Recoil and power corrections in high-x_T direct photon production

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We study a class of nonperturbative corrections to single-inclusive photon cross sections at measured transverse momentum p_T, in the large-x_T limit. We develop an extension of the joint (threshold and transverse momentum) resummation formalism, appropriate for large x_T, in which there are no kinematic singularities associated with recoil, and for which matching to fixed-order and to threshold resummation at next-to-leading logarithm (NLL) is straightforward. Beyond NLL, we find contributions that can be attributed to recoil from initial-state radiation. Associated power corrections occur as inverse powers of p_T^2 and are identified from the infrared structure of integrals over the running coupling. They have significant energy-dependence and decrease from typical fixed-target to collider energies. Energy conservation, which is incorporated into joint resummation, moderates the effects of perturbative recoil and power corrections for large x_T.

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I. INTRODUCTION

To study the interplay of perturbative and nonperturbative dynamics in processes involving hadronic states it is natural to begin with observables whose perturbative analysis is well understood. For certain observables, perturbation theory not only provides predictions at leading power in a large momentum scale, but also characterizes power corrections in that scale. This can come about, for example, through nonconvergent perturbative expansions that exhibit sensitivity to the strong-coupling and/or vacuum structure of the theory [1].

Relying on perturbative resummations, this approach has had phenomenological successes in the description of a variety of inclusive and semi-inclusive cross sections. These include average and differential event shapes, primarily but not exclusively in e^+e^- annihilation [2–9], and electroweak annihilation cross sections at measured transverse momentum [10–13]. The value of the event shape or lepton pair transverse momentum provides a second scale in the cross section, and varying this scale changes the relative importance of perturbative and nonperturbative dynamics. Thus, the transition between perturbative and nonperturbative QCD is in principle available for study in these observables.

In this paper, we adopt this general philosophy and employ the joint resummation [14,15] of threshold [16] and transverse momentum [12] enhancements to study power corrections in the hard-scattering scale p_T for single-particle inclusive (1PI) cross sections in the large $x_T^2 = 4p_T^2/S$ region. Using direct photon production as an example, we will show that these corrections exhibit significant x_T dependence, which moderates both perturbative and nonperturbative recoil at large x_T compared to estimates based on transverse momentum resummation alone. These conclusions are made possible by a simplification of the joint resummation formalism that is specific to the $x_T \to 1$ limit.

Direct photon production was originally envisioned as a relatively straightforward process with which to test fixed-order perturbative calculations and to determine the gluon distribution [17–19]. The extensive data on direct photon production [20–24], however, has turned out to be more complex than was perhaps expected. Presumably for this reason it has inspired varied theoretical and phenomenological studies [25–37]. Nevertheless, for this benchmark process important questions remain unresolved. In particular, it has been argued that fixed-target data for direct photon production in the lower p_T range (roughly below 5 GeV) are difficult to reconcile with collinear-factorized NLO cross sections [30,32]. Additionally, threshold resummation [34–36] appears to explain the data only for larger p_T.

This difficulty has motivated the use of k_T-dependent, or unintegrated, parton distributions combined with recoiling partonic $2 \to 2$ subprocesses [31]. Information on the partonic transverse momenta in such distributions may come from resummed perturbation theory [12,13,31], and/or from comparisons to data [11,30,38,39], including Drell-Yan, photon and hadron pair cross sections. Probably the simplest approach is to assume a Gaussian dependence $\exp[-k_T^2/\langle k_T^2\rangle]$ [18]. As we review below, perturbative resummations predict logarithmic p_T dependence for the parameter $\langle k_T^2\rangle$. They also imply that $\langle k_T^2\rangle$ depends on the parton flavor.

The use of unintegrated distributions requires an extension of collinear factorization [40]. In particular, a technical challenge in the case of light particle production is the potential for an artificial infrared singularity when the total transverse momentum of the initial-state partons is comparable to the observed p_T [18]. One way to avoid this singularity is to impose strong ordering in transverse momenta, as in [31], a procedure which requires definition beyond leading logarithm. A related approach is described in detail in Ref. [33], based on a specific implementation of

k_T-resummation. As presented in [33], however, fits in this formalism favor fixed $\langle k_T^2 \rangle$, with no indication of the p_T-dependence implied by k_T resummation. Other studies, however, seem to imply that $\langle k_T^2 \rangle$ is S-dependent [30]. In summary, it remains unclear how much of what we interpret as recoil, or parton transverse momentum, is perturbative and how much nonperturbative. Here we come back to this question in the context of a generalized resummation formalism.

Resummed perturbation theory for 1PI cross sections was extended in [14,15,41] using joint resummation. Joint resummation systematically combines singular behavior at zero transverse momentum for initial-state partons with that at partonic threshold, where the initial state partonic invariant mass $\hat{s} = x_a x_b S$ is just large enough to produce the observed final-state. This method was applied to Z and Higgs production in [42,43], where no kinematic singularities arise, because the produced electroweak state is massive and the transverse momentum of the lepton pair is directly observed. In [42], some implications for the specific forms of power corrections were also pointed out. Although the joint formalism was applied to high-p_T photon production in [41], its application was hampered by the same infrared singularity mentioned above, associated with the production of a massless particle. As noted in [41], the complexity of a simultaneous resummation in transverse momentum and energy above threshold appears to make impractical a matching of the sort developed for transverse momentum resummation alone in [33].

In this paper, we extend this work, and revisit logarithmic and power corrections to the direct photon cross section in the joint resummation formalism. Compared to previous work, however, we use the kinematics of the large-x_T limit to reformulate joint resummation, taking into account recoil effects in the partonic subprocess while avoiding a kinematic singularity. The resulting resummed cross section reduces to threshold resummation at next-to-leading logarithm (NLL) and can be matched to finite-order and threshold-resummed cross sections in a straightforward fashion. Beginning at NNLL, the cross section also includes a contribution that can be identified as the finite residue left from the cancellation of the transverse momentum singularities of real and virtual gluons radiated in the initial state. Enhancements to the cross section associated with final state interactions are treated only to leading power in this paper, and appear in the same manner as in threshold resummation. In another paper we will argue that the results found here are not changed qualitatively by these effects.

The parameters that control power corrections associated with joint resummation at partonic threshold are found to be related to parameters familiar from the transverse momentum distributions in electroweak annihilation. The power corrections also inherit significant energy dependence. For large x_T, both perturbative recoil and nonperturbative power corrections to the predictions of threshold resummation are suppressed by the phase space restrictions built into joint resummation. This effect is important, however, only for x_T near one, or equivalently for large values of its conjugate Mellin moment variable N. For smaller x_T or N of order unity an analysis based on k_T resummation alone may be appropriate, but should be matched to the results of joint resummation in the large x_T region.

We begin Sec. II with a brief summary of the joint resummation formula as developed for direct photon cross sections, and exhibit the kinematic singularity. In the next subsection, the cross section is expressed as a double inverse transform. This is followed by a simple reformulation that eliminates the kinematic singularity and reduces the jointly resummed cross section to a single transform that extends threshold resummation for the direct photon cross section. The resulting Sudakov exponents of joint resummation are analyzed in Sec. III, where we identify the form of the recoil and power corrections to the direct photon cross section that are implied by joint resummation. We explore the phenomenology of these corrections in Sec. IV, exhibit the suppression of power corrections for large S at fixed p_T, and briefly discuss possible subdominant corrections not directly associated with partonic threshold. We conclude with a summary, and a brief discussion of possible implications for an eventual global treatment of single-photon and single-hadron cross sections.

