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Zero mode effect in the 1^{-+} four quark states

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We calculate the masses of the 1^{-+} four quark states which decay dominantly into $\rho\pi$ and $\eta\pi$, respectively, by QCD sum rules approach. We include the zero mode contribution and find it plays an important role in the sum rules. We predict that the masses of the states $\eta\pi$ and $\rho\pi$ both are 1.4–1.5 GeV. This is close to the experimental candidates $\pi_1(1370)$ and $\pi_1(1440)$.

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I. INTRODUCTION

1^{-+} exotic mesons have been identified for several years. In 1997, a group at Brookhaven National Laboratory first claimed an isopin vector meson $\pi_1(1370)$ with quantum number 1^{-+} in the channel $\pi^- p \to \pi^- \eta p$ [1]. This state has been confirmed further by VES and CBar [2]. It could be identified as a hybrid at first sight. However, all theoretical calculations show this seems not the case. The lattice QCD and the flux tube model predict the 1^{-+} mass is around 1.9 GeV [3,4], which is much heavier than the $\pi_1(1370)$. QCD sum rule also predicts it should be larger than 1.6 GeV [5]. Besides, the decays of the 1^{-+} hybrid also have been studied in the context of various models [4,6] and also appear to be in disagreement with the experimental data of the $\pi_1(1370)$. For instance, the flux model predicts the 1^{-+} hybrid dominantly decays into $f_1\pi$ and $b\pi$ and QCD sum rule's calculation shows (although differs from that of the flux model) $f_1\pi$ and $\rho\pi$ are the hybrid's dominant decay channels. Most recently, Klempt gave a stronger argument based on the SU(3) flavor symmetry to rule out the possibility that the $\pi_1(1370)$ could be a hybrid [7]. Because the $\pi_1(1370)$ is seen in the channel $\eta\pi$ but not $\eta'\pi$, the $\pi_1(1370)$ must be a member of the SU(3) decuplet. Therefore, it could not be a $\bar{q}qg$ hybrid.

An arugment is given by Close that the lowest 1^{-+} four quark state should be in the combination $|0^- 1^+\rangle$, such as πf_1 or πb_1, because they are in S wave [8]. However , both of them are heavier than the $\pi_1(1370)$ and consequently cannot be seen in the $\pi_1(1370)$ decay. Although $\eta\pi$ and $\rho\pi$ are in P wave, the combinations $\eta\pi$, $\rho\pi$, πf_1, and πb_1 belong to the same order of orbital excitation for a four quark system. Their masses should not be quite different. $\eta\pi$ is already seen in the $\pi_1(1370)$ decay. But why not for $\rho\pi$? Actually, when $\pi^0\eta$ does not appear in BNL's reaction $\pi^- p \to \pi^- \eta p$, some authors conclude that the 1^{-+} $\pi_1(1370)$ might not exist [9]. Klempt's answer is that in the t channel there is no ρ exchange. That means the $\pi_1(1370)$ couples $\rho\pi$ very weakly. Then how about the $\pi_1(1440)$? Which is another 1^{-+} state seen in the $\rho\pi$ channel and a little bit heavier than the $\pi_1(1370)$ [10]. Is the $\pi_1(1440)$ the same as the $\pi_1(1370)$? If yes, it contradicts to Klempt's statement. If no, how does one explain the two states with the same quantum number and only 70 MeV mass interval?

In order to investigate this problem, we calculate the masses of the 1^{-+} four states which decay dominantly into $\rho\pi$ and $\eta\pi$, respectively, from QCD sum rule approach. We find the direct instanton effect plays an important role. The direct instanton effect is very large in the sum rules of the state $\eta\pi$ while it is proportional to the light quark mass square for the case $\rho\pi$. This might hint the different structure of states $\rho\pi$ and $\eta\pi$. The predicted masses of $\rho\pi$ and $\eta\pi$ are both 1.4–1.5 GeV, which is close to the experimental candidates $\pi_1(1370)$ and $\pi_1(1440)$.

