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Dissipative neutrino oscillations in randomly fluctuating matter

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The generalized dynamics describing the propagation of neutrinos in randomly fluctuating media is analyzed: It takes into account matter-induced, decoherence phenomena that go beyond the standard Mikheyev-Smirnov-Wolfenstein (MSW) effect. A widely adopted density fluctuation pattern is found to be physically untenable: A more general model needs to be instead considered, leading to flavor changing effective neutrino-matter interactions. They induce new, dissipative effects that modify the neutrino oscillation pattern in a way amenable to a direct experimental analysis.

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I. INTRODUCTION

When a neutrino propagates in a constant distribution of matter, coherent forward scattering phenomena can affect its time evolution. Despite the smallness of the cross section induced by the neutrino interaction with the medium, these matter effects can significantly modify the oscillation pattern, through the so-called Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [1,2].

However, forward scattering phenomena are just the simplest matter-induced effects that can occur to a neutrino when the medium is allowed to fluctuate. In this case, the neutrino can be viewed as an open system, i.e., a subsystem immersed in an external environment (the medium) [3–7]; its time evolution, obtained from the total neutrino + matter dynamics by eliminating (i.e., integrating over) the matter degrees of freedom, is no longer unitary: It takes into account possible exchanges of entropy and energy between the neutrino and the fluctuating medium.

In many physical situations, one can safely ignore the details of the matter dynamics and use an effective description of the medium as a classical, random external field. Quite in general, any environment can be modeled in this way, provided the characteristic decay time of the associated correlations is sufficiently small with respect to the typical evolution time of the subsystem. In the case of relativistic neutrinos, this time scale can be roughly identified with the vacuum oscillation length: We shall therefore consider media that fluctuate on time scales shorter than this. It has been recently pointed out that the interior of the sun could indeed satisfy such a condition [8], as likely as the earth mantel. Thus, a neutrino created in the sun or moving through the earth would effectively see a random fluctuating distribution of scattering centers and therefore be subjected to stochastic, incoherent interaction with the medium. In this situation, correlations in the medium play a fundamental role: They are responsible for the generation of new matter effects, beyond the MSW ones, leading to irreversibility and loss of quantum coherence.

The effects of fluctuating matter on neutrino propagation have been first discussed in [9–11], and recently reconsidered in [12–14]. However, all these analyses deal with a simple density fluctuation pattern, naturally suggested by the standard MSW treatment. Further, these fluctuations are assumed to be exactly δ correlated; this is a highly idealized description of the environment that, e.g., for heat baths, can be attained only in the limit of infinite temperature.

Instead, in the following a more realistic exponentially damped form for the correlation functions in the medium will be adopted. Limiting for simplicity the discussion to the analysis of the oscillations of two species of neutrinos, we shall see that the effects induced by matter fluctuations can be fully described in terms of a limited number of phenomenological parameters. They affect the oscillation pattern in a very distinctive way that is amenable to a direct experimental study.

On the other hand, when the simplified density fluctuation hypothesis considered in [9–14] is adopted, a single constant is sufficient to parametrize the new matter effects. However, this approximation appears physically untenable, since by adopting it certain transition probabilities take unacceptable negative values; this serious inconsistency can be cured only by allowing more general matter fluctuations, pointing towards the presence of flavor changing neutrino-matter interactions.[1]

As a final remark, it is interesting to point out that the dissipative effects induced by a randomly fluctuating me-

[1]Although within a mean-field (MSW) approach, this possibility has recently been reconsidered in [15–17] and found compatible with present experimental data.

dium on neutrino oscillations involve in general the CP-violating phase that is present in the mixing matrix for Majorana neutrinos. Therefore, contrary to the vacuum case, matter oscillation experiments can provide, at least in principle, a way to distinguish between Dirac and Majorana neutrinos.

II. MASTER EQUATION

In discussing the mixing of two neutrino species, we shall adopt the familiar effective description in terms of a two-dimensional Hilbert space [18–22]; the flavor states, that we shall conventionally call $|\nu_e\rangle$ and $|\nu_\mu\rangle$, will be chosen as basis states. With respect to this basis, the physical neutrino states are then represented by density matrices R, i.e., by Hermitian 2×2 matrices, with nonnegative eigenvalues and unit trace. Their time evolution equation can be cast in a standard Liouville–von Neumann form [23,24]:

$$\frac{\partial R(t)}{\partial t} = -i[H_0, R(t)] + L_t[R(t)]. \tag{2.1}$$

The first piece on the right-hand side describes the propagation of the neutrinos in vacuum; in the chosen basis, the effective Hamiltonian H_0 takes the standard form:

$$H_0 = \omega \vec{n} \cdot \vec{\sigma}, \tag{2.2}$$

where $\omega = \Delta m^2/4E$, Δm^2 being the square mass difference of the two mass eigenstates and E the average neutrino energy, while the unit vector $\vec{n} = (\sin 2\theta, 0, -\cos 2\theta)$ contains the dependence on the mixing angle, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ being the vector of Pauli matrices. The additional contribution $L_t[R]$ takes into account the presence of matter. As explained above, we shall consider the case of a rapidly fluctuating medium, which can be described by classical stochastic fields. Its action on the traveling neutrinos can then be expressed via the commutator with a time-dependent Hermitian matrix $V(t)$,