II. SELF-CONSISTENT RECOIL IN JOINT RESUMMATION

A. Partonic recoil in direct photon production

Joint resummation [14,41] is an extension of threshold [16] and transverse momentum resummations [12,13] that unifies these two formalisms. So far, at the phenomenological level it has been applied primarily to the single electroweak boson (mass Q) production cross sections at low transverse momentum, $Q_T \ll Q$ [42,43]. In this case, threshold resummation is associated with corrections of the form $[\alpha_s^n/(1-z)]\ln^{2n-1}(1-z)$, with $z = Q^2/\hat{s}$, where $\sqrt{\hat{s}}$ is the invariant mass of the partonic pair that annihilates into the observed boson. Such corrections are "implicit" in the sense that they contribute to the hadronic cross section only after convolution with the parton distribution functions, and hence give nonlogarithmic, although potentially significant, contributions to the cross section. Singular corrections in Q_T, on the other hand, are explicit in the cross sections themselves, appearing as terms like $\alpha_s^n/Q_T^2 \ln^{2n-1}(Q_T/Q)$ directly for the measured spectrum.

For single-particle inclusive cross sections such as direct photon production at measured $p_T \gg \Lambda_{\mathrm{QCD}}$ the situation is slightly different. To leading order in the hard-scattering, incoming partons produce a photon-parton system, which subsequently evolves into a photon-jet pair. At higher orders in α_s, the pair recoils against unobserved soft gluon radiation with total transverse momentum Q_T, in much the same way as for a single electroweak boson. When only the photon is observed, Q_T is integrated and singularities at $Q_T/p_T = 0$ cancel, analogously to singularities at $1 - z = 0$ in threshold resummation. Thus in the direct photon cross section at measured p_T, both transverse momentum and threshold singularities are implicit rather than explicit. Nevertheless, small-Q_T gluon radiation can play a significant role in the cross section. Applied to direct photon production, joint resummation attempts to estimate the effects of these soft emissions systematically [14,41].

For an observed photon of momentum p_T, the photon transverse momentum in the pair center-of-mass is

$$\mathbf{p}'_T = \mathbf{p}_T - \mathbf{Q}_T/2. \tag{1}$$

In the limit that $Q_T/p_T \ll 1$ the cross section is a convolution [41] of the resummed distribution in Q_T with a hard-scattering function evaluated at photon momentum p'_T. Nonzero pair momentum Q_T, if in the direction of the observed photon, decreases the scale of the hard-scattering, and can thus enhance the cross section. As emphasized in Ref. [41], however, when Q_T grows to the order of p_T, this approximation generates kinematic singularities. Their effect is non-negligible because the fall-off in soft gluon transverse momenta has a powerlike perturbative tail. In [41], we dealt with the kinematic singularity in a rather crude way by cutting off the resummed Q_T spectrum at a convenient scale $Q_T = \bar{\mu}$:

$$p_T^3 \frac{d\sigma^{\rm res}_{AB\to\gamma X}}{dp_T} = \int dQ^2 d^2Q_T p_T^3 \frac{d\sigma^{\rm res}_{AB\to\gamma X}}{dQ^2 d^2Q_T dp_T} \theta(\bar{\mu} - Q_T), \tag{2}$$

where Q is the invariant mass of the photon-parton pair. At threshold, the latter is fixed by

$$Q = 2p_T. \tag{3}$$

The scale $\bar{\mu}$ in Eq. (2) may be regarded as a matching scale. Ideally, at $Q_T \sim \bar{\mu} < p_T$, one would replace the resummed cross section $p_T^3 d\sigma^{\rm res}_{AB\to\gamma X}/dQ^2 d^2Q_T dp_T$ by the fixed-order (NLO) one, which does not have the kinematic singularity. In practice, this becomes a very complicated procedure, and it is more convenient to derive a jointly resummed cross section that does not require a cutoff. We will show below that this may be achieved by applying an additional, self-consistent approximation that is exact at partonic threshold. To do so, we must recall the explicit form of the cross section derived in Refs. [14,41].

B. The double inverse transform

Integrated over rapidities, the jointly resummed direct photon cross section is written in terms of Mellin moments of the $\overline{\rm MS}$ parton distributions, $\tilde{\phi}_{a/H}(N,\mu) \equiv \int_0^1 dx x^{N-1} \phi_{a/H}(x,\mu)$, as

$$\begin{aligned} p_T^3 \frac{d\sigma^{\rm (resum)}_{AB\to\gamma X}}{dp_T} = {}& \sum_{ab} \frac{p_T^4}{8\pi S^2} \int_C \frac{dN}{2\pi i} \tilde{\phi}_{a/A}(N,\mu) \tilde{\phi}_{b/B}(N,\mu) \\ & \times \int_0^1 d\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|M_{ab}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}} \\ & \times C^{ab\to\gamma c}[\alpha_s(\mu), \tilde{x}_T^2] \\ & \times \int \frac{d^2Q_T}{(2\pi)^2} \Theta(\bar{\mu} - Q_T) \left(\frac{S}{4p_T'^2}\right)^{N+1} \\ & \times P_{ab}\left(N, Q_T, \frac{2p_T}{\tilde{x}_T}, \mu\right), \end{aligned} \tag{4}$$

where μ is the factorization and renormalization scale, and the $|M_{ab}|^2$ are squared amplitudes for the partonic processes $ab \to \gamma c$. The variable $\tilde{x}_T^2$ is defined by

$$\tilde{x}_T^2 \equiv \frac{1}{\cosh^2 \tilde{\eta}}, \tag{5}$$

where $\tilde{\eta}$ is the rapidity of the direct photon in the center-of mass of the hard-scattering. At partonic threshold, or equivalently large values of the moment variable N, $\tilde{\eta}$ is forced to unity. For this reason, we will approximate

$$2p_T/\tilde{x}_T \sim 2p_T \equiv Q \tag{6}$$

in the functions P_{ab} in Eq. (4), where dependence on p_T is logarithmic. The contour C in Eq. (4) and the b integral in (7) below define the inverse transforms from N, b space to z and Q_T. These contour integrals were described in detail in Refs. [41,42].

The functions P_{ab} in Eq. (4) were derived in Ref. [41] and provide Q_T dependence at fixed N. Each P_{ab} is itself the Fourier transform of the exponentiated logarithmic dependence on N and b,

$$P_{ab}(N, Q_T, Q, \mu) = \int d^2b e^{-ib\cdot Q_T} \exp[E_{ab\to\gamma c}(N, b, Q, \mu)], \tag{7}$$

where the $E_{ab\to\gamma c}$ are "Sudakov" exponents that we will specify explicitly below. They can be split into initial and final-state contributions, where, as shown in [41], all b-dependence comes from the initial-state,

$$E_{ab\to\gamma c}(N, b, Q, \mu) = E^{\rm IS}_{ab}(N, b, Q, \mu) + E^{\rm FS}_{abc}(N, Q, \mu). \tag{8}$$

The N-independent coefficients $C^{ab\to\gamma c}$ contain the effects of hard virtual corrections and are perturbative series of the form $C^{ab\to\gamma c} = 1 + \frac{\alpha_s}{\pi} C^{ab\to\gamma c(1)} + \ldots$. To next-to-leading logarithmic accuracy one needs the first order terms which may be found in [35], and are given in the Appendix below. In this paper, we concentrate on the initial state exponent, which contains all leading logarithmic effects and all b dependence.

