Combined nonrelativistic constituent quark model and heavy quark effective theory study of semileptonic decays of Λ_b and Ξ_b baryons

C. Albertus,[1] E. Hernández,[2] and J. Nieves[1]

[1]*Departamento de Física Moderna, Universidad de Granada, E-18071 Granada, Spain*
[2]*Grupo de Física Nuclear, Facultad de Ciencias, E-37008 Salamanca, Spain*
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We present the results of a nonrelativistic constituent quark model study of the semileptonic decays $\Lambda_b^0 \to \Lambda_c^+ l^- \bar{\nu}_l$ and $\Xi_b^0 \to \Xi_c^+ l^- \bar{\nu}_l$ ($l = e, \mu$). We work on coordinate space, with baryon wave functions recently obtained from a variational approach based on heavy quark symmetry. We develop a novel expansion of the electroweak current operator, which supplemented with heavy quark effective theory constraints, allows us to predict the baryon form factors and the decay distributions for all q^2 (or equivalently w) values accessible in the physical decays. Our results for the partially integrated longitudinal and transverse decay widths, in the vicinity of the $w = 1$ point, are in excellent agreement with lattice calculations. Comparison of our integrated Λ_b-decay width to experiment allows us to extract the V_{cb} Cabbibo-Kobayashi-Maskawa matrix element for which we obtain a value of $|V_{cb}| = 0.040 \pm 0.005(\text{stat})^{+0.001}_{-0.002}(\text{theory})$ also in excellent agreement with a recent determination by the DELPHI Collaboration from the exclusive $\bar{B}_d^0 \to D^{*+} l^- \bar{\nu}_l$ decay. Besides for the $\Lambda_b(\Xi_b)$-decay, the longitudinal and transverse asymmetries and the longitudinal to transverse decay ratio are $\langle a_L \rangle = -0.954 \pm 0.001(-0.945 \pm 0.002)$, $\langle a_T \rangle = -0.665 \pm 0.002(-0.628 \pm 0.004)$, and $R_{L/T} = 1.63 \pm 0.02(1.53 \pm 0.04)$, respectively.

I. INTRODUCTION

The understanding of the nonperturbative strong interaction effects in the exclusive $b \to c$ semileptonic transition is necessary for the determination of the cb (V_{cb}) Cabbibo-Kobayashi-Maskawa (CKM) matrix element from the experimentally measured rates and distributions. A considerable amount of work has been carried out in the meson sector, where the ideas of heavy quark symmetry (HQS) [1] and heavy quark effective theory (HQET) [2] were first developed. In the theoretical side, there exist lattice calculations [3–6] and a large variety of other theoretical analysis (HQET, dispersive bounds, quark model, sum rules, etc.) [7–12]. From the experimental point of view there was also an important activity, and CLEO and Belle collaborations have recent measurements of $B \to D^*$ decays [13–15].

The discovery of the Λ_b baryon at CERN [16], the discovery of most of the charmed baryons of the SU(3) multiplet on the second level of the SU(4) lowest 20-plet [17], and the recent measure of the semileptonic decay of the Λ_b^0 [18] make the study of the weak interactions of heavy baryons timely. Experimental knowledge of the Λ_b semileptonic decay can lead to an independent estimate of V_{cb} if the effects of the strong interaction in the decay are understood. There exists an abundant amount of literature on the subject [19–34]. Almost all theoretical approaches applied to the meson sector also have been explored for baryons. A common drawback in most of these studies is the impossibility of describing

the decay distributions for all q^2 (q is the four momentum transferred to the leptons in the decay) accessible values in the physical decay. Thus, lattice calculations and HQET based approaches lead to reliable predictions in the neighborhood of $q^2_{\max} = (m_{\Lambda_b} - m_{\Lambda_c})^2$, conventional sum rule approaches are more reliable near $q^2 = 0$, while traditional nonrelativistic constituent quark models (NRCQM's) cannot predict differential decay rates far from $q^2_{\max}$.

HQS allows theoretical control of the nonperturbative aspects of the calculation around the infinite quark mass limit. The classification of the weak decay form factors of heavy baryons has been simplified greatly in HQET [35]. In addition, the $\Lambda_{Q=b,c}$, $\Xi_{Q=b,c}$ baryons have a particularly simple structure in that they are composed of a heavy quark and light degrees of freedom with zero angular momentum. At leading order in an expansion on the heavy quark mass only one universal form factor, the Isgur-Wise function, is required to describe the $\Lambda_b \to \Lambda_c$ semileptonic decay. In next to leading order, $1/m_Q$ [36], one more universal function and one mass parameter are introduced [37]. However, HQS does not determine the universal form factors and the mass parameter, and one still needs to employ some other nonperturbative methods.

In this work we determine the nonperturbative corrections to the electroweak $\Lambda_b \to \Lambda_c$ matrix element by using different NRCQM's. We use a spectator model with only one-body current operators, and work in coordinate space, with baryon wave functions recently obtained from a HQS

based variational[1] approach [38]. We propose a novel expansion of the electroweak current operator, which allows us to predict the decay distributions for all q^2 values accessible in the physical decay. Thus, we keep up to first order terms in the internal (small) heavy quark momentum within the baryon but all orders in the transferred (large) momentum $\vec{q}$. Some preliminary results were presented in [40]. Now, we shall further impose $\mathcal{O}(1/m_Q)$ accuracy HQET constraints among the form factors to improve on the spectator model results. The paper is organized as follows. In Sec. II we introduce the form factors and their relation to the differential decay width. Those form factors carry all nonperturbative QCD corrections to the semileptonic Λ_b and Ξ_b decays. In Sec. III, we relate baryon wave function with form factors and introduce the heavy quark internal momentum expansion (Sec. III D). A brief summary of the HQET predictions for these decays is outlined in Sec. IV, while our results and main conclusions are presented in Secs. V and VI, respectively. Finally, in the Appendix A some detailed formulas can be found.

II. DIFFERENTIAL DECAY WIDTH AND FORM-FACTORS

We will focus on the $\Lambda_b(p) \to \Lambda_c(p')l(k')\bar{\nu}_l(k)$ reaction, where p, p', k and k' are the four-momenta of the involved particles. The generalization to the study of the Ξ_b baryon semileptonic decay is straightforward. In the Λ_b rest frame (LAB frame), the differential decay width reads

$$d\Gamma = 8|V_{cb}|^2 m_{\Lambda_c} G^2 \frac{d^3 p'}{2E'_{\Lambda_c}(2\pi)^3} \frac{d^3 k}{2E_{\nu_l}(2\pi)^3} \frac{d^3 k'}{2E'_l(2\pi)^3}$$
$$\times (2\pi)^4 \delta^4(p - p' - k - k') L^{\alpha\beta} W_{\alpha\beta}, \quad (1)$$

where[2] $m_{\Lambda_c} = 2285$ MeV, and $G = 1.1664 \times 10^{-11}$ MeV^{-2} is the Fermi decay constant. L and W are the leptonic and hadronic tensors, respectively. The leptonic tensor is given by [in our convention, we take $\epsilon_{0123} = +1$ and the metric $g^{\mu\nu} = (+, -, -, -)$]:

$$L_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma} k \cdot k' + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k^\beta. \quad (2)$$

The hadronic tensor includes all sorts of nonleptonic vertices and corresponds to the charged electroweak $\Lambda_b \to \Lambda_c$ transition. It is given by

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[1]In Ref. [38], we developed a rather simple method to solve the nonrelativistic three-body problem for baryons with a heavy quark, where we have made full use of the consequences of HQS for that system. Thanks to HQS, the method proposed provides us with simple wave functions, while the results obtained for the spectrum and other observables compare quite well with the lengthy Faddeev calculations done in [39].

[2]We also take $m_{\Lambda_b} = 5624$ MeV, $m_{\Xi_b} = 5800$ MeV, and $m_{\Xi_c} = 2469$ MeV.

$$W^{\mu\sigma} = \frac{1}{2} \sum_{r,s} \langle \Lambda_c; \vec{p}', s|j_{cc}^\mu(0)|\Lambda_b; \vec{p}, r\rangle$$
$$\times \langle \Lambda_c; \vec{p}', s|j_{cc}^\sigma(0)|\Lambda_b; \vec{p}, r\rangle^*, \quad (3)$$

where r and s are helicity indices and baryon states are normalized so that $\langle \vec{p}, r|\vec{p}', s\rangle = (2\pi)^3 (E/m)\delta^3(\vec{p} - \vec{p}')\delta_{rs}$. Finally the charged current is given by

$$j_{cc}^\mu = \overline{\Psi}_c \gamma^\mu(1 - \gamma_5)\Psi_b, \quad (4)$$

with Ψ_c and Ψ_b quark fields. The nonperturbative strong interaction effects are contained in the matrix elements of the weak current j_{cc}^μ which can be written in terms of six invariant form factors F_i, G_i with $i = 1, 2, 3$, as follows

$$\langle \Lambda_c; \vec{p}', s|j_\mu^{cc}(0)|\Lambda_b; \vec{p}, r\rangle = \bar{u}_{\Lambda_c}^{(s)}(\vec{p}')\{\gamma_\mu(F_1 - \gamma_5 G_1)$$
$$+ v_\mu(F_2 - \gamma_5 G_2)$$
$$+ v'_\mu(F_3 - \gamma_5 G_3)\}u_{\Lambda_b}^{(r)}(\vec{p}), \quad (5)$$

where u_{Λ_c} and u_{Λ_b} are dimensionless Λ_c and Λ_b Dirac spinors, normalized to $\bar{u}u = 1$, and $v_\mu = p_\mu/m_{\Lambda_b}$ ($v'_\mu = p'_\mu/m_{\Lambda_c}$) is the four velocity of the Λ_b (Λ_c) baryon. The form factors are functions of the velocity transfer $w = v \cdot v'$ or equivalently of $q^2 = (p - p')^2 = m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - 2m_{\Lambda_b} m_{\Lambda_c} w$. In the decay $\Lambda_b(p) \to \Lambda_c(p')l(k')\bar{\nu}_l(k)$ and for massless leptons, the variable q^2 ranges from 0 (smallest transfer), which corresponds to $w = w_{max} = (m_{\Lambda_b}^2 + m_{\Lambda_c}^2)/2m_{\Lambda_b} m_{\Lambda_c} \approx 1.434$, to $q_{max}^2 = (m_{\Lambda_b} - m_{\Lambda_c})^2$ (highest transfer, final Λ_c at rest), which corresponds to $w = 1$.

