and decays into a meson, as schematically shown in Fig. 1, is defined by the $\gamma^*\gamma M$ vertex,

$$\Gamma_\mu = -ie^2 F_{\gamma^*\gamma \rightarrow M}(Q^2) \epsilon_{\mu\nu\rho\sigma} p_M^\nu \epsilon^\rho q^\sigma, \quad (2)$$

in which q is the momentum of the off-shell photon, $Q^2 = -q^2 = \mathbf{q}_\perp^2 - q^+ q^- = \mathbf{q}_\perp^2$ is the squared four-momentum transfer of the virtual photon, and ϵ is the polarization vector of the on-shell photon. We choose the light-cone

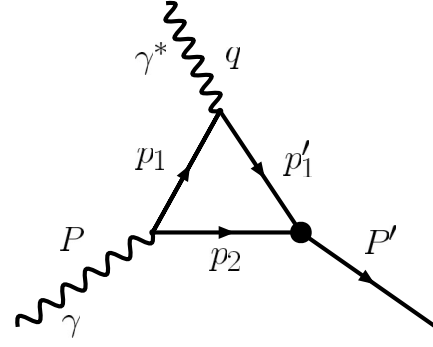


FIG. 1. The diagram for the contribution to the transition form factor $F_{\gamma^*\gamma \rightarrow M}$. The arrows indicate the direction that the particles are moving.

frame

$$\begin{cases} P &= (P^+, \frac{q^2 + \mathbf{q}_\perp^2}{P^+}, \mathbf{0}_\perp), \\ P' &= (P'^+, \frac{M^2}{P'^+}, \mathbf{q}_\perp), \\ q &= (0, \frac{Q^2}{P^+}, \mathbf{q}_\perp), \\ p_1 &= (xP^+, \frac{\mathbf{k}_\perp^2 + m^2}{xP^+}, \mathbf{k}_\perp) \\ p_2 &= [(1-x)P^+, \frac{\mathbf{k}_\perp^2 + m^2}{(1-x)P^+}, -\mathbf{k}_\perp], \\ p_1' &= (xP'^+, \frac{\mathbf{k}'_\perp^2 + m^2}{xP'^+}, \mathbf{k}'_\perp). \end{cases} \quad (3)$$

Instead of calculating the diagram directly, we introduce the quark-antiquark wave function of the photon [3] by calculating the matrix elements of

$$\frac{\bar{u}(p_1^+, p_1^-, \mathbf{p}_{1\perp})}{\sqrt{p_1^+}} \gamma \cdot \epsilon \frac{v(p_2^+, p_2^-, \mathbf{p}_{2\perp})}{\sqrt{p_2^+}}, \quad (4)$$

which are the numerators of the wave functions corresponding to each constituent spin S^z configuration. The two boson polarization vectors in light-cone gauge are $\epsilon^\mu = (\epsilon^+ = 0, \epsilon^-, \epsilon_\perp)$, where $\epsilon_{\perp 1,2} = \mp \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} \pm \hat{\mathbf{y}})$. To satisfy the Lorentz condition $k_{\text{photon}} \cdot \epsilon = 0$, the polarizations have the relation $\epsilon^- = (2\epsilon_\perp \cdot \mathbf{k}_\perp)/k^+$ with k_{photon} , thus we have

$$\begin{cases} \Psi_R^\dagger(x, \mathbf{k}_\perp, \uparrow, \downarrow) = -\frac{\sqrt{2}(k_1 + ik_2)}{1-x} \varphi_\gamma, & [I^z = +1] \\ \Psi_R^\dagger(x, \mathbf{k}_\perp, \downarrow, \uparrow) = +\frac{\sqrt{2}(k_1 + ik_2)}{x} \varphi_\gamma, & [I^z = +1] \\ \Psi_R^\dagger(x, \mathbf{k}_\perp, \uparrow, \uparrow) = -\frac{\sqrt{2}m}{x(1-x)} \varphi_\gamma, & [I^z = 0] \\ \Psi_R^\dagger(x, \mathbf{k}_\perp, \downarrow, \downarrow) = 0, \end{cases} \quad (5)$$

in which

$$\varphi_\gamma = \frac{e_q}{D} = \frac{e_q}{\lambda^2 - [(m^2 + \mathbf{k}_\perp^2)/x] - [(m^2 + \mathbf{k}_\perp^2)/(1-x)]}, \quad (6)$$

where λ is the photon mass and equal to zero. Each

configuration satisfies the spin sum rule: $J^z = S_q^z + S_{\bar{q}}^z + l^z = +1$. Therefore, the quark-antiquark Fock-state for the photon ($J^z = +1$) has four possible spin combinations:

$$|\Psi_\gamma(P^+, \mathbf{P}_\perp)\rangle = \int \frac{d^2\mathbf{k}_\perp dx}{16\pi^3} [\Psi_R^\dagger(x, \mathbf{k}_\perp, \uparrow, \downarrow)|xP^+, \mathbf{k}_\perp, \uparrow, \downarrow\rangle + \Psi_R^\dagger(x, \mathbf{k}_\perp, \downarrow, \uparrow)|xP^+, \mathbf{k}_\perp, \downarrow, \uparrow\rangle + \Psi_R^\dagger(x, \mathbf{k}_\perp, \uparrow, \uparrow)|xP^+, \mathbf{k}_\perp, \uparrow, \uparrow\rangle + \Psi_R^\dagger(x, \mathbf{k}_\perp, \downarrow, \downarrow)|xP^+, \mathbf{k}_\perp, \downarrow, \downarrow\rangle]. \quad (7)$$