Threshold resummation is recovered from Eq. (4) by setting b to zero in the exponents $E_{ab\to\gamma c}$. In this case, the b

integral produces a delta function that sets $p'_T = p_T$. Then the exponent $E_{ab\to\gamma c}$ reverts to its threshold resummed form, and $S/4p'^2_T \to 1/x_T^2$. Recoil enhances the cross section Eq. (4) because even for $Q_T \ll p_T$, the ratio $S/4p'^2_T$ can be larger than $S/4p_T^2$. For large enough Q_T, the factor $S/4p'^2_T$ can diverge, and a cutoff is required, as discussed above. This momentum configuration, however, requires $Q_T \sim 2p_T$, and hence is far outside the region where resummation is applicable. This problem is not due to our approximation in the region of interest, but to our extrapolation beyond that region. When Q_T approaches p_T in magnitude, the factorization between gluon emission and hard-scattering fails. For large N, however, the "profile" functions P_{ab} vanish once $Q_T > Q/N \ll p_T$. The P_{ab}'s vanish for moderate Q_T because the exponents $E_{ab\to\gamma c}$ develop large (negative) logarithms once $bQ/N > 1$ (see below, Eq. (17)). This ensures that for $Q_T > Q/N$ the exponential $\exp[-ib \cdot Q_T]$ oscillates on a smaller scale than the size of the region where $\exp[E_{ab\to\gamma c}]$ is nonvanishing. Put another way, because the widths of the profile functions in b space are of order N/Q, their Fourier transforms to Q_T space have widths of order Q/N. Numerical examples for the Q_T integrand in Eq. (2) were given in Ref. [41], which show the fall-off of the profile function for increasing Q_T, followed by the kinematic singularity as Q_T increases to the order of p_T. We conclude that for large N the true enhancement due to recoil is insensitive to modifications of the integrand above $Q_T \sim p_T/N \ll p_T$. Since large N corresponds to $x_T \to 1$, we expect a suppression of recoil effects in this limit. In addition, N is conjugate to k_0/p_T, where k_0 is the energy of initial state radiation [14,41]. The relation $Q_T < p_T/N$ is thus equivalent to the restriction that the total transverse momentum of initial state radiation is less than its energy. We will use this observation shortly.

C. Elimination of the kinematic singularity

Given that all-order recoil effects enhance the jointly resummed cross section from values of Q_T such that $Q_T < p_T/N$, it is only in this region that we are required to maintain accurate expressions for leading Q_T behavior (that is, $1/Q_T^2$ times logarithms). In fact, to construct the jointly resummed expression in Eq. (4), we have neglected corrections that are nonsingular at $Q_T = 0$ and $1 - z = 0$. This means that we do not in general have control over corrections suppressed by powers of Q_T/p_T, and also that we are free to change the resummed expression at this level of accuracy. Such a change will only affect the result from the region of Q_T beyond the range that gives enhancement. These modifications will not produce logarithms, and we can adjust for them by matching to the cross section at fixed order.

In summary, we are free to choose an extrapolation that does not produce spurious singularities at large Q_T and which does not change the singularity structure at $Q_T = 0$. This may be done in such a way that the resulting resummed expression remains accurate to NLL in the variables N and b.

In this spirit, we make the following approximation, accurate to corrections that are suppressed by factors of Q_T/p_T:

$$\left(\frac{S}{4(\vec{p}_T - \frac{1}{2}\vec{Q}_T)^2}\right)^{N+1} = (x_T^2)^{-N-1} \exp\left\{(N+1)\vec{Q}_T \cdot \vec{p}_T/p_T^2 \times \left[1 + \mathcal{O}\left(\frac{NQ_T^2}{p_T^2}\right)\right]\right\}. \tag{9}$$

Notice that the exponent reaches order unity at just those values of Q_T for which the profile function begins to decrease. Replacing the singular power dependence on Q_T with the exponential, we retain the leading behavior at low Q_T, but eliminate the kinematic singularity, as desired. Again we emphasize that suppression for $Q_T > p_T/N$ is a reflection of energy conservation.

Let us now study the effect of the approximation in Eq. (9). Consider for the moment $N + 1 = -i\mathcal{N}$ with $\mathcal{N}$ fixed and real. Using Eq. (7), we may then replace the second line of Eq. (4) according to

$$\begin{aligned} &C^{ab\to\gamma c} \int \frac{d^2Q_T}{(2\pi)^2} \Theta(\bar{\mu} - Q_T)\left(\frac{S}{4p'^2_T}\right)^{N+1} P_{ab}\left(N, Q_T, \frac{2p_T}{\tilde{x}_T}, \mu\right) \\ &\longrightarrow C^{ab\to\gamma c}(x_T^2)^{-N-1} \int d^2b \exp[E_{ab\to\gamma c}(N, b, Q, \mu)] \\ &\quad\times \int \frac{d^2Q_T}{(2\pi)^2} \mathrm{e}^{-i\mathcal{N}\vec{Q}_T\cdot\vec{p}_T/p_T^2 - i\vec{b}\cdot\vec{Q}_T}. \end{aligned} \tag{10}$$

Here we have extended the Q_T integral to infinity. The integral may then be performed, and gives $\delta^{(2)}(\vec{b} + \mathcal{N}\vec{p}_T/p_T^2)$. Using this delta function to perform the b integral in (10), and inserting the result back into Eq. (4), we find

$$\begin{aligned} p_T^3 \frac{d\sigma^{(\mathrm{resum})}_{AB\to\gamma X}}{dp_T} = &\sum_{ab} \frac{p_T^4}{8\pi S^2} \int_C \frac{dN}{2\pi i} \tilde{\phi}_{a/A}(N, \mu)\tilde{\phi}_{b/B}(N, \mu) \\ &\times \int_0^1 d\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|M_{ab}(\tilde{x}_T^2)|^2}{\sqrt{1 - \tilde{x}_T^2}} \\ &\times C^{ab\to\gamma c}(x_T^2)^{-N-1} \\ &\times \exp\left[E_{ab\to\gamma c}\left(N, -i\frac{N+1}{p_T}, Q, \mu\right)\right]. \end{aligned} \tag{11}$$

This expression for the direct photon cross section is similar to the result for pure threshold resummation, except for the additional b dependence, which has become dependence on the combination $(N + 1)/p_T$ in the exponent. Although we have derived this form for imaginary values of $N + 1$, it can be analytically continued to any N, and we use (11) as the result of the Q_T integral in the high-x_T

jointly resummed cross section. Recoil is self-consistently taken into account through the exponential in (9), which is accurate up to power corrections as shown. There are no kinematic singularities at large Q_T. For Q_T competitive with p_T, of course, the approximations we have made fail, but in this region the profile function is small.

D. Matching

Matching is now straightforward for the stabilized cross section, Eq. (11), and can be handled as for the threshold-resummed cross section. We simply expand the exponents to NLO (for example) in terms of $\alpha_s(p_T)$, and replace these approximate expressions with the exact hard-scattering cross sections at that order.

We emphasize that within our new treatment we have been able to perform both the Q_T and the b integrals. This is a great advantage for phenomenological applications, since now the evaluation of the cross section is technically equivalent to that of a standard threshold-resummed one. In fact, as we show below, Eq. (11) is identical to normal threshold resummation to NLL, but differs at NNLL through a well-defined set of terms that can be identified uniquely as recoil effects. The fact that the NLL threshold logarithms are unchanged by recoil is an important consistency check of our approach because these logarithms are uniquely specified in the perturbative single-inclusive cross section.