II. SUM RULES FOR 1^{-+} FOUR QUARK STATES

The main task for the 1^{-+} mass prediction in the QCD sum rule approach is to calculate the current-current correlator

$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iqx}\langle 0|Tj_\mu(x)j_\nu^\dagger(0)|0\rangle = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi_v(s^2) + \frac{q^\mu q^\nu}{q^2}\Pi_s(s^2), \quad (1)$$

where the interpolated current $j_\mu(x)$ has the quantum number 1^{-+}. In this paper, we just focus on $\pi_1(1370)$ and $\pi_1(1440)$. These two states are seen only in the channel $\pi\eta$ and $\pi\rho$, respectively. They look like molecule states. We need to construct the 1^{-+} four quark currents with such property, for instance,

$$j_{1\mu} = \bar{q}(x)\gamma_5\sigma q(x)\bar{q}(x)\gamma_5\gamma_\mu q(x), \quad (2)$$

for the $\eta\pi$ state [where σ is the isospin matrix and $q = 2^{-1/4}(u, d)^T$], and

$$j_{2\mu\nu} = \epsilon_{\mu\nu\rho\sigma}(\bar{u}\gamma_5\gamma^\rho d\bar{d}\gamma^\sigma u - \bar{d}\gamma_5\gamma^\rho u\bar{u}\gamma^\sigma d), \quad (3)$$

for the $\rho\pi$ state. We cannot find a dimension six 1^{-+} current for the $\rho\pi$ state. But $j_{2\mu\nu}$ indeed annihilates a 1^{-+} $\rho\pi$ state.

These currents do not exactly represent the molecule states. For instance, $j_{1\mu}$ can both easily decay into $\pi\eta(\eta')$ and πf_1 if its mass permits. We cannot avoid such property, but other channels, such as $b_1\pi$ and $\rho\pi$, are indeed suppressed in $j_{1\mu}$'s decay.

Using the standard operator-product expansion (OPE) method [11], we get (up to irrelevant polynomials in q^2)

$$\mathrm{Im}\Pi(s)_{1v}^{(\mathrm{OPE})} = \frac{11}{1\,179\,648}\frac{s^4}{\pi^5} + \frac{\langle\alpha_s G^2\rangle}{8192}\frac{s^2}{\pi^4} - \frac{\langle\bar{q}q\rangle^2}{128}\frac{s}{\pi}, \quad (4)$$

$$\mathrm{Im}\Pi(s)_{2v}^{(\mathrm{OPE})} = \frac{1}{30\,720\pi^5}s^4 - \frac{\langle\alpha_s G^2\rangle}{768\pi^4}s^2 + \frac{5}{324}\frac{s}{\pi^2}\alpha_s(s) \times \langle\bar{q}q\rangle^2[1 + 6\gamma_E + 12\ln(s/\mu^2)], \quad (5)$$

where γ_E is Euler's constant, and we will use

$$\alpha_s(q^2) = \frac{4\pi}{9\ln(q^2/\mu^2)},$$

We also have ignored the two quark condensate since it always accompanies with the mass of light quark, thus it is less important to compare with these gluon condensates. In the correlator of the current $j_{2\mu\nu}$, the leading order four quark condensate and the three gluon condensate vanish. The next leading order four quark condensate is very important. This situation is similar to that of the meson ρ.

In order to perform QCD sum rules for the 1^{-+} four quark states, we also should know something about the meson spectral density. Usually the spectral density $\rho_v(s) = \mathrm{Im}\Pi_v(s)$ is defined via the standard dispersion relation

$$\Pi_v(q^2) = \frac{(q^2)^n}{\pi}\int_0^\infty ds\frac{\rho_v(s)}{s^n(s-q^2)} + \sum_{k=0}^{n-1}a_k(q^2)^k, \quad (6)$$

where the a_k are appropriate subtraction constants to render Eq. (6) finite.

