$B_d^0(t) \to \pi^+\pi^-$ and $B_s^0(t) \to K^+K^-$ decays: A tool to measure new-physics parameters

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If physics beyond the standard model is present in B decays, experimental measurements seem to suggest that it principally affects those processes with significant $\bar{b} \to \bar{s}$ penguin amplitudes. It was recently argued that, in general, such new-physics (NP) effects can be parametrized in terms of a single NP amplitude $\mathcal{A}^q$ and phase Φ_q, for $q = u, d, s, c$. In this paper, we show that the study of the decays $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$ allows one to measure the NP parameters $\mathcal{A}^u$ and Φ_u. We examine the implications for this method of the latest experimental results on these decays. If NP is found in $B_s^0(t) \to K^+K^-$, it can be partially identified through measurements of $B_s^0(t) \to K^0\bar{K}^0$.

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To date, theoretical analyses of CP violation in the B system have generally concentrated on two subjects. First, many different methods have been proposed for extracting the CP-violating angles α, β and γ of the unitarity triangle [1] within the standard model (SM) [2]. Second, there have been numerous studies of new-physics (NP) signals through measurements of CP violation in B decays. We now have many ways of detecting the presence of NP. However, there has been relatively little work on the third and final ingredient, which is to find ways of *measuring* the NP parameters. If this can be done it might be possible to identify the new-physics, before the production of new particles at high-energy colliders.

A first step in this direction was taken in Ref. [3], in which it was shown that one can reduce the number of NP parameters to a manageable level, and measure them. The knowledge of these parameters then allows one to partially identify the new-physics. The argument goes as follows.

At present, we have a number of experimental hints of new-physics. First, within the SM, the CP asymmetry in $B_d^0(t) \to J/\psi K_S$ ($\sin 2\beta$) is equal to that measured in other decays dominated by the quark-level transition $\bar{b} \to \bar{s}s\bar{s}$. However, Belle finds a discrepancy of 2.2σ between the CP asymmetry in $B_d^0(t) \to \phi K_S$ and that in $B_d^0(t) \to J/\psi K_S$ [4]. In addition, the *BABAR* measurement of the CP asymmetry in $B_d^0(t) \to \eta' K_S$ differs from that in $B_d^0(t) \to J/\psi K_S$ by 3.0σ [5]. Second, ratios of various $B \to \pi K$ branching ratios, which are equal in the SM [6], are found to differ from one another by 1.6σ [7]. Furthermore, Belle finds a 2.4σ discrepancy with the SM in $B \to \pi K$ direct asymmetries: $A_{CP}(K^+\pi^-) \neq A_{CP}(K^\pm\pi^0)$, a result confirmed by *BABAR* [4]. Third, *BABAR* has measured a nonzero triple-product asymmetry in $B \to \phi K^*$ at 1.7σ [8]. However, this effect is expected to vanish in the SM [9]. Note that all of these new-physics signals should be viewed with some skepticism. None of them is statistically significant, and the two experiments have not yet converged on any of the above measurements. For example, while *BABAR* finds $\sin 2\beta[B_d^0(t) \to \eta' K_S] = 0.27 \pm 0.14 \pm 0.03$ [10], Belle finds $\sin 2\beta[B_d^0(t) \to \eta' K_S] = 0.65 \pm 0.18 \pm 0.04$ [11], which is consistent with the value of $\sin 2\beta$ found in $B_d^0(t) \to J/\psi K_S$. Still, these hints are intriguing, since all involve B decays which receive large contributions from $\bar{b} \to \bar{s}$ penguin amplitudes. In this paper (as in Ref. [3]), we therefore adopt the point of view that there is new-physics, and that it contributes significantly only to those decays with sizeable $\bar{b} \to \bar{s}$ penguin amplitudes, but does not affect decays involving $\bar{b} \to \bar{d}$ penguins.

Consider now a $B \to f$ decay with a $\bar{b} \to \bar{s}$ penguin. The NP operators are assumed to be roughly the same size as the SM $\bar{b} \to \bar{s}$ penguin operators, so that the new effects are important. There are many potential NP operators. At the quark level, these take the form $\mathcal{O}_{\rm NP}^{ij,q} \sim \bar{s}\Gamma_i b \bar{q} \Gamma_j q$ ($q = u, d, s, c$), where the $\Gamma_{i,j}$ represent Lorentz structures, and color indices are suppressed. The NP contributes to $B \to f$ through the matrix elements $\langle f | \mathcal{O}_{\rm NP}^{ij,q} | B \rangle$. Each of the matrix elements can have different weak phases and in principle each can also have a different strong phase. However, it was argued in Ref. [3] that all NP strong phases are negligible, which leads to a great simplification: one can combine all NP matrix elements into a single NP amplitude, with a single weak phase:

$$\sum \langle f | \mathcal{O}_{\rm NP}^{ij,q} | B \rangle = \mathcal{A}^q e^{i\Phi_q}, \tag{1}$$

where $q = u, d, s, c$. (In general, the $\mathcal{A}^q$ and Φ_q will be process-dependent. The NP phase Φ_q will be the same for all $\bar{b} \to \bar{s}q\bar{q}$ decays only if all NP operators for the same quark-level process have the same weak phase.)