The differential decay rates from transversely (Γ_T) and longitudinally (Γ_L) polarized W's, are given, neglecting lepton masses, by (the total width is $\Gamma = \Gamma_L + \Gamma_T$) [41]

$$\frac{d\Gamma_T}{dw} = \frac{G^2|V_{cb}|^2}{12\pi^3} m_{\Lambda_c}^3 \sqrt{w^2 - 1} q^2 \{(w - 1)|F_1(w)|^2 + (w + 1)|G_1(w)|^2\},$$

$$\frac{d\Gamma_L}{dw} = \frac{G^2|V_{cb}|^2}{24\pi^3} m_{\Lambda_c}^3 \sqrt{w^2 - 1} \{(w - 1)|\mathcal{F}^V(w)|^2 + (w + 1)|\mathcal{F}^A(w)|^2\}, \quad (6)$$

$$\mathcal{F}^{V,A}(w) = [(m_{\Lambda_b} \pm m_{\Lambda_c})F_1^{V,A} + (1 \pm w)(m_{\Lambda_c} F_2^{V,A} + m_{\Lambda_b} F_3^{V,A})],$$

$$F_i^V \equiv F_i(w), \quad F_i^A \equiv G_i(w), \quad i = 1, 2, 3,$$

where in the last expression the $+(-)$ sign goes together with the $V(A)$ upper index. The polar angle distribution reads [41]:

$$\frac{d^2\Gamma}{dw d\cos\theta} = \frac{3}{8}\left(\frac{d\Gamma_T}{dw} + 2\frac{d\Gamma_L}{dw}\right)\{1 + 2\alpha' \cos\theta + \alpha'' \cos^2\theta\}, \quad (7)$$

where θ is the angle between $\vec{k}'$ and $\vec{p}'$ measured in the $W_{\text{off shell}}$ rest frame, and α' and α'' are asymmetry parame-

ters which can be expressed as

$$\alpha' = \frac{d\Gamma_T}{dw} \frac{da_T}{dw} \Big/ \left(\frac{d\Gamma_T}{dw} + 2\frac{d\Gamma_L}{dw}\right),$$

$$\frac{da_T}{dw} = -\frac{G^2|V_{cb}|^2}{6\pi^3} \frac{m_{\Lambda_c}^3}{\frac{d\Gamma_T}{dw}} q^2(w^2 - 1)F_1(w)G_1(w),$$

$$(8)$$

$$\alpha'' = \left(\frac{d\Gamma_T}{dw} - 2\frac{d\Gamma_L}{dw}\right) \Big/ \left(\frac{d\Gamma_T}{dw} + 2\frac{d\Gamma_L}{dw}\right). \quad (9)$$

There are other asymmetry parameters if the successive hadronic cascade decay $\Lambda_c \to a + b$, where a ($J_a = 1/2$) and b ($J_b = 0$) are hadrons, is considered. Two new angles are usually defined—Θ_Λ the angle between the Λ_c momentum in the Λ_b rest frame and the a hadron momentum in the Λ_c rest frame, and χ the relative azimuthal angle between the decay planes defined by the three-momenta of the l, ν leptons and the three-momenta of the a, b hadrons. The decay distributions with respect to these two angles read [41]:

$$\frac{d^2\Gamma}{dw\,d\cos\theta_\Lambda} \propto 1 + P_L \alpha_\Lambda \cos\theta_\Lambda,$$

$$\frac{d^2\Gamma}{dw d\chi} \propto 1 - \frac{3\pi^2}{32\sqrt{2}} \gamma\alpha_\Lambda \cos\chi,$$

$$(10)$$

where α_Λ is the asymmetry parameter in the Λ_c hadronic decay (for the nonleptonic decays $\Lambda_c \to \Lambda\pi$ and $\Lambda_c \to \Sigma\pi$ one has: $\alpha_{\Lambda_c^+ \to \Lambda\pi^+} = -0.94^{+0.24}_{-0.08}$ [42], -0.96 ± 0.42 [43], and $\alpha_{\Lambda_c^+ \to \Sigma^+\pi^0} = -0.45 \pm 0.32$ [42]), and P_L (longitudinal polarization of the daughter baryon Λ_c) and γ are given by

$$P_L = \left(\frac{d\Gamma_T}{dw}\frac{da_T}{dw} + \frac{d\Gamma_L}{dw}\frac{da_L}{dw}\right) \Big/ \left(\frac{d\Gamma_T}{dw} + \frac{d\Gamma_L}{dw}\right),$$

$$\frac{da_L}{dw} = -\frac{G^2|V_{cb}|^2}{12\pi^3} \frac{m_{\Lambda_c}^3}{\frac{d\Gamma_L}{dw}} (w^2 - 1)\mathcal{F}^V(w)\mathcal{F}^A(w),$$

$$(11)$$

$$\gamma = \left(\frac{G^2|V_{cb}|^2}{6\sqrt{2}\pi^3} m_{\Lambda_c}^3 \sqrt{q^2}\sqrt{w^2 - 1}\{(w + 1)\mathcal{F}^A(w)G_1(w)\right.$$

$$\left. - (w - 1)\mathcal{F}^V(w)F_1(w)\}\right) \Big/ \left(\frac{d\Gamma_T}{dw} + \frac{d\Gamma_L}{dw}\right). \quad (12)$$

The asymmetry parameters introduced in Eqs. (8) and (9) and Eqs. (11) and (12) are functions of the velocity transfer w. On averaging over w, the numerators and denominators are integrated separately and thus we have

$$\langle a_T \rangle = -\frac{G^2|V_{cb}|^2}{6\pi^3} \frac{m_{\Lambda_c}^3}{\Gamma_T} \int_0^{w_{\max}} q^2(w^2 - 1)F_1(w)G_1(w)dw,$$

$$(13)$$

$$\langle a_L \rangle = -\frac{G^2|V_{cb}|^2}{12\pi^3} \frac{m_{\Lambda_c}^3}{\Gamma_L} \int_0^{w_{\max}} (w^2 - 1)\mathcal{F}^V(w)\mathcal{F}^A(w)dw,$$

$$(14)$$

$$\langle \gamma \rangle = \frac{G^2|V_{cb}|^2}{6\sqrt{2}\pi^3} \frac{m_{\Lambda_c}^3}{\Gamma} \int_0^{w_{\max}} \sqrt{q^2}(w^2 - 1)^{1/2}\{(w + 1)$$

$$\times \mathcal{F}^A(w)G_1(w) - (w - 1)\mathcal{F}^V(w)F_1(w)\}dw, \quad (15)$$

$$\langle \alpha' \rangle = \frac{\langle a_T \rangle}{1 + 2R_{L/T}}, \qquad \langle \alpha'' \rangle = \frac{1 - 2R_{L/T}}{1 + 2R_{L/T}},$$

$$\langle P_L \rangle = \frac{\langle a_T \rangle + R_{L/T}\langle a_L \rangle}{1 + R_{L/T}}, \qquad R_{L/T} = \frac{\Gamma_L}{\Gamma_T}.$$

$$(16)$$

III. BARYON WAVE FUNCTIONS AND FORM FACTORS

Baryon wave functions are taken from our previous work in Ref. [38], where different nonrelativistic Hamiltonians (H) for the three quark (q, q', Q, with[3] $q, q' = l$ or s and $Q = c$ or b) system of the type

$$H = \sum_{i=q,q',Q} \left(m_i - \frac{\vec{\nabla}_{x_i}^2}{2m_i}\right) + V_{qq'} + V_{Qq} + V_{Qq'}, \quad (17)$$

were used. In the above equation $m_q, m_{q'}$, and m_Q are constituent quark masses, and the quark-quark interaction terms V_{ij} depend on the quark spin-flavor quantum numbers and the quark coordinates ($\vec{x}_1, \vec{x}_2$, and $\vec{x}_h$ for the q, q', and Q quarks, respectively, see Fig. 1).

