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Exact string theory model of closed timelike curves and cosmological singularities

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We study an exact model of string theory propagating in a space-time containing regions with closed timelike curves (CTCs) separated from a finite cosmological region bounded by a big bang and a big crunch. The model is an nontrivial embedding of the Taub-NUT geometry into heterotic string theory with a full conformal field theory (CFT) definition, discovered over a decade ago as a heterotic coset model. Having a CFT definition makes this an excellent laboratory for the study of the stringy fate of CTCs, the Taub cosmology, and the Milne/Misner-type chronology horizon which separates them. In an effort to uncover the role of stringy corrections to such geometries, we calculate the complete set of α' corrections to the geometry. We observe that the key features of Taub-NUT persist in the exact theory, together with the emergence of a region of space with Euclidean signature bounded by timelike curvature singularities. Although such remarks are premature, their persistence in the exact geometry is suggestive that string theory is able to make physical sense of the Milne/Misner singularities and the CTCs, despite their pathological character in general relativity. This may also support the possibility that CTCs may be viable in some physical situations, and may be a natural ingredient in pre-big bang cosmological scenarios.

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I. INTRODUCTION AND MOTIVATION

The Taub-NUT space-time [1,2] is an interesting one. We can write a metric for it as follows:

$$ds^2 = -f_1(dt - l\cos\theta d\phi)^2 + f_1^{-1}dr^2 + (r^2 + l^2)(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where

$$f_1 = 1 - 2\frac{Mr + l^2}{r^2 + l^2}. \quad (2)$$

The angles θ and ϕ are the standard angles parameterizing an S^2 with ranges $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$. In addition to simple time translation invariance, the metric has an SO(3) invariance acting as rotations on the S^2. To preserve $d\xi = dt - l\cos\theta d\phi$, a time translation must also accompany a general rotation. This makes t periodic with period $4l\pi$, which can be deduced by asking for there to be no conical singularities in the North or South Poles. The coordinate t is fibered over the S^2 making a squashed S^3, and the full invariance is under an SU(2) action on this space.

There are two very different regions of this space-time, as one moves in r, distinguished by the sign of $f_1(r)$. The regions are separated by the loci (with S^3 topology)

$$r_\pm = M \pm \sqrt{M^2 + l^2}, \quad (3)$$

where f_1 vanishes. They are, in a sense, horizons. The metric is singular there, but there exist extensions the nature of which is subtle in general relativity (for a review, see Ref. [3]). One of the things which we will discuss in detail later is the fact that the string theory provides an extremely natural extension.

The region $r_- < r < r_+$ has $f_1(r) < 0$. The coordinate r plays the role of time, and the geometry changes as a function of time. This is the ‘‘Taub’’ cosmology, and spatial slices have the topology of an S^3. The volume of the universe begins at $r = r_-$ at zero, it expands to a maximum value, and then contracts to zero again at $r = r_+$. This is a classical ‘‘big bang’’ followed by a classical ‘‘big crunch.’’

On either side of this Taub region, $f_1(r) > 0$. The coordinate t plays the role of time, and we have a static spatial geometry, but since t is periodic, it is threaded by closed timelike curves. Constant radial slices have the topology of an S^3 where the time is a circle fibered over the S^2. These regions are called the ‘‘NUT’’ regions.

It is fascinating to note that the Taub and NUT regions are connected. There are geodesics which can pass from one region to another, and analytic extensions of the metric can be written down [3]. The geometry is therefore interesting, since it presents itself as a laboratory for the study of a cosmology which naturally comes capped with regions containing CTCs. Classical physics would seem to suggest that one can begin within the cosmological region and after waiting a finite time, find that the universe contained closed timelike loops.

