PHYSICAL REVIEW D **71,** 011504(R) (2005)

Photon polarization in $B \to X\gamma$ in the standard model

Benjamín Grinstein,[1] Yuval Grossman,[2,3,4] Zoltan Ligeti,[5] and Dan Pirjol[6]

[1]*Department of Physics, University of California at San Diego, La Jolla, California 92093, USA*
[2]*Department of Physics, Technion-Israel Institute of Technology, Technion City, 32000 Haifa, Israel*
[3]*Physics Department, Boston University, Boston, Massachusetts 02215, USA*
[4]*Jefferson Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[5]*Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720, USA*
[6]*Center for Theoretical Physics, Massachusetts Institute for Technology, Cambridge, Massachusetts 02139, USA*

(Received 17 December 2004; published 26 January 2005)

The standard model prediction for the $\overline{B}^0 \to X_{s,d}\gamma$ decay amplitude with a right-handed photon is believed to be tiny, suppressed by $m_{s,d}/m_b$, compared to the amplitude with a left-handed photon. We show that this suppression is fictitious: In inclusive decays, the ratio of these two amplitudes is only suppressed by $g_s/(4\pi)$, and in exclusive decays by $\Lambda_{\rm QCD}/m_b$. The suppression is not stronger in $\overline{B}^0 \to X_d\gamma$ decays than it is in $\overline{B}^0 \to X_s\gamma$. We estimate that the time dependent *CP* asymmetries in $B \to K^*\gamma$, $\rho\gamma$, $K_S\pi^0\gamma$, and $\pi^+\pi^-\gamma$ are of order 0.1 and that they have significant uncertainties.

DOI: 10.1103/PhysRevD.71.011504 PACS numbers: 13.20.He

I. INTRODUCTION

The standard model (SM) predicts that photons are mainly left-handed in $b \to q\gamma$ $(q = s, d)$ decay (and right-handed in $\bar{b} \to \bar{q}\gamma$). We define the ratio

$$r_q e^{i(\phi_q+\delta_q)} \equiv \frac{A_R}{A_L} \equiv \frac{A(\overline{B} \to f_q\gamma_R)}{A(\overline{B} \to f_q\gamma_L)}, \tag{1}$$

where ϕ is a weak phase and δ is a strong phase. It is usually stated that $r_q = m_q/m_b \ll 1$ in the SM [1]. (Throughout this paper $\overline{B}$ refers to $\overline{B}^0$ or B^- that contain a b quark, and r, ϕ and δ depend on the final state, f.) New physics can modify this prediction, and therefore several methods have been proposed to measure the photon helicity [1,2].

In $B \to f\gamma$, where f is a *CP* eigenstate, since γ_L and γ_R cannot interfere, the time dependent *CP* asymmetry,

$$\frac{\Gamma[\overline{B}^0(t) \to f\gamma] - \Gamma[B^0(t) \to f\gamma]}{\Gamma[\overline{B}^0(t) \to f\gamma] + \Gamma[B^0(t) \to f\gamma]} = S_{f\gamma}\sin(\Delta m t) - C_{f\gamma}\cos(\Delta m t), \tag{2}$$

is sensitive to r. In the SM, ϕ_s and $C_{f_s\gamma}$ are suppressed by $|(V_{ub}V_{us})/(V_{tb}V_{ts})|$, and to first order in $r_s(\ll 1)$

$$S_{f_s\gamma} = -2r_s\cos\delta_s\sin2\beta. \tag{3}$$

The first measurements of such *CP* asymmetries were carried out recently [3,4],

$$S_{K^*\gamma} = \begin{cases} +0.25 \pm 0.63 \pm 0.14 & \text{BABAR} \\ -0.79^{+0.63}_{-0.50} \pm 0.10 & \text{BELLE,} \end{cases} \tag{4}$$

yielding a world average $S_{K^*\gamma} = -0.28 \pm 0.45$. At a super-$B$ factory the statistical error with 50 ab^{-1} data is estimated to be $\delta(S_{K^*\gamma}) = 0.04$ [5]. The Belle Collaboration also measured the *CP* asymmetry $S_{K_S\pi^0\gamma} = -0.58^{+0.46}_{-0.38} \pm 0.11$, integrating over the invariant mass range $0.6\ \text{GeV} < m_{K_S\pi^0} < 1.8\ \text{GeV}$ [6]. It will also be possible to measure this *CP* asymmetry in $B \to \pi^+\pi^-\gamma$ [7], and maybe even with additional pions [8].