The quark-antiquark Fock-state wave function of the pion is also derived [3] by using the relativistic field theory treatment of the interaction vertex along with the idea in [17,18]. In the light-cone frame of the pion,

$$\begin{cases} P &= (P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp), \\ p_1 &= (xP^+, \frac{\mathbf{p}_{1\perp}^2 + m^2}{xP^+}, \mathbf{p}_{1\perp}), \\ p_2 &= [(1-x)P^+, \frac{\mathbf{p}_{2\perp}^2 + m^2}{(1-x)P^+}, \mathbf{p}_{2\perp}], \end{cases} \quad (8)$$

we can obtain the four components of the spin wave function by calculating the matrix elements of

$$\frac{\bar{v}(p_2^+, p_2^-, -\mathbf{k}_\perp)}{\sqrt{p_2^+}} \gamma_5 \frac{u(p_1^+, p_1^-, \mathbf{k}_\perp)}{\sqrt{p_1^+}}, \quad (9)$$

from which we have

$$\begin{cases} \frac{\bar{v}_1}{\sqrt{p_2^+}} \gamma_5 \frac{u_1}{\sqrt{p_1^+}} &= -\frac{2mP^+}{4mx(1-x)P^{+2}}, \\ \frac{\bar{v}_1}{\sqrt{p_2^+}} \gamma_5 \frac{u_1}{\sqrt{p_1^+}} &= +\frac{2mP^+}{4mx(1-x)P^{+2}}, \\ \frac{\bar{v}_1}{\sqrt{p_2^+}} \gamma_5 \frac{u_1}{\sqrt{p_1^+}} &= +\frac{2(k_1 + ik_2)P^+}{4mx(1-x)P^{+2}}, \\ \frac{\bar{v}_1}{\sqrt{p_2^+}} \gamma_5 \frac{u_1}{\sqrt{p_1^+}} &= +\frac{2(k_1 - ik_2)P^+}{4mx(1-x)P^{+2}}, \end{cases} \quad (10)$$

where m is the mass of the quark. After the normalization, we can obtain the light-cone representation for the spin structure of the pion, which is the minimal Fock state of the pion light-cone wave function:

$$\begin{cases} \Psi_{\pi L}(x, \mathbf{k}_\perp, \uparrow, \downarrow) &= -\frac{m}{\sqrt{2(m^2 + \mathbf{k}_\perp^2)}} \varphi_\pi, \quad [l^z = 0] \\ \Psi_{\pi L}(x, \mathbf{k}_\perp, \downarrow, \uparrow) &= +\frac{m}{\sqrt{2(m^2 + \mathbf{k}_\perp^2)}} \varphi_\pi, \quad [l^z = 0] \\ \Psi_{\pi L}(x, \mathbf{k}_\perp, \uparrow, \uparrow) &= +\frac{k_1 + ik_2}{\sqrt{2(m^2 + \mathbf{k}_\perp^2)}} \varphi_\pi, \quad [l^z = -1] \\ \Psi_{\pi L}(x, \mathbf{k}_\perp, \downarrow, \downarrow) &= +\frac{k_1 - ik_2}{\sqrt{2(m^2 + \mathbf{k}_\perp^2)}} \varphi_\pi, \quad [l^z = +1] \end{cases} \quad (11)$$

in which we may employ the Brodsky-Huang-Lepage (BHL) prescription [19],

$$\varphi_\pi(x, \mathbf{k}) = A \exp\left[-\frac{1}{8\beta^2} \frac{\mathbf{k}_\perp^2 + m^2}{x(1-x)}\right], \quad (12)$$

for the momentum space wave function, which is a non-relativistic solution of the Bethe-Salpeter equation in an instantaneous approximation in the rest frame for the meson. Each configuration satisfies the spin sum rule: $J^z = S_q^z + S_{\bar{q}}^z + l^z = 0$. Hence, the Fock expansion of the two-particle Fock state for the pion has these four possible spin combinations:

$$\begin{aligned} \langle \Psi_\pi(P^+, \mathbf{P}_\perp = \mathbf{0}_\perp) | &= \int \frac{d^2\mathbf{k}_\perp dx}{16\pi^3} \\ &\times [\Psi_{\pi L}(x, \mathbf{k}_\perp, \uparrow, \downarrow) \langle xP^+, \mathbf{k}_\perp, \uparrow, \downarrow | \\ &+ \Psi_{\pi L}(x, \mathbf{k}_\perp, \downarrow, \uparrow) \langle xP^+, \mathbf{k}_\perp, \downarrow, \uparrow | \\ &+ \Psi_{\pi L}(x, \mathbf{k}_\perp, \uparrow, \uparrow) \langle xP^+, \mathbf{k}_\perp, \uparrow, \uparrow | \\ &+ \Psi_{\pi L}(x, \mathbf{k}_\perp, \downarrow, \downarrow) \langle xP^+, \mathbf{k}_\perp, \downarrow, \downarrow |]. \end{aligned} \quad (13)$$

There are two higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components in the expression of the light-cone spin wave function of the pion besides the ordinary helicity ($\lambda_1 + \lambda_2 = 0$) components. Such higher helicity components come from the Melosh-Wigner rotation in the light-cone quark model [20,21], and the same effect plays an important role in understanding the proton ‘‘spin puzzle’’ in the nucleon case [22,23]. One may also state that these higher helicity components contain contribution from an orbital angular moment from a relativistic viewpoint [24].

In addition, we would like to add some more remarks on the Gaussian-type wave function of the BHL prescription that we employ above. As a matter of fact, the Gaussian wave function is a nonrelativistic solution of the Bethe-Salpeter equation in an instantaneous approximation in the rest frame of the meson as the space wave function. The BHL wave function equation (12) is an extension from a nonrelativistic wave function into a relativistic form by using the Brodsky-Huang-Lepage Ansatz [19], and we can consider it as an approximate wave function that respects the Lorentz invariance in the light-cone formalism. However, it works phenomenologically well in a lot of calculations (e.g., [3,21,25,26]). Moreover, Donnachie, Gravelis, and Shaw [25] indicated that the other four possible different space wave functions have similar analytical properties to the BHL wave function when the

parameter β is small (the β is equal to P_F in their paper; the small β is corresponding to the ρ and ϕ meson cases). However, they also illustrated that the BHL wave function is better than the other four wave functions in the high β situation (for the J/ψ meson). Hence, it gives us the idea that the BHL wave function may be an appropriate choice for us right now. (Noticing that the space wave functions for the vector mesons are the same with those for the pseudoscalar mesons, the argument made by Donnachie *et al.* is also valid for π^0 and other pseudoscalar mesons.)