It is an extremely natural question to ask whether or not this is an artifact of classical physics, a failure of general relativity to protect itself from the apparent pathologies with which such time machines seem to be afflicted. This leads to a closer examination of the neighborhood of the

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loci $f_1(r) = 0$ located at $r = r_{\pm}$, which we shall call (adopting common parlance) "chronology horizons." For small $\tau = r - r_-$, we see that $f_1 = -c\tau$, where c is a constant, and we get for the (τ, ξ) plane

$$ds^2 = -(c\tau)^{-1} d\tau^2 + c\tau d\xi^2, \quad (4)$$

which is the metric of a two-dimensional version of the "Milne" Universe, or "Misner space" [4]. It is fibered over the S^2.

There is an early study of cosmological singularities of this type in a semiclassical quantum treatment, reported on in Ref. [5]. There, the vacuum stress-energy tensor for a conformally coupled scalar field in the background is computed, and it diverges at $\tau = 0$. This is taken by some as an encouraging sign that a full theory of quantum gravity might show that the geometry is unstable to matter fluctuations and the appropriate back-reaction should give a geometry which is modified at the boundaries between the Taub and NUT regions. In fact, this is the basis of the "chronology protection conjecture" of Ref. [6], which suggests (using Taub-NUT as a one of its key examples) that the full physics will conspire to forbid the creation of CTCs in a space-time that does not already have them present, i.e., the Misner geometry of the chronology horizon is destroyed and replaced by a nontraversable region[1]. The expectations of a full theory of quantum gravity in this regard are (at least) twofold: (1) It should prescribe exactly what types of matter propagate in the geometry, and; (2) It should give a prescription for exactly how the geometry is modified, incorporating any back-reaction of the matter on the geometry in a self-consistent way.

Since the papers of Ref. [5,6], a lot has happened in fundamental physics. In particular, it is much clearer that there is a quantum theory of gravity on the market. It should allow us to study the questions above[2]. Of course, we are referring to string theory (including its not yet fully defined nonperturbative completion in terms of M-theory). While the theory has yet to be developed to the point where we can address the physics of space-time backgrounds in as dextrous a way as possible, there are many questions which we can ask of the theory, and in certain special cases, we can study certain space-time backgrounds in some detail.

In fact, as we will recall in the next section, the Taub-NUT space-time can be embedded into string theory in a way that allows its most important features to be studied in a very controlled laboratory, an *exact* conformal field theory [19]. It is therefore not just accessible as a solution to the leading order in an expansion in small α' (the inverse string tension), but to all orders and beyond. Leading order captures only the physics of the massless modes of the string, (the low-energy limit) and so any back-reaction effecting the geometry *via* high-energy effects cannot be studied in this limit. With the full conformal field theory one can in principle extract the complete geometry, including all the effects of the infinite tower of massive string states that propagate in it. We do this in the present paper and extract the fully corrected geometry. We observe that the key features of the geometry *survive* to all orders in α', even though placed in a string theory setting without any special properties to forbid corrections. This result means that a large family of high-energy effects which could have modified the geometry are survived by the full string theory. The string seem to propagate in this apparently pathological geometry with no trouble at all. It is of course possible that the new geometry we find is unstable to the presence of a test particle or string, but this type of effect does not show up in the CFT in this computation. Such test-particle effects are important to study[3] in order to understand the complete fate of the geometry by studying its stability against fluctuations. Our work here yields the fully corrected geometry in which such probe computations should be carried out in this context. More properly, the probe computation should be done in the full conformal field theory, in order to allow the string theory to respond fully to the perturbation. The conformal field theory discussed here is a complete laboratory for such studies, and as it describes the Taub-NUT geometry, it provides the most natural stringy analogue of this classic geometry within which to answer many interesting questions[4].

In Sec. II, we recall the stringy Taub-NUT metric discovered in Ref. [19], and write it in a new coordinate which gives it a natural extension exhibiting the Taub and NUT regions and their connection via Misner space. We also recall the work of Refs. [63–65] which demonstrates how to obtain the low-energy metric as a stringy embedding by starting with the standard Taub-NUT metric of

[1]Even staying within relativity, there are many who take an alternative view, by, e.g., showing that a nondivergent stress tensor can be obtained by computing in a different vacuum, thus calling into the question the need for such a conjecture. See, for example, Refs. [7–16] and for a recent stringy example, see Ref. [17].