$$L_t[R(t)] = -i[V(t), R(t)], \qquad V(t) = \vec{V}(t) \cdot \vec{\sigma}, \tag{2.3}$$

whose components $V_1(t)$, $V_2(t)$, $V_3(t)$ form a real, stationary Gaussian stochastic field $\vec{V}(t)$; they are assumed to have in general a nonzero constant mean and translationally invariant correlations:

$$\hat{W}_{ij}(t-s) \equiv \langle V_i(t)V_j(s)\rangle - \langle V_i(t)\rangle\langle V_j(s)\rangle, \qquad i, j = 1, 2, 3. \tag{2.4}$$

Since the generalized Hamiltonian $V(t)$ in (2.3) involves stochastic variables, the density matrix $R(t)$, solution of the equation of motion (2.1), is also stochastic. Instead, we are interested in the behavior of the reduced density matrix $\rho(t) \equiv \langle R(t)\rangle$ which is obtained by averaging over the noise; it is $\rho(t)$ that describes the effective evolution of the neutrinos in the medium and allows the computation of relevant transition probabilities. By making the additional assumption that neutrinos and noise be decoupled at $t = 0$, so that the initial state is $\rho(0) \equiv \langle R(0)\rangle = R(0)$, a condition very well satisfied in typical situations, an effective master equation for $\rho(t)$ can be derived by going to the interaction representation, where we set:

$$\tilde{R}(t) = e^{itH_0}R(t)e^{-itH_0}, \qquad \tilde{\vec{\sigma}}(t) = e^{itH_0}\vec{\sigma}e^{-itH_0}, \qquad \tilde{L}_t[\ ] \equiv -i[\vec{V}(t)\cdot\tilde{\vec{\sigma}}(t),\ ]. \tag{2.5}$$

By averaging $\tilde{R}(t)$ over the noise, we get the reduced density matrix $\tilde{\rho}(t) \equiv \langle \tilde{R}(t)\rangle$ in the interaction representation: It is convenient to operate on the standard series expansion of $\tilde{R}(t)$, so that

$$\tilde{\rho}(t) = \mathcal{N}_t[\tilde{\rho}(0)] \equiv \sum_{k=0}^{\infty} N_t^{(k)}[\tilde{\rho}(0)] \tag{2.6a}$$

where the terms $N_t^{(k)}$ are explicitly given by

$$N_t^{(k)}[\tilde{\rho}(0)] = \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{k-1}} ds_k \times \langle \tilde{L}_{s_1}\tilde{L}_{s_2}\cdots\tilde{L}_{s_k}\rangle[\tilde{\rho}(0)], \tag{2.6b}$$

with $N_t^{(0)} = 1$ the identity, $N_t^{(0)}[\tilde{\rho}(0)] = \tilde{\rho}(0)$. The resulting series is a sum over multiple integrals of correlators $\langle \tilde{L}_{s_1}\tilde{L}_{s_2}\cdots\tilde{L}_{s_k}\rangle$, that is of averages over the noise of successive commutators with respect to the stochastic operators $V(t) = \vec{V}(t)\cdot\vec{\sigma}$ at different times. The density matrix $\tilde{\rho}(0)$ is not averaged over due to the assumption on the initial state $\rho(0) = \tilde{\rho}(0)$.

In order to arrive at a more manageable time evolution, we use a technique [23], of which we give a brief account below, that leads to a so-called convolutionless master equation and is based on the hypothesis of weak coupling between system and stochastic environment. The first step is to write the formal inverse of the map $\mathcal{N}_t$ in (2.6a), i.e., $\tilde{\rho}(0) = \mathcal{N}_t^{-1}[\tilde{\rho}(t)]$, so that

$$\mathcal{N}_t^{-1} = (1 + N_t^{(1)} + N_t^{(2)} + \cdots)^{-1} = 1 - N_t^{(1)} - N_t^{(2)} - (N_t^{(1)})^2 - \cdots, \tag{2.7}$$

where only terms containing up to two-point correlation functions have been indicated. Further, denoting with $\dot{N}_t^{(k)}$ the time derivative of $N_t^{(k)}$, it follows that the reduced density matrix in the interaction representation satisfies the equation of motion

$$\frac{\partial \tilde{\rho}(t)}{\partial t} = \frac{\partial \mathcal{N}_t}{\partial t}[\tilde{\rho}(0)] = \frac{\partial \mathcal{N}_t}{\partial t}\mathcal{N}_t^{-1}[\tilde{\rho}(t)] = \{\dot{N}_t^{(1)} + (\dot{N}_t^{(2)} - \dot{N}_t^{(1)}N_t^{(1)}) + \cdots\}[\tilde{\rho}(t)]. \tag{2.8}$$

Since the interaction of the traveling neutrinos with the medium is weak, one can focus the attention on the dominant terms of the previous expansion, neglecting all contributions higher than the second-order ones. Further, since the characteristic decay time of correlations in the medium is by assumption much smaller than the typical time scale of the system, the memory effects implicit in (2.8) should not be physically relevant and the use of the Markovian approximation justified. This is implemented in practice by extending to infinity the upper limit of the integrals appearing in $\dot{N}^{(2)}$ and $N^{(1)}$ [3–5].