The final resummed cross section thus has a form that is closely related to matched threshold resummation [42]:

$$p_T^3 \frac{d\sigma_{AB}^{\mathrm{res}}}{dp_T} = \sum_{ab} \int_C \frac{dN}{2\pi i} \tilde{\sigma}_{ab}^{(0)}(N) \times C^{ab\to\gamma c}(x_T^2)^{-N-1} e^{E_{ab\to\gamma c}[N,-i(N+1/p_T),Q,\mu]} + p_T^3 \frac{d\sigma_{AB}}{dp_T}^{NLO} - p_T^3 \frac{d\sigma_{AB}}{dp_T}^{\mathrm{res}|_{\alpha_s^2}}, \tag{12}$$

where $\sigma_{ab}^{(0)}(N)$ is the moment of the lowest-order cross section,

$$\tilde{\sigma}_{ab}^{(0)}(N) = \frac{p_T^4}{8\pi S^2} \tilde{\phi}_{a/A}(N,\mu)\tilde{\phi}_{b/B}(N,\mu) \times \int_0^1 d\tilde{x}_T^2 (\tilde{x}_T^2)^N \frac{|M_{ab}(\tilde{x}_T^2)|^2}{\sqrt{1-\tilde{x}_T^2}}, \tag{13}$$

and where the final terms in (12) express our matching to the fixed-order (NLO, $\mathcal{O}(\alpha_s^2)$) cross section $p_T^3 \frac{d\sigma_{AB}}{dp_T}^{NLO}$ by taking out the $\mathcal{O}(\alpha_s^2)$ expansion of the perturbative part of the resummed cross section, $p_T^3 \frac{d\sigma_{AB}}{dp_T}^{\mathrm{res}|_{\alpha_s^2}}$.

III. PERTURBATIVE AND NONPERTURBATIVE EXPONENTS

A. Resummed perturbative recoil

To clarify the relationship between joint and threshold resummation and the implications of our new treatment of recoil, we review the N and b dependence of the resummed exponent at NLL found in [14]. To all orders, the NLL initial-state logarithms in N and b are generated from an integral, derived using the eikonal nature of soft gluon emission, that extends down to zero scale in the running coupling. It may be written in a convenient form as

$$E_{ab}^{\mathrm{IS}}(N,b,Q,\mu=Q) = \int_0^{Q^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i[\alpha_s(k_T)] \left[J_0(bk_T) K_0\left(\frac{2Nk_T}{Q}\right) + \ln\left(\frac{\bar{N}k_T}{Q}\right)\right], \tag{14}$$

where $Q \equiv 2p_T$ (see Eq. (3)) is the minimal center of mass energy of the partonic subprocess. Here and below, we define

$$\bar{N} = N e^{\gamma_E}. \tag{15}$$

The anomalous dimensions $A_a(\alpha_s)$ have the familiar expansion $A_a(\alpha_s) = \sum_n (\alpha_s/\pi)^n A_a^{(n)}$, with

$$A_a^{(1)} = C_a A_a^{(2)} = \frac{1}{2} C_a K \equiv \frac{1}{2} C_a \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9} T_R N_f \right], \tag{16}$$

where $C_q = C_F$ for quarks and $C_g = C_A$ for gluons. The presence of the Bessel function $K_0(2Nk_T/Q)$ reflects the conservation of energy that must be imposed to resum threshold and k_T enhancements simultaneously. For the analogous exponent in k_T resummation, the function $K_0(2Nk_T/Q)$ in Eq. (14) is replaced by $-\ln(k_T/Q)$ and $\ln(\bar{N}k_T/Q)$ by $\ln(k_T/Q)$, and the k_T integral produces logarithms of bQ for any $b > 1/Q$. In joint resummation, however, b must be greater than N/Q to produce logarithms. As a result, for $N \to \infty$ the profile function in Q_T space decreases once $Q_T > Q/N$, as discussed in Sec. IIB above.

Starting from Eq. (14) we isolate the effect of perturbative recoil by separating it from the corresponding exponent for threshold resummation. Since threshold resummation is already accurate to NLL in the transform variable N [14], for consistency recoil must appear first at the next logarithmic order, and it does. As we shall see, however, its influence on the 1PI cross section need not be negligible in perturbation theory. In addition, the integral over the anomalous dimension $A[\alpha_s(k_T)]$ through the infrared region suggests a specific set of nonperturbative corrections, whose effects we will also study. For initial-state radiation, the form of contributions beyond NLL accuracy is given in [14]. Threshold logarithms associated with final-state interactions beyond NLL will be the subject of a separate investigation. We will argue that they respect the pattern for power corrections found here.

As directed by Eq. (11), we now set $b = -i(N+1)/p_T$ in Eq. (14), noting the Bessel function relation $J_0(iz) = I_0(z)$. We then reorganize the equation as

$$\begin{aligned} E_{ab}^{\rm IS}\Big(N, b = -i\frac{N+1}{p_T}, Q, \mu = Q\Big) &= \int_0^{4p_T^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i[\alpha_s(k_T)]\Big[K_0\Big(\frac{Nk_T}{p_T}\Big) + \ln\Big(\frac{\bar{N}k_T}{2p_T}\Big)\Big] + \int_0^{4p_T^2} \frac{dk_T^2}{k_T^2} \sum_{i=a,b} A_i[\alpha_s(k_T)] \\ &\quad \times \Big[I_0\Big(\frac{(N+1)k_T}{p_T}\Big) - 1\Big] K_0\Big(\frac{Nk_T}{p_T}\Big) \\ &\equiv E_{ab,\rm thr}^{\rm IS}(N, p_T) + \delta E_{ab,\rm rec}(N, p_T), \end{aligned} \tag{17}$$

where we have again used $Q = 2p_T$. We have identified the first term on the right-hand-side of Eq. (17) with the exponent for threshold resummation for initial-state logarithmic behavior in N [14].

The second term on the right side of (17) is the recoil correction. It now amounts simply to an N-dependent correction to the threshold-resummed cross section. As required by the self-consistency of NLL threshold resummation, this expression is free of NLL logarithms in N, because for small arguments z,

$$I_0(z) \sim 1 + \frac{z^2}{4}, \quad K_0(z) \sim -\ln\Big[\frac{z e^{\gamma_E}}{2}\Big]\Big(1 + \frac{z^2}{4}\Big) + \frac{z^2}{4}. \tag{18}$$

On the other hand, as $z \equiv Nk_T/p_T$ becomes large with $\mathrm{Re}(z) > 0$, $I_0(z)$ increases as $e^z/\sqrt{2\pi z}$, while $K_0(z)$ decreases as $e^{-z}/\sqrt{(2z/\pi)}$, so that

$$I_0(z)K_0(z) \to \frac{1}{2z} \quad [\mathrm{Re(z)} > 0]. \tag{19}$$

At fixed coupling, and replacing $N+1$ by N in I_0 in Eq. (17), the net result is a convergent, N-independent integral, equal to $(C_a + C_b) \times (\alpha_s/2\pi)\, \zeta(2)$, a modest but still significant contribution in the exponent. Although not characterized by a large logarithm, this result is the unique, finite remainder left from the cancellation between real and virtual emission after the Q_T integral, at leading power in N and α_s in the exponent. As such, it is not a generic finite correction, but the soft tail of an infrared safe integration. Although finite-order-by-order in perturbation theory, such contributions are subject to N-dependent corrections associated with the running of the coupling.

For large values of N, we can readily estimate the effect of the running coupling to perturbative recoil, by noting that the combination $(I_0 - 1)K_0$ becomes sharply peaked near $k_T = p_T/N$, with a width that is asymptotically negligible compared to the scale on which the coupling runs. As a result, to NNLL, we may isolate perturbative recoil including the running of the coupling by the expression,

$$\delta E_{ab,rec}^{\rm NNLL} = (C_a + C_b)\frac{\alpha_s(4p_T^2/\bar{N}^2)}{2\pi}\zeta(2). \tag{20}$$

We will use this expression below to estimate the effects of perturbative recoil.

B. Nonperturbative corrections from threshold

The recoil exponent $\delta E_{ab,\rm rec}(N, p_T)$ in Eq. (17) provides an estimate of perturbative recoil, and is also a guide to nonperturbative power corrections. The most basic observation about these corrections is that they factorize and exponentiate, in much the same manner as for event shapes in e^+e^- annihilation and for the transverse momentum distributions of electroweak boson production. This follows from the form of the resummed exponent, in which the entire dependence on the running coupling is through a single, integrated scale, k_T. We emphasize that a similar result holds for the full eikonal exponent to all logarithmic order. Indeed, the same underlying nonperturbative parameters that appear in Drell-Yan cross sections will appear in power corrections to direct photon cross sections. As noted in [42], power corrections from threshold and transverse momentum resummations are separately additive in the exponent. As we shall see, this leads to an extra power correction compared to estimates based on transverse momentum resummations alone [12,13].