For the physical state of π^0 , one should also take into account the color and flavor degrees of freedom [1,2],

$$|\Psi_{\pi^0}\rangle = \sum_a \frac{\delta_b^a}{\sqrt{n_c}} \frac{1}{\sqrt{2}} [|u^a \bar{u}^b\rangle - |d^a \bar{d}^b\rangle], \quad (14)$$

where a and b are color indices, $n_c = 3$ is the number of colors, and now $|q^a \bar{q}^b\rangle$ contains the full spin structure shown above. So, we can get the photon-meson transition form factor of the pion:

$$\begin{aligned} F_{\gamma\gamma^*\rightarrow\pi}(Q^2) &= \frac{\Gamma^+}{-ie^2(\epsilon_\perp \times \mathbf{q}_\perp) p_\pi'^-} \\ &= 2\sqrt{3}(e_u^2 - e_d^2) \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \varphi_\pi(x, \mathbf{k}_\perp) \left\{ \frac{m}{x\sqrt{m^2 + \mathbf{k}_\perp^2}} \right. \\ &\quad \times \left[\frac{1}{-\lambda^2 + [(m^2 + \mathbf{k}_\perp^2)/x] + [(m^2 + \mathbf{k}_\perp^2)/(1-x)]} \right] + (1 \leftrightarrow 2) \Big\} \end{aligned} \quad (15)$$

$$= 4\sqrt{3}(e_u^2 - e_d^2) \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\varphi_\pi(x, \mathbf{k}_\perp) \frac{m}{x\sqrt{m^2 + \mathbf{k}_\perp^2}} \times \frac{1}{(m^2 + \mathbf{k}_\perp^2)/x - (m^2 + \mathbf{k}_\perp^2)/(1-x)} \right], \quad (16)$$

in which $\mathbf{k}_\perp' = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ after considering the Drell-Yan-West assignment [27], and λ ($=0$) is the mass of the photon.

III. THE η - η' MIXING SCHEMES

In fact, there are two popular mixing schemes for η and η' . Feldmann *et al.* [9] suggested the mixing scheme based on the quark flavor basis $q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$,

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \quad (17)$$

and

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} f_q & 0 \\ 0 & f_s \end{pmatrix}, \quad (18)$$

where ϕ is the mixing angle. The $q\bar{q}$ - $s\bar{s}$ mixing only introduces one mixing angle in the mixing of the decay constants.

On the other hand, people also use the mixing scheme based on η_8 and η_0 mixing for η and η' . In the SU(3) quark model, the physical η and η' states dominantly consist of a flavor SU(3) octet $\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and a singlet $\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$, respectively. The usual mixing scheme reads

$$\begin{cases} |\eta\rangle &= \cos\theta|\eta_8\rangle - \sin\theta|\eta_0\rangle, \\ |\eta'\rangle &= \sin\theta|\eta_8\rangle + \cos\theta|\eta_0\rangle, \end{cases} \quad (19)$$

in which θ is the mixing angle. For the calculation of the

decay constants of the η_8 and η_0 mixing, Feldmann-Kroll indicate that the two mixing angle scheme could be better than their former investigations [28]:

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_8 \\ \sin\theta_8 & \cos\theta_8 \end{pmatrix} \begin{pmatrix} f_8 & 0 \\ 0 & f_0 \end{pmatrix}. \quad (20)$$

In addition, one could find that these two schemes could be related by the following equation through the mixing angles, finally:

$$\theta = \phi - \arctan \frac{1}{\sqrt{3}}. \quad (21)$$

From the point view of the chiral symmetry and the SU(3) symmetry as well as their breaking mechanisms, we find that the η_8 and η_0 mixing scheme may be more reasonable and physical.

First of all, let us have a brief review on the chiral symmetry and its breaking which have an underlying relationship with the π^0 , η , and η' mesons[6]. In the chiral symmetry limit, it is well known that the Lagrangian has the $SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$ symmetry, but the absence of this symmetry in the ground state (the QCD vacuum) leads to the chiral symmetry spontaneously breaking into $SU(3) \times U(1)_B$ symmetry. Because there are eight spontaneously broken continuous symmetries (there are nine when taking into account the chiral anomaly which is associated with the $U(1)_A$ symmetry breaking), there are eight massless Goldstone Bosons (which finally are identified as the meson octet) and one massive particle (which is known as η_0) according to Goldstone's theorem

and the chiral anomaly, respectively. The massless octet includes the meson π_8^0 and η_8 . Together with η_0 , they mix into massive mesons π^0 , η , and η' during the explicit SU(3) symmetry breaking after introducing the quark mass term into the Lagrangian.

From the above discussion, we may reach a physical intuitive idea that it is natural and straightforward to use the η_0 and η_8 mixing scheme as a direct result of the SU(3) symmetry breaking if we assume that the π_8^0 does not mix with η_0 and η_8 at all. From this point of view, the introduction of η_0 and η_8 is more reasonable than η_q and η_s .

Moreover, since it is well known that pion, kaon, and η_8 belong to the same group of octet mesons in the SU(3) symmetry limit, their parameters should be the same except for the quark masses. In this sense, one may relate the decay constants of η and η' to the pion and kaon in the η_8 - η_0 mixing scheme. The CLEO Collaboration [4] reported their pole fit results as $\Lambda_\pi = 776 \pm 10 \pm 12 \pm 16$ MeV, $\Lambda_\eta = 774 \pm 11 \pm 16 \pm 22$ MeV, and $\Lambda_{\eta'} = 859 \pm 9 \pm 18 \pm 20$ MeV. These results imply that the nonperturbative properties of π and η are very similar. In addition, the absolute value of θ is small and $\cos\theta|\eta_8\rangle$ is the leading order in the η_8 - η_0 mixing scheme of the η . They are consistent with the basic physical intuition that both π and η_8 are in the SU_f(3) octet and are pseudomassless Goldstone particles. Therefore, that is why the authors of [29] take the parameters of η_8 as equal to the pion, such as $b_8 = b_\pi$ in their paper. From a strict sense, if the pion, kaon, and η_8 are in the same group of octet mesons, the mass of m_q , m_s , and $\beta_8 = \beta_\pi$ in the BHL wave function should be the same in the calculations of the π , η , and η' transition form factors.