The purpose of this paper is to study the SM prediction for r. (For earlier attempts to go beyond the naive estimate, see Refs. [9,10].) We find that r is only suppressed by $g_s/(4\pi)$ in inclusive $b \to X\gamma$ decay, and by $\Lambda_{\rm QCD}/m_b$ in exclusive $\overline{B} \to \overline{K}^*\gamma$ and $\rho\gamma$ decay.

To understand the origin of such effects, recall that the effective Hamiltonian for $b \to s\gamma$ is [11]

$$H_{\rm eff} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_{i=1}^{8}C_i(\mu)O_i(\mu). \tag{5}$$

For our discussion the operators directly relevant are

$$O_2 = (\bar{c}\gamma^\mu P_L b)(\bar{s}\gamma_\mu P_L c), \qquad O_7 = \frac{e}{16\pi^2}\bar{s}\sigma^{\mu\nu}F_{\mu\nu}(m_bP_R + m_sP_L)b, \tag{6}$$

where $P_{R,L} = (1 \pm \gamma_5)/2$, and we neglect the m_sP_L part of O_7 hereafter. At the parton level, as long as $b \to s\gamma$ is a two-body decay (either from the leading contribution of O_7 or subleading virtual contributions from $O_{i\neq7}$), the left-handed s quark is back-to-back to a photon. Then the two-body kinematics implies that only γ_L is allowed. This argument does not apply to multibody final states, such as $b \to s\gamma+$ gluons.

The m_bP_R part of the leading operator O_7 contributes only to A_L to all orders in the strong interaction. To prove this, note that the electromagnetic tensor for $\gamma_{L,R}$ is $F^{L,R}_{\mu\nu} = \frac{1}{2}(F_{\mu\nu} \pm i\tilde{F}_{\mu\nu})$, where $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\lambda}F^{\rho\lambda}$, and O_7 can be written in terms of $m_bF^L_{\mu\nu}$. Thus, independent of hadronic physics, the photon from O_7 is left-handed. This argument only applies for O_7. Indeed, we find by explicit calculation that other operators produce right-handed photons once QCD corrections are included.

II. INCLUSIVE $B \to X_s\gamma$ AND $B \to X_d\gamma$

In this section we estimate r from an inclusive calculation. The result can only be trusted if several hadronic final states are allowed to contribute, and r for specific final states cannot be obtained from this calculation.

The leading contribution to the inclusive $\overline{B} \to X_s\gamma_R$ rate is of order α_s. It arises from bremsstrahlung contributions to the matrix elements of operators $O_{i\neq 7}$. The dominant contribution comes from O_2 shown in Fig. 1. The corresponding amplitude was calculated in Refs. [12,13]. We find that it yields equal rates for left- and right-handed photons at order α_s, at any point in the $b \to sg\gamma$ Dalitz plot.

Because of the complicated m_c-dependence of the double differential rate, $d\Gamma_{22}^{(\mathrm{brem})}/dE_\gamma dE_g$, we integrate over E_g and E_γ numerically. To reduce the large sensitivity to the scale of α_s, we include the known order $\alpha_s^2\beta_0$ contribution to $\Gamma_{22}^{(\mathrm{brem})}$ [14]. (At this order the equality of the decay rates to left- and right-handed photons is violated by less than 1%, and can be neglected.)

Using the "effective" Wilson coefficients at leading order [15], $C_2(m_b) = 1.1$ and $C_7(m_b) = -0.31$, $\alpha_s(m_b) = 0.22$ with $m_b = 4.8$ GeV, and $m_c = 1.4$ GeV, we obtain

$$\frac{\Gamma_{22}^{(\mathrm{brem})}}{\Gamma_0} \simeq 0.025, \qquad \Gamma_0 = \frac{G_F^2|V_{tb}V_{ts}^*|^2\alpha_{\mathrm{em}}C_7^2m_b^5}{32\pi^4}. \quad (7)$$

This result corresponds to integrating the numerator over $x \equiv 2E_\gamma/m_b > 0.75$ (that is, roughly, $E_\gamma > 1.8$ GeV). This result includes also the $\mathcal{O}(\alpha_s^2\beta_0)$ correction, which is sizable, indicating that the relevant scale of α_s may be well below m_b; without including it the result in Eq. (7) would be 0.015. Thus, we find at lowest order in g_s

$$\langle r_s\rangle|_{x>0.75} = \sqrt{\Gamma_{22}^{(\mathrm{brem})}/(2\Gamma_0)} \simeq 0.11, \quad (8)$$

decreasing only slowly with a stiffer cut on x.