Using the additivity of the nonperturbative corrections, we write for the full exponent

$$\begin{aligned} E_{ab\to\gamma c}(N, p_T) &= E_{abc}^{\rm PT} + \delta E_{ab}^{\rm np}, \\ E_{abc}^{\rm PT} &= E_{abc,\rm thr}^{\rm PT} + \delta E_{ab,\rm rec}^{\rm NNLL}, \end{aligned} \tag{21}$$

where $\delta E_{ab}^{\rm np}$ accounts for nonperturbative contributions from low scales in k_T, of order $\Lambda_{\rm QCD}$. The full perturbative threshold exponent at NLL, $E_{abc,\rm thr}^{\rm PT}$, with initial- and final-state contributions, was derived in [35–37,44]. As noted above, in this study we derive nonperturbative and recoil corrections associated with initial-state radiation only.

For small to moderate values of N, the integral in $E_{ab}^{\rm IS}(N, p_T)$ is perturbatively dominated. Nonperturbative corrections are generated by treating Nk_T/p_T as a small parameter in both the threshold and recoil exponents of Eq. (17). Expanding the integrands of both $\delta E_{ab,\rm rec}(N, p_T)$ and $E_{ab,\rm thr}^{\rm IS}$ in Eq. (17) for small k_T^2, we parametrize the resulting $1/p_T^2$ terms as

$$\delta E_{ab}^{\rm np} = \frac{(N+1)^2 + N^2}{4p_T^2} \sum_{i=a,b} \left[\lambda_i^{1,1} + \lambda_i^{1,0} \ln\left(\frac{2p_T}{\kappa \bar{N}}\right)\right] + \frac{N^2}{4p_T^2} \sum_{i=a,b} \lambda_i^{1,0} \quad (N\kappa/p_T < 1), \tag{22}$$

where the N^2 terms come from the threshold (K_0) integral in Eq. (17), while the $(N+1)^2$ term is from the recoil (I_0) term. The logarithm in both cases arises from the expansion of the function $K_0(2Nk_T/p_T)$, and, as noted above, its presence can be traced to the imposition of energy conservation in joint resummation. The constants $\lambda_i^{m,n}$ in Eq. (22) are interpreted as the nonperturbative content of moments of the running coupling [3–5], with indices in a notation inspired by [4]. More specifically, these are moments of the anomalous dimensions $A_a[\alpha_s(k_T)]$, [6]

$$\lambda_a^{m,n} = \int_0^{\kappa^2} dk_T^2 (k_T^2)^{m-1} A_a[\alpha_s(k_T)] \ln^n\left(\frac{k_T}{\kappa}\right) = \frac{C_a}{\pi} \int_0^{\kappa^2} dk_T^2 (k_T^2)^{m-1} \alpha_s(k_T) \ln^n\left(\frac{k_T}{\kappa}\right) + \ldots. \tag{23}$$

In Eq. (23), the upper limit κ for the k_T^2 integral, which also appears as the scale in logarithms of p_T in Eq. (22), is a factorization scale. To isolate a truly nonperturbative coupling, as in Ref. [4], we could subtract perturbative contributions to the λ's to the order corresponding to our level of resummation. Since this process does not change the p_T and N dependence of the expressions, and because the nonperturbative parameters appear in the same manner here as in electroweak annihilation [12], it is not necessary to provide such an analysis for our purposes.

As in the case of electroweak bosons [10,11,45], and in contrast to event shapes [2,3,5,7], only even powers result from the expansion of the Bessel functions in (17), the first of which has been displayed in Eq. (22). For N not too large, that is, for $N\Lambda_{\rm QCD} \ll p_T$ we expect only one or two power corrections to be significant, but for larger N, the resummed cross section should be supplemented by a function with a more general N-dependence [6,7]. In the following section, we will study the phenomenological implications of such dependence.

C. The full exponent

Summarizing our results so far, the full exponent is the sum of a perturbative threshold exponent, perturbative recoil and nonperturbative corrections,

$$E_{ab\to\gamma c}\left(N, i\frac{N+1}{p_T}, Q, \mu\right) = E_{abc,\rm thr}^{\rm PT}(N, p_T) + \delta E_{ab,\rm rec}^{\rm NNLL}(N, p_T) + \delta E_{ab}^{\rm np}, \tag{24}$$

where the nonperturbative exponent δE_{ab}^{np} is given in (22) above and the NNLL recoil correction $\delta E_{ab,\rm rec}^{\rm NNLL}$ by (20). $E_{ab,\rm thr}(N, p_T)$ is the full exponent for threshold resummation in prompt-photon production, including initial and final-state contributions [35–37,44] (see the Appendix).

IV. PHENOMENOLOGY OF POWER CORRECTIONS

The expressions derived above provide useful information on the phenomenology of power corrections associated with soft gluon emission. First, p_T dependence enters through even powers, with a leading nonperturbative coefficient that is identical to that encountered as the coefficient of b^2 in electroweak annihilation. For comparison, the latter may be written in terms of the same parameters $\lambda_q^{m,n}$ as

$$\delta E_{q\bar{q}}^{\rm np}({\rm Drell-Yan}) = -\frac{b^2}{4} \sum_{i=q,\bar{q}} \left[\lambda_i^{1,1} + \lambda_i^{1,0} \ln\left(\frac{Q}{\kappa}\right)\right]. \tag{25}$$

In contrast, for the single-particle inclusive cross section in joint resummation the nonperturbative corrections in Eq. (22) possess highly nontrivial N-dependence, from recoil directly, as well as from threshold resummation. This implies that these power corrections inherit nontrivial energy-dependence, and we may expect their effects to change with the overall energy.

To see the qualitative energy-dependence implied by the nonperturbative exponents derived above, we note that the Mellin moment N and the variable $\ln x_T^2$ are in a conjugate relationship, exhibited in the inverse transform (12),

$$N \leftrightarrow \frac{1}{\ln(1/x_T^2)}. \tag{26}$$

Identifying these quantities in the nonperturbative power correction of Eq. (22), and recalling that $x_T = 2p_T/\sqrt{S}$, we immediately see that the nonperturbative exponent is suppressed not only by a power of p_T, but also by a power of $\ln S$ at fixed p_T,

$$\delta E_{ab}^{\rm np} \leftrightarrow \frac{\lambda_a^{1,0} + \lambda_b^{1,0}}{4p_T^2 \ln^2(\frac{S}{4p_T^2})} \ln\left[2p_T \ln\left(\frac{S}{4p_T^2}\right)\right]. \tag{27}$$

Even though this expression is eventually to be convoluted with the hard-scattering cross sections and the parton distributions, we may conclude that at fixed p_T, the importance of power corrections will decrease as $\sqrt{S}$ increases. At the same time, as x_T approaches unity, the coefficient of $1/p_T^2$ diverges, and the nominal power correction may dominate at the edge of phase space. Even as this coefficient diverges, however, the logarithm in the numerator eventually changes sign, so that for x_T close enough to unity the enhancement becomes a suppression. This is not an accident, because the presence of the factor of N in the logarithm reflects energy conservation, which is respected by joint resummation. To give a realistic estimate of the behavior of these nonperturbative corrections, we return to moment space.