By returning to the Schrödinger representation, one finally obtains [24]

$$\frac{\partial \rho(t)}{\partial t} = -i[H, \rho(t)] + L[\rho(t)], \tag{2.9a}$$

where

$$H = H_0 + H_1 + H_2 \equiv \vec{\Omega} \cdot \vec{\sigma}, \tag{2.9b}$$

$$L[\rho] = \frac{1}{2} \sum_{i,j=1}^{3} C_{ij}[2\sigma_i \rho \sigma_j - \{\sigma_j \sigma_i, \rho\}]. \tag{2.9c}$$

The effective Hamiltonian in matter, H, differs from the one in vacuum, H_0, by first-order terms [coming from the piece $\dot{N}^{(1)}$ in (2.8)] depending on the noise mean values:

$$H_1 = \langle \vec{V}(t) \rangle \cdot \vec{\sigma}, \tag{2.10}$$

and by second-order contributions [coming from the second-order terms in (2.8)],

$$H_2 = \sum_{i,j,k=1}^{3} \epsilon_{ijk} C_{ij} \sigma_k, \tag{2.11}$$

involving the noise correlations (2.4) through the time-independent combinations:

$$C_{ij} = \sum_{k=1}^{3} \int_0^{\infty} dt \hat{W}_{ij}(t) U_{kj}(-t), \tag{2.12}$$

where the 3×3 orthogonal matrix $U(t)$ is defined by the following transformation rule: $e^{itH_0}\sigma_i e^{-itH_0} = \sum_{j=1}^{3} U_{ij}(t)\sigma_j$. On the other hand, the contribution $L[\rho]$ in (2.9c) is a time-independent, trace-preserving linear map involving the symmetric coefficient matrix $C_{ij} \equiv C_{ij} + C_{ji}$. It introduces irreversibility, inducing in general dissipation and loss of quantum coherence. Altogether, Eq. (2.9) generates a semigroup of linear maps, $\Gamma_t : \rho(0) \mapsto \rho(t) \equiv \Gamma_t[\rho(0)]$, for which composition is defined only forward in time: $\Gamma_t \circ \Gamma_s = \Gamma_{t+s}$, with $t, s \geq 0$; this is a very general physical requirement that should be satisfied by all Markovian open system dynamics.[2] The set of maps Γ_t is usually referred to as a quantum dynamical semigroup [3–7].

The typical observable that is accessible to the experiments is the probability $\mathcal{P}_{\nu_e \to \nu_\mu}(t)$ for having a transition to a neutrino of type ν_μ at time t, assuming that the neutrino has been generated as ν_e at $t = 0$. In the language of density matrices, it is given by

$$\mathcal{P}_{\nu_e \to \nu_\mu}(t) \equiv \mathrm{Tr}[\rho_{\nu_e}(t) \rho_{\nu_\mu}], \tag{2.13}$$

where $\rho_{\nu_e}(t)$ is the solution of (2.9) with the initial condition given by the matrix $\rho_{\nu_e}(0) = \rho_{\nu_e} \equiv |\nu_e\rangle\langle\nu_e|$, while $\rho_{\nu_\mu} = 1 - \rho_{\nu_e}$. By expanding the neutrino density matrix in terms of the Pauli matrices and the identity σ_0, $\rho = [\sigma_0 + \vec{\rho} \cdot \vec{\sigma}]/2$, the linear equation (2.9a) reduces to a diffusion equation for the components ρ_1, ρ_2, ρ_3 of the vector $\vec{\rho}$:

$$\frac{\partial \vec{\rho}(t)}{\partial t} = -2\mathcal{H} \vec{\rho}(t); \tag{2.14}$$

the entries of 3×3 matrix $\mathcal{H}$ can be expressed in terms of the coefficients Ω_i and C_{ij} appearing in the Hamiltonian and noise contribution in (2.9b) and (2.9c) [26]:

$$\mathcal{H} = \begin{bmatrix} a & b + \Omega_3 & c - \Omega_2 \\ b - \Omega_3 & \alpha & \beta + \Omega_1 \\ c + \Omega_2 & \beta - \Omega_1 & \gamma \end{bmatrix}, \tag{2.15}$$

with $a = C_{22} + C_{33}$, $\alpha = C_{11} + C_{33}$, $\gamma = C_{11} + C_{22}$, $b = -C_{12}$, $c = -C_{13}$, $\beta = -C_{23}$. The solution of (2.14) involves the exponentiation of the matrix $\mathcal{H}$,

$$\vec{\rho}(t) = \mathcal{M}(t) \vec{\rho}(0), \qquad \mathcal{M}(t) = e^{-2\mathcal{H}t}, \tag{2.16}$$

so that the transition probability in (2.13) can be rewritten as

$$\begin{aligned} \mathcal{P}_{\nu_e \to \nu_\mu}(t) &= \frac{1}{2}\Big[1 + \sum_{i,j=1}^{3} \rho^i_{\nu_e} \rho^j_{\nu_\mu} \mathcal{M}_{ij}(t)\Big] \\ &= \frac{1}{2}[1 - \mathcal{M}_{33}(t)]. \end{aligned} \tag{2.17}$$