A. Intrinsic Hamiltonian

We briefly outline here the procedure followed in [38]. To separate the center of mass free motion, we went to the heavy quark frame ($\vec{R}, \vec{r}_1, \vec{r}_2$), where $\vec{R}$ and $\vec{r}_1$ ($\vec{r}_2$) are the center of mass position in the LAB frame and the relative position of the q (q') quark with respect to the heavy Q quark. In this frame, the Hamiltonian reads

$$H = -\frac{\vec{\nabla}_{\vec{R}}^2}{2M_{tot}} + H^{int}, \quad (18)$$

$$H^{int} = \sum_{i=q,q'} h_i^{sp} + V_{qq'}(\vec{r}_1 - \vec{r}_2, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_Q} + \sum_{i=q,q',Q} m_i,$$

$$(19)$$

$$h_i^{sp} = -\frac{\vec{\nabla}_i^2}{2\mu_i} + V_{Qi}(\vec{r}_i, spin), \qquad i = q, q', \quad (20)$$

where M_{tot} is the sum of quark masses, ($m_q + m_{q'} + m_Q$), $\mu_{q,q'} = (1/m_{q,q'} + 1/m_Q)^{-1}$, and $\vec{\nabla}_{1,2} = \partial/\partial_{\vec{r}_1, \vec{r}_2}$. The intrinsic Hamiltonian H^{int} describes the dynamics of the baryon and we used a variational approach to solve it [44]. H^{int} consists of the sum of two single particle Hamiltonians (h_i^{sp}), which describe the dynamics of the light quarks in the mean field created by the heavy quark, plus the light—light interaction term, which includes the Hughes-Eckart term ($\vec{\nabla}_1 \cdot \vec{\nabla}_2$). In Ref. [38], several quark-

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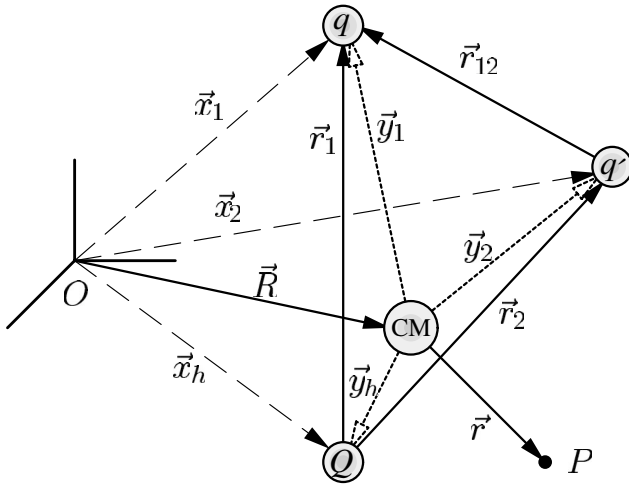[3] l denotes a light quark of flavor u or d.

FIG. 1.   Definition of different coordinates used through this work.

quark interactions, fitted to the meson spectra, were used to predict charmed and bottom baryon masses and some static electromagnetic properties. Furthers details can be found there.

B. $\Lambda_{b,c}$ and $\Xi_{b,c}$ wave functions and HQS

To solve the intrinsic Hamiltonian of Eq. (19), a HQS inspired variational approach was used in Ref. [38]. HQS is an approximate $SU(N_F)$ symmetry of QCD, N_F being the number of heavy flavors. This symmetry appears in systems containing heavy quarks with masses much larger than any other energy scale ($\eta = \Lambda_{\rm QCD}, m_u, m_d, m_s, \ldots$) controlling the dynamics of the remaining degrees of freedom. For baryons containing a heavy quark, and up to corrections of the order $\mathcal{O}(\frac{\eta}{m_Q})$, HQS guarantees that the heavy baryon light degrees of freedom quantum numbers (spin, orbital angular momentum, and parity) are always well defined. We took advantage of this fact in Ref. [38] in choosing the family of variational wave functions. Assuming that the ground states of the baryons are in s-wave and a complete symmetry of the wave function under the exchange of the two light quarks (u, d, s) flavor, spin, and space degrees of freedom [SU(3) quark model], the wave functions read (I and $S_{\rm light}^\pi$ are the isospin and the spin parity of the light degrees of freedom)[4]

   (i) Λ-type baryons: $I = 0$, $S_{\rm light}^\pi = 0^+$

$$\left| \Lambda_Q; J = \frac{1}{2}, M_J \right\rangle = \{|00\rangle_I \otimes |00\rangle_{S_{\rm light}}\}$$
$$\times \Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) \otimes |Q; M_J\rangle, \tag{21}$$

where the spatial wave function, since we are assuming s-wave baryons, can only depend on the relative distances r_1, r_2, and $r_{12} = |\vec{r}_1 - \vec{r}_2|$. In addition $\Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) = \Psi_{ll}^{\Lambda_Q}(r_2, r_1, r_{12})$ to guarantee a complete symmetry of the wave function under the exchange of the two light quarks (u, d) flavor, spin, and space degrees of freedom. Finally M_J is the baryon total angular momentum third component.[5]

   (ii) Ξ-type baryons: $I = \frac{1}{2}$, $S_{\rm light}^\pi = 0^+$

$$\left| \Xi_Q; J = \frac{1}{2}, M_J; M_T \right\rangle$$
$$= \frac{1}{\sqrt{2}} \{|ls\rangle \Psi_{ls}^{\Xi_Q}(r_1, r_2, r_{12}) - |sl\rangle$$
$$\times \Psi_{sl}^{\Xi_Q}(r_1, r_2, r_{12})\} \otimes |00\rangle_{S_{\rm light}} \otimes |Q; M_J\rangle, \tag{23}$$

where the isospin third component of the baryon M_T is that of the light quark l ($1/2$ or $-1/2$ for the u or the d quark, respectively).

The spatial wave function[6] $\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})$ was determined in [38], by use of the variational principle $\delta\langle B_Q | H^{\rm int} | B_Q \rangle = 0$, and can be easily reconstructed from Tables X and XI of that reference.

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[5]Note, that SU(3) flavor symmetry [SU(2), in the case of the Λ_Q baryon] would also allow for a component in the wave function of the type

$$\sum_{M_S M_Q} \left(\frac{1}{2} 1 \frac{1}{2} | M_Q M_S M_J\right)\{|00\rangle_I \otimes |1 M_S\rangle_{S_{\rm light}}\}\Theta_{ll}^{\Lambda_Q}(r_1, r_2, r_{12})$$
$$\otimes |Q; M_Q\rangle, \tag{22}$$

with $\Theta_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) = -\Theta_{ll}^{\Lambda_Q}(r_2, r_1, r_{12})$ (for instance terms of the type $r_1 - r_2$), and where the real numbers $(j_1 j_2 j | m_1 m_2 m) = \langle j_1 m_1 j_2 m_2 | j m \rangle$ are Clebsch-Gordan coefficients. This component is forbidden by HQS in the limit $m_Q \to \infty$, where $S_{\rm light}$ turns out to be well defined and set to zero for Λ_Q-type baryons. The most general SU(2) Λ_Q wave function will involve a linear combination of the two components, given in Eqs. (21) and (22). Neglecting $\mathcal{O}(\eta/m_Q)$ corrections, HQS imposes an additional constraint, which justifies the use of a wave function of the type of that given in Eq. (21) with the obvious simplification of the three-body problem. Within a spectator model for the Λ_b decay, in which the light degrees of freedom remain unaltered, and due to the orthogonality in the spin space, taking into account the $S_{\rm light} = 1$ components of the Λ_Q wave functions would lead to $\mathcal{O}(\eta^2/m_Q^2)$ corrections to the transition form factors of Eq. (5).

[6]Its normalization is given by

$$1 = \int d^3 r_1 \int d^3 r_2 |\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})|^2$$
$$= 8\pi^2 \int_0^{+\infty} dr_1 r_1^2 \int_0^{+\infty} dr_2 r_2^2 \int_{-1}^{+1} d\mu |\Psi_{qq'}^{B_Q}(r_1, r_2, r_{12})|^2 \tag{24}$$

where μ is the cosine of the angle formed by $\vec{r}_1$ and $\vec{r}_2$.

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[4]An obvious notation has been used for the isospin-flavor ($|I, M_I\rangle_I$, $|ls\rangle$ or $|sl\rangle$) and spin ($|S, M_S\rangle_{S_{\rm light}}$) wave functions of the light degrees of freedom.

C. The $\langle \Lambda_c; \vec{p}', s | j_\mu^{cc}(0) | \Lambda_b; \vec{p}, r \rangle$ and $\langle \Xi_c; \vec{p}', s | j_\mu^{cc}(0) | \Xi_b; \vec{p}, r \rangle$ matrix elements

We will first focus on the $\Lambda_b \to \Lambda_c$ matrix element. Within a NRCQM and considering only one-body current operators (spectator approximation) we have in the Λ_b rest frame

$$\langle \Lambda_c; \vec{p}', s | j_{cc}^\alpha(0) | \Lambda_b; \vec{0}, r \rangle = \sqrt{\frac{E'_{\Lambda_c}}{m_{\Lambda_c}}} \int d^3q_1 d^3q_2 d^3q_h d^3q'_h \sqrt{\frac{m_b}{E_b(\vec{q}_h)}} \sqrt{\frac{m_c}{E_c(\vec{q}'_h)}} [\bar{u}_c^{(s)}(\vec{q}'_h) \gamma^\alpha (1 - \gamma_5) u_b^{(r)}(\vec{q}_h)]$$

$$\times [\phi_{\vec{p}'}^{\Lambda_c}(\vec{q}_1, \vec{q}_2, \vec{q}'_h)]^* \phi_{\vec{0}}^{\Lambda_b}(\vec{q}_1, \vec{q}_2, \vec{q}_h), \tag{25}$$

with $\vec{p}' = \vec{p} - \vec{q} = -\vec{q}$ and u_c and u_b charm and bottom quark Dirac spinors. The wave functions in momentum space appearing in the above equation are the Fourier transformed of those in coordinate space

$$\phi_{\vec{P}}^{\Lambda_Q}(\vec{q}_1, \vec{q}_2, \vec{q}_h) = \int \frac{d^3x_1}{(2\pi)^{3/2}} \frac{d^3x_2}{(2\pi)^{3/2}} \frac{d^3x_h}{(2\pi)^{3/2}} e^{-i(\vec{q}_1 \cdot \vec{x}_1 + \vec{q}_2 \cdot \vec{x}_2 + \vec{q}_h \cdot \vec{x}_h)} \psi_{\vec{P}}^{\Lambda_Q}(\vec{x}_1, \vec{x}_2, \vec{x}_h), \tag{26}$$

where the spatial wave function of the Λ_Q baryon with total momentum $\vec{P}$ [see Eq. (18)] is given by