For the dominant form of the nonperturbative exponent at moderate N and p_T, we are guided by Eq. (22). For larger N, however, all power corrections in N may become relevant. To account for this, we introduce a function of N/p_T that generalizes this expression. We will refer to this as a shape function by analogy to the discussion of [6]. We tailor the N-dependence to the behavior of the Bessel functions of Eq. (17) for large and small values of their arguments, which both depend on the combination N/p_T. Matching to the small-N behavior of the Bessel functions in (18) and to the large-N behavior in (19), we modify Eq. (22),

$$\delta E_{ab}^{\rm np} = \mu_0 \frac{C_a + C_b}{\pi} \frac{(N+1)^2 + N^2}{4p_T^2} \frac{\ln(1 + \frac{2p_T}{NQ_0})}{(1 + \frac{Q_0 N}{p_T})^2}. \quad (28)$$

Here the scale μ_0 is a parameter of dimension mass squared, which can be thought of as the integral of $A_a[\alpha_s(k_T)](\pi/C_a)$ over k_T^2 with unit weight in (23). The overall factors of color charges in Eq. (28) reflect the proportionality of the coefficients A_a to C_a. The parameter Q_0 is a scale whose value accounts for nonlogarithmic terms. If $\lambda^{1,1} = 0$ in (23), we may identify Q_0 with the scale κ in Eq. (23). This is the result found in Ref. [4] with $\kappa = 2$ GeV.

To estimate the impact of these nonperturbative corrections for large x_T, we are aided by our experience with electroweak boson production and with perturbative resummation. As can be seen from Eq. (23), the nonperturbative parameters in our approach are related to moments of the strong coupling, which suggests some form of universality for them. In a study of Z production at the Tevatron we estimated nonperturbative effects; the value obtained may be translated into $[(\lambda_q^{1,0} + \lambda_{\bar{q}}^{1,0})/4] \times \ln(m_Z/\kappa) \approx 0.8\ \mathrm{GeV}^2$ in Eq. (28), or $\lambda_q^{1,0} \sim \lambda_{\bar{q}}^{1,0} \sim 0.4\ \mathrm{GeV}^2$, consistent with the result quoted in [4] for $\kappa = 2$ GeV. This implies for the parameter μ_0 in (28),

$$\mu_0(\mathrm{Drell-Yan}) = \frac{\pi \lambda_q^{1,0}}{C_F} \sim 1\ \mathrm{GeV}^2. \quad (29)$$

The nonperturbative exponent δE_{ab}^{np} in Eq. (28) with this value of μ_0 is our best estimate of the N-dependent exponent at large N. Substituted into the full exponent (24) it provides a measure of power corrections at large x_T.

To illustrate the influence of these power corrections, we compute the ratio of the cross section with threshold resummation plus the nonperturbative term (28) to the cross section with threshold resummation alone. We do this for several cases that are directly relevant for comparison with experiments: for pBe scattering with fixed-target beam energies $E = 530$ GeV and $E = 800$ GeV (E706 [22]), for pp and $\bar{p}p$ scattering with beam energy $E = 315$ GeV (UA6 [21]), and for pp scattering at $\sqrt{s} = 63$ GeV (R806 [46]). We use the GRV set of parton distributions from Ref. [47]. We normalize all our results to the threshold-resummed cross section, i.e., Eq. (12) with $\delta E_{\rm recoil}^{\rm NNLL} = \delta E_{\rm np} = 0$. This is advantageous because the dependence on the factorization and renormalization scales is small in this cross section, and because our recoil and nonperturbative corrections have been defined relatively to it. Note that as implied by Eq. (4) our resummation is done for the cross section integrated over all photon rapidities. In principle, we should account for the finite ranges of rapidity covered in the various experiments, which could be done using the techniques developed in Ref. [37]. However, as implied by the results of [37], the dependence on rapidity will be very weak in the ratios we consider here and can be neglected for simplicity. Our results are always matched to the NLO cross section as described after Eq. (12). We do not take into account a photon fragmentation contribution to the cross section.

Shown in Fig. 1 are results for the energies discussed above, as functions of x_T. The enhancements exhibited in the figure are both small and for the most part relatively flat. We have included values of x_T far from unity, especially for R806, to illustrate the point that these power corrections decrease with energy. For RHIC and CDF energies, the effects of (28) are practically negligible, of the order of just a few tenths of a percent. We emphasize, however, that this result applies only to extrapolations to small x_T of the expressions derived for $x_T \to 1$, and is not necessarily representative of the true behavior of the cross section at low x_T.

The moderation of the x_T dependence of the cross section at large x_T in Fig. 1 associated with energy con-

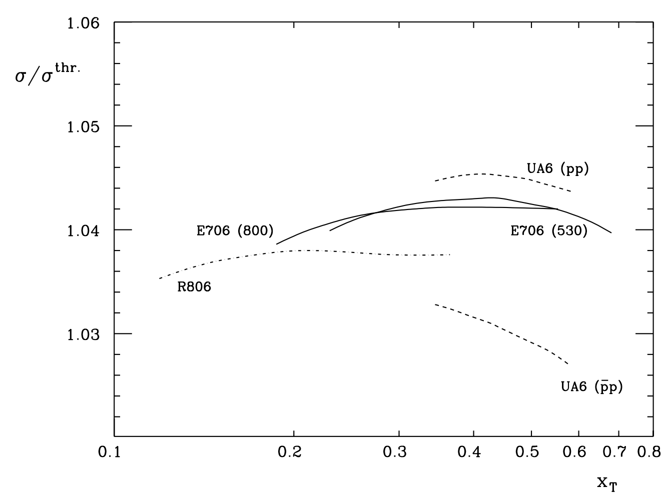

FIG. 1. Ratios of direct photon cross sections computed with threshold resummation and nonperturbative shape function (28) to cross sections with threshold resummation only. The curves are given as functions of x_T for kinematics relevant for comparison to fixed-target and ISR experiments (see text).

servation is illustrated by comparison to Fig. 2, which shows the analogous ratios when the nonperturbative coefficient is allowed to reflect the $b^2 \leftrightarrow (N+1)^2/p_T^2$ dependence that is characteristic of k_T resummation, starting from Eq. (25), rather than (28). The shape function then has the same overall quadratic N-dependence as (28), but lacks the N-dependence in the logarithm and the denominator that reflects the influence of the K_0 function in (17). We thus have

$$\delta E_{ab}^{\rm np} = \mu_0 \frac{C_a + C_b}{\pi} \frac{(N+1)^2}{4p_T^2} \ln\left(\frac{2p_T}{\kappa}\right). \quad \text{(Fig.2)} \quad (30)$$

Relative to Fig. 1, these curves show both strong enhancements and marked upturns toward increasing x_T.

To complete this discussion, we consider two additional variations of the cross sections computed with Eq. (28). So far, we have ignored the term in (24) associated with perturbative recoil, Eq. (20). It is probable that the incorporation of recoil at NNLL would affect the values of the nonperturbative exponents. Indeed, since both NNLL recoil and power corrections are derived from the same starting expression, Eq. (14), there is a serious potential for double counting. On the one hand, for small values of NQ_0/p_T the recoil integral in Eq. (14) is dominated by k_T that are outside the soft region $k_T \leq Q_0$. On the other hand, once $N \geq p_T/Q_0$, the integration region that gives rise to the result (20) overlaps the power corrections almost entirely. Nevertheless, it is interesting to test the influence of the corrections suggested by Eq. (20). To correct for double counting, at least partially, we use a modified estimate for recoil, which has the property that for small N it approaches (20), while it vanishes for large NQ_0/p_T,

$$\delta \bar{E}_{ab,\rm rec}^{\rm SUB} = (C_a + C_b)\frac{\alpha_s(4p_T^2/\bar{N}^2)}{\pi} \times \frac{\zeta(2)}{2}\left[1 - \frac{2}{\zeta(2)}\left(\frac{NQ_0}{2p_T}\right)^2 \frac{\ln\left(1 + {\rm e}^{1/2-\gamma_E}\frac{2p_T}{NQ_0}\right)}{1 + {\rm e}^{1/2-\gamma_E}\frac{NQ_0}{\zeta(2)p_T}}\right]. \quad (31)$$