Indeed, taking the standard form of the Pauli matrices with respect to the orthonormal basis $|\nu_e\rangle = \binom{1}{0}$ and $|\nu_\mu\rangle = \binom{0}{1}$, then $\rho_{\nu_e} = (1 + \sigma_3)/2$ and $\rho_{\nu_\mu} = (1 - \sigma_3)/2$.

When correlations in the medium are negligible, i.e., the combination in (2.4) is vanishingly small, Eq. (2.9) describes standard (MSW) matter effects, for the presence of matter is signaled solely by the shift H_1 in the effective Hamiltonian. In this case, the neutrino-medium interaction

[2]Notice that the procedure of averaging transition probabilities over random matter profiles as performed in [25] is not compatible with this basic evolution law.

is dominated by coherent forward scattering, and, in the absence of flavor changing effects, the stochastic vector field in (2.3) results oriented along the third axis, whence $H_1 = A\sigma_3$, where $A \equiv \langle V_3(t)\rangle = G_F n_e/\sqrt{2}$ gives the extra energy contribution that electron neutrinos receive when traveling in ordinary matter (G_F is the Fermi constant, while n_e represents the electron number density in the medium). As a consequence, the transition probability in (2.17) can be expressed in terms of a modified frequency ω_M and mixing angle θ_M in matter,

$$\begin{aligned} \omega_M &= \omega[\sin^2 2\theta + (1 - A/A_R)^2 \cos^2 2\theta]^{1/2}, \\ \sin 2\theta_M &= \frac{\omega}{\omega_M}\sin 2\theta, \end{aligned} \tag{2.18}$$

$A_R = \omega\cos 2\theta$ being the value of A at resonance. In fact, the assumption of negligible correlations amounts to considering in Eq. (2.14) a matrix $\mathcal{H}$ of the form

$$\mathcal{H} = \begin{pmatrix} 0 & \Omega_3 & 0 \\ -\Omega_3 & 0 & \Omega_1 \\ 0 & -\Omega_1 & 0 \end{pmatrix}, \qquad \Omega_1 = \omega\sin(2\theta), \qquad \Omega_3 = A - \omega\cos(2\theta). \tag{2.19}$$

This matrix can be easily exponentiated as in (2.16),

$$\mathcal{M}(t) = \begin{pmatrix} \frac{\Omega_1^2 + \Omega_3^2\cos(2\omega_M t)}{\omega_M^2} & -\frac{\Omega_3}{\Omega_1}\sin(2\omega_M t) & \frac{\Omega_1\Omega_3}{\omega_M^2}[1-\cos(2\omega_M t)] \\ -\frac{\Omega_3}{\Omega_1}\sin(2\omega_M t) & \cos(2\omega_M t) & -\frac{\Omega_1}{\Omega_1}\sin(2\omega_M t) \\ \frac{\Omega_1\Omega_3}{\omega_M^2}[1-\cos(2\omega_M t)] & -\frac{\Omega_1}{\Omega_1}\sin(2\omega_M t) & \frac{\Omega_3^2 + \Omega_1^2\cos(2\omega_M t)}{\omega_M^2} \end{pmatrix}, \tag{2.20}$$

whence the explicit form of the element $\mathcal{M}_{33}(t)$ yields the familiar expression:

$$\mathcal{P}_{\nu_e\to\nu_\mu}(t) = \sin^2 2\theta_M \sin^2 \omega_M t. \tag{2.21}$$

The situation can significantly change for neutrinos immersed in a random medium; while traveling in it, they encounter matter fluctuations, whose correlations $\hat{W}_{ij}(t-s)$ determine the dissipative contribution in (2.9c). In a typical bath at finite temperature, the correlation functions assume an exponentially damped form; therefore, one can generically write

$$\hat{W}_{ij}(t-s) = W_{ij} e^{-\lambda_{ij}|t-s|}, \tag{2.22}$$

with W_{ij} and λ_{ij} time-independent, real coefficients, with $\lambda_{ij} \geq 0$. Further, as discussed before, the stochastic medium fluctuates on time intervals much shorter than the typical neutrino "free" evolution time scale $1/\omega$, so that the decay parameters λ_{ij} must be much larger than the vacuum frequency ω. This fact allows neglecting all contributions higher than the first-order one in the ratio ω/λ_{ij} while evaluating the coefficients C_{ij} in (2.12). For generic correlations as in (2.22), these coefficients, and therefore the entries of the matrix $\mathcal{H}$ in (2.15), are all nonvanishing. However, the parameters a, b, c, α, β, γ describing matter decoherence effects are not all free: As we shall see, physical consistency requires them to satisfy certain inequalities; in turn, these constraints reflect some fundamental characteristics of the matter-neutrino interactions.