While this argument—that the new-physics strong phases are negligible—is quite general, there are still

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ways of getting around this result. For example, it does not hold if the NP is quite light, or if there is a significant enhancement of certain matrix elements. The reader should therefore be aware of these caveats.

In Ref. [12], a number of methods for measuring the $\mathcal{A}^q$ and Φ_q were examined. Here we analyze another method. It consists of using $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decays. These two decays, which are related by flavor SU(3), have been used in the past to both obtain information about SM parameters [13,14] and to detect the presence of new-physics [15,16]. In all cases, one has to address the size of SU(3)-breaking effects [17]. In the present paper, we show that measurements of these two decays actually allow one to *measure* the NP parameters $\mathcal{A}^u$ and Φ_u.

Consider the decay $B_s^0 \to K^+K^-$ within the SM. It is governed by the quark-level process $\bar{b} \to \bar{s}u\bar{u}$, and in terms of diagrams [18], the amplitude receives several contributions:

$$A(B_s^0 \to K^+K^-) = -T' - P' - E' - PA' - \frac{2}{3}P_{\rm EW}^{\prime C}. \quad (2)$$

In the above, the amplitude is written in terms of a color-favored tree amplitude T', a gluonic penguin amplitude P', an exchange amplitude E', a penguin annihilation amplitude PA', and a color-suppressed electroweak penguin amplitude $P_{\rm EW}^{\prime C}$. (The primes on the amplitudes indicate a $\bar{b} \to \bar{s}$ transition.) These various contributions can be grouped into two types. There are charged-current contributions, proportional to $V_{ub}^* V_{us}$, and the penguin-type contributions $V_{ub}^* V_{us} P_u' + V_{cb}^* V_{cs} P_c' + V_{tb}^* V_{ts} P_t'$. (Here the charged-current term includes T' and E', while the penguin term includes P', PA' and $P_{EW}^{\prime C}$.) The unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix can be used to eliminate $V_{tb}^* V_{ts}$, so that the penguin-type contributions become $V_{ub}^* V_{us}(P_u' - P_t') + V_{cb}^* V_{cs}(P_c' - P_t')$. The amplitude for $B_s^0 \to K^+K^-$ can then be written as [13]

$$\begin{aligned} A(B_s^0 \to K^+K^-) &= V_{ub}^* V_{us}(A_{CC}^{\prime u} + A_{\rm pen}^{\prime ut}) + V_{cb}^* V_{cs} A_{\rm pen}^{\prime ct} \\ &= \left|\frac{V_{us}}{V_{ud}}\right| C' e^{i\theta_{C'}} \left(e^{i\gamma} + \left|\frac{V_{cs}V_{ud}}{V_{us}V_{cd}}\right| d' e^{i\theta'}\right). \end{aligned} \quad (3)$$

(Magnitudes of CKM matrix elements, such as $|V_{us}/V_{ud}|$, have been included to permit easy comparison with the amplitude for the decay $B_d^0 \to \pi^+\pi^-$ [13,14]). Here, $A_{\rm pen}^{\prime it} \equiv P_i' - P_t'$ ($i = u, c$), γ is a SM CP-violating phase [1,13]

$$\begin{aligned} C' e^{i\theta_{C'}} &\equiv |V_{ub}^* V_{ud}|(A_{\rm CC}^{\prime u} + A_{\rm pen}^{\prime ut}), \\ d' e^{i\theta'} &\equiv \frac{1}{R_b}\left(\frac{A_{\rm pen}^{\prime ct}}{A_{CC}^{\prime u} + A_{\rm pen}^{\prime ut}}\right), \end{aligned} \quad (4)$$

with $R_b = |(V_{ub}^* V_{ud})/(V_{cb}^* V_{cd})|$. In the above, θ' is the relative strong phase between $A_{\rm pen}^{\prime ct}$ and $(A_{CC}^{\prime u} + A_{\rm pen}^{\prime ut})$, and $\theta_{C'}$ is the strong phase associated with C'. (If one works entirely within the SM, $\theta_{C'}$ is unimportant.)