The value of $\cos\delta_s$ is physical, as it enters $S_{f_s\gamma}$ in Eq. (3). Yet it cannot be estimated from the inclusive calculation. The reason is that the dominant contribution to A_R comes from the $b \to s\gamma g$ amplitude generated by O_2, while to A_L from the $b \to s\gamma$ decay generated by O_7. These are different final states, for which one can choose the phase conventions independently. These amplitudes can still contribute to the same hadronic final states and interfere once hadronization effects are included. Thus, the relevant phase for any final state is determined by the

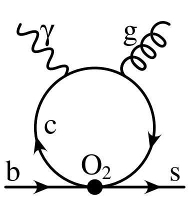

FIG. 1. Leading contribution to $\overline{B} \to X_s\gamma_R$. A second diagram with the gluon and the photon exchanged is not shown.

hadronization processes and cannot be extracted from the inclusive calculation. Comparing the absorptive and dispersive parts of the inclusive result, we find $\cos^2\delta_s \simeq 0.3$ with small variation over $0.75 < x < 1$. The only conclusion we can draw here is that we expect the strong phase to be generically large.

Next we discuss inclusive $B \to X_d\gamma$. In Eq. (5), operators multiplying the suppressed Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{ub}V_{us}^*$ were neglected. The analogous terms are important in $b \to d\gamma$, since in this decay the u quark loop is not CKM suppressed compared to the c and t loops. The $b \to d\gamma$ effective Hamiltonian is of the form

$$H_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{td}^*\sum_{i=1}^{8}C_i(\mu)O_i'(\mu) - \frac{4G_F}{\sqrt{2}}V_{ub}V_{ud}^*\sum_{i=1}^{2}C_i(\mu)[O_i'(\mu) - O_{i,u}'(\mu)], \quad (9)$$

where prime denotes that in the standard operator basis in Eq. (6) the s quark is replaced by d, and the u subscript means that c and $\bar{c}$ are replaced by u and $\bar{u}$; for example, $O_{2,u}' = (\bar{u}\gamma^\mu P_L b)(\bar{d}\gamma_\mu P_L u)$. It is useful to define

$$r_d e^{i(\phi_d+\delta_d)} = r_s e^{i\delta_s} + \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}r_d^u e^{i\delta_d^u}, \quad (10)$$

where the first [second] term comes from the contribution of the top [bottom] line in Eq. (9) to A_R. The first term is the same as in $b \to s$ decay (recall that we neglected m_s).

For the u quark loop, taking the limit of the quark mass to zero, the calculation of Ref. [14] simplifies considerably, and we can obtain analytic results

$$\frac{1}{\Gamma_0}\frac{d\Gamma_{22}^{(u,\mathrm{brem})}}{dx} = \frac{C_2^2}{C_7^2}\left\{\frac{\alpha_s}{27\pi}2x(2-x) + \frac{\alpha_s^2\beta_0}{27\pi^2}\times\left[\frac{x(44-19x+18x^2-24x^3)}{12} - \frac{x(2-x)}{2}\ln[x(1-x)] + \frac{2x^3}{3}\ln x\right]\right\}, \quad (11)$$

where α_s is evaluated at the scale m_b in the $\overline{\mathrm{MS}}$ scheme. For the difference of the rates to γ_R and γ_L we obtain

$$\frac{1}{\Gamma_0}\frac{d[\Gamma_{22}^{(u,R)} - \Gamma_{22}^{(u,L)}]}{dx} = \frac{C_2^2}{C_7^2}\frac{\alpha_s^2\beta_0}{27\pi^2}\frac{x^2(3-3x+4x\ln x)}{6}. \quad (12)$$

This difference integrates to zero, and it gives a slight γ_L enhancement near $x = 1$. We obtain for $x > 0.75$

$$\Gamma_{22}^{(\mathrm{u,brem})}/\Gamma_0 \simeq 0.030. \quad (13)$$

The absolute values of the amplitudes corresponding to the c and u loops [the squares of which yield Eqs. (7) and (13)] are comparable to each other. In the absence of strong phases this would result in a cancellation and lead to a very small r_d^u. Note that, as in the $b \to s\gamma$ case, we cannot predict the sign of $\cos\delta_d^u$. Moreover, the values of r_d^u and δ_d^u are sensitive to the difference between the strong phases of the c and u loops, for which we do not consider the perturbative result reliable. In particular, the matrix element of $O'_{2,u}$ in $b \to d\gamma$ may have sizable long distance contributions. In any event, the short distance calculation predicts that the strong phase vanishes for the u loop contribution, while it is sizable for the c loop. Therefore, the cancellation in r_d^u is unlikely to be effective. The important point is that we expect $r_d \sim r_d^u \sim r_s$.