We shall now discuss some interesting cases of the master equation (2.9), corresponding to specific physical realizations of the medium through which the neutrinos propagate.

III. GENERALIZED MSW DYNAMICS

The simplest instance of a stochastic medium corresponds to ordinary matter with density fluctuations, where only the propagation of electron neutrinos is affected. It generalizes the familiar MSW mean-field treatment by adding to it decoherence effects. In this case, the stochastic Hamiltonian in (2.3) becomes diagonal and, without loss of generality, only the stochastic field $V_3(t)$ can be taken to be nonvanishing; the neutrinos are still forward scattered by the medium, although no longer in a coherent way. This situation is discussed in [9–14], where however the density fluctuations in the medium are taken to be exactly δ-function correlated. This is a highly idealized assumption, that can hardly be reproduced in ordinary conditions. Instead, the much more realistic exponential ansatz (2.22) will be used here, where the only nonvanishing correlation strength and decay constant are $W_{33} \equiv W$ and $\lambda_{33} \equiv \lambda$, respectively.

The noise contributions in (2.9) can be explicitly computed; within our approximation, one finds that only the entries C_{23} and C_{33} of the coefficient matrix in (2.9c) are nonvanishing,

$$a = C_{33} = \frac{2W}{\lambda}, \qquad \beta = -C_{23} = \frac{\omega W}{\lambda^2}\sin 2\theta, \tag{3.1}$$

while the Hamiltonian contribution H_1 is proportional to σ_3 (the standard MSW piece) and H_2 to σ_1:

$$\Omega_1 = \omega\left(1 + \frac{W}{\lambda^2}\right)\sin 2\theta, \qquad \Omega_2 = 0, \qquad \Omega_3 = -\omega\left(1 - \frac{A}{A_R}\right)\cos 2\theta. \tag{3.2}$$

Surprisingly, the dynamics generated by (2.9), or equivalently (2.14), with these coefficients appears to be physically unacceptable.

As mentioned at the beginning, any density matrix must be a positive operator (i.e., its eigenvalues should be nonnegative) in order to represent a physical state: Its eigenvalues have the physical meaning of probabilities. Therefore, any time evolution needs to preserve this property, otherwise an initial state would not be mapped to another state at a later time. This is precisely what happens when the neutrino evolution in the medium is modeled by (2.9) with dissipative parameters as in (3.1). In fact, the probability $\mathcal{P}(t)$ for having a transition from an initial neutrino state $\rho(0)$ to its orthogonal state $\rho_\perp \equiv 1 - \rho(0)$ at a later time t is given by the first equality in (2.17), with the substitutions $\rho_{\nu_e} \to \rho(0)$, $\rho_{\nu_\mu} \to \rho_\perp$. Since $\mathcal{P}$ is initially zero, its time derivative must be positive at $t = 0$, otherwise we would have physically unacceptable negative transition probabilities as soon as $t > 0$. A simple computation gives $\dot{\mathcal{P}}(0) = \sum_{i,j=1}^{3} \rho(0)_i \mathcal{H}_{ij} \rho(0)_j \geq 0$, and since this must be true for any initial state, physical consistency requires the symmetric part of the matrix $\mathcal{H}$ in (2.15) to be positive. One easily sees that this is impossible with the assignment in (3.1).[3]

In the case of a δ-correlated medium, the parameter β identically vanishes and no inconsistencies arise; however, as mentioned before, this choice is not supported by strong physical motivations and appears just a mathematically convenient simplification. By naively relaxing the δ-correlated assumption, one ends up with the simple stochastic system discussed above, which turns out to be seriously flawed. As a consequence, modeling matter fluctuations only in terms of electron density is physically untenable and indicates that in order to consistently describe neutrino oscillations in random matter more complex situations need to be analyzed, involving a richer covariance structure than with $\vec{V}(t) = [0, 0, V_3(t)]$.

Alternatively, instead of the random matter model one may question the approximations used in deriving the master equation (2.9), and precisely the weak-coupling hypothesis and the Markovian limit. However, the first assumption appears rather well satisfied in the case of the neutrinos, as they interact very weakly with matter, while the Markovian approximation is justified by the physically motivated choice of rapidly decaying matter correlations: $\lambda \gg \omega$. In reality, once a slightly generalized model of random medium is adopted, the master equation (2.9) results perfectly adequate to consistently treat decoherence phenomena in neutrino-matter oscillations.

[3] An example of the emergence of negative transition probabilities is explicitly provided in the appendix.