Of course, since $B_s^0 \to K^+K^-$ involves a $\bar{b} \to \bar{s}$ penguin amplitude, one must include the effects of new-physics. In this case, only the u-quark NP parameters appear [see Eq. (1)]. Including these, the full amplitude for $B_s^0 \to K^+K^-$ can be written

$$\begin{aligned} A(B_s^0 \to K^+K^-) = &\left|\frac{V_{us}}{V_{ud}}\right| C' e^{i\theta_{C'}} \\ &\times \left(e^{i\gamma} + \left|\frac{V_{cs}V_{ud}}{V_{us}V_{cd}}\right| d' e^{i\theta'}\right) + \mathcal{A}^u e^{i\Phi_u}. \end{aligned} \quad (5)$$

The amplitude for the CP-conjugate process, $\overline{B}_s^0 \to K^+K^-$, can be obtained from the above by simply changing the signs of the weak phases γ and Φ_u.

There are a total of three measurements which can be made of $B_s^0(t) \to K^+K^-$: the total branching ratio, the direct CP asymmetry, and the mixing-induced CP asymmetry. However, these measurements depend on eight theoretical parameters: C', d', $\theta_{C'}$, θ', $\mathcal{A}^u$, Φ_u, γ, and the B_s^0–$\overline{B}_s^0$ mixing phase, ϕ_s.

Two of these parameters can be measured independently. First, the weak phase γ can be obtained from B decays which are unaffected by new-physics in $\bar{b} \to \bar{s}$ penguin amplitudes. For instance, it can be obtained from $B^\pm \to DK$ decays [19]. Alternatively, the angle α can be extracted from $B \to \pi\pi$ [20], $B \to \rho\pi$ [21] or $B \to \rho\rho$ decays [22]. β is already known from measurements of $B_d^0(t) \to J/\psi K_S$ [23], so that γ can be obtained using $\gamma = \pi - \beta - \gamma$. Finally, it is also possible to take γ from a fit to the various unitarity-triangle measurements. (The present preferred value is $\gamma = 62°$ [23,24].)

The second parameter is ϕ_s, the phase of B_s^0–$\overline{B}_s^0$ mixing. However, its measurement may pose theoretical problems. The standard way to measure ϕ_s is through CP violation in $B_s^0(t) \to J/\psi\eta$ (or $B_s^0(t) \to J/\psi\phi$). However, there is a potential problem here: this decay receives NP contributions from $O_{\rm NP}^c \sim \bar{s}b\bar{c}c$ operators (the Lorentz and color structures have been suppressed), so that there may be effects from these NP operators in the measurement of ϕ_s.

The solution to this problem can be found by considering B_d^0–$\overline{B}_d^0$ mixing. The phase of this mixing is unaffected by new-physics and thus takes its SM value, β. The canonical way to measure this angle is via CP violation in $B_d^0(t) \to J/\psi K_S$. However, this decay also receives NP contributions from $O_{\rm NP}^c$ operators. On the other hand, the value of β extracted from $B_d^0(t) \to J/\psi K_S$ is in line with SM expectations. This strongly suggests that any $O_{\rm NP}^c$ contributions to this decay are quite small, say $\lesssim 20\%$. Now, the nonstrange part of the η wave function has a negligible contribution to $\langle J/\psi\eta|O_{\rm NP}^c|B_s^0\rangle$. Thus, this matrix element can be related by flavor SU(3) to $\langle J/\psi K_S|O_{\rm NP}^c|B_d^0\rangle$ (up to a mixing angle). That is, both matrix elements are very small. In other words, we do not expect significant $O_{\rm NP}^c$ contributions to $B_s^0(t) \to J/\psi\eta$, so that ϕ_s can be measured through CP violation in this decay, even in the presence of NP. (Note that, in general, NP which affects $\bar{b} \to \bar{s}$ transitions will also contribute to B_s^0–$\overline{B}_s^0$ mixing, i.e., one will have NP operators of the form $\bar{s}b\bar{b}s$. In this case, the phase of B_s^0–$\overline{B}_s^0$ mixing may well differ from its SM value ($\simeq 0$) due to the presence of NP.)

Assuming that γ and ϕ_s are measured independently, the $B_s^0(t) \to K^+K^-$ amplitude still depends on six unknown theoretical quantities: C', d', $\theta_{C'}$, θ', $\mathcal{A}^u$ and Φ_u. With only three experimental measurements, one cannot extract the theoretical parameters. In particular, we cannot obtain the NP quantities $\mathcal{A}^u$ and Φ_u.