Therefore, we employ the intuitive η_8 - η_0 mixing scheme in the calculations of the π , η , and η' transition form factors by using the uniform parameters, which shows that the SU(3) symmetry limit works well in this paper.

In practice, we utilize the SU_f(3) broken form of wave functions for flavor octet η_8 and singlet η_0 :

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d})\phi_8^q(x, \mathbf{k}_\perp) - \frac{2}{\sqrt{6}}s\bar{s}\phi_8^s(x, \mathbf{k}_\perp), \quad (22)$$

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d})\phi_0^q(x, \mathbf{k}_\perp) + \frac{1}{\sqrt{3}}s\bar{s}\phi_0^s(x, \mathbf{k}_\perp), \quad (23)$$

in which we use the Gaussian wave function of the BHL prescription:

$$\phi_8^q(x, \mathbf{k}_\perp) = A_8 \exp\left[-\frac{m_q^2 + \mathbf{k}_\perp^2}{8\beta_8^2 x(1-x)}\right], \quad (24)$$

$$\phi_8^s(x, \mathbf{k}_\perp) = A_8 \exp\left[-\frac{m_s^2 + \mathbf{k}_\perp^2}{8\beta_8^2 x(1-x)}\right], \quad (25)$$

$$\phi_0^q(x, \mathbf{k}_\perp) = A_0 \exp\left[-\frac{m_q^2 + \mathbf{k}_\perp^2}{8\beta_0^2 x(1-x)}\right], \quad (26)$$

$$\phi_0^s(x, \mathbf{k}_\perp) = A_0 \exp\left[-\frac{m_s^2 + \mathbf{k}_\perp^2}{8\beta_0^2 x(1-x)}\right], \quad (27)$$

and $q\bar{q}$ and $s\bar{s}$ are the spin parts of the wave functions which are similar to the pion with all possible spin states.

Moreover, it is convenient to use the method for the η_8 - η_0 mixing scheme which was developed by Cao-Signal [8] in obtaining the mixing angle θ and the decay constants. In this treatment, we can get θ , f_8 , and f_0 directly without involving θ_8 and θ_0 . In the η_8 - η_0 mixing scheme, we have

$$F_{\gamma\gamma^*\rightarrow\eta}(Q^2) = F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2)\cos\theta - F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2)\sin\theta, \quad (28)$$

$$F_{\gamma\gamma^*\rightarrow\eta'}(Q^2) = F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2)\sin\theta + F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2)\cos\theta. \quad (29)$$

While for the π^0 case, we have

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{\pi\alpha^2 m_{\pi^0}^3}{4} |F_{\gamma\gamma^*\rightarrow\pi}(0)|^2, \quad (30)$$

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{\alpha^2 m_{\pi^0}^3}{64\pi^3} \frac{1}{f_\pi^2}. \quad (31)$$

Generalizing these equations to η_8 and η_0 , we have

$$\begin{aligned} \Gamma(\eta \rightarrow \gamma\gamma) &= \frac{\pi\alpha^2 m_\eta^3}{4} |F_{\gamma\gamma^*\rightarrow\eta}(0)|^2 \\ &= \frac{\alpha^2 m_\eta^3}{64\pi^3} \left(\frac{\cos\theta}{\sqrt{3}f_8} - \frac{2\sqrt{2}\sin\theta}{\sqrt{3}f_0} \right)^2, \end{aligned} \quad (32)$$

$$\begin{aligned} \Gamma(\eta' \rightarrow \gamma\gamma) &= \frac{\pi\alpha^2 m_{\eta'}^3}{4} |F_{\gamma\gamma^*\rightarrow\eta'}(0)|^2 \\ &= \frac{\alpha^2 m_{\eta'}^3}{64\pi^3} \left(\frac{\sin\theta}{\sqrt{3}f_8} + \frac{2\sqrt{2}\cos\theta}{\sqrt{3}f_0} \right)^2. \end{aligned} \quad (33)$$

Thus we could get

$$\rho_1 = \frac{F_{\gamma\gamma^*\rightarrow\eta}(0)}{F_{\gamma\gamma^*\rightarrow\eta'}(0)} = \frac{\tan\theta_{08} - \tan\theta}{1 + \tan\theta_{08} \times \tan\theta}, \quad (34)$$

$$= \tan(\theta_{08} - \theta), \quad (35)$$

in which we let $\tan\theta_{08} = f_0/(\sqrt{8}f_8)$. Along with the same idea, by taking the $Q^2 \rightarrow \infty$ limit, we could have

$$\begin{aligned}\rho_2 &= \frac{F_{\gamma\gamma^*\rightarrow\eta}(Q^2\rightarrow\infty)}{F_{\gamma\gamma^*\rightarrow\eta'}(Q^2\rightarrow\infty)} \\ &= \frac{F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2\rightarrow\infty)\cos\theta - F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2\rightarrow\infty)\sin\theta}{F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2\rightarrow\infty)\sin\theta + F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2\rightarrow\infty)\cos\theta} \\ &= \frac{1 - 8\tan\theta_{08}\times\tan\theta}{\tan\theta + 8\tan\theta_{08}},\end{aligned}\quad (36)$$

in which we have $\lim_{Q^2\rightarrow\infty}Q^2F_8(Q^2) = 2f_8/\sqrt{3}$ and $\lim_{Q^2\rightarrow\infty}Q^2F_0(Q^2) = (4\sqrt{2}f_0)/\sqrt{3}$. CLEO [4] proposed that the $\gamma\gamma^* \rightarrow M$ transition form factors could be approximated by

$$F_{\gamma\gamma^*\rightarrow M}(Q^2) = F_{\gamma\gamma^*\rightarrow M}(0) \times \frac{1}{1 + Q^2/\Lambda_M^2}; \quad (37)$$