IV. DIAGONAL CORRELATIONS

When the components of the stochastic field $\vec{V}(t)$ are all nonvanishing, the noise Hamiltonian in (2.3) is no longer diagonal: In this case, while traveling in the medium, all neutrino species undergo incoherent scatterings, in general involving not exclusively the forward direction; this may happen only in the presence of flavor changing interactions. However, as a minimal extension of the previously treated case, we shall assume $V_1(t)$ and $V_2(t)$ to have zero mean, so that the Hamiltonian correction H_1 contains only the standard MSW contribution, and further take the correlation functions in (2.22) to be diagonal:

$$\hat{W}_{ij}(t-s) = W_i e^{-\lambda_i |t-s|} \delta_{ij}. \tag{4.1}$$

In addition, for simplicity we shall consider situations for which the ratios W_i/λ_i are all equal to a common factor $\mathcal{W} > 0$; in this case, the parameters appearing in (2.15) take the form

$$a = \alpha = \gamma = 4\mathcal{W}, \qquad b = 2\omega\mathcal{W}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\cos 2\theta,$$
$$c = 0, \qquad \beta = 2\omega\mathcal{W}\left(\frac{1}{\lambda_3} - \frac{1}{\lambda_2}\right)\sin 2\theta, \tag{4.2a}$$

$$\Omega_1 = \omega\left[1 + 2\mathcal{W}\left(\frac{1}{\lambda_2} + \frac{1}{\lambda_3}\right)\right]\sin 2\theta, \qquad \Omega_2 = 0,$$
$$\Omega_3 = -\omega\left[1 - \frac{A}{A_R} + 2\mathcal{W}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)\right]\cos 2\theta, \tag{4.2b}$$

and the master equation (2.9) can be exactly integrated. Notice that the request of positivity of $\rho(t)$ for any $t \geq 0$ now requires $\alpha^2 \geq b^2 + \beta^2$, a condition that is always satisfied by the original hypothesis of fast decaying matter-correlations: $\lambda_i \gg \omega$. Even more, this inequality guarantees not only the positivity of the evolution generated by (2.9), but actually a stronger attribute, that of "complete positivity" [3–5]. This property is crucial in assuring the consistency of any generalized, dissipative dynamics in all possible physical conditions and should always be imposed in place of simple positivity to avoid possible inconsistencies in the treatment [26]; it is reassuring that it emerges naturally from our simple model of random matter, without the need of further assumptions.

The transition probability $\mathcal{P}_{\nu_e \to \nu_\mu}$ in (2.17) can be explicitly computed and cast in the simple form:

$$\mathcal{P}_{\nu_e \to \nu_\mu}(t) = \tfrac{1}{2}(1 - e^{-2\alpha t}) + e^{-2\alpha t}\sin^2 2\tilde{\theta}\sin^2 \Omega t, \tag{4.3}$$

where $\Omega = [\Omega_1^2 + \Omega_3^2 - b^2 - \beta^2]^{1/2}$ is the modified oscillation frequency, while $\sin^2 2\tilde{\theta} = (\Omega_1^2 - \beta^2)/\Omega^2 \leq 1$ defines a new mixing angle. In comparison with the standard result in (2.21), one sees that the presence of a random medium introduces exponential damping terms and further modifies the neutrino effective masses and mixing properties; a resonance enhancement is still present for $A = A_R$, but its effectiveness is reduced by the damping factors. This is even more dramatic at large times, where the decoherence effects dominate: The neutrino state ρ is driven to the totally mixed state $\sigma_0/2$ and the transition probability approaches its asymptotic $1/2$ value.

These conclusions apply to neutrinos traveling in uniform random media. When this is not the case, the neutrino total time evolution results from the composition of arbitrarily many partial evolutions corresponding to media with uniform properties, but in general of different thicknesses; then, the complete evolution matrix $\mathcal{M}(t)$ as defined in (2.16) will be the result of the composition of the corresponding ones pertaining to the various media (a simple example is given in the appendix). Nevertheless, for slowly varying conditions, this composition can be well approximated by its adiabatic expression, obtained by the instantaneous diagonalization of the now time-dependent matrix $\mathcal{H}$ in (2.15) and the assumption that the neutrino states evolve as one of its eigenstates.[4] Within this approximation and neglecting fast oscillating terms, the averaged transition probability can be cast in the following form:

$$\mathcal{P}_{\nu_e \to \nu_\mu}(t) = \frac{1}{2}\left[1 - \frac{e^{-8\mathcal{W}t}}{\mathcal{R}}\left(1 - \frac{A}{A_R} + \frac{4\mathcal{W}}{\lambda_2}\right)\cos 2\theta\right], \tag{4.4}$$

with

$$\mathcal{R} = \left\{\left[1 + \frac{4\mathcal{W}}{\lambda_2}\left(1 + \frac{\lambda_2}{\lambda_3} + \frac{4\mathcal{W}}{\lambda_3}\right)\right]\tan^2 2\theta + \left(1 - \frac{A}{A_R}\right) \times\left[1 - \frac{A}{A_R} + \frac{4\mathcal{W}}{\lambda_2}\left(1 + \frac{\lambda_2}{\lambda_1} + \frac{4\mathcal{W}}{\lambda_1}\right)\right]\right\}^{1/2}. \tag{4.5}$$