The necessary additional information can be obtained by considering measurements of $B_d^0 \to \pi^+\pi^-$. Within the SM, this decay is related by flavor SU(3) to $B_s^0 \to K^+K^-$. However, because it is described at the quark level by $\bar{b} \to \bar{d}u\bar{u}$, it does not receive NP contributions. Its amplitude can be written in terms of diagrams as

$$A(B_d^0 \to \pi^+\pi^-) = -T - P - E - PA - \frac{2}{3}P_{\rm EW}^C, \tag{6}$$

where the diagrams are written without primes to indicate a $\bar{b} \to \bar{d}$ transition.

Similar to $B_s^0 \to K^+K^-$, the amplitude for $B_d^0 \to \pi^+\pi^-$ can be written in terms of pieces proportional to $V_{ub}^*V_{ud}$ and $V_{cb}^*V_{cd}$ [13–16]:

$$\begin{aligned} A(B_d^0 \to \pi^+\pi^-) &= V_{ub}^*V_{ud}(A_{CC}^u + A_{\rm pen}^{ut}) + V_{cb}^*V_{cd}A_{\rm pen}^{ct} \\ &= Ce^{i\theta_C}(e^{i\gamma} - de^{i\theta}), \end{aligned} \tag{7}$$

where $A_{\rm pen}^{it} \equiv P_i - P_t$ ($i = u, c$), and

$$\begin{aligned} Ce^{i\theta_C} &\equiv |V_{ub}^*V_{ud}|(A_{\rm CC}^u + A_{\rm pen}^{ut}), \\ de^{i\theta} &\equiv \frac{1}{R_b}\left(\frac{A_{\rm pen}^{ct}}{A_{CC}^u + A_{\rm pen}^{ut}}\right). \end{aligned} \tag{8}$$

Since the strong phase θ_C cannot be measured (it is just an overall phase), the $B_d^0 \to \pi^+\pi^-$ amplitude depends on the four quantities γ, C, d and θ.

As with $B_s^0(t) \to K^+K^-$, there are three measurements to be made in $B_d^0(t) \to \pi^+\pi^-$: the total branching ratio, the direct CP asymmetry, and the mixing-induced CP asymmetry. Thus, assuming that γ has been measured independently, the three $B_d^0(t) \to \pi^+\pi^-$ measurements allow the extraction of C, d and θ. However, assuming a perfect flavor SU(3) symmetry, we have $C' = C$, $d' = d$ and $\theta' = \theta$. With this information, the number of unknown theoretical quantities in $B_s^0(t) \to K^+K^-$ is reduced to three: $\theta_{C'}$ $\mathcal{A}^u$ and Φ_u. The three measurements of $B_s^0(t) \to K^+K^-$ therefore permit us to obtain the parameters $\mathcal{A}^u$ and Φ_u.

This discussion shows that the measurements of $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$, along with the independent determinations of γ and ϕ_s, allow one to *measure* the NP quantities $\mathcal{A}^u$ and Φ_u. The knowledge of these parameters, along with those obtained via other methods [12] will allow us to rule out many NP models and thus partially identify the new-physics.

Of course, in practice, one has to examine the theoretical precision with which $\mathcal{A}^u$ and Φ_u can be obtained. In particular, one has to take into account the SU(3)-breaking effects in relating C', d' and θ' to C, d and θ. The ratio $|C'/C|$ has recently been calculated using QCD sum rules [25]:

$$\left|\frac{C'}{C}\right| = 1.76^{+0.15}_{-0.17}. \tag{9}$$

However, the effect of SU(3) breaking in relating the other parameters has not yet been computed. In Ref. [16], the impact of U-spin breaking was explored by allowing the parameters to vary in certain ranges: $d'/d = 1.0 \pm 0.2$, and $\theta' - \theta = 0° \pm 40°$. These uncertainties must be included in the extraction of $\mathcal{A}^u$ and Φ_u. (Note: the assumed size of SU(3) breaking in d'/d and $\theta' - \theta$ is smaller than that found for $|C'/C|$ in Eq. (9). This is because all factorizable SU(3)-breaking effects cancel in the ratio d'/d, leaving only nonfactorizable corrections. SU(3) breaking in $|C'/C|$ is expected to be larger since both factorizable and nonfactorizable terms are present.)

Suppose now that new-physics has been found through measurements of $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$, i.e., $\mathcal{A}^u$ and Φ_u have been seen to be nonzero. Below we show how measurements of $B_s^0 \to K^0\bar{K}^0$ can be used to partially identify the NP.