$$\psi_{\vec{P}}^{\Lambda_Q}(\vec{x}_1, \vec{x}_2, \vec{x}_h) = \frac{e^{i\vec{P} \cdot \vec{R}}}{(2\pi)^{3/2}} \Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}), \tag{27}$$

with $\Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12})$ defined in the previous subsection. The actual calculations are done in coordinate space, and we find

$$\langle \Lambda_c; -\vec{q}, s | j_{cc}^\alpha(0) | \Lambda_b; \vec{0}, r \rangle = \sqrt{\frac{E'_{\Lambda_c}}{m_{\Lambda_c}}} \int d^3r_1 d^3r_2 e^{i\vec{q} \cdot (m_q \vec{r}_1 + m_{q'} \vec{r}_2)/M_{tot}^c} [\Psi_{ll}^{\Lambda_c}(r_1, r_2, r_{12})]^*$$

$$\times \left\{ \sqrt{\frac{m_b}{E_b(\vec{l})}} \sqrt{\frac{m_c}{E_c(\vec{l}')}} [\bar{u}_c^{(s)}(\vec{l}') \gamma^\alpha (1 - \gamma_5) u_b^{(r)}(\vec{l})] \right\} \Psi_{ll}^{\Lambda_b}(r_1, r_2, r_{12}), \tag{28}$$

with the operators $\vec{l} = i\vec{\nabla}_{\vec{r}_1} + i\vec{\nabla}_{\vec{r}_2}$ and $\vec{l}' = \vec{l} - \vec{q}$ acting on the Λ_b intrinsic wave function. Finally, the flavor of the light quarks (q, q') are *up* and *down* and $M_{tot}^c = m_u + m_d + m_c$, with $m_u = m_d$ as dictated by SU(2)-isospin symmetry.

The $\Xi_b \to \Xi_c$ matrix element is easily obtained from the results above, by using $\Psi_{ls}^{\Xi_Q}$ and m_s instead of $\Psi_{ll}^{\Lambda_Q}$ and $m_{q'} = m_u = m_d$, respectively.

D. Heavy quark internal momentum expansion and form factor equations

Taking $\vec{q}$ in the positive z direction and by comparing both sides of Eq. (28) for the spin flip $\alpha = 1$ or 2 and spin nonflip $\alpha = 0$ and $\alpha = 3$ components, all form factors $F's$ and $G's$ can be found. The main problem lies on the operatorial nature of the right hand side of Eq. (28), which requires of some approximations to make its evaluation feasible. Nonrelativistic expansions of the involved momenta in Eq. (28) are usually performed [28], but this is only justified near q_{max}^2. With the Λ_b baryon at rest, $\vec{l}$ in Eq. (28) is an internal momentum which is much smaller than any of the heavy quark masses. On the other hand, the transferred momentum $\vec{q}$, which coincides, up to a sign, with the total momentum carried out by the Λ_c baryon, can

be large (note that $|\vec{q}| = m_{\Lambda_c} \sqrt{w^2 - 1}$ and at $q^2 = 0$, $|\vec{q}(w = w_{max})| \approx m_{\Lambda_b}/2$). We have expanded the right-hand side of Eq. (28), neglecting second order terms in $\vec{l}$ but keeping all orders in $\vec{q}$. For instance, this expansion for the charm quark energy gives: $E_c(\vec{l}') \approx E_c(\vec{q}) \times [1 - \vec{l} \cdot \vec{q}/E_c^2(\vec{q})] + \mathcal{O}(\vec{l}^2/m_Q^2)$ with $E_c(\vec{q}) \equiv E_c = (m_c^2 + \vec{q}^2)^{1/2}$. Thanks to this novel expansion of the electroweak current operator, in which $\vec{q}$ is exactly treated, we are able to predict the decay distributions for all q^2 values accessible in the physical decays, improving in this manner on the existing NRCQM calculations. Finally, we get the form factors from two (vector and axial) subsets of three equations with three unknowns ($F's$ and $G's$). For the $\Lambda_b \to \Lambda_c$ transition, these equations are compiled in Table I. The *hat* form factors and the dimensionless baryon integrals ($\mathcal{I}$ and $\mathcal{K}$) appearing in the table are given by

$$\hat{F}_i(w) = \left(\frac{E'_{\Lambda_c} + m_{\Lambda_c}}{2E'_{\Lambda_c}} \right)^{1/2} \left(\frac{2E_c}{E_c + m_c} \right)^{1/2} F_i(w),$$

$$\hat{G}_i(w) = \left(\frac{E'_{\Lambda_c} + m_{\Lambda_c}}{2E'_{\Lambda_c}} \right)^{1/2} \left(\frac{2E_c}{E_c + m_c} \right)^{1/2} G_i(w), \tag{29}$$

$$i = 1, 2, 3,$$

$$\mathcal{I}(w) = \int d^3r_1 d^3r_2 e^{i\vec{q} \cdot (m_q \vec{r}_1 + m_{q'} \vec{r}_2)/M_{tot}^c} [\Psi_{ll}^{\Lambda_c}(r_1, r_2, r_{12})]^* \Psi_{ll}^{\Lambda_b}(r_1, r_2, r_{12}), \tag{30}$$

TABLE I. Equations used to determine the $\Lambda_b \to \Lambda_c$ transition form factors. The *hat* form factors and baryon integrals ($\mathcal{I}$ and $\mathcal{K}$) are given in Eqs. (29)–(31).

Vector	
$\alpha = 0$, spin nonflip	$\hat{F}_1 + \hat{F}_2 + \frac{E'_{\Lambda_c}}{m_{\Lambda_c}}\hat{F}_3 = \mathcal{I} + \frac{\vec{q}^2\mathcal{K}}{2(E_c+m_c)}\left(\frac{m_c}{E_c^2} - \frac{1}{m_b}\right)$
$\alpha = 3$, spin nonflip	$\frac{|\vec{q}|}{E'_{\Lambda_c}+m_{\Lambda_c}}\hat{F}_1 + \frac{|\vec{q}|}{m_{\Lambda_c}}\hat{F}_3 = \frac{|\vec{q}|\mathcal{I}}{E_c+m_c} - \frac{|\vec{q}|\mathcal{K}}{2}\left(\frac{m_c}{E_c^2} + \frac{1}{m_b}\right)$
$\alpha = 2$, spin flip	$\frac{|\vec{q}|}{E'_{\Lambda_c}+m_{\Lambda_c}}\hat{F}_1 = \frac{|\vec{q}|\mathcal{I}}{E_c+m_c} - \frac{|\vec{q}|\mathcal{K}}{2}\left(\frac{m_c}{E_c^2} - \frac{1}{m_b}\right)$
Axial	
$\alpha = 0$, spin nonflip	$\frac{|\vec{q}|}{E'_{\Lambda_c}+m_{\Lambda_c}}\left(-\hat{G}_1 + \hat{G}_2 + \frac{E'_{\Lambda_c}}{m_{\Lambda_c}}\hat{G}_3\right) = -\frac{|\vec{q}|\mathcal{I}}{E_c+m_c} + \frac{|\vec{q}|\mathcal{K}}{2}\left(\frac{m_c}{E_c^2} + \frac{1}{m_b}\right)$
$\alpha = 3$, spin nonflip	$\hat{G}_1 - \frac{\vec{q}^2}{m_{\Lambda_c}(E'_{\Lambda_c}+m_{\Lambda_c})}\hat{G}_3 = \mathcal{I} + \frac{\vec{q}^2\mathcal{K}}{2(E_c+m_c)}\left(\frac{m_c}{E_c^2} - \frac{1}{m_b}\right)$
$\alpha = 1$, spin flip	$\hat{G}_1 = \mathcal{I} + \frac{\vec{q}^2\mathcal{K}}{2(E_c+m_c)}\left(\frac{m_c}{E_c^2} + \frac{1}{m_b}\right)$

$$\mathcal{K}(w) = \frac{1}{\vec{q}^2}\int d^3r_1 d^3r_2\, e^{i\vec{q}\cdot(m_q\vec{r}_1+m_{q'}\vec{r}_2)/M_{\text{tot}}^c}\left[\Psi_{ll}^{\Lambda_c}(r_1,r_2,r_{12})\right]^*[\vec{l}\cdot\vec{q}]\Psi_{ll}^{\Lambda_b}(r_1,r_2,r_{12}). \tag{31}$$

For degenerate transitions ($m_b = m_c = m_Q$), the baryon factors $\mathcal{I}(w)$ and $\mathcal{K}(w)$ are related, i.e., $2\mathcal{K}(w)/\mathcal{I}(w) = (m_q + m_{q'})/(m_q + m_{q'} + m_Q)$, as can be deduced from a integration by parts in Eq. (31). By means of a partial wave expansion and after a little of Racah algebra, the integrals get substantially simplified. Explicit expressions can be found in the Appendix A.

Baryon number conservation implies that $F(1) = \sum_i F_i(1) = 1$ in the limit of equal baryon states. The first equation of Table I leads to $\sum_i F_i(1) = \mathcal{I}(1)$, since $w = 1$ implies $|\vec{q}| = 0$. Besides, $\mathcal{I}(1)$ accounts for the overlap between the charmed and bottom baryon wave functions and therefore it takes the value one for equal baryon states, accomplishing exact baryon number conservation. In general, vector current conservation for degenerate transitions imposes the restriction $F_2(w) = F_3(w)$, which is violated within the spectator approximation assumed in this work. Thus, for instance, at zero recoil, we find $F_2(1) - F_3(1) = 1 - m_{\Lambda_Q}/M_{\text{tot}}$, and therefore we do not get vector current conservation because of baryon binding terms. Two body currents induced by interquark interactions are needed to conserve the vector current.