After Borel transforming the spectral density, we get the sum rule:

$$R_i(M_B^2) = \frac{1}{\pi}\int_0^{s_0} e^{-s/M_B^2}s^i\rho_v(s)ds, \quad (7)$$

where $i = 1, 2$, and the quantity R_k represents the QCD prediction, and the threshold s_0 separates the contribution from higher excited states and the QCD continuum.

In the single narrow resonance scenario, the lowest-lying resonance mass can be obtained from ratios

$$m_v^2 = \frac{\int_0^{s_0} e^{-s/M_B^2}s^2\rho_v(s)ds}{\int_0^{s_0} e^{-s/M_B^2}s\rho_v(s)ds}. \quad (8)$$

Thus we can use Eq. (8) to predict the mass of the 1^{-+} four quark state.

The various QCD parameters that will be used in the phenomenological analysis of Eq. (8) are

$$\langle\alpha_s G^2\rangle = 0.08\ \mathrm{GeV}^4, \quad (9)$$

$$\langle\bar{q}q\rangle = -(0.24\ \mathrm{GeV})^3, \quad (10)$$

$$\mu = 0.2\ \mathrm{GeV}. \quad (11)$$

In Fig. 1, we show the mass of 1^{-+} four quark state referred to the Borel parameter M_B.

The sum rules of the $\eta\pi$ state are not stable (at small Bore parameter the spectrum density is negative) and the predicted mass of the $\eta\pi$ is heavier than that of the $\rho\pi$. This is contradictory to the experiments about the $\pi_1(1370)$ and the $\pi_1(1440)$. This problem arises because we have not taken the direct instanton effect into account yet. In history, QCD sum rules based on OPE gave a good description of vector mesons, such as ρ, ω, J/ψ and so on but failed in scalar or pseudoscalar mesons. It was found later that the direct instanton effect is large in the scalar (pseudoscalar) channel but little in the vector channel. However, this statement is only valid in the quark antiquark system. For a four quark system, we can see the direct instanton effect could be also very important in the vector channel.

The calculation of the direct instanton effect is via the so-called zero mode (ψ_0), which is a classical solution of the Dirac equation (given by 't Hooft) in the background field of instantons: $\not{D}\psi_0 = 0$.

Expanding the quark propagator in the background field of instanton for small quark mass, we get [14]:

$$S(x, y) = \frac{\psi_0(x)\psi_0^\dagger(y)}{im} + S^{nzm}(x, y) + m\Delta(x, y) + \cdots, \quad (12)$$

where $\Delta(x, y)$ is the propagator of a scalar quark.

The first term of the expansion is known as the zero mode part of the propagator in the instanton field, which reads

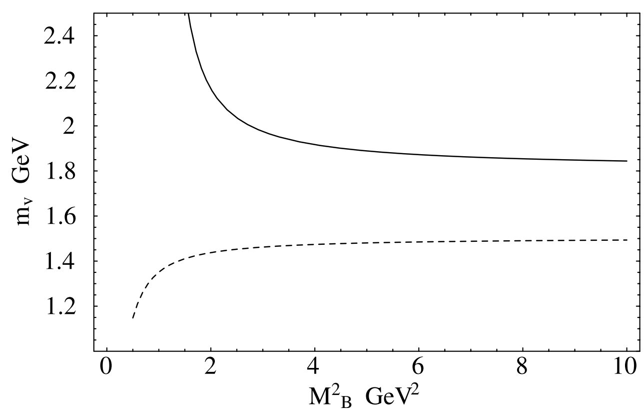

FIG. 1. Sum rules for the $\eta\pi$ and $\rho\pi$ states, where $s_0 =$ 3 GeV. (The solid lines correspond to the sum rules for $\eta\pi$ while the dashed lines the sum rules for the $\rho\pi$).

$$S^{zm}(x, y; z) = \frac{(\not{x} - \not{z})\gamma_\mu \gamma_\nu (\not{y} - \not{z})}{8m^* i}\left[\tau_\mu^- \tau_\nu^+ \frac{1-\gamma_5}{2}\right] \times \phi(x-z)\phi(y-z), \tag{13}$$

where

$$\phi(x) = \frac{\rho}{\pi}\frac{1}{|x|(x^2+\rho^2)^{3/2}},$$

and $\tau_\mu^\pm = (\tau, \mp i)$.