With respect to standard, familiar expressions, the action of the stochastic medium is signaled by the presence in the second term of a modified weight and a damping factor; these additional contributions depend on the ratios of the three matter-correlation decay constants λ_i and the corresponding strength $\mathcal{W}$. Although in the weak-coupling regime one expects $\mathcal{W} \ll \omega$, the decay constant ratios need not be small. Therefore, the behavior of (4.4) as a function of the neutrino energy can sensibly differ from the one obtained in absence of decoherence effects (concrete examples are shown in Fig. 1).

Of particular interest is the application of (4.4) to the solar neutrino case, where Δm^2 and θ can be taken to assume the best fit values obtained in recent data analysis (e.g., see [27,28] and references therein); thanks to the availability of a larger decoherence parameter space, the electron surviving probability $\mathcal{P}_{\nu_e \to \nu_e} \equiv 1 - \mathcal{P}_{\nu_e \to \nu_\mu}$ is found to differ not only from the standard, noiseless expression, but also from those obtained with δ-correlated fluctuating matter as reported in [13,14] (for a comparison, see Fig. 1). These results, together with the still present uncertainties in the fluctuating behavior of the solar matter, appear to open concrete possibilities for an experimental study of matter-induced effects in neutrino oscillations that go beyond the standard MSW phenomenology.

[4]Possible hoppings among the instantaneous eigenstates can also be easily included; for simplicity, we ignore them here.

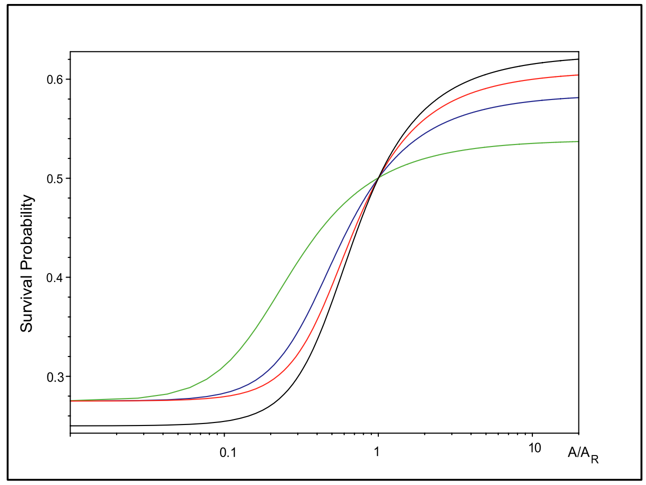

FIG. 1 (color online). Behavior of electron neutrino mean survival probability $\mathcal{P}_{\nu_e \to \nu_e}$ as a function of the neutrino energy (through the ratio A/A_R), for $\sin^2 2\theta \simeq 0.8$, density dominated matter fluctuations, $\lambda_1, \lambda_2 \gg \lambda_3$, and different correlation strengths, $\mathcal{W}/\lambda_2 \simeq 10^{-4}$–$10^{-3}$. The lower starting (black) curve corresponds to the case of noiseless matter (standard MSW effect), while the remaining (colored) ones show the effect of the stochastic fluctuations. The initial gap among the group of curves is due to the presence of the decoherence driven damping factor.

V. DISCUSSION

In the most general situation, the correlations in the stochastic medium have the form (2.22) and thus all the entries of the matrix C_{ij} in (2.12) result nonvanishing; as a consequence, all second-order pieces in the effective Hamiltonian (2.9b) as well as in the dissipative part (2.9c) will contribute to the master equation (2.9a). Further, the first-order mean-field approximation in (2.10) will no longer be diagonal, taking into account the presence of possible flavor changing interactions [15–17].

Nevertheless, even in this very general case, the corresponding matrix $\mathcal{H}$ in (2.15) cannot result totally generic: As already pointed out, the positivity of the evolved state $\rho(t)$ must be preserved under all circumstances; this is guaranteed by the mentioned condition of complete positivity of the evolution generated by (2.9). This property requires the positivity of the matrix C_{ij} in (2.9c) and as a consequence imposes certain inequalities among the dissipative parameters in (2.15) (see [29,30] for explicit expressions). These conditions are certainly of help in restricting the parameter space needed to describe a totally generic random medium.