The corresponding Ξ_b decay quantities are obtained from the above expressions by means of the substitutions mentioned at the end of Sec. III C. Note that $\mathcal{I}$ and $\mathcal{K}$ depend on both the heavy and light flavors, hence, and for the sake of clarity, from now on we will use the notation $\mathcal{I}_\Lambda^{cb}$ or $\mathcal{I}_\Xi^{cb}$ for the Λ_b and Ξ_b decays and a similar notation for the $\mathcal{K}$ factors.

IV. HQET AND FORM FACTORS

When all energy scales relevant in the problem are much smaller than the heavy quark masses, HQS is an excellent tool to understand charm and bottom physics. Close to zero recoil ($w = 1$) and at leading order in the heavy quark mass expansion, only one universal (independent of the heavy flavors) form factor, the Isgur-Wise function[7] (ξ^{ren}) is required to describe the $\Lambda_b \to \Lambda_c$ semileptonic decay. To next order, $1/m_Q$, one more universal (χ^{ren}) function and one mass parameter ($\bar{\Lambda}$) are introduced. These functions, and also the form factors, depend on the heavy baryon light cloud flavor, and thus in general they will be different for Ξ^- transitions, though one expects small deviations thanks to the SU(3)-flavor symmetry.

We compile here some useful results from Refs. [7,45], where more details can be found. Including $1/m_Q$ corrections the $\Lambda_b \to \Lambda_c$ form factors factorize in the form

$$F_i(w) = N_i(w)\hat{\xi}_{cb}(w) + \mathcal{O}(1/m_Q^2),$$
$$G_i(w) = N_i^5(w)\hat{\xi}_{cb}(w) + \mathcal{O}(1/m_Q^2), \qquad i = 1, 2, 3, \tag{32}$$

$$\hat{\xi}_{cb}(w) = \xi^{\text{ren}}(w) + \left(\frac{\bar{\Lambda}}{2m_b} + \frac{\bar{\Lambda}}{2m_c}\right)\left[2\chi^{\text{ren}}(w) + \frac{w-1}{w+1}\xi^{\text{ren}}(w)\right], \tag{33}$$

where the coefficients N_i, N_i^5 contain both radiative ($\hat{C}_i, \hat{C}_i^5$)[8] and $1/m_Q$ corrections. $\bar{\Lambda}$ is the binding energy of the heavy quark in the corresponding Λ baryon ($\bar{\Lambda} = m_{\Lambda_Q} - m_Q$) and, because of the dependence on the heavy quark masses, $\hat{\xi}_{cb}$ is no longer a universal form factor. The function $\chi^{\text{ren}}(w)$ arises from higher-dimension operators in the HQET Lagrangian and vanishes at zero recoil. Both functions $\hat{\xi}_{cb}$ and ξ^{ren} are normalized to one at zero recoil.

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[7]Note that, though called in the same manner, because of the different light cloud, this function is different to that entering in the study of $B \to D$ and $B \to D^*$ semileptonic transitions.

[8]They are known up to order $\alpha_s^2(z\ln z)^n$, where $z = m_c/m_b$ is the ratio of the heavy-quark masses and $n = 0, 1, 2$.

TABLE II. Correction factors (taken from Ref. [7]) for the $\Lambda_b \rightarrow \Lambda_c$ decay form factors.

w	N_1	N_2	N_3	$\sum_i N_i$	N_1^5	N_2^5	N_3^5
1.00	1.49	-0.36	-0.10	1.03	0.99	-0.42	0.15
1.11	1.40	-0.32	-0.09	0.99	0.94	-0.37	0.13
1.22	1.32	-0.30	-0.09	0.93	0.91	-0.34	0.12
1.33	1.26	-0.27	-0.08	0.91	0.88	-0.31	0.11
1.44	1.20	-0.25	-0.07	0.88	0.85	-0.28	0.10

The numerical values of the correction factors N_i, N_i^5 depend on the value of $\bar{\Lambda}$, which is not precisely known. We reproduce here (Table II) Table 4.1 of Ref. [7], where these correction factors are given for all baryon velocity transfer w accessible in the $\Lambda_b \rightarrow \Lambda_c l \bar{\nu}_l$ decay.[9] The parameters $\bar{\Lambda}/2m_b$ and $\bar{\Lambda}/2m_c$ were set to 0.07 and 0.24, respectively. At zero recoil, Luke's theorem [36] protects the quantities $F(w) = \sum_i F_i(w)$ and $G_1(w)$ from $\mathcal{O}(1/m_Q)$ corrections

$$F(1) = \sum_i F_i(1) = \eta_V + \mathcal{O}(1/m_Q^2),$$
$$G_1(1) = \eta_A + \mathcal{O}(1/m_Q^2),$$
(34)

where η_V and η_A are entirely determined by short distance corrections [i.e., $N_1^5(1) = \hat{C}_1^5(1)$ and $\sum_i N_i(1) = \sum_i \hat{C}_i(1)$] which are in principle well known since they are computed using perturbative QCD techniques. The second relation might be used to extract a model independent (up to $1/m_Q^2$ corrections) value of $|V_{cb}|$ from the measurement of semileptonic Λ_b decays near zero recoil, where the rate is governed by the form factor G_1. From Eq. (6), one finds

$$\lim_{w \rightarrow 1} \frac{1}{\sqrt{w^2 - 1}} \frac{d\Gamma}{dw} = \frac{G^2 |V_{cb}|^2}{4\pi^3} m_{\Lambda_c}^3 (m_{\Lambda_b} - m_{\Lambda_c})^2 \eta_A^2$$
$$+ \mathcal{O}(1/m_c^2).$$
(35)

V. RESULTS

To obtain the wave functions for the Λ_Q and Ξ_Q baryons, we will use different NRCQM interactions whose details can be found in Ref. [38]. Following the notation of this reference, we will refer to them as AL1, AL1χ, AL2, AP1, AP2, and BD. Their free parameters had been adjusted in the meson sector [46–48]. The potentials considered differ in the form factors used for the hyperfine terms, the power of the confining term[10] ($p = 1$, as suggested by lattice QCD calculations [50], or $p = 2/3$ which for mesons gives the correct asymptotic Regge trajectories [51]), or the use of a form factor in the one gluon exchange

Coulomb potential [52]. All of them provide reasonable and similar masses and static properties for Λ_Q, Σ_Q, Σ_Q^*, Ξ', Ξ_Q', Ξ_Q^*, Ω_Q, and Ω_Q^* baryons [38].

For the Λ_b decay we will pay special attention to the AL1 and AL1χ interquark potentials. The AL1 potential is based on a phenomenological interquark interaction which includes a term with a shape and a color structure determined from the one gluon exchange contribution and a confinement potential. The second model (AL1χ) includes the same heavy quark/light quark potential as the AL1 model, while the light quark/light quark is built from the SU(2) chirally inspired quark-quark interaction of Ref. [53] which includes a pattern of spontaneous chiral symmetry breaking and that was applied with great success to the meson sector in Ref. [48].

From the experimental side, the Λ_b semileptonic branching fraction into the exclusive semileptonic mode was measured in DELPHI to be [18]

$$Br(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l) = [5.0^{+1.1}_{-0.8}(\text{stat})^{+1.6}_{-1.2}(\text{syst})]. \quad (36)$$

A remark is in order here; the perturbative QCD corrections have been neglected in Ref. [18], i.e., the correction factors N_i, N_i^5 are computed with $\hat{C}_1 = \hat{C}_1^5 = 1$ and $\hat{C}_{2,3} = \hat{C}_{2,3}^5 = 0$, and a functional form of the type

$$\hat{\xi}_{cb}(w) = e^{-\hat{\rho}^2(w-1)}, \quad (37)$$

is also assumed in that reference, where it is also found that[11]

$$\hat{\rho}^2 = 2.0^{+0.8}_{-1.1}, \quad (38)$$

where all uncertainties quoted in Ref. [18] have been added in quadratures. On the other hand, the branching fraction given by the Particle Data Group is [17]

$$Br(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l + \text{anything}) = (9.2 \pm 2.1)\%, \quad (39)$$

which is hardly consistent to that quoted in Eq. (36). Nevertheless, none of the values quoted in Eqs. (36) and (39) correspond to direct measurements. We will assume here an error weighted averaged value[12] of those given in

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[9]Note the values for those correction factors are somewhat different from the ones quoted in Ref. [45].

[10]The force which confines the quarks is still not well understood, although it is assumed to come from long-range nonperturbative features of QCD [49].

[11]Note that $-\hat{\rho}^2$ is not the slope at the origin of the universal Isgur-Wise function $\xi^{ren}(w)$ introduced in Eq. (33).

[12]We add in quadratures the statistical and systematic uncertainties quoted in Eq. (36).
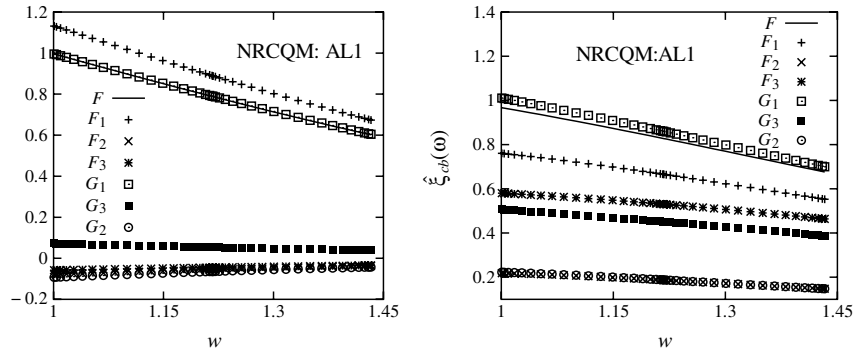
FIG. 2.   NRCQM $\Lambda_b \rightarrow \Lambda_c$ form factors (left) and $\hat{\xi}_{cb}$ function (right) from the AL1 interquark interaction.