Now let us consider the case of the current $j_{1\mu}$. The leading contributions of nonzero mode have been already obtained in Eq. (4). From Eq. (12) we know that the zero mode part is the dominating part of the full propagator since for light quark, m^* is very small. So Eq. (4) is not a complete correlation function. We need several new diagrams which include zero mode contribution.

First we must include Fig. 2, in which the pseudoscalar loop receives zero mode contributions. In the single instanton approximation, the result is

$$\Pi_{ps}^{\mathrm{SIA}}(x) = -\frac{6n\rho^4}{m^{*2}\pi^2}\int_0^1 dy \frac{y^2(1-y)^2}{[x^2 y(1-y)+\rho^2]^4}, \tag{14}$$

where the effective mass is $m^* = \pi\rho(2n/3)^{1/2}$ according to the mean field estimate [12]. This function is well defined. It vanishes as x goes to infinity, meanwhile it is finite when x goes to zero. In order to simplify our calculation, we expand this function about the point $x = 0$ and only preserve the leading order, that is, we only preserve the constant term. Combining with the pseudovector loop contribution, we finally get the zero mode contribution for Fig. 2.

The pseudovector loop does not receive contribution of zero modes, though it does receive a contribution from the interference between the zero mode part and the lead mass correction; we ignore it since the effect is not very important.

Besides Fig. 2, there are still several diagrams, showing in Fig. 3, which also receive zero mode contribution. The contribution of Fig. 3 must be doubled since the arrows of the fermion lines have two directions.

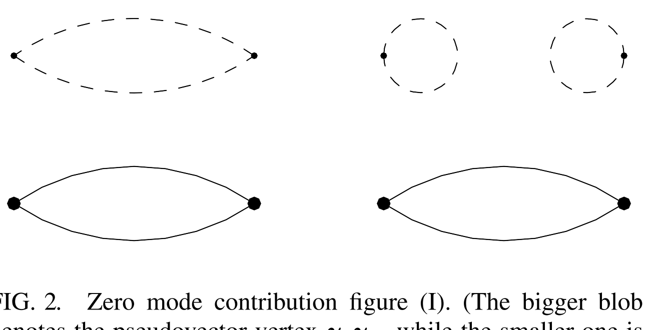

FIG. 2. Zero mode contribution figure (I). (The bigger blob denotes the pseudovector vertex $\gamma_5\gamma_\mu$, while the smaller one is the pseudoscalar vertex γ_5. The dashed lines mean zero mode propagator. The coordinates of the left two vertices are zero while the right ones x.)

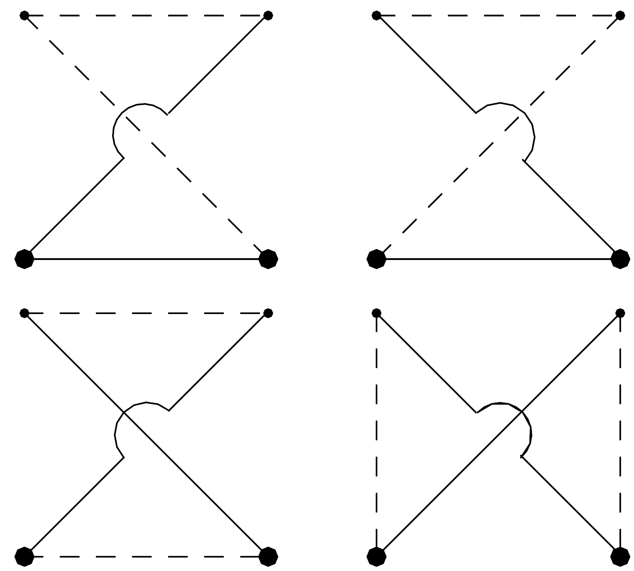

FIG. 3. Zero mode contribution figure (II).