thus we obtain

$$\rho_2 = \rho_1 \frac{\Lambda_\eta^2}{\Lambda_{\eta'}^2}. \quad (38)$$

Finally one obtains

$$\tan\theta = \frac{-9(\rho_1 + \rho_2) + \sqrt{81(\rho_1 - \rho_2)^2 + 32(\rho_1\rho_2 + 1)^2}}{2(8 - \rho_1\rho_2)}, \quad (39)$$

$$\frac{f_0}{f_8} = \sqrt{8}\tan(\theta + \arctan\rho_1), \quad (40)$$

and gets

$$f_8 = \frac{1}{4\sqrt{3}\pi^2[F_{\gamma\gamma^*\rightarrow\eta}(0)\cos\theta + F_{\gamma\gamma^*\rightarrow\eta'}(0)\sin\theta]}, \quad (41)$$

$$f_0 = \frac{\sqrt{8}}{4\sqrt{3}\pi^2[F_{\gamma\gamma^*\rightarrow\eta'}(0)\sin\theta - F_{\gamma\gamma^*\rightarrow\eta}(0)\cos\theta]}, \quad (42)$$

by using the above results.

IV. $\gamma^*\gamma \rightarrow \eta$ AND $\gamma^*\gamma \rightarrow \eta$ TRANSITION FORM FACTORS

There have been many different approaches to discuss the photon-meson transition form factors of light pseudo-scalar mesons π^0 , η , and η' , such as the light-cone perturbation theory by Cao-Huang-Ma [2,29], the light-front quark model by Hwang and Choi-Ji [26], the QCD sum rule calculation by Radyushkin-Ruskov [30], and also other approaches [31]. We now perform a systematic calculation of these transition form factors in the light-cone framework just presented in Sec. II. The advantage of this new framework is that the predictions should be applicable at both low- and high-energy scales.

Similar to the pion transition form factor and from Eqs. (22) and (23), we can get η_8 and η_0 transition form factors:

$$\begin{aligned}F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2) &= 4(e_u^2 + e_d^2) \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \\ &\times \frac{m_q}{x\sqrt{m_q^2 + \mathbf{k}_\perp'^2}} \phi_8^q(x, \mathbf{k}_\perp') \frac{x(1-x)}{m_q^2 + \mathbf{k}_\perp'^2} \\ &- 8e_s^2 \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \frac{m_s}{x\sqrt{m_s^2 + \mathbf{k}_\perp'^2}} \\ &\times \phi_8^s(x, \mathbf{k}_\perp') \frac{x(1-x)}{m_s^2 + \mathbf{k}_\perp'^2},\end{aligned}\quad (43)$$

$$\begin{aligned}F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2) &= 4\sqrt{2}(e_u^2 + e_d^2) \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \\ &\times \frac{m_q}{x\sqrt{m_q^2 + \mathbf{k}_\perp'^2}} \phi_0^q(x, \mathbf{k}_\perp') \frac{x(1-x)}{m_q^2 + \mathbf{k}_\perp'^2} \\ &+ 4\sqrt{2}e_s^2 \int \frac{dx d^2\mathbf{k}_\perp}{16\pi^3} \frac{m_s}{x\sqrt{m_s^2 + \mathbf{k}_\perp'^2}} \\ &\times \phi_0^s(x, \mathbf{k}_\perp') \frac{x(1-x)}{m_s^2 + \mathbf{k}_\perp'^2},\end{aligned}\quad (44)$$

in which $\mathbf{k}_\perp' = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ after considering the Drell-Yan-West assignment, and then we get $F_{\gamma\gamma^*\rightarrow\eta}(Q^2)$ and $F_{\gamma\gamma^*\rightarrow\eta'}(Q^2)$ in the η_8 - η_0 mixing scheme,

$$F_{\gamma\gamma^*\rightarrow\eta}(Q^2) = F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2)\cos\theta - F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2)\sin\theta, \quad (45)$$

$$F_{\gamma\gamma^*\rightarrow\eta'}(Q^2) = F_{\gamma\gamma^*\rightarrow\eta_8}(Q^2)\sin\theta + F_{\gamma\gamma^*\rightarrow\eta_0}(Q^2)\cos\theta. \quad (46)$$

A. Numerical calculations

First of all, we would like to determine the mixing angle θ and decay constants of f_8 and f_0 by employing Eq. (39) to Eq. (42) with two different sets of experimental data which may cast some light on the clarification of the obvious current disagreement between the former Primakoff experiments and collider results in the measurements of $\Gamma(\eta \rightarrow \gamma\gamma)$, and then give more reasonable predictions on the mixing angle θ . From the Particle Data Group book [32], we get

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.74 \pm 0.54 \text{ eV}, \quad (47)$$

$$\Gamma(\eta \rightarrow \gamma\gamma) = 0.46 \pm 0.04 \text{ keV}, \quad (48)$$

$$\Gamma(\eta' \rightarrow \gamma\gamma) = 4.29 \pm 0.15 \text{ keV}, \quad (49)$$

and

$$m_{\pi^0} = 134.9766 \pm 0.0006 \text{ MeV}, \quad (50)$$

$$m_\eta = 547.30 \pm 0.12 \text{ MeV}, \quad (51)$$

$$m_{\eta'} = 957.78 \pm 0.14 \text{ MeV}. \quad (52)$$

We can get $\theta = -14.7^\circ \pm 2.0^\circ$, $f_0 = 1.13 \pm 0.08 f_\pi$, and $f_8 = 0.97 \pm 0.07 f_\pi$. However, $\Gamma(\eta \rightarrow \gamma\gamma) = 0.511 \pm 0.026 \text{ keV}$ if we do not include the Primakoff production measurement of $\eta \rightarrow \gamma\gamma$ [$\Gamma(\eta \rightarrow \gamma\gamma) = 0.324 \pm 0.046 \text{ keV}$] which obviously disagrees with other collider measurements. Therefore, we obtain $\theta = -16.1^\circ \pm 1.5^\circ$, $f_0 = 1.11 \pm 0.08 f_\pi$, and $f_8 = 0.95 \pm 0.07 f_\pi$. Moreover, we find that the mixing angle $\phi = \theta + \arctan \frac{1}{\sqrt{3}} = 38.6^\circ$ is compatible with [9] which gives the phenomenological value of the mixing angle $\phi = 39.3^\circ \pm 1.0^\circ$ from eight decay and scattering processes. The mixing independent ratio R can be defined as follows:

$$R \equiv \frac{M_\pi^3}{\Gamma(\pi \rightarrow \gamma\gamma)} \left[\frac{\Gamma(\eta \rightarrow \gamma\gamma)}{M_\eta^3} + \frac{\Gamma(\eta' \rightarrow \gamma\gamma)}{M_{\eta'}^3} \right] \quad (53)$$

$$= \frac{1}{3} \left(\frac{f_\pi^2}{f_8^2} + 8 \frac{f_\pi^2}{f_0^2} \right). \quad (54)$$