Eqs. (36) and (39)

$$\langle \mathrm{Br}(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l) \rangle_{\mathrm{avg}} = (6.8 \pm 1.3)\%. \qquad (40)$$

The total Λ_b^0 width is given by its lifetime $\tau_{\Lambda_b^0} = 1.229 \pm 0.080$ ps [17] and thus one finds

$$\Gamma(\Lambda_b^0 \rightarrow \Lambda_c^+ l^- \bar{\nu}_l) = (5.5 \pm 1.4)10^{10} \text{ s}^{-1}. \qquad (41)$$

Besides, data from Z decays in DELPHI have been searched for $\bar{B}_d^0 \rightarrow D^{*+} l^- \bar{\nu}_l$ decays. These events are used to measure the CKM matrix element $|V_{cb}|$ [15]

$$|V_{cb}| = 0.0414 \pm 0.0012(\mathrm{stat}) \pm 0.0021(\mathrm{syst})$$

$$\pm 0.0018(\mathrm{theory}). \qquad (42)$$

Let us first examine the bare NRCQM predictions without including HQET constraints.

A. NRCQM form factors

In Fig. 2 we present the $\Lambda_b \rightarrow \Lambda_c$ form factors obtained from the AL1 interquark interaction (left) and also the predictions for the $\hat{\xi}_{cb}$ function (right) as extracted from any of the form factors ($\hat{\xi}_{cb} = F_i/N_i, G_i/N_i^5, F/\sum_i N_i, i = 1, 2, 3$) shown in the left panel. The correction factors N_i, N_i^5 are taken from Table II. Several comments are in order:

(i) As expected from HQS, the form factors $F_2, F_3, G_2,$ and G_3 are significantly smaller than the dominant ones F_1 and G_1.

(ii) Recalling the discussion of Sec. III D on vector current conservation for degenerate transitions, one must conclude that the NRCQM predictions for the F_2 and F_3 form factors are not reliable at all, since their sizes are comparable to the expected theoretical uncertainties $\mathcal{O}(1 - m_{\Lambda_Q}/M_{\mathrm{tot}})$ affecting them. Presumably, one should draw similar conclusions for the axial G_2 and G_3 form factors. As clearly seen in Fig. 2 the $\hat{\xi}_{cb}(w)$ functions obtained from the $F_2, F_3, G_2,$ and G_3 form factors substantially differ among themselves and are in complete disagreement to those obtained from the F and G_1 form factors.

(iii) NRCQM predictions for the vector F and axial G_1 form factors are much more reliable, and lead to similar $\hat{\xi}_{cb}$ functions, with discrepancies smaller than around 4%. Such discrepancies can be attributed either to $\mathcal{O}(1/m_Q^2)$ corrections, not included in $\hat{\xi}_{cb}$, or to deficiencies of the NRCQM. Lattice results of Ref. [19] for these two form factors, though have large errors, are in good agreement with the results shown in Fig. 2.

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B. HQET and NRCQM combined analysis

To improve the NRCQM results, we proceed as follows. We assume the NRCQM estimate of the vector form factor F ($F = F_1 + F_2 + F_3$) to be correct for the whole range of velocity transfers accessible in the physical decay[13] and use it to obtain the flavor depending $\hat{\xi}_{cb}$ function. Now by using Eq. (32) and the HQET coefficients N_i, N_i^5 compiled in Table II, we reconstruct the rest of form factors, in terms of which we can predict the longitudinal and transverse differential decay widths and the asymmetry parameters defined in Sec. II. We will estimate the theoretical error of the present analysis by accounting for the spread of the results obtained when all calculations are repeated by determining $\hat{\xi}_{cb}$ from the NRCQM G_1 form factor and/or by using different interquark interactions.

1. Λ_b decay

Results of our HQET improved NRCQM analysis for the Λ_b decay are compiled in Fig. 3 and Tables III and IV. In the first of the tables, we give the total and partially integrated semileptonic decay widths, split into the contributions to the rate from transversely (Γ_T) and longitudinally (Γ_L) polarized W's, and the value of the flavor depending $\hat{\xi}_{cb}(w)$ function and its derivatives at zero recoil, together with our estimates for the uncertainties of the present analysis. We also compare, when possible, with the

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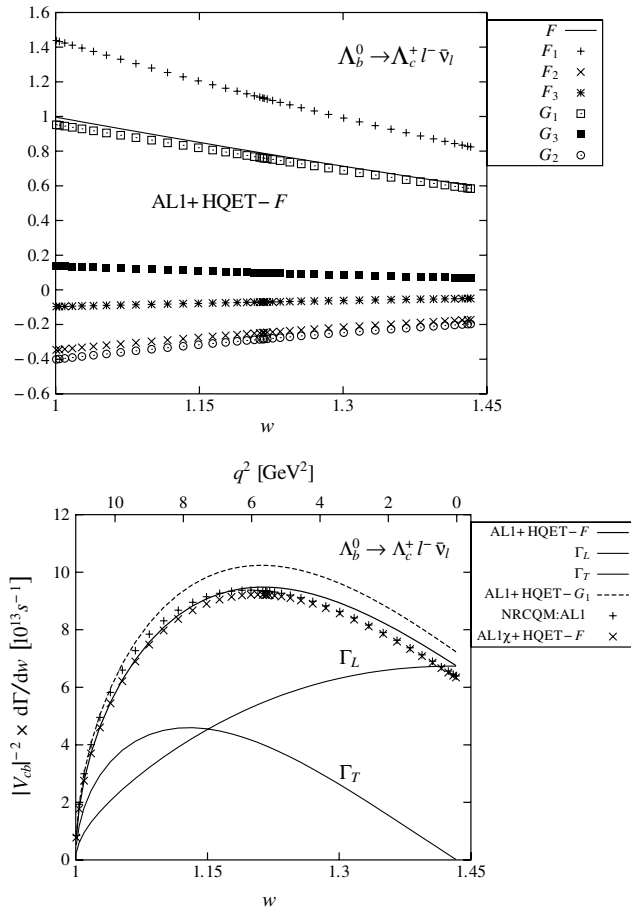[13]Let us remind here that the NRCQM gives correctly $F(1)$ in the case of degenerate transitions.

FIG. 3. $\Lambda_b \to \Lambda_c$ form factors (top) and differential decay width (bottom) constrained by means of the HQET relations of Eq. (32). AL1 and AL1χ interquark interactions have been used, and HQET $- F$ and HQET $- G_1$ stand for HQET models where $\hat{\xi}_{cb}$ is determined from the NRCQM predictions for the F and G_1 form factors, respectively. For the AL1 $+$ HQET $- F$ model, longitudinal and transverse differential decay distributions are plotted as well. For comparison in the bottom panel, we also show the bare NRCQM decay width distribution obtained from the form factors plotted in the left panel of Fig. 2.

lattice results of Ref. [19]. In the second of the tables, we compile our predictions for the w-averaged asymmetry parameters defined in Eqs. (13)–(16). Our results compare exceptionally well to those obtained by Cardarelli and Simula from a light-front constituent quark model [30]. On the other hand, we should mention that the NRCQM described in Sec. VA leads to similar (discrepancies of around 2–3%) differential decay rates, as can be appreciated in Fig. 3. From the discussion in Sec. VA, this fact should be considered as an accident. For the w-averaged asymmetry parameters given in Table IV, discrepancies are in general higher, being of the order of 20% for the $\langle a_T \rangle$ and $\langle \alpha' \rangle$ asymmetries.

From our theoretical determination of the total semileptonic width in Table III and the experimental estimate in

Eq. (41), we get

$$|V_{cb}| = 0.040 \pm 0.005(\text{stat})^{+0.001}_{-0.002}(\text{theory}), \qquad (43)$$

in remarkable agreement with the recent determination of this parameter from $\bar{B}^0_d \to D^{*+} l^- \bar{\nu}_l$ decays [Eq. (42)]. The experimental uncertainties on the Λ_b semileptonic branching ratio turn out to be the major source of error in the present determination of $|V_{cb}|$, being the theoretical error in both Eqs. (42) and (43) comparable in size. We point out nevertheless that our determination of $|V_{cb}|$ is based on a NRCQM description of the baryon and as such it is not model independent. From a conceptual point of view a determination of $|V_{cb}|$ based on Eq. (35) would be preferred in a nonrelativistic approach as both baryons are at rest in the $w \to 1$ limit. Unfortunately the lack of enough experimental data in that region prevents such a calculation. Note nevertheless that our partially integrated width $(\hat{\Gamma}_L + \hat{\Gamma}_T)$ is in good agreement with the lattice results of Ref. [19] for w values up to $w \approx 1.20$ ($|\vec{q}| \approx 0.66 m_{\Lambda_c}$) where lattice calculations are reliable, and that our total width $\Gamma = 3.46^{+0.27}_{-0.11}|V_{cb}|^2 10^{13} \text{ s}^{-1}$ agrees with the value of $\Gamma = 3.1 \pm 1.0|V_{cb}|^2 10^{13} \text{ s}^{-1}$ obtained in Ref. [25] using QCD sum rules.

With respect to the $1/m_Q$-corrected Isgur-Wise function $\hat{\xi}_{cb}(w)$ our results show a clear departure[14] from a single exponential [see Eq. (37)] functional form, and instead, in the velocity transfer range accessible in the physical decay, it is rather well described by a rank three polynomial in powers of $(w - 1)$. In what $\hat{\xi}'_{cb}(1)$ respects, our estimate lies in the lower end of the range of Eq. (38). As mentioned above, the perturbative QCD corrections were neglected in Ref. [18]. If we do not include the short distance contributions when relating the NRCQM AL1 F form factor and the HQET $\hat{\xi}_{cb}(w)$ function, the slope of this latter function becomes larger (in absolute value), i.e., $\hat{\xi}'_{cb}(1) = -0.99$, in closer agreement with the DELPHI estimate. Besides, the assumption in Ref. [18] of the functional form of Eq. (37) leads also to larger, in absolute value, slopes. Thus, to get a semileptonic decay width of $3.46 |V_{cb}|^2 10^{13} \text{ s}^{-1}$ (our prediction for AL1 $+$ HQET $- F$), a value of $\hat{\rho}^2 = 1.20$ is required.[15]

Finally, we would like to remark the minor differences, of the order of a few percent, existing between the AL1 and AL1χ NRCQM based predictions for the decay distributions. This is a common feature, when the different interquark interactions studied in Ref. [38] are considered. All results are compiled in Table III.