The crucial difference between the time dependent CP asymmetries in $B \to f_s\gamma$ and $B \to f_d\gamma$ is that in the latter case, naively, there are two very strong suppression factors. First, considering O_7 only, $S_{f_d\gamma}$ is suppressed by m_d/m_b. Second, the phase of the dominant decay amplitude, $V_{tb}V_{td}^*$, cancels the phase of the $B^0 - \overline{B}^0$ mixing amplitude, yielding another strong suppression of $S_{f_d\gamma}$. Both of these suppressions are fictitious, since the $V_{ub}V_{ud}^*(O'_2 - O'_{2,u})$ contributions lift both the m_d/m_b suppression, just like in the $b \to s\gamma$ case discussed earlier, and also the suppression coming from the cancellation of the mixing and the decay phases. The leading contribution to $S_{f_d\gamma}$ is proportional to r_d^u, which gives a contribution to A_R with weak phase γ. Using the fact that the phase of the mixing amplitude is 2β, and that of A_L is β, we obtain at leading order

$$S_{f_d\gamma} = -2\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| r_d^u \cos\delta_d^u \sin(\beta + \gamma). \qquad (14)$$

This result is independent at first order in r_d^u of the small direct CP violation in $b \to d\gamma$ in the SM.

A model dependent way to connect the inclusive calculations with exclusive $B \to K^*\gamma$ or $\rho\gamma$ is the Ali-Greub model [16] obtained by smearing the inclusive rate with a model shape function and integrating over $E_\gamma > (m_B^2 - 1\ \text{GeV}^2)/(2m_B)$ (i.e., attributing the rate to $m_X < 1$ GeV to the K^* or ρ). Restricting the shape function parameters to reproduce $\mathcal{B}(B \to K^*\gamma)$, we obtain $r_{K^*} \sim 0.025$, with little sensitivity to the model parameters. This is comparable in magnitude but independent of the m_s/m_b contribution. This model also yields $r_\rho \sim r_{K^*}$, which is much larger than m_d/m_b. To compare with the "more inclusive" measurement [6], we computed $\langle r_s\rangle_{m_{X_s}<1.8\ \text{GeV}} \sim 0.055$ using simple shape function models. Given the model uncertainties, this figure should be taken as a rough estimate.

III. EXCLUSIVE $B \to K^*\gamma$ AND $B \to \rho\gamma$

We consider next the photon polarization in exclusive $\overline{B} \to \overline{K}^*\gamma$ and $\rho\gamma$ decays using soft-collinear effective theory (SCET) [17]. We prove that O_{1-6} contribute only to A_L to all orders in α_s at leading order in Λ_{QCD}/m_b (this is viewed as of the same order as m_{K^*}/E_{K^*}). We identify several types of subleading SCET operators that give right-handed photons, and yield $r_{K^*,\rho}$ of order Λ_{QCD}/m_b.

Using SCET, a factorization theorem for heavy-to-light form factors has been proven at leading order in Λ_{QCD}/m_b [18–20]. There are two contributions to the form factors of the same order in Λ_{QCD}/m_b: a nonfactorizable (or soft or form-factor) part and factorizable (or hard scattering) part.

The exclusive radiative decays considered here were analyzed in an expansion in Λ_{QCD}/m_b in [9,21–23], and recently in SCET [24]. The nonfactorizable part receives its dominant contribution from O_7 and gives rise to A_L only. The operator O_2 enters at leading order only via factorizable contributions.

When operators $O_{i\neq 7}$ are included, the hierarchy of the relevant scales gets rather complicated, since $m_b^2 > m_c^2 \sim \Lambda_{\text{QCD}}m_b > \Lambda_{\text{QCD}}^2$. Reference [24] assumed that the $c\bar{c}$ loop is dominated by hard loop momenta and can be integrated out near the scale m_b. We adopt a simpler approach, by neglecting the charm mass and assuming that its effects can be included as a perturbation using the formalism of Ref. [25] without encountering singularities. This assumption is borne out by explicit one- and two-loop calculations.