After combining all contributions of Figs. 2 and 3, we finally get:

$$\mathrm{I\,m}\Pi_{1v}^{(\mathrm{zm})}(s) = \frac{21}{320}\frac{s}{\pi^5\rho^6}, \tag{15}$$

where all zero mode contributions preserve to leading order.

Combining Eq. (4) and Eq. (15) and using the new parameters $\rho = 1/0.6$ GeV we finally get Fig. 4 , which shows the mass of the resonance is 1.4–1.5 GeV.

Because the zero mode flips helicity of the quark, if all vertices of the current are vector type, the direct instanton effect via the zero mode is proportional to the light quark mass (or higher power), because it needs the quark mass to flip helicity. Therefore for the second current, the direct

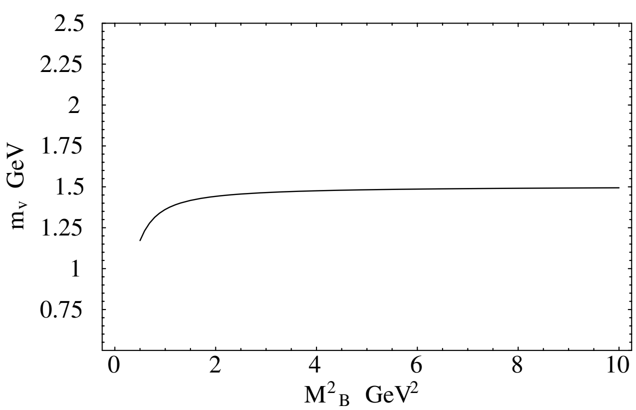

FIG. 4. Sum rules for the $\eta\pi$ includes zero mode contributions, where $s_0 = 3$ GeV.

instanton contribution from the zero mode is proportional to the light quark mass square. We neglect it.

III. DISCUSSION AND SUMMARY

After taking account of the direct instanton contribution, we find a dramatic result that the sum rules for the $\eta\pi$ become quite stable. From Fig. 4, we read the mass of the $\eta\pi$ is around 1.4–1.5 GeV. Because the correlator of the current $j_{2\mu\nu}$ does not get a big direct instanton contribution, the mass of $\rho\pi$ is still 1.4–1.5 GeV. The mass of the $\eta\pi$ is slightly lighter than that of the $\rho\pi$. This is quite consistent with the experimental data of the $\pi_1(1370)$ and $\pi_1(1440)$. Moreover, the different impact of the direct instanton contribution on the correlators of $j_{1\mu}$ and $j_{2\mu\nu}$ probably hints the different structure of states $\eta\pi$ and $\rho\pi$. One might think the difference of the direct instanton contribution is compensated by the difference of the four quark condensate, because they have the same dimension. But we also note that the sign of the four dimension gluonic condensate of these two correlators is opposite. All of these hints the mixing between the $\eta\pi$ and $\rho\pi$ might be small. This result should not be quite surprising. In our opinion, because a four quark system has much more degree of freedom than a two quark system, the spectrum of the four quark system should be more crowded. This has already been confirmed in Ref. [13], where the authors find there are several four quark states with the same quantum number and some of them are almost degenerate. Besides, we need to mention that we only consider the two flavor case. If we include s quark, the mass prediction could be slightly different. For instance, it is nature that the state $\eta'\pi$ might be 200–300 MeV heavier. However, such calculation is more complicated. Besides, our calculation assumes the states have the molecule structure. If states have the different structure, such as the diquark structure, the prediction probably is different. We will discuss such cases in another paper.

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