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[14]Note for instance that $\hat{\xi}''_{cb}(1)$ and $\hat{\xi}'''_{cb}(1)$ have changed signs with respect to those deduced from Eq. (37).

[15]Note also that both approaches provide dΓ/dw distributions which are quite similar making it difficult for experimentalists to decide which one is preferred.

TABLE III. Λ_b semileptonic decay: Theoretical predictions for totally and partially ($\hat{\Gamma}_{L,T} = \int_1^{w_0} dw \frac{d\Gamma_{L,T}}{dw}$) integrated decay widths, in units of $|V_{cb}|^2 10^{13}$ s^{-1} and for $\hat{\xi}_{cb}(w)$ and its derivatives at zero recoil. The meaning of columns two to eight is the same as in Fig. 3, with the obvious changes due to the use of different interquark interactions. In the ninth column (Theor. Avg.) we give our final results with theoretical uncertainties obtained from the spread of the results shown in the table. Finally in the last column we compile the Lattice QCD results of Ref. [19].

	HQET − F AL1	HQET − G₁ AL1	HQET − F AL1χ	HQET − F AL2	HQET − F AP1	HQET − F AP2	HQET − F BD	Theor. Avg.	Lattice Ref. [19]
Γ	3.46	3.73	3.35	3.57	3.50	3.60	3.49	$3.46^{+0.27}_{-0.11}$	$\cdots$
Γ_L	2.14	2.31	2.07	2.22	2.18	2.25	2.16	$2.14^{+0.17}_{-0.07}$	$\cdots$
Γ_T	1.31	1.42	1.28	1.34	1.33	1.36	1.32	$1.31^{+0.11}_{-0.03}$	$\cdots$
$\hat{\Gamma}_L, w_0$									
1.10	0.23	0.25	0.22	0.22	0.23	0.23	0.23	$0.23^{+0.02}_{-0.01}$	$0.23^{+0.03}_{-0.02}$
1.15	0.43	0.47	0.42	0.43	0.43	0.43	0.43	$0.43^{+0.04}_{-0.01}$	$0.44^{+0.08}_{-0.06}$
1.20	0.68	0.73	0.66	0.69	0.68	0.68	0.68	$0.68^{+0.05}_{-0.02}$	$0.71^{+0.17}_{-0.13}$
1.25	0.96	1.04	0.94	0.98	0.97	0.97	0.96	$0.96^{+0.08}_{-0.02}$	$1.0^{+0.3}_{-0.2}$
1.30	1.26	1.37	1.23	1.29	1.28	1.28	1.27	$1.26^{+0.11}_{-0.03}$	$1.4^{+0.5}_{-0.4}$
1.35	1.59	1.71	1.54	1.63	1.61	1.61	1.60	$1.59^{+0.12}_{-0.05}$	$\cdots$
$\hat{\Gamma}_L, w_0$									
1.10	0.34	0.37	0.34	0.34	0.34	0.34	0.34	$0.34^{+0.03}_{-0.00}$	$0.34^{+0.06}_{-0.04}$
1.15	0.57	0.62	0.56	0.58	0.57	0.57	0.57	$0.57^{+0.05}_{-0.01}$	$0.53^{+0.16}_{-0.14}$
1.20	0.79	0.86	0.78	0.80	0.80	0.80	0.79	$0.79^{+0.07}_{-0.01}$	$0.7^{+0.3}_{-0.3}$
1.25	0.98	1.06	0.96	1.00	0.99	0.99	0.99	$0.98^{+0.08}_{-0.02}$	$0.8^{+0.6}_{-0.5}$
1.30	1.14	1.23	1.11	1.16	1.15	1.15	1.14	$1.14^{+0.09}_{-0.03}$	$0.8^{+0.9}_{-0.8}$
1.35	1.24	1.35	1.22	1.27	1.26	1.26	1.25	$1.24^{+0.11}_{-0.02}$	$\cdots$
$\hat{\xi}_{cb}(1)$	0.97	1.01	0.97	0.97	0.97	0.97	0.97	$0.97^{+0.04}_{-0.00}$	0.99 ± 0.01
$-\hat{\xi}'_{cb}(1)$	0.58	0.65	0.64	0.52	0.58	0.52	0.56	$0.58^{+0.07}_{-0.06}$	1.1 ± 1.0
$-\hat{\xi}''_{cb}(1)$	0.73	0.59	0.72	0.79	0.63	0.70	0.82	$0.73^{+0.09}_{-0.14}$	$\cdots$
$\hat{\xi}'''_{cb}(1)$	2.3	2.0	2.6	2.3	1.8	1.9	2.5	$2.3^{+0.3}_{-0.5}$	$\cdots$

TABLE IV. Theoretical predictions for the w-averaged asymmetry parameters defined in Eqs. (13)–(16). We quote central values from the AL1 + HQET − F model and the theoretical uncertainties have been determined as in Table III.

	$R_{L/T}$	$\langle a_T \rangle$	$\langle a_L \rangle$	$\langle P_L \rangle$	$\langle \alpha' \rangle$	$\langle \alpha'' \rangle$	$\langle \gamma \rangle$
$\Lambda_b^0 \to \Lambda_c^+ l^- \bar{\nu}_l$	1.63 ± 0.02	-0.665 ± 0.002	-0.954 ± 0.001	-0.844 ± 0.003	-0.156 ± 0.001	-0.531 ± 0.004	0.439 ± 0.004
$\Xi_b^0 \to \Xi_c^+ l^- \bar{\nu}_l$	1.53 ± 0.04	-0.628 ± 0.004	-0.945 ± 0.002	-0.820 ± 0.004	-0.154 ± 0.001	-0.508 ± 0.008	0.475 ± 0.006

2. Ξ_b decay

Results of our HQET improved NRCQM analysis for the Ξ_b decay are compiled in Tables IV and V. As in the Λ_b-decay case, the decay parameters do not depend significantly on the potential, among those considered in this work. This fact allows us to make precise theoretical predictions, which nicely agree to the lattice results of Ref. [19]. On the other hand, we find small SU(3) deviations, and thus as a matter of example we find

$$\frac{\Gamma(\Xi_b \to \Xi_c l \bar{\nu}_l)}{\Gamma(\Lambda_b \to \Lambda_c l \bar{\nu}_l)} = 0.86^{+0.10}_{-0.07}, \qquad (44)$$

which will naturally fit within SU(3) symmetry expectations.

Finally, in Fig. 4 we plot the $\Lambda_b \to \Lambda_c$ and $\Xi_b \to \Xi_c$ $1/m_Q$-corrected Isgur-Wise functions, $\hat{\xi}_{cb}(w)$ from the AL1 + HQET − F model. We see there the size of possible SU(3) symmetry violations as a function of the velocity transfer w. The zero recoil slope, in absolute value, is significantly larger for the $\Xi_b \to \Xi_c$ transition than for the $\Lambda_b \to \Lambda_c$ one. A similar behavior is also found in the meson sector in the $B \to D_{(s)}, D^*_{(s)}$ decays. Lattice calculations show that the slope at zero recoil of the mesonic Isgur-Wise function is larger in magnitude in the case where the spectator quark is a strange one [4].

TABLE V.   As in Table III, but for Ξ_b baryon semileptonic decay.

	HQET $-F$	HQET $-G_1$	HQET $-F$	HQET $-F$	HQET $-F$	HQET $-F$	Theor.	Lattice
	AL1	AL1	AL2	AP1	AP2	BD	Avg.	Ref. [19]
Γ	2.96	3.21	3.04	2.97	3.12	3.08	$2.96^{+0.25}_{-0.00}$	$\cdots$
Γ_L	1.79	1.94	1.85	1.80	1.90	1.88	$1.79^{+0.15}_{-0.00}$	$\cdots$
Γ_T	1.17	1.27	1.19	1.17	1.22	1.21	$1.17^{+0.10}_{-0.00}$	$\cdots$
$\hat{\Gamma}_L, w_0$								
1.10	0.27	0.29	0.27	0.27	0.27	0.27	$0.27^{+0.02}_{-0.00}$	$0.28^{+0.02}_{-0.03}$
1.15	0.49	0.54	0.50	0.49	0.50	0.50	$0.49^{+0.05}_{-0.00}$	$0.54^{+0.07}_{-0.08}$
1.20	0.75	0.82	0.76	0.75	0.77	0.77	$0.75^{+0.07}_{-0.00}$	$0.86^{+0.14}_{-0.16}$
1.25	1.03	1.12	1.05	1.03	1.07	1.06	$1.03^{+0.09}_{-0.00}$	$1.2^{+0.2}_{-0.3}$
1.30	1.31	1.42	1.34	1.31	1.37	1.36	$1.31^{+0.11}_{-0.00}$	$1.7^{+0.4}_{-0.4}$
$\hat{\Gamma}_T, w_0$								
1.10	0.39	0.43	0.39	0.39	0.40	0.40	$0.39^{+0.04}_{-0.00}$	$0.38^{+0.05}_{-0.05}$
1.15	0.63	0.68	0.63	0.63	0.64	0.64	$0.63^{+0.05}_{-0.00}$	$0.58^{+0.13}_{-0.15}$
1.20	0.83	0.90	0.84	0.83	0.85	0.85	$0.83^{+0.07}_{-0.00}$	$0.7^{+0.3}_{-0.3}$
1.25	0.99	1.08	1.01	0.99	1.02	1.02	$0.99^{+0.09}_{-0.00}$	$\cdots$
1.30	1.10	1.19	1.12	1.10	1.14	1.13	$1.10^{+0.09}_{-0.00}$	$\cdots$
$\hat{\xi}_{cb}(1)$	0.97	1.01	0.96	0.97	0.97	0.97	$0.97^{+0.04}_{-0.01}$	0.99 ± 0.01
$-\hat{\xi}'_{cb}(1)$	1.14	1.22	1.06	1.15	1.02	1.05	1.14 ± 0.08	1.4 ± 0.8
$-\hat{\xi}''_{cb}(1)$	0.13	0.00	0.30	0.03	0.30	0.37	$0.13^{+0.24}_{-0.16}$	$\cdots$
$\hat{\xi}'''_{cb}(1)$	4.6	4.5	4.7	4.3	4.3	4.7	$4.6^{+0.1}_{-0.3}$	$\cdots$