The Hamiltonian in Eq. (5) is matched in SCET$_\text{I}$ onto

$$H_{\text{eff}} = -\frac{G_F V_{tb}V_{ts}^* e}{\sqrt{2}\pi^2} E_\gamma[c(\omega)\bar{s}_{n,\omega}\mathcal{A}^\perp m_b P_L b_v + b_{1L}(\omega_i)O^{(1L)}(\omega_i) + b_{1R}(\omega_i)O^{(1R)}(\omega_i) + \cdots], \qquad (15)$$

where the ω's are the usual collinear label momenta. The relevant modes are soft quarks and gluons with momenta $k_s \sim \Lambda$ and two types of collinear quarks and gluons along n and $\bar{n}$ (including charm, which can be soft or collinear). Note that the approach of Ref. [24], treating $m_c \sim m_b$, will likely require additional modes. We take the photon momentum as $q_\mu = E_\gamma \bar{n}_\mu$, the collinear s quark to move along n_μ, and $\mathcal{A}_\mu^\perp$ denotes the transverse photon field. The operator in the first line in Eq. (15) occurs at leading order in the expansion parameter, $\lambda = \sqrt{\Lambda/m_b}$, and its Wilson coefficient is dominated by C_7, $c(\omega) = C_7 + \mathcal{O}[\alpha_s(m_b)]$. The operators in the second line,

$$\begin{aligned} O^{(1L)}(\omega_1, \omega_2) &= \bar{s}_{n,\omega_1}\mathcal{A}^\perp\left[\frac{1}{\bar{n}\cdot\mathcal{P}} ig\mathcal{B}_n^\perp\right]_{\omega_2} P_R b_v, \\ O^{(1R)}(\omega_1, \omega_2) &= \bar{s}_{n,\omega_1}\left[\frac{1}{\bar{n}\cdot\mathcal{P}} ig\mathcal{B}_n^\perp\right]_{\omega_2}\mathcal{A}^\perp P_R b_v, \end{aligned} \qquad (16)$$

are the only SCET$_\text{I}$ operators suppressed by λ that couple to a transverse photon and are allowed by power counting and s chirality. Here $ig\mathcal{B}_n^\nu \equiv W^\dagger[\bar{n}\cdot iD_c, iD_{c\perp}^\nu]W$ is the collinear gluon field tensor; for the remaining notations see Ref. [26]. The operators $O^{(1L)}$ and $O^{(1R)}$ couple only to $\gamma_{L,R}$, respectively. Their Wilson coefficients are

$$\begin{aligned} b_{1L}(\omega_1, \omega_2) &= C_7 + C_2/3 + \mathcal{O}[C_{3-6}, \alpha_s(m_b)], \\ b_{1R}(\omega_1, \omega_2) &= -C_2/3 + \mathcal{O}[C_{3-6}, \alpha_s(m_b)]. \end{aligned} \qquad (17)$$

Although the operators in the first and second lines of Eq. (15) are of different orders in λ, after matching onto SCET$_\text{II}$, they contribute at the same order in Λ_{QCD}/m_b to the $B \to K^*\gamma$ amplitude. The leading order $c(\omega)$ term gives the nonfactorizable contribution, which only contributes to A_L, while $O^{(1L)}$ and $O^{(1R)}$ give the factorizable contributions through time-ordered products with the ultrasoft-collinear Lagrangian, $\mathcal{L}_{\xi q}^{(1)}$ [27]. After matching onto SCET$_{\rm II}$, the expressions for $T^{(1L,R)} = \int d^4x T\{O^{(1L,R)}(\omega_i), i\mathcal{L}_{\xi q}^{(1)}(x)\}$ are given schematically by

$$T^{(1L)} \to \int dk_+ J_\perp(\omega_i^{(l)}, k_+) \otimes [(\bar{q}Y_n)_{k_+} \not{n} \mathcal{A}^\perp \gamma_\perp^\alpha P_R (Y_n^\dagger b_v)] \times (\bar{s}_{n,\omega_1'} \not{\bar{n}} \gamma_\perp^\alpha q_{n,\omega_2'}), \quad (18)$$

$$T^{(1R)} \to \int dk_+ J_\parallel(\omega_i^{(l)}, k_+) \otimes [(\bar{q}Y_n)_{k_+} \not{n} \mathcal{A}^\perp P_R (Y_n^\dagger b_v)] \times (\bar{s}_{n,\omega_1'} \not{\bar{n}} P_L q_{n,\omega_2'}), \quad (19)$$

where $J_{\perp,\parallel}$ are jet functions that have expansions in $\alpha_s(\sqrt{\Lambda_{\rm QCD} m_b})$. The operator in Eq. (18) contributes to $B \to K^*\gamma$ at leading order,