The current experimental value of R which was given in the proposal of the PrimEx Collaboration [6] at JLab is $R_{\text{exp}} = 2.5 \pm 0.5(\text{stat}) \pm 0.5(\text{syst})$. We can get $R = 2.45$ and $R = 2.54$, respectively, by using the above two sets of parameters. With the latter set of the fitted value of the mixing angle θ and decay constants of f_8 and f_0 as the input parameters, we can fix the remaining seven parameters by the following nine constraints.

In the formulas of the transition form factor $F_{\gamma\gamma^* \rightarrow P}(Q^2)$ ($P = \pi, \eta_8, \eta_0$), the parameters are the normalization constants A_π, A_8 and A_0 , the harmonic scale $\beta_\pi = \beta_8$ and β_0 , and the quark masses $m_q = m_u = m_d$ and m_s . In order to take a numerical calculation of the transition form factor $F_{\gamma\gamma^* \rightarrow M}(Q^2)$ and compare it with the available experimental data, we need to employ nine constraints to fix those seven parameters above. Thus, we can determine all these seven parameters in the transition form factor uniquely.

1. The decay widths of π, η , and η' [4,32]:

$$\begin{aligned} F_{\pi\gamma}(0) &= \sqrt{\frac{4}{\alpha^2 \pi M_\pi^3}} \Gamma(\pi \rightarrow \gamma\gamma) \\ &= 0.274 \pm 0.010 \text{ GeV}^{-1}, \quad 0.274 \text{ GeV}^{-1}, \end{aligned} \quad (55)$$

$$\begin{aligned} F_{\eta\gamma}(0) &= \sqrt{\frac{4}{\alpha^2 \pi M_\eta^3}} \Gamma(\eta \rightarrow \gamma\gamma) \\ &= 0.273 \pm 0.009 \text{ GeV}^{-1}, \quad 0.277 \text{ GeV}^{-1}, \end{aligned} \quad (56)$$

$$\begin{aligned} F_{\eta'\gamma}(0) &= \sqrt{\frac{4}{\alpha^2 \pi M_{\eta'}^3}} \Gamma(\eta' \rightarrow \gamma\gamma) \\ &= 0.342 \pm 0.006 \text{ GeV}^{-1}, \quad 0.343 \text{ GeV}^{-1}. \end{aligned} \quad (57)$$

2. The $Q^2 \rightarrow \infty$ limiting behavior of $Q^2 F_{\gamma\gamma^* \rightarrow P}(0) F_{\gamma\gamma^* \rightarrow P}(Q^2)$ [1,8,33]:

$$\lim_{Q^2 \rightarrow \infty} \pi^2 Q^2 F_{\gamma\gamma^* \rightarrow \pi}(0) F_{\gamma\gamma^* \rightarrow \pi}(Q^2) = \frac{1}{2}, \quad 0.49, \quad (58)$$

$$\lim_{Q^2 \rightarrow \infty} 3\pi^2 Q^2 F_{\gamma\gamma^* \rightarrow \eta_8}(0) F_{\gamma\gamma^* \rightarrow \eta_8}(Q^2) = \frac{1}{2}, \quad 0.48, \quad (59)$$

$$\lim_{Q^2 \rightarrow \infty} 3\pi^2 Q^2 F_{\gamma\gamma^* \rightarrow \eta_0}(0) F_{\gamma\gamma^* \rightarrow \eta_0}(Q^2) = 4, \quad 3.99. \quad (60)$$

3. The $Q^2 \rightarrow \infty$ limiting behavior of $Q^2 F_{\gamma\gamma^* \rightarrow P}(Q^2)$ [1,8,33]:

$$\begin{aligned} \lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma\gamma^* \rightarrow \pi}(Q^2) &= 2f_\pi = 184.8 \pm 0.2 \text{ MeV}, \\ &184.8 \text{ MeV}, \end{aligned} \quad (61)$$

$$\begin{aligned} \lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma\gamma^* \rightarrow \eta_8}(Q^2) &= \frac{2}{\sqrt{3}} f_8 = 101 \pm 7 \text{ MeV}, \\ &95 \text{ MeV}, \end{aligned} \quad (62)$$

$$\begin{aligned} \lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma\gamma^* \rightarrow \eta_0}(Q^2) &= \frac{4\sqrt{2}}{\sqrt{3}} f_0 = 334 \pm 15 \text{ MeV}, \\ &332 \text{ MeV}, \end{aligned} \quad (63)$$

in which the weak decay constant $f_\pi = 92.4 \text{ MeV}$ is defined [34] by $\pi \rightarrow \mu\nu$ decay.

These constraints are not completely independent, but are necessary since some of them are free from uncertainties; for example, Eqs. (59) and (60) are free from the decay constants f_0 and f_8 . Combined with consideration of other properties of the pion [3], we can obtain $m_q = 200 \text{ MeV}$, $m_s = 550 \text{ MeV}$, $\beta_\pi = \beta_8 = 410 \text{ MeV}$, $\beta_0 = 475 \text{ MeV}$, $A_\pi = 0.0475 \text{ MeV}^{-1}$, $A_8 = 0.0331 \text{ MeV}^{-1}$, and $A_0 = 0.0440 \text{ MeV}^{-1}$. Among these seven parameters, three of them (m_q , A_π , and β_π) are the same in our previous work [3] and have already been fixed; only the other four are new parameters. These three parameters satisfy Eqs. (55), (58), and (61) very well. Then we fix the four new parameters by using Eqs. (56), (57), (59), and (60). Since the parameter fixing scheme is somehow unique, numerical results of these parameters do not have much room to vary, and not surprisingly we find these fixed seven parameters give very good prediction for Eqs. (62) and (63). Reversely, we can compute the values of the above nine constraints by using the above seven fixed parameters, and we also provide the fitted values at the end of each equation. Therefore, after this simple parameter fixing scheme, we could start to calculate the transition form factor for these mesons.