VI. CONCLUDING REMARKS

We have identified two of the main deficiencies of the NRCQM description of the semileptonic decay of the Λ_b and Ξ_b baryons: (i) A standard momentum expansion of the electroweak current is totally unappropriated, far from the zero recoil point. (ii) Within the usual spectator model approximation, with only one-body current operators, the vector part of the electroweak charged current is not conserved for degenerate transitions. Both drawbacks prevent

FIG. 4. $\Lambda_b \to \Lambda_c$ and $\Xi_b \to \Xi_c$ $1/m_Q$-corrected Isgur-Wise functions $\hat{\xi}_{cb}(w)$ from the AL1 + HQET $-F$ model.

NRCQM's to make reliable predictions of form factors and totally integrated decay rates. In the present work we have solved both deficiencies, and thus we have developed a novel expansion for the electroweak current operator, where all orders on the transferred momentum $\vec{q}$ are kept. To improve on the second of the mentioned deficiencies, we have also implemented HQET constraints among the form factors. In addition to other desirable features, we would restore in this way, vector current conservation for degenerate transitions.

Our HQET improved NRCQM analysis leads to an accurate and reliable description of the Λ_b semileptonic decay. Thus, we determine the $1/m_Q$-corrected Isgur-Wise function which governs this process and, thanks to the branching fraction values quoted in Refs. [17,18], extract the modulus of the cb CKM matrix element [Eq. (43)]. Our determination of $|V_{cb}|$ comes out in total agreement with that obtained from semileptonic B $\to$ D* decays [Eq. (42)], and if it suffers from larger uncertainties that the latter one is because of a poorer experimental measurement of the semileptonic branching fraction for the Λ_b case. We also give various w-averaged asymmetry parameters, which determine the angular distribution of the decay.

In respect to the Ξ_b-semileptonic decay, we also find an accurate and reliable description of the various physical magnitudes which govern this transition and find SU(3)
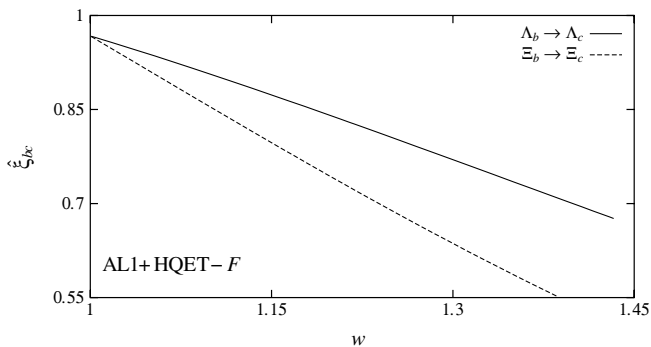
symmetry deviations of the order of 15%. At zero recoil, the $1/m_Q$-corrected Isgur-Wise function slope, in absolute value, is significantly larger for the $\Xi_b \to \Xi_c$ transition than for the $\Lambda_b \to \Lambda_c$ one.

APPENDIX A: EVALUATION OF THE I AND $\mathcal{K}$ INTEGRALS

We use a partial wave expansion of the Λ_b, Λ_c, Ξ_b, and Ξ_c wave functions,

$$\Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}) = \sum_{l=0}^{+\infty} f_l^Q(r_1, r_2) P_l(\mu),$$

$$\Psi_{ls}^{\Xi_Q}(r_1, r_2, r_{12}) = \sum_{l=0}^{+\infty} g_l^Q(r_1, r_2) P_l(\mu), \qquad Q = c, b, \tag{A1}$$

where μ is the cosine of the angle between the vectors $\vec{r}_1$ and $\vec{r}_2$, being $r_{12} = (r_1^2 + r_2^2 - 2r_1 r_2 \mu)^{1/2}$, and P_l Legendre polynomials of rank l. Therefore, the radial functions $f_l^Q(r_1, r_2)$ and $g_l^Q(r_1, r_2)$ are obtained from their corresponding wave function by means of

$$f_l^Q(r_1, r_2) = \frac{2l+1}{2} \int_{-1}^{+1} d\mu P_l(\mu) \Psi_{ll}^{\Lambda_Q}(r_1, r_2, r_{12}),$$

$$g_l^Q(r_1, r_2) = \frac{2l+1}{2} \int_{-1}^{+1} d\mu P_l(\mu) \Psi_{ls}^{\Xi_Q}(r_1, r_2, r_{12}), \tag{A2}$$

where r_{12} depend on r_1, r_2, and μ. In terms of integrals of the above functions, the baryon factor I_Λ^{cb} reads (we recall that for Λ_b decay, $|\vec{q}| = m_{\Lambda_c} \sqrt{w^2 - 1}$),

$$I_\Lambda^{cb}(w) = (4\pi)^2 \sum_l \sum_{l'} \sum_{l''} (-1)^{l''} (ll'l''|000)^2$$

$$\times \int_0^{+\infty} dr_1 r_1^2 j_{l''}(x_1) \int_0^{+\infty} dr_2 r_2^2 j_{l''}(x_2)$$

$$\times [f_l^c(r_1, r_2)]^* f_{l'}^b(r_1, r_2), \tag{A3}$$

where the flavor of the light quarks (q, q') are *up* and *down*

$$x_1 = \frac{m_q |\vec{q}|}{M_{\text{tot}}^c} r_1, \qquad x_2 = \frac{m_{q'} |\vec{q}|}{M_{\text{tot}}^c} r_2, \tag{A4}$$

and $M_{\text{tot}}^c = m_u + m_d + m_c$, with $m_u = m_d$. Besides, $(ll'l''|000)$ is a Clebsh-Gordan coefficient and j_l are spherical Bessel's functions.

On the other hand, the baryon factor $\mathcal{K}$ can be computed as

$$\mathcal{K}_\Lambda^{cb}(w) = \frac{16\pi^2}{\sqrt{3}|\vec{q}|} \sum_l \sum_{l'} \sum_{l''} \sum_{l'''} \sum_{L=l'+1, l'-1} (-1)^{l+L} i^{l''+l'''+1} ((2L+1)(2l''+1)(2l'''+1))^{1/2} (lLl''|000)(l'll'''|000)$$

$$\times (l'''l''1|000) W(l'''l1L; l'l'') \int_0^{+\infty} dr_1 r_1^2 \int_0^{+\infty} dr_2 r_2^2 \Big\{ j_{l''}(x_1) j_{l'''}(x_2) [f_l^c(r_1, r_2)]^* \Omega_L [f_{l'}^b(r_1, r_2)]$$

$$+ j_{l''}\Big(\frac{m_{q'}}{m_q} x_1\Big) j_{l'''}\Big(\frac{m_q}{m_{q'}} x_2\Big) [f_l^c(r_1, r_2)]^* \Omega_L [f_{l'}^b(r_1, r_2)] \Big\}, \tag{A5}$$

where $W(...)$ are Racah coefficients, and the differential operators Ω_L are defined as

$$\Omega_{L=l'+1} = -\Big(\frac{l'+1}{2l'+1}\Big)^{1/2} \Big[\frac{\partial}{\partial r_1} - \frac{l'}{r_1}\Big],$$

$$\Omega_{L=l'-1} = \Big(\frac{l'}{2l'+1}\Big)^{1/2} \Big[\frac{\partial}{\partial r_1} + \frac{l'+1}{r_1}\Big]. \tag{A6}$$

Note that $\mathcal{K}$ remains finite in the limit $|\vec{q}| \to 0$, since one cannot take the orders (l'' and l''') of both Bessel functions to be 0 due to the Clebsh-Gordan coefficient $(l'''l''1|000)$.

For Ξ_b decay, I_Ξ^{cb} and $\mathcal{K}_\Xi^{cb}$ are obtained from Eqs. (A3)–(A6) by replacing f-type radial wave functions by g-type ones and taking $m_{q'} = m_s$.

Finally, in the $m_b > m_c >> m_q, m_{q'}$ limit and in the neighborhood of $w = 1$, the I and $\mathcal{K}$ baryon factors behave like $\mathcal{O}(1)$ and $\mathcal{O}(m_q/m_c, m_{q'}/m_c)$, respectively.[16]

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[16]This is trivial, since in this limit for the I integral case, the $l'' = 0$ contribution becomes the dominant one, while for the $\mathcal{K}$ factor, the $l'' = l''' = 0$ contribution is forbidden by the Clebsh-Gordan $(l'''l''1|000)$. Thus, for this latter baryon factor and in the mentioned limit, the leading contributions are the $l'' = 1$, $l''' = 0$ and $l'' = 0$, $l''' = 1$ ones.

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