$$A_L = \frac{G_F V_{tb} V_{ts}^* e}{\sqrt{2}\pi^2} \frac{m_B^3}{2} \left[c(m_B) \zeta_\perp^{BK^*} + \frac{f_B f_{K^*}^\perp}{m_B} \int dx dt dk_+ \times b_{1L}[m_B(1-t), m_B t] J_\perp(x, t, k_+) \phi_{K^*}^\perp(x) \phi_B^+(k_+) \right], \quad (20)$$

where all nonperturbative matrix elements are defined as in [26]. The operator in Eq. (19) only gives rise to longitudinal K^* and therefore does not contribute to $B \to K^*\gamma$. This proves that O_2 contributes at leading order in $\Lambda_{\rm QCD}/m_b$ to $\overline{B} \to \overline{K}^*\gamma_L$ via Eq. (18), but its contribution to $\overline{B} \to \overline{K}^*\gamma_R$ vanishes at this order. (Interestingly, it would contribute to $\overline{B}^* \to \overline{K}^{(*)}\gamma_R$ at leading order.) The same proof also holds for the other four-quark operators, O_{1-6}.

This result agrees with the $\mathcal{O}(\alpha_s)$ computation [21,22], and extends it to all orders in α_s. This is important, since $\alpha_s(\mu)$ at the hard-collinear scale $\mu^2 \sim \Lambda m_b$ may or may not be perturbative [18,28,29]. The suppression of A_R can be understood from a simple helicity argument. Since the s quark is left-handed, the K^* cannot be right-handed, unless additional right-handed gluons end up in the K^*. Such higher Fock states' contributions are power suppressed.

The $\overline{B} \to \overline{K}^*\gamma_R$ amplitude does arise at subleading order in $\Lambda_{\rm QCD}/m_b$. There are several sources of such corrections. For example, (i) time-ordered products of $O^{(1R)}$ with the subleading collinear Lagrangian $\mathcal{L}_{\xi\xi}^{(n\geq 1)}$, which lead to factorizable contributions similar to Eq. (18) containing an explicit factor $\alpha_s(\sqrt{\Lambda_{\rm QCD} m_b})$; (ii) higher order terms in the SCET$_{\rm I}$ Lagrangian in Eq. (15), e.g., the $\mathcal{O}(\lambda^2)$ operator

$$O^{(2R)}(\omega) = \int \frac{dx_+}{2\pi} dk_- e^{-ix_+k_-/2} \kappa\left(\frac{2E_\gamma k_-}{m_c^2}\right) \times \bar{s}_{n,\omega}[Y_{\bar{n}}^\dagger i\not{D}^{\rm us} Y_{\bar{n}}]\left(\frac{\bar{n}x_+}{2}\right) \mathcal{A}^\perp P_R b_v, \quad (21)$$

where the gauge invariant operator $[Y_{\bar{n}}^\dagger i\not{D}^{\rm us} Y_{\bar{n}}](x)$ contains Wilson lines of the ultrasoft fields in the $\bar{n}$ direction. $O^{(2R)}$ is obtained by matching the graph in Fig. 1 with one ultrasoft gluon, and its Wilson coefficient is related to b_{1R} in Eq. (17) by reparametrization invariance [30]. The $C_2/3$ terms in Eq. (17) correspond to $m_c = 0$, when $\kappa(z) = 1/2$. In this case the k_- and x_+ integrals in Eq. (21) can be performed trivially. In Eq. (21) we kept $m_c \neq 0$ to exhibit the analogy with the nonperturbative corrections to inclusive $b \to s\gamma$ decay [31,32]. For $m_c \neq 0$, additional nonlocality is introduced by $\kappa(z) = 1/2 - 2\arctan[\sqrt{z/(4-z)}]^2/z$ with $z = 2E_\gamma k_-/m_c^2$ [12]. [For $m_c \neq 0$, the $C_2/3$ terms in Eq. (17) should be multiplied by $2\kappa(2E_\gamma \omega_2/m_c^2)$.] Expanding $O^{(2R)}$ in x_+ reproduces the series of operators proportional to $(m_b \Lambda_{\rm QCD}/m_c^2)^n$ in Eq. (5) in [32]. Time-ordered products of $O^{(2R)}$ with the ultrasoft-collinear Lagrangian, $\mathcal{L}_{\xi q}^{(2)}$, give $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ soft contributions to A_R, with no α_s suppression compared to A_L.