The SM amplitude for $B_s^0 \to K^+K^-$ is given in Eq. (2) in terms of the diagrams T', P', E', PA' and $P_{\rm EW}^{\prime C}$. However, it is believed that $|E'|$, $|PA'|$, $|P_{\rm EW}^{\prime C}| \ll |P'|$. Neglecting these small pieces–the theoretical error is expected to be at the level of $\sim$5%–the decay amplitude becomes

$$A(B_s^0 \to K^+K^-) \simeq -T' - P'. \tag{10}$$

This can be written as in Eq. (5) (repeated here for convenience):

$$\begin{aligned} A(B_s^0 \to K^+K^-) = {}& \left|\frac{V_{us}}{V_{ud}}\right| C'e^{i\theta_{C'}} \\ & \times \left(e^{i\gamma} + \left|\frac{V_{cs}V_{ud}}{V_{us}V_{cd}}\right| d'e^{i\theta'}\right) + \mathcal{A}^u e^{i\Phi_u}, \end{aligned} \tag{11}$$

where we have included the new-physics contribution. Here the SM parameters (C', d', etc.) are related only to the diagrams T' and P':

$$\begin{aligned} \left|\frac{V_{us}}{V_{ud}}\right| C'e^{i\theta_{C'}}e^{i\gamma} &\equiv T' + V_{ub}^*V_{us}(P_u' - P_t'), \\ \left|\frac{V_{us}}{V_{ud}}\right| C'd'e^{i(\theta_{C'}+\theta')} &\equiv V_{cb}^*V_{cs}(P_c' - P_t'). \end{aligned} \tag{12}$$

One can similarly neglect the small amplitudes in $B_d^0 \to \pi^+\pi^-$, so that

$$A(B_d^0 \to \pi^+\pi^-) \simeq -T - P = Ce^{i\theta_C}(e^{i\gamma} - de^{i\theta}), \quad (13)$$

where again the SM parameters are related to T and P only. The method described earlier still applies. Assuming that γ and ϕ_s are known independently, measurements of $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$ permit the extraction of all theoretical quantities: the SM parameters, as well as $\mathcal{A}^u$ and Φ_u. This is equivalent to measuring T', $V_{ub}^* V_{us}(P'_u - P'_t)$ and $V_{cb}^* V_{cs}(P'_c - P'_t)$, along with the NP parameters.

Now, the SM amplitude for $B_s^0 \to K^0\bar{K}^0$ is given by

$$A(B_s^0 \to K^0\bar{K}^0) = P' + PA' - \frac{1}{3}P_{\rm EW}^{\prime C}. \quad (14)$$

As above, we can neglect PA' and $P_{\rm EW}^{\prime C}$. P' also contains a piece proportional to $V_{ub}^* V_{us}$. However, $|V_{ub}^* V_{us}(P'_u - P'_t)| \ll |V_{cb}^* V_{cs}(P'_c - P'_t)|$ since $|V_{ub}^* V_{us}/V_{cb}^* V_{cs}| \simeq 2\%$. We therefore retain only the piece $V_{cb}^* V_{cs}(P'_c - P'_t)$. New-physics can affect this decay as well, but here only the d-quark NP parameters appear:

$$A(B_s^0 \to K^0\bar{K}^0) = V_{cb}^* V_{cs}(P'_c - P'_t) + \mathcal{A}^d e^{i\Phi_d}. \quad (15)$$

Note that $V_{cb}^* V_{cs}(P'_c - P'_t)$ is already known from the $B_s^0(t) \to K^+K^-$ analysis.

The measurement of $B_s^0(t) \to K^0\bar{K}^0$ now allows us to partially identify the new-physics. If we assume that the NP is isospin-conserving then $\mathcal{A}^d = \mathcal{A}^u$ and $\Phi_d = \Phi_u$. In this case, the $B_s^0(t) \to K^+K^-$ analysis gives us *all* of the theoretical parameters in $A(B_s^0 \to K^0\bar{K}^0)$, and we can predict the values of the various measurable quantities in this decay (the total branching ratio, the direct CP asymmetry, and the mixing-induced CP asymmetry). We can then make measurements of $B_s^0(t) \to K^0\bar{K}^0$ and compare the results with the theoretical predictions. If they disagree, we will have ruled out isospin-conserving NP.

Particular types of new-physics also allow us to establish the amplitude for $B_s^0 \to K^0\bar{K}^0$. For example, suppose that Z-mediated $\bar{b} \to \bar{s}$ flavor-changing neutral currents are present. In this case, we know how the d-quark NP parameters are related to the u-quark NP parameters. In particular, $\Phi_d = \Phi_u$ and $\mathcal{A}^d = \mathcal{A}^u \times c_d(Z)/c_u(Z)$, where $c_d(Z)$ and $c_u(Z)$ are the (known) Z couplings to $\bar{d}d$ and $\bar{u}u$, respectively. Thus, for this model of NP, once again we know the full $B_s^0 \to K^0\bar{K}^0$ amplitude, and can compare experimental measurements with theoretical predictions. A disagreement (beyond the level of the theoretical uncertainty) will allow us to rule out this type of NP.