The results are in good agreement with the experimental data which we have listed above. Moreover, it is interesting to notice that the masses of the light-flavor quarks (the up

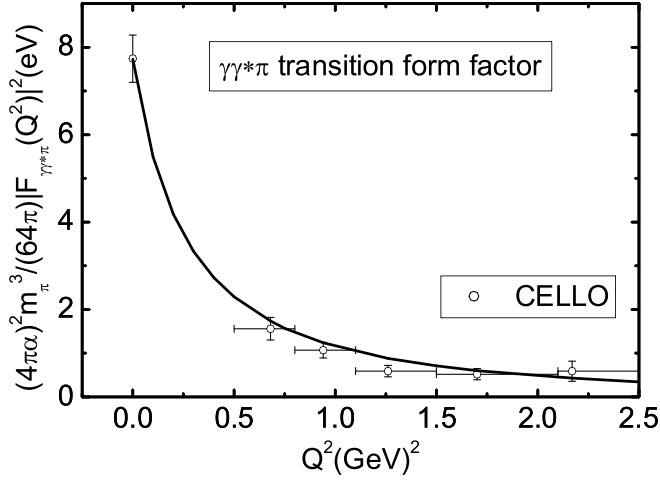


FIG. 2. Theoretical prediction of $(4\pi\alpha)^2 \times (m_\pi^3/64\pi)|F_{\gamma\gamma^*\pi}(Q^2)|^2$ calculated with the pion wave function in the BHL prescription compared with the experimental data. The data for the transition form factor are taken from Ref. [5].

quarks and down quarks) from the above constraints are just in the correct range (e.g., 200 ~ 300 MeV) of the constituent quark masses from more general considerations. Naturally, the transition form factor results emerging from this assumption are in quite good agreement with the experimental data.

Figure 2 indicates that the theoretical values of the photon-pion ($\gamma\gamma^* \rightarrow \pi$) transition form factors in the case of low Q^2 fit the experimental data well. One may consider this work as a light-cone version of the relativistic quark model [20,21], which should be valid in the low-energy scale about $Q^2 \leq 2 \text{ GeV}^2$. However, it is also physically in accordance with the light-cone perturbative QCD approach [1,2], which is applicable at the high-energy scale of $Q^2 > 2 \text{ GeV}^2$. The reason is that it is not

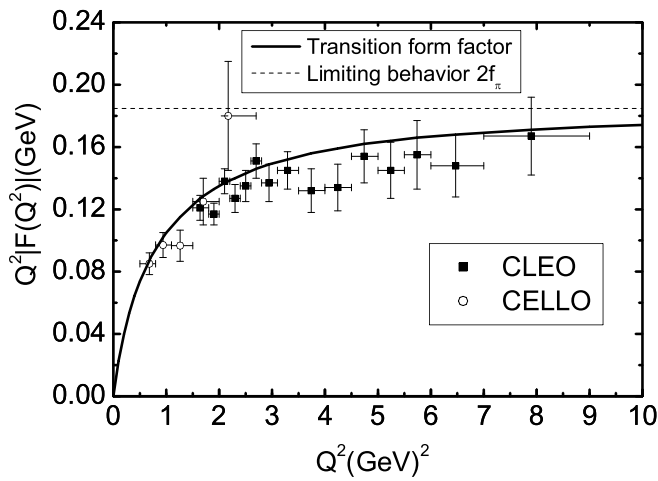


FIG. 3. Theoretical prediction of $Q^2 |F_{\gamma\gamma^*\pi}(Q^2)|$ calculated with the pion wave function in the BHL prescription compared with the experimental data. The data for the transition form factor are taken from Refs. [4,5].

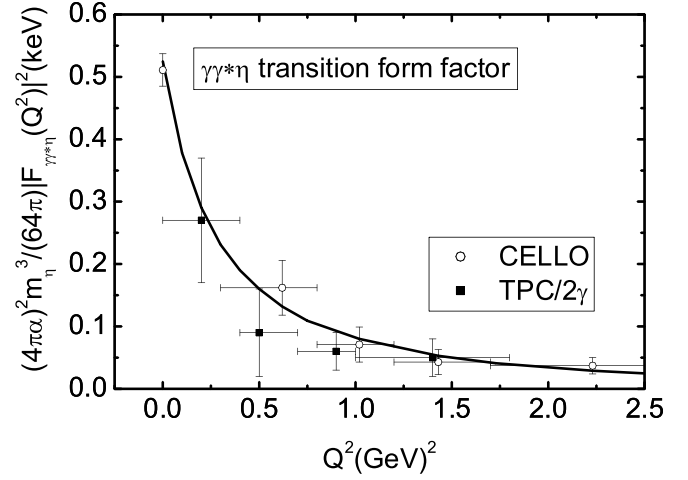


FIG. 4. Theoretical prediction of $(4\pi\alpha)^2 \times (m_\eta^3/64\pi)|F_{\gamma\gamma^*\eta}(Q^2)|^2$ compared with the experimental data in the low-energy scale. The data for the transition form factor are taken from Refs. [5,37].

necessary to incorporate the hard-gluon exchange between the quark and antiquark of the meson, which should be generally considered at high Q^2 for exclusive processes, in the light-cone perturbative QCD approach for the pion-photon transition form factor [1,2]. As a result, it is no wonder that our predictions for the transition form factor at high Q^2 also agree with the experimental data at the high-energy scale in Fig. 3.