A complete study of these subleading contributions is rather involved and we leave it for future work. It seems unlikely to us that a cancellation could result in a suppression of r_{K^*} and r_ρ to order $\Lambda_{\rm QCD}^2/m_b^2$. This leads to the dimensional estimate

$$r_{K^*} \sim \frac{1}{3} \frac{C_2}{C_7} \frac{\Lambda_{\rm QCD}}{m_b} \sim 0.1. \quad (22)$$

This effect dominates over the m_s piece of O_7 for r_{K^*}. The estimate for r_ρ is more involved because of the contributions with different weak phases,

$$r_\rho \sim r_{K^*}\left[1 + \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\left(C_{\rm loop}\frac{m_c^2}{m_b^2} + C_{\rm WA} 4\pi \frac{\Lambda_{\rm QCD}}{m_b}\right)\right], \quad (23)$$

The $C_{\rm loop}$ term comes from the noncancellation of the c and u loops, and we expect it to have a numerically large coefficient. The $C_{\rm WA}$ term arises from weak annihilation, whose contribution to A_R at order $\Lambda_{\rm QCD}/m_b$ vanishes [9]. The latter contributes significantly only in $B^\pm$, while in B^0 it is color suppressed. Thus, we expect that the SM prediction for $S_{\rho\gamma}$ is not much smaller than it is for $S_{K^*\gamma}$.

For higher mass one-body hadronic final states, A_R still vanishes at leading order, but the suppression by m_X/E_X is expected to be less effective as m_X increases (although there is no evidence for this in the $B \to DX$ data [33]). Thus, the SM value of r is expected to depend on the final state. For high-mass and multibody final states A_R may arise, formally, at leading order in $\Lambda_{\rm QCD}/m_b$. For example, $\overline{B} \to \overline{B}^*\pi_{\rm (soft)}$ followed by $\overline{B}^* \to \overline{K}^{(*)}\gamma_R$ can give rise to $\overline{B}^0 \to K_S\pi^0\gamma_R$ with $m_{K_S\pi^0} \sim \sqrt{\Lambda_{\rm QCD} m_b}$, without a $\Lambda_{\rm QCD}/m_b$ suppression. Therefore, averaging the results of $B \to K^*\gamma$ [3,4] with $B \to K_S\pi^0\gamma$ [6] is not free from theoretical uncertainties.

IV. DISCUSSION AND SUMMARY

We studied the standard model prediction for the ratio of amplitudes $r = A_R/A_L$ to produce right- vs left-handed photons in $B \to X_{s,d}\gamma$. Considering only O_7, $r_{s,d} =$

$m_{s,d}/m_b$; however, including the other operators, $O_{i\neq 7}$, A_R becomes much larger than this naive estimate, and hadronic physics gives rise to sizable uncertainties. The time dependent CP asymmetries also become sensitive to the strong phase.

In inclusive $B \to X_s\gamma$ and $X_d\gamma$ decays, A_R is only suppressed by $g_s/(4\pi)$. We calculated r inclusively, and found it to be of order 0.1 depending on the cut on the photon energy [see Eq. (8)]. While this calculation is reliable, it cannot be easily compared to data. If one restricts the hadronic final state, such as in the measurement of the time dependent CP asymmetry $S_{K_S\pi^0\gamma}$ with a range of $K_S\pi^0$ invariant masses, then that can no longer be calculated using inclusive methods. Still, our results indicate that it would be hard to argue that a measurement of $S_{K_S\pi^0\gamma} \sim 0.1$ cannot be due to SM physics.

In exclusive $\overline{B} \to \overline{K}^*\gamma$ and $\rho\gamma$ decays, we proved using SCET that A_R vanishes at lowest order in the $\Lambda_{\rm QCD}/m_b$ expansion to all orders in α_s. The leading contribution to A_R is formally of order $\Lambda_{\rm QCD}/m_b$, but numerically it is enhanced by C_2/C_7 [see Eqs. (22) and (23)]. The result depends on unknown hadronic matrix elements, which give rise to a sizable uncertainty in the SM prediction.

Both the inclusive and the exclusive calculations predict that r_s and r_d are comparable to each other in the SM. Thus, we also expect the time dependent CP asymmetries in $\overline{B}^0 \to f_d\gamma$ to be comparable to those in $\overline{B}^0 \to f_s\gamma$ (except for a modest suppression by $|V_{ub}/V_{td}|$), in contrast to what has been widely believed. We estimate these asymmetries in decays such as $B \to K^*\gamma$, $\rho\gamma$, $K_S\pi^0\gamma$, and $\pi^+\pi^-\gamma$ to be of order 0.1, with large uncertainties.