Finally, we examine the latest experimental results on $B_d^0(t) \to \pi^+\pi^-$ and $B_s^0(t) \to K^+K^-$, and discuss their implications for the method of measuring new-physics parameters outlined earlier. We use data from *BABAR* [26], Belle [27] and CDF [28].

We begin by presenting the SM expressions for $\langle {\rm BR}(B_s^0 \to K^+K^-)\rangle$, $A_{\rm dir}(B_s^0 \to K^+K^-)$ and $A_{\rm mix}(B_s^0 \to K^+K^-)$ [13]:

$$\langle {\rm BR}(B_s^0 \to K^+K^-)\rangle^{\rm SM} = g_{ps}\epsilon C'^2 \Delta, \quad (16)$$

$$A_{\rm dir}^{\rm SM} = \frac{1}{\Delta}(2\tilde{d}' \sin\gamma \sin\theta'), \quad (17)$$

$$A_{\rm mix}^{\rm SM} = \frac{1}{\Delta}[\sin(2\gamma + \phi_s^{\rm SM}) + 2\tilde{d}' \cos\theta' \sin(\gamma + \phi_s^{\rm SM}) + \tilde{d}'^2 \sin\phi_s^{\rm SM}]. \quad (18)$$

In the above, $\epsilon \equiv |V_{us}/V_{ud}|^2$, $\tilde{d}' \equiv |(V_{cs}V_{ud})/(V_{us}V_{cd})|d'$, $\Delta = 1 + 2\tilde{d}' \cos\gamma \cos\theta' + \tilde{d}'^2$ and $g_{ps} = \tau_{Bs}\sqrt{1 - 4m_K^2/m_{B_s^0}^2}/(16\pi m_{B_s^0})$.

In the presence of new-physics, the deviations of these observables from their SM expressions are given by

$$\langle {\rm BR}(B_s^0 \to K^+K^-)\rangle = \langle {\rm BR}(B_s^0 \to K^+K^-)\rangle^{\rm SM} \cdot (1 + \mathcal{B}^{\rm NP}), \quad (19)$$

$$A_{\rm dir}(B_s^0 \to K^+K^-) = \frac{A_{\rm dir}^{\rm SM} + \mathcal{D}^{\rm NP}}{1 + \mathcal{B}^{\rm NP}}, \quad (20)$$

$$A_{\rm mix}(B_s^0 \to K^+K^-) = \frac{A_{\rm mix}^{\rm SM} \cdot \cos\delta\phi_s^{\rm NP} + \mathcal{M}^{\rm NP}}{1 + \mathcal{B}^{\rm NP}}, \quad (21)$$

where $\phi_s = \phi_s^{\rm SM} + \delta\phi_s^{\rm NP}$ [29], and

$$\mathcal{B}^{\rm NP} = \frac{1}{\Delta}\{z^2 + 2z[\cos\theta_{C'} \cos(\Phi_u - \gamma) + \tilde{d}' \cos\Phi_u \cos(\theta_{C'} + \theta')]\}, \quad (22)$$

$$\mathcal{D}^{\rm NP} = \frac{1}{\Delta}\{2z[\tilde{d}' \sin\Phi_u \sin(\theta_{C'} + \theta') + \sin\theta_{C'} \sin(\Phi_u - \gamma)]\}, \quad (23)$$

$$\mathcal{M}^{\rm NP} = \frac{1}{\Delta}\{z^2 \sin(2\Phi_u + \phi_s) + 2z[\cos\theta_{C'} \sin(\Phi_u + \phi_s + \gamma) + \tilde{d}' \cos(\theta_{C'} + \theta') \sin(\Phi_u + \phi_s)] + [\cos(2\gamma + \phi_s^{\rm SM}) + 2\tilde{d}' \cos\theta' \cos(\gamma + \phi_s^{\rm SM}) + \tilde{d}'^2 \cos\phi_s^{\rm SM}] \sin\delta\phi_s^{\rm NP}\}, \quad (24)$$

with $z = \mathcal{A}^u/(\sqrt{\epsilon}C')$. In order to measure these NP functions, one needs predictions for $A_{\rm dir}^{\rm SM}(B_s^0 \to K^+K^-)$, $A_{\rm mix}^{\rm SM}(B_s^0 \to K^+K^-)$, and ${\rm BR}^{\rm SM}(B_s^0 \to K^+K^-)$. As described earlier, this information can be obtained from measurements of $B_d^0(t) \to \pi^+\pi^-$. To be precise, these measurements allow one to extract d, θ, and C, which then allow us to obtain the SM predictions for the $B_s^0 \to K^+K^-$ observables, given a prescription for the inclusion of U-spin breaking effects.