The specific form of the subtraction has been chosen to reproduce the integral of the recoil term in Eq. (17) over the range $0 \leq k_T \leq Q_0$ at fixed coupling. For N fixed, Eq. (31) is a leading power contribution, with power-suppressed corrections, which, however, conspire to cancel the leading term when $NQ_0/p_T \gg 1$. Figure 3 shows the same sets of ratios as in Fig. 1, including now $\delta \bar{E}_{ab,\rm rec}^{\rm SUB}$ in addition to $\delta E_{ab}^{\rm np}$, Eq. (28). To avoid double-counting with the $C^{ab\to\gamma c}$ coefficents in the cross section, Eq. (11), we subtract the leading term $(C_a + C_b)\alpha_s(4p_T^2)\zeta(2)/2\pi$ from the latter. We see a substantial increase compared to the pure power corrections, in addition to a moderate slope toward large x_T. Evidently, the enhancement associated with the scale of the coupling dominates the cancellation in (31) at large N, at least throughout the experimental ranges shown. We do not take the level of this enhancement too literally, given our rough treatment of double-counting, but conclude that it does demonstrate the possible importance of nonleading logarithms, even when multiplied by powers of $1/N$ [25], and their interplay with the magnitudes of the parameters of power corrections.

Finally, we illustrate the possible influence of terms that are nonleading by a power in N. Although we have derived Eq. (11) only for $x_T \to 1$, the form is sufficiently general that it can be extrapolated to any value of x_T. Clearly, as we leave the kinematic regions where large N dominates, terms that are nonleading by powers are expected to become more and more important. Indeed, nonleading terms may be generated from the low-scale limit of partonic

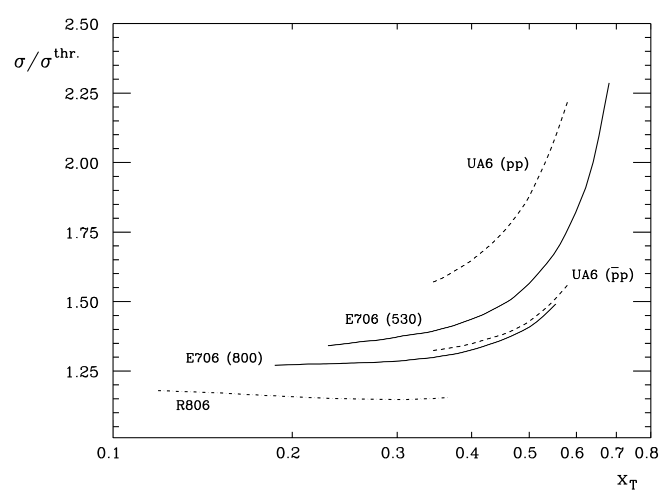

FIG. 2. Same as Fig. 1, but for the k_T-inspired nonperturbative shape function (30).

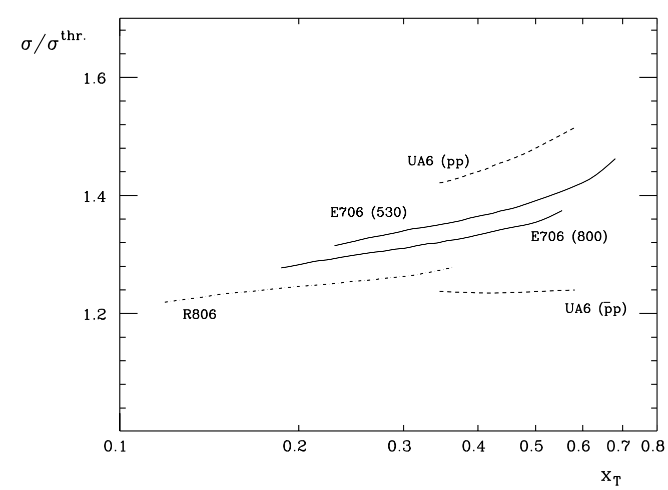

FIG. 3. Same as Fig. 1, but including the subtracted NNLL recoil exponent (31).

evolution. Thinking of the pervasive upturn of experimental cross sections relative to NLO noted long ago [26], we assume a phenomenological parametrization for the N-dependent nonperturbative exponents that behaves as N/p_T^2 for large N, is related to the splitting functions, and enhances the cross section at low x_T. The simplest ansatz of this sort is the following modification of the quark-gluon exponent (only),

$$\delta\bar{\mathrm{E}}_{\mathrm{np}}^{(gq)} = \delta\bar{\mathrm{E}}_{\mathrm{np}}^{(g\bar{q})} = \delta\mathrm{E}_{\mathrm{np}}^{(gq)} + \mu_1 \frac{C_A}{4\pi} \frac{(N+1)^2}{p_T^2} \frac{1}{N-1}, \qquad \delta\bar{\mathrm{E}}_{\mathrm{np}}^{(q\bar{q})} = \delta\mathrm{E}_{\mathrm{np}}^{(q\bar{q})}. \tag{32}$$

The parameter μ_1 is defined by analogy to μ_0 in (28) and (22), but does not have a direct or indirect interpretation in terms of resummed perturbation theory.

In Fig. 4 we show the same ratios, but now computed with the modified shape functions (32), choosing $\mu_1 = \mu_0 = 1\ \mathrm{GeV}^2$. These ratios indeed show a noticeable upturn toward small x_T. We observe, however, that the magnitudes of the enhancements are nowhere near those necessary to describe the low-x_T direct photon data, especially of E706 [22]. Since we are now considering terms that are subleading at large N, we also make exploratory calculations at higher energies, relevant to comparisons with the collider experiments at Tevatron ($\sqrt{s} = 1800$ GeV) and RHIC ($\sqrt{s} = 200$ GeV). As one can see, rather sharp upturns at $p_T \lesssim 5$ GeV are a distinct possibility here, if our ansatz in Eq. (32) is realistic. We finally note, without claims of physics significance, that it is possible to provide a qualitatively successful (χ^2 per degree of freedom approximately 1.5) "global" fit of direct photon data from E706, UA6, R806, and even CDF, with the ansatz (32), but only for values of μ_1 in the range of 10 GeV^2, which implies $\mu_1 C_A/4\pi \sim 2\ \mathrm{GeV}^2$. The origin of such a large scale, occuring as roughly $2\ \mathrm{GeV}^2/p_T^2$, is at the least not obvious. On the other hand, it could be simply an artifact of using Eq. (11) outside the region where the exponent $E(N, b)$ in (17), evaluated at $b = -i(N+1)/p_T$, has a straightforward interpretation. Even more serious, however, are the potential consequences of such corrections for pion production at collider energies. These issues can only be clarified by further work. We again emphasize that we can offer no specific justification for the form (32), beyond the analogy to the singularity at $N = 1$ in the gluon-gluon splitting function.

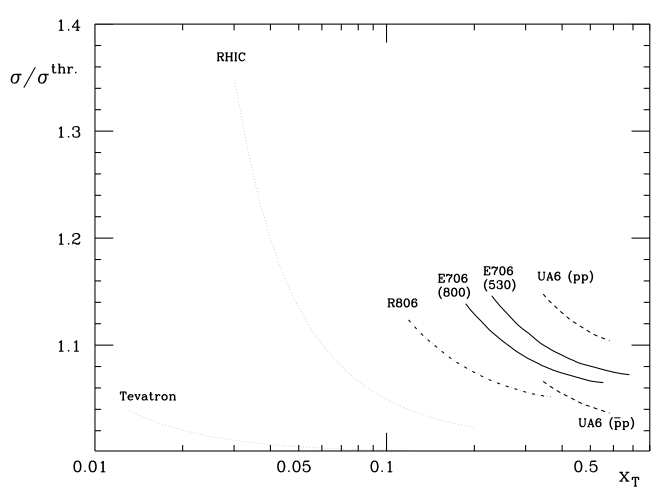

FIG. 4. Same as Fig. 1, but for a nonperturbative function with terms nonleading in N as given by Eq. (32). We also show results at $\sqrt{s} = 1800$ GeV and $\sqrt{s} = 200$ GeV, relevant for comparisons with Tevatron and RHIC data.