Even with these constraints, no simple, exact analytic expressions for the transition probability $\mathcal{P}_{\nu_e\to\nu_\mu}(t)$ in (2.17) can in general be given. However, as discussed before, second-order matter contributions to $\mathcal{H}$ are small with respect to the vacuum frequency ω; therefore, in solving (2.14) one can integrate the Hamiltonian dynamics exactly, while treating the dependence on $a, b, c, \alpha, \beta, \gamma$ in perturbation theory.[5] In this way manageable, approximate expressions for the transition probabilities can be obtained. Having now at disposal a larger parameter space, their form involves multiple damping factors and oscillation phases, showing possible larger deviations from the standard behavior. In addition, notice that in order to describe neutrino mixing in a generic random medium two mixing angles, $\hat{\theta}$ and $\hat{\varphi}$, are in general needed: They parametrize the components of the unit vector $\Omega_i/\Omega \equiv (\cos\hat{\varphi}\sin 2\hat{\theta}, \sin\hat{\varphi}\sin 2\hat{\theta}, -\cos 2\hat{\theta})$, with $\Omega = |\vec{\Omega}|$, which identifies the effective Hamiltonian $H = \vec{\Omega}\cdot\vec{\sigma}$ in (2.9b).

Actually, in the presence of Majorana neutrinos, also in vacuum the most general mixing matrix involves two angles, θ and φ, so that the explicit expression of the free effective Hamiltonian H_0 in terms of these angles is as for H above. Although for oscillations in vacuum involving only two species of neutrinos the angle φ disappears from all observables, this is no longer true in the presence of matter-induced decoherence effects. Indeed, one can directly check that the transition probabilities explicitly depend on φ, unless the dissipative parameters a, b, c, α, β, γ are all zero; at least in principle, it is therefore possible to distinguish between Dirac and Majorana neutrinos by studying their oscillations in random matter. The detailed analysis of such dependence is certainly beyond the scope of the present investigation and thus, in order to keep the treatment as simple as possible, in the previous discussions we have tacitly assumed the neutrinos traveling in matter to be of Dirac type, setting $\varphi = 0$ from the beginning.

As a final remark, let us mention that master equations of the type (2.9) generate the most general open system dynamics compatible with a semigroup composition law and the requirement of complete positivity, and as such can be applied to model in a physically consistent way a wide range of phenomena [3–7]. In particular, they have been recently used in order to describe dissipative effects induced at low energies by the dynamics of fundamental objects (strings and branes) at a very high scale, typically the Planck mass [29,30]. These string induced decoherence effects may modify the pattern of neutrino oscillations, and in principle interfere with the phenomena described above. Nevertheless, besides being very small, they affect in equal manner all types of neutrinos, so that they can be isolated from the matter-induced effects by analyzing data taken in different experimental conditions.

[5]The effects of the Hamiltonian corrections to the free motion are in general not small, in particular, near resonance; this is why no approximation is allowed in the evolution generated by the effective Hamiltonian (2.9b).

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APPENDIX

In order to show that negative probabilities arise in experimental accessible observables once the naive model of density fluctuating matter discussed in the text is adopted, one needs to combine neutrino propagation in vacuum with that in the medium. Consider a neutrino, created as ν_e, that propagates for time t' in vacuum, then enters the random medium in which it stays for a time t, and is finally detected after having traveled again in vacuum for a further time t''. The probability $\mathcal{P}_{\nu_e\to\nu_\mu}(\tau)$ of finding a neutrino of type ν_μ at the final time $\tau = t' + t + t''$ can be expressed as in (2.17), where the total transition matrix $\mathcal{M}(\tau)$ is now the product of three terms, $\mathcal{M}(\tau) = \mathcal{M}_0(t')\cdot\mathcal{M}(t)\cdot\mathcal{M}_0(t'')$, the middle representing the propagation in the medium with parameters as in (3.1) and (3.2), while the outer two the free motion in vacuum, generated by the Hamiltonian (2.2).

When the vacuum evolution time t' is chosen to be very short, such that $\sin\omega t' = \beta/(2a\sin 2\theta) = \omega/4\lambda$, the state of the neutrino entering the medium is $\rho_- = [\sigma_0 - \vec{\rho}_-\cdot\vec{\sigma}]/2$, where $\vec{\rho}_- = -\mathcal{M}_0(t')\vec{\rho}_{\nu_e}(0)$ coincides with the eigenvector of the dissipative part of $\mathcal{H}$ relative to its negative eigenvalue. Similarly, with the same choice also for t'', one finds $\mathcal{M}_0(-t'')\vec{\rho}_{\nu_\mu}(0) = \vec{\rho}_-$, so that when exiting the medium the neutrino is found in the state $\rho_+ = 1 - \rho_-$, orthogonal to ρ_-. With these conditions, one has $\mathcal{P}_{\nu_e\to\nu_\mu}(\tau) = \mathcal{P}_{-\to+}(t)$, and near resonance, one explicitly finds

$$\mathcal{P}_{\nu_e\to\nu_\mu}(\tau) = \frac{1}{2}\left[1 - e^{-at}\left(\cos 2\Omega t + D\frac{\sin 2\Omega t}{\Omega}\right)\right],$$

where $D = [a^2/4 + \beta^2]^{1/2}$ and $\Omega = [\omega^2 - D^2]^{1/2}$; this expression indeed assumes unphysical negative values for sufficiently small times: $\mathcal{P}_{\nu_e\to\nu_\mu}(\tau) \simeq (a/2 - D)t$.

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