The present experimental situation of the mixing-induced and direct CP asymmetry of $B_d^0 \to \pi^+\pi^-$ is still quite uncertain [26,27]:

$$A_{\rm dir}(B_d^0 \to \pi^+\pi^-) = \begin{cases} -0.09 \pm 0.15 \pm 0.04 & (BABAR) \\ -0.58 \pm 0.15 \pm 0.07 & (\text{Belle}) \end{cases} \quad (25)$$

$$A_{\rm mix}(B_d^0 \to \pi^+\pi^-) = \begin{cases} 0.30 \pm 0.17 \pm 0.03 & (BABAR) \\ 1.00 \pm 0.21 \pm 0.07 & (\text{Belle}) \end{cases} \quad (26)$$

$$\langle \text{BR}(B_d^0 \to \pi^+\pi^-)\rangle = \begin{cases} (4.7 \pm 0.6 \pm 0.2) \times 10^{-6} & (BABAR) \\ (4.4 \pm 0.6 \pm 0.3) \times 10^{-6} & (\text{Belle}) \end{cases} \quad (27)$$

In light of this, we perform the analysis separately for the *BABAR* and Belle data.

The range of values for d and θ are obtained as in [14] and corresponds to the shaded regions in Fig. 1(a) (*BABAR*) and Fig. 1(b) (Belle). The CKM angle γ is taken to be in the range $\gamma(\text{degrees}) = 62^{+10}_{-12}$ [23] and the B_d^0–$\bar{B}_d^0$ weak mixing phase is set to $\phi_d = 48°$ [23].

From the *BABAR* experimental values (within $\pm 1\sigma$) we find two separate sets of solutions for d and θ which are compatible with $A_{\rm mix}(B_d^0 \to \pi^+\pi^-)$ and $A_{\rm dir}(B_d^0 \to \pi^+\pi^-)$. The first solution gives an unconstrained range for θ, while the allowed values for d are:

$$d \in (0, 0.47) \quad (28)$$

The value of the parameter C can be obtained from Eqs. (16), (27), and (28), and lies in the following range:

$$C \in (1.7, 2.8) \times 10^{-8}\ \text{GeV}. \quad (29)$$

The second solution has $d > 2.5$, which is strongly disfavored by QCD-factorization arguments [30] which predicts a value around 0.3 [31]. For this reason, we do not consider this second solution further. Moreover, in [14] it was shown that these large values of d give rise to complex solutions when the ratio of neutral channels are taken into account and are therefore unphysical.

Turning to the Belle data, we obtain a very large range for d, which also includes quite unrealistic values. However, since these values are not split into two regions, here we will take the complete range. The allowed values for the hadronic parameters with the Belle data are:

$$d \in (0.51, 3.95), \qquad \theta \in (102°, 147°), \quad (30)$$

and the corresponding range for C is given by:

$$C \in (0.5, 1.9) \times 10^{-8}\ \text{GeV}. \quad (31)$$

The predicted values for the mixing-induced and direct CP asymmetries for $B_s^0(t) \to K^+K^-$ in the SM are given in Table I on the basis of the *BABAR* and Belle data. These ranges take into account U-spin breaking effects, as described in Eq. (9) and the discussion following it. They also include the correlations between d and θ for each value of γ implied by the shaded regions in Figs. 1(a) and 1(b). While the prediction from present *BABAR* data is compatible with all possible values for the $B_s^0(t) \to K^+K^-$ asymmetries, Belle gives quite constrained ranges. In Ref. [14] the average value of these asymmetries with previous data was also computed.

At present, the observables $A_{\rm dir}(B_s^0 \to K^+K^-)$, $A_{\rm mix}(B_s^0 \to K^+K^-)$ have not yet been measured, so we cannot give estimates for $\mathcal{D}^{\rm NP}$ and $\mathcal{M}^{\rm NP}$. As for $\text{BR}(B_s^0 \to K^+K^-)$, it is more useful to present the ratio of branching ratios of $B_s^0 \to K^+K^-$ and $B_d^0 \to \pi^+\pi^-$: $R_d^{s\rm SM} \equiv BR(B_s^0 \to K^+K^-)/BR(B_d^0 \to \pi^+\pi^-)$ [16]. This ratio, which has been measured experimentally [28], does not require in the SM the knowledge of C and C' individually, but only the ratio, which can be taken from Eq. (9). Our prediction for this quantity in the SM is:

$$0.1 \leq R_{dBABAR}^{s\rm SM} \leq 13.9,\ 3.6 \leq R_{d\rm Belle}^{s\rm SM} \leq 85.5. \quad (32)$$

This can be compared with a recent experimental measurement at CDF [28]:

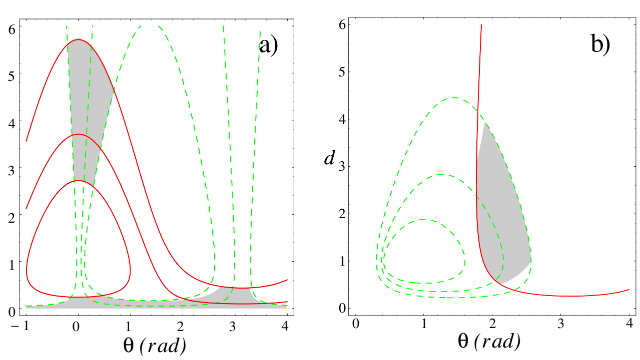

FIG. 1 (color online). (a) The dashed lines show the allowed region in the $d-\theta$ plane using only *BABAR* experimental data on $A_{\rm dir}(B_d^0 \to \pi^+\pi^-)$. We allow for $\pm 1\sigma$ deviations of $A_{\rm dir}(B_d^0 \to \pi^+\pi^-)$ and γ. The solid lines show the same for the measured value of $A_{\rm mix}(B_d^0 \to \pi^+\pi^-)$, also at $\pm 1\sigma$. The shaded regions are those values consistent with both observables for each value of γ. (b) Same for Belle experimental data.

TABLE I. Predicted ranges for the $B_s^0(t) \to K^+K^-$ CP asymmetries in the SM using Eqs. (16)–(18). U-spin breaking effects have been included as explained in the text.

Observable	*BABAR*	Belle
$A_{\rm dir}^{\rm SM}(B_s^0 \to K^+K^-)$	$(-1, 1)$	$(-0.02, 0.18)$
$A_{\rm mix}^{\rm SM}(B_s^0 \to K^+K^-)$	$(-1, 1)$	$(-0.19, 0.05)$

$$\frac{f_d}{f_s}\frac{BR(B_d^0 \to \pi^+\pi^-)}{BR(B_s^0 \to K^+K^-)} = 0.48 \pm 0.14, \qquad (33)$$

which implies

$$R_d^s = 7.7 \pm 2.5. \qquad (34)$$

We therefore see that, with the present data, the SM prediction for R_d^s overlaps with the experimental range for both experiments *BABAR* and Belle. There is therefore no evidence for a nonzero $\mathcal{B}^{\rm NP}$. However, the very large errors leave much room for new-physics. These errors will be dramatically reduced with improved experimental measurements for the $B_d^0 \to \pi^+\pi^-$ observables.

To summarize, present data hints at new-physics in $\bar{b} \to \bar{s}$ penguin amplitudes, but there is no evidence for NP in decays involving $\bar{b} \to \bar{d}$ penguins. We therefore assume that the NP is present only in $\bar{b} \to \bar{s}$ transitions. Recently it was shown that the NP strong phases are negligible [3], in which case all NP effects can be parametrized in terms of a single amplitude $\mathcal{A}^q$ and phase Φ_q for each quark-level decay $\bar{b} \to \bar{s}q\bar{q}$ $(q = u, d, s, c)$. In this paper we have shown that one can *measure* the NP parameters $\mathcal{A}^u$ and Φ_u using the decays $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$. Because these processes are related by flavor SU(3), there is some theoretical uncertainty in the method. We have shown explicitly how this technique can be implemented experimentally.

We have also shown that, given the presence of nonzero $\mathcal{A}^u$ and Φ_u in $B_s^0(t) \to K^+K^-$, measurements of the decay $B_s^0(t) \to K^0\bar{K}^0$ can be used to partially identify the NP. This latter decay involves the NP parameters $\mathcal{A}^d$ and Φ_d. Within certain classes of NP models, there is a known relationship between $\mathcal{A}^u$ and Φ_u and $\mathcal{A}^d$ and Φ_d. This allows us to *predict* the values of quantities in $B_s^0(t) \to K^0\bar{K}^0$, given measurements of $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$. Should the experimental values of these quantities disagree with the predictions, this would rule out these NP models.

Finally, we have discussed the implications of present data for the method of measuring $\mathcal{A}^u$ and Φ_u in $B_s^0(t) \to K^+K^-$ and $B_d^0(t) \to \pi^+\pi^-$. Although we have measurements of the various $B_d^0(t) \to \pi^+\pi^-$ observables, albeit with large (and sometimes inconsistent) errors, only the branching ratio for $B_s^0 \to K^+K^-$ has been measured. The comparison of the branching ratios for these two processes does not suggest the presence of NP. However, the errors are still very large, leaving much space for this NP. In the future, these errors will be greatly reduced with improved experimental measurements.

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