V. CONCLUSIONS

We have presented an analysis of recoil and power corrections from initial state radiation in single inclusive direct photon cross sections at large x_T. In this limit, we resum logarithmic corrections in N and simultaneously control logarithmic and power corrections in NQ_T/p_T, where Q_T is a measure of partonic transverse momentum. Our new treatment avoids any kinematic singularity when Q_T is large. The resulting expression is equivalent to threshold resummation at NLL in perturbation theory, with NNLL recoil effects. We have also shown that we may exponentiate power corrections of the form NQ_T/p_T.

In the large x_T region, leading power corrections enter in moment space as powers of $(N/p_T)^2$, with the leading term multiplied by a logarithm of the form $\ln(p_T/NQ_0)$. We have observed that at large x_T power corrections are suppressed relative to expectations based on k_T resummation alone. This suppression is attributable to phase space restrictions on initial state radiation near partonic threshold. This result raises the possibility of a link between a matched k_T resummation similar to that of Ref. [33], at relatively low x_T, and a joint resummation at large x_T. We have presented our analysis for initial-state radiation, which includes all k_T dependence in joint resummation. A detailed discusssion including the role of final-state radiation will be given elsewhere.

Looking beyond direct photon production, we anticipate that similar analyses may shed light on single-hadron and jet production. A simple, but possibly significant observation is that in single-hadron cross sections, the relevant scale for power corrections associated with partonic threshold and transverse momentum is $\hat{s}$, the total partonic c.m. energy squared. Because $\hat{s} \geq z^{-2}(4p_T^2)$, with z the momentum fraction associated with fragmentation, nonperturbative effects that are inverse powers of $\hat{s}$ are suppressed by factors of z^2 when expressed in terms of p_T^2. Issues such as these will be relevant to an effort to tie together perturbative and nonperturbative effects in the full range of inclusive hadronic reactions.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix we provide the explicit forms of the exponents $E_{ab\to\gamma c}$, as given in [35]. According to Eq. (8) the exponent is split up into pieces associated with initial and final state contributions. According to Eq. (17), within our treatment of recoil, the initial-state exponent becomes $E^{\rm IS}_{ab,{\rm thr}}(N, p_T) + E_{ab,{\rm rec}}(N, p_T)$. One has

$$E^{\rm IS}_{ab,{\rm thr}}(N, p_T) = \sum_{i=a,b}\left[\frac{1}{\alpha_s(\mu^2)} h_i^{(0)}(\lambda) + h_i^{(1)}(\lambda, 2p_T, \mu, \mu_F)\right], \quad \text{(A1)}$$

where

$$h_i^{(0)}(\lambda) = \frac{A_i^{(1)}}{2\pi b_0^2}[2\lambda + (1-2\lambda)\ln(1-2\lambda)], \quad \text{(A2)}$$

and

$$h_i^{(1)}(\lambda, 2p_T, \mu, \mu_F) = \frac{A_i^{(1)} b_1}{2\pi b_0^3}\left[\frac{1}{2}\ln^2(1-2\lambda) + 2\lambda + \ln(1-2\lambda)\right] + \frac{1}{2\pi b_0}\left[-\frac{A_i^{(2)}}{\pi b_0} + A_i^{(1)}\ln\left(\frac{4p_T^2}{\mu^2}\right)\right][2\lambda + \ln(1-2\lambda)] - \frac{A_i^{(1)}}{\pi b_0}\lambda\ln\left(\frac{4p_T^2}{\mu_F^2}\right). \quad \text{(A3)}$$

For completeness, we have distinguished between the renormalization scale μ and the factorization scale μ_F. The $A_i^{(1)}$ are as in Eq. (16), and we have defined

$$\lambda = b_0\alpha_s(\mu^2)\ln\bar{N}, \quad b_0 = \frac{11C_A - 4T_R N_F}{12\pi}, \quad b_1 = \frac{17C_A^2 - 10C_A T_R N_F - 6C_F T_R N_F}{24\pi^2}. \quad \text{(A4)}$$

For the NNLL exponent $E_{ab,{\rm rec}}(N, p_T)$ we obtain

$$E^{\rm NNLL}_{ab,{\rm rec}} = (C_a + C_b)\frac{\alpha_s(\mu^2)}{2\pi(1-2\lambda)}\zeta(2). \quad \text{(A5)}$$

The exponent for the final state reads:

$$E^{\rm FS}_{abc}(N, 2p_T, \mu) = \frac{1}{\alpha_s(\mu^2)} f_c^{(0)}(\lambda) + f_c^{(1)}(\lambda, 2p_T, \mu) + g^{(1)}_{abc}(\lambda), \quad \text{(A6)}$$

with

$$f_a^{(0)}(\lambda) = 2h_a^{(0)}(\lambda/2) - h_a^{(0)}(\lambda), \quad \text{(A7)}$$

$$f_a^{(1)}(\lambda, 2p_T, \mu) = 2h_a^{(1)}(\lambda/2, 2p_T, \mu, 2p_T) - h_a^{(1)}(\lambda, 2p_T, \mu, 2p_T) + \frac{A_a^{(1)}\ln 2}{\pi b_0}[\ln(1-2\lambda) - \ln(1-\lambda)] - \frac{B_a^{(1)}}{\pi b_0}\ln(1-\lambda), \quad \text{(A8)}$$

$$g^{(1)}_{q\bar{q}g}(\lambda) = -\frac{C_A}{\pi b_0}\ln(1-2\lambda)\ln 2, \quad \text{(A9)}$$

$$g^{(1)}_{qgq}(\lambda) = -\frac{C_F}{\pi b_0}\ln(1-2\lambda)\ln 2. \quad \text{(A10)}$$

Here,

$$B_q^{(1)} = \frac{3}{4}C_F, \quad B_g^{(1)} = \frac{\beta_0}{4}. \quad \text{(A11)}$$

Finally, the coefficients $C^{ab\to\gamma c}$ of Eq. (4) read [35]:

$$C^{q\bar{q}\to\gamma g} = 1 + \frac{\alpha_s}{\pi}\left[-\frac{1}{2}(2C_F - C_A)\ln 2 + \frac{1}{2}K - K_q + 2\zeta(2)\left(2C_F - \frac{1}{2}C_A\right) + \frac{5}{4}(2C_F - C_A)\ln^2 2 + \frac{3}{2}C_F\ln\frac{2p_T^2}{\mu_F^2} - \pi b_0\ln\frac{2p_T^2}{\mu^2}\right], \quad \text{(A12)}$$

$$C^{qg\to\gamma q} = 1 + \frac{\alpha_s}{\pi}\left[-\frac{1}{10}(C_F - 2C_A)\ln 2 - \frac{1}{2}K_q + \frac{\zeta(2)}{10}(2C_F + 19C_A) + \frac{1}{2}C_F\ln^2 2 + \left(\frac{3}{4}C_F + \pi b_0\right)\ln\frac{2p_T^2}{\mu_F^2} - \pi b_0\ln\frac{2p_T^2}{\mu^2}\right], \quad \text{(A13)}$$

where

$$K = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{10}{9}T_R N_f, \; K_q = \left(\frac{7}{2} - \frac{\pi^2}{6}\right)C_F. \quad \text{(A14)}$$

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