To conclude, our main result is that the standard model prediction for A_R/A_L is of order 0.1, with sizable uncertainty. A measurement of it much above this level would indicate new physics. To draw conclusions from a smaller value would require a more complete analysis and knowledge of hadronic matrix elements including strong phases. More effort in this direction would be welcome.

ACKNOWLEDGMENTS

We thank Andy Cohen and Iain Stewart for helpful discussions. Z. L. thanks the particle theory group at Boston University for its hospitality while part of this work was completed. This work was supported in part by the DOE under Grant No. DE-FG03-97ER40546 (B. G.), by DOE Contract No. DE-AC03-76SF00098 (Z. L.), by DOE Grant No. DOE-FG03-97ER40546, and NSF Grant No. PHY-9970781 (D. P.).

[1] D. Atwood *et al.*, Phys. Rev. Lett. **79**, 185 (1997).

[2] T. Mannel and S. Recksiegel, Acta Phys. Pol. B **28**, 2489 (1997); Y. Grossman and D. Pirjol, J. High Energy Phys. 06 (2000) 029; M. Gronau *et al.*, Phys. Rev. Lett. **88**, 051802 (2002); G. Hiller and A. Kagan, Phys. Rev. D **65**, 074038 (2002); M. Gronau and D. Pirjol, Phys. Rev. D **66**, 054008 (2002).

[3] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **93**, 201801 (2004).

[4] Belle Collaboration, K. Abe *et al.*, hep-ex/0409049.

[5] SuperKEKB Physics Working Group Collaboration, A. G. Akeroyd *et al.*, hep-ex/0406071.

[6] Belle Collaboration, K. Abe *et al.*, hep-ex/0411056.

[7] D. Atwood *et al.*, hep-ph/0410036.

[8] O. Long (private communication).

[9] B. Grinstein and D. Pirjol, Phys. Rev. D **62**, 093002 (2000).

[10] L. M. Sehgal and J. van Leusen, Phys. Lett. B **591**, 235 (2004).

[11] B. Grinstein *et al.*, Phys. Lett. B **202**, 138 (1988); Nucl. Phys. **B339**, 269 (1990).

[12] A. Ali and C. Greub, Z. Phys. C **49**, 431 (1991).

[13] N. Pott, Phys. Rev. D **54**, 938 (1996).

[14] Z. Ligeti *et al.*, Phys. Rev. D **60**, 034019 (1999).

[15] A. J. Buras *et al.*, Nucl. Phys. **B424**, 374 (1994).

[16] A. Ali and C. Greub, Phys. Lett. B **259**, 182 (1991).

[17] C. W. Bauer *et al.*, Phys. Rev. D **63**, 014006 (2001); C. W. Bauer *et al.*, Phys. Rev. D **63**, 114020 (2001); C. W. Bauer and I. W. Stewart, Phys. Lett. B **516**, 134 (2001); C. W. Bauer *et al.*, Phys. Rev. D **65**, 054022 (2002).

[18] C. W. Bauer *et al.*, Phys. Rev. D **67**, 071502 (2003).

[19] M. Beneke and T. Feldmann, Nucl. Phys. **B685**, 249 (2004).

[20] B. O. Lange and M. Neubert, Nucl. Phys. **B690**, 249 (2004).

[21] M. Beneke *et al.*, Nucl. Phys. **B612**, 25 (2001).

[22] S. W. Bosch and G. Buchalla, Nucl. Phys. **B621**, 459 (2002).

[23] A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C **23**, 89 (2002).

[24] J. G. Chay and C. Kim, Phys. Rev. D **68**, 034013 (2003).

[25] A. K. Leibovich *et al.*, Phys. Lett. B **564**, 231 (2003).

[26] D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 094005 (2003); **69**, 019903(E) (2004).

[27] M. Beneke *et al.*, Nucl. Phys. **B643**, 431 (2002).

[28] C. W. Bauer *et al.*, Phys. Rev. D **70**, 054015 (2004).

[29] R. J. Hill *et al.*, J. High Energy Phys. 07 (2004) 081; T. Becher and R. J. Hill, J. High Energy Phys. 10 (2004) 055.

[30] A. V. Manohar *et al.*, Phys. Lett. B **539**, 59 (2002).

[31] M. B. Voloshin, Phys. Lett. B **397**, 275 (1997).

[32] Z. Ligeti *et al.*, Phys. Lett. B **402**, 178 (1997).

[33] Z. Ligeti *et al.*, Phys. Lett. B **507**, 142 (2001); C. W. Bauer *et al.*, Phys. Rev. D **67**, 014010 (2003).