Figs. 4 and 5 show that our predictions for the $\gamma^*\gamma \rightarrow \eta$ transition form factors agree with the experimental data in the low- and high-energy scale, respectively. In addition, the numerical results of the $\gamma^*\gamma \rightarrow \eta'$ transition form factor also give a good fit of the experiments both in the low- and moderately high-energy scale in Figs. 6 and 7. The prediction that we have made for the low Q^2 (0.001 – 0.5 GeV^2) behaviors of the photon-meson transition form

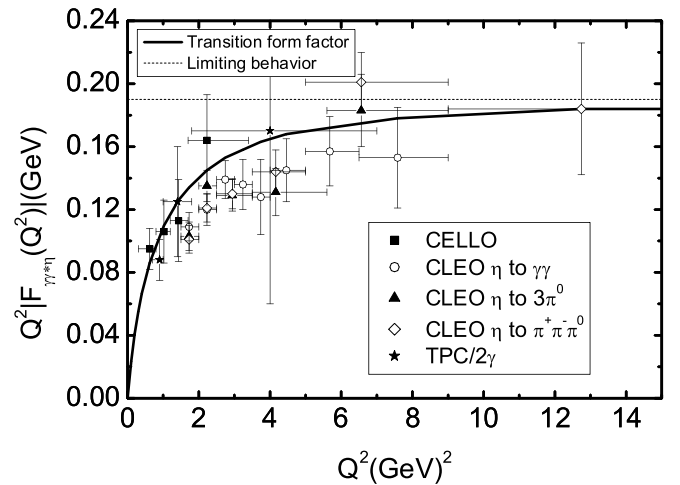


FIG. 5. Theoretical prediction of $Q^2 |F_{\gamma\gamma^*\eta}(Q^2)|$ compared with the experimental data. The data for the transition form factor are taken from Refs. [4,5,37].

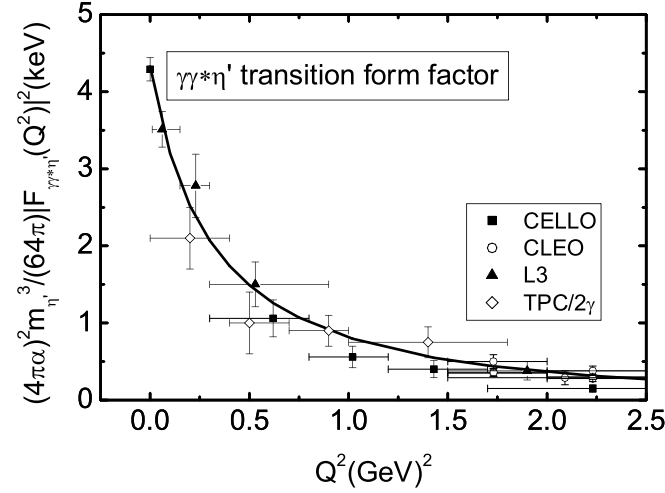


FIG. 6. Theoretical prediction of $(4\pi\alpha)^2 \times (m_\eta^3/64\pi)|F_{\gamma\gamma^*\eta'}(Q^2)|^2$ compared with the experimental data in the low-energy scale. The data for the transition form factor are taken from Refs. [4,5,37,38].

factors of π^0 , η , and η' are measurable in the $e + A(\text{Nucleus}) \rightarrow e + A + M$ process via the Primakoff effect at JLab and DESY.

Generally speaking, the medium to high Q^2 behavior of the transition form factors should include leading-twist order (so-called pQCD picture) and next-to-leading order corrections [35,36], but we only take the leading order into account in this literature. However, we find that our results for the leading order of the transition form factors fit the experimental data at small Q^2 well and are also physically consistent with the light-cone pQCD approach at large Q^2 .

V. CONCLUSION

The light-cone formalism provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom, and for the application of perturbative QCD to exclusive processes. With the minimal Fock-state expansions of the pion and photon wave functions from the light-cone representation of the spin structure of the pseudoscalar meson and photon vertices, we investigate the photon-meson transition form

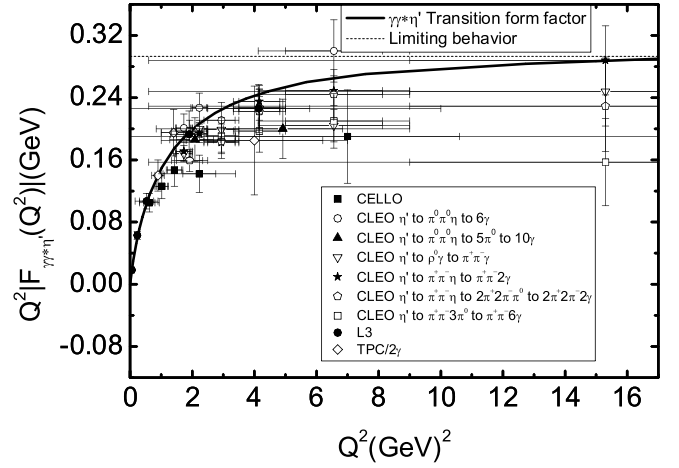


FIG. 7. Theoretical prediction of $Q^2|F_{\gamma\gamma^*\eta'}(Q^2)|$ compared with the experimental data. The data for the transition form factor are taken from Refs. [4,5,37,38].

factors by adopting the Drell-Yan-West assignment to get the light-cone framework that works at both low Q^2 and high Q^2 . We employ the experimental values of the decay widths of π , η , and η' , the limiting behavior of $\lim_{Q^2 \rightarrow \infty} Q^2 F_{\gamma^*\gamma \rightarrow M}(Q^2) F_{\gamma^*\gamma \rightarrow M}(0)$ ($M = \pi, \eta_8, \eta_0$), and the limiting behavior of $Q^2 F_{\gamma^*\gamma \rightarrow M}(Q^2)$ as the nine constraints to fix those seven parameters in the π , η_8 , and η_0 wave functions. With the fixed π , η_8 , and η_0 wave functions, we find that our numerical predictions for the photon-meson transition form factors of light pseudoscalar mesons π , η , and η' agree with the experimental data at both the low- and high-energy scales, in a wide region compared to previous studies.

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