[2]Leaving aside the question of CTCs, cosmological singularities of Misner-type have recently become relevant in the context of cosmologies inspired by string- (and M-) theory. See for example Ref. [18]

[3]They have been found for the leading order geometry in its form as an orbifold of Minkowski space by a Lorentz boost [20–24].

[4]There are a number of other interesting conformal field theories (and studies thereof) which have been presented, which at low energy describe geometries which although are not Taub-NUT spacetimes, do share many of the key features in local patches. Some of them are listed in Refs. [25–40]. Refs. [39,40] also contain useful comments and literature survey. There are also many papers on the properties of string theory in spacetimes with CTCs, such as the BMPV [41] spacetime [42–54] and the Gödel [55] space-time [56–62].

Eq. (1). It is the ''throat'' or ''near-horizon'' region of this space-time that was discovered in Ref. [19], where an exact conformal field theory (a ''heterotic coset model'') can be constructed which encodes the full stringy corrections. We review the conformal field theory construction in Secs. III A and III B, where the Lagrangian definition is reviewed. Happily, the extension of the throat geometry we present in Sec. II (described by the same conformal field theory) contains all the interesting features: the Taub region with its big bang and big crunch cosmology, the NUT regions with their CTCs, and the Misner space behavior which separates them. Therefore we have a complete string theory laboratory for the study of the properties of Taub-NUT, allowing us to address many of the important questions raised in the relativity community. For example, questions about the analytic extension from the NUT to the Taub regions are put to rest by the fact that the full conformal field theory supplies a natural extension *via* the structure of $SL(2, \mathbb{R})$ (Sec. II). Further, having the full conformal field theory means that we can construct the α' corrections to the low-energy metric, and we do so in Sec. III F, capturing *all* of the corrections, after constructing an exact effective action in Secs. III D and III E. We analyze the exact metric in Sec. III G, and end with a discussion in Sec. IV, noting that there are many questions that can be answered in this laboratory by direct computation in the fully defined model.

II. STRINGY TAUB-NUT

Taub-NUT space-time, being an empty-space solution to the Einstein equations, is trivially embedded into string theory with no further work. It satisfies the low-energy equations of motion of any string theory, where the dilaton is set to a constant and all the other background fields are set to zero. This is not sufficient for what we want to do, since we want to have a means of getting efficient computational access to the stringy corrections to the geometry. A new embedding must be found which allows such computational control.

This was achieved some time ago. An exact conformal field theory describing the Taub-NUT space-time (in a certain throat or ''near-horizon'' limit) was constructed in Ref. [19]. This CFT will be described in the next section. The geometry comes with a nontrivial dilaton and antisymmetric tensor field, together with some electric and magnetic fields. The string theory is heterotic string theory. This model is in fact the earliest nontrivial embedding of Taub-NUT into string theory, and uses a novel construction known as ''heterotic coset models'' in order to define the theory [19,66–68]. The technique was discovered as a method of naturally defining (0, 2) conformal field theories, i.e., backgrounds particularly adapted to yielding minimally supersymmetric vacua of the heterotic string. That aspect will not be relevant here, since we will not tune the model in order to achieve space-time supersymmetry.

The low-energy metric of the stringy Taub-NUT space-time was presented in Ref. [19] as (in string frame)

$$ds^2 = k\left\{d\sigma^2 - \frac{\cosh^2\sigma - 1}{(\cosh\sigma + \delta)^2}(dt - \lambda\cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2\right\}, \tag{5}$$

where $0 \le \sigma \le \infty$, $\delta \ge 1$, $\lambda \ge 0$. The dilaton behaves as

$$\Phi - \Phi_0 = -\tfrac{1}{2}\ln(\cosh\sigma + \delta), \tag{6}$$

and there are other fields which we will discuss later. This is in fact the NUT region of the geometry, and $\sigma = 0$ is a Misner horizon. We note here that the embedding presents a natural analytic extension of this model which recovers the other NUT region and the Taub cosmology as well; replace $\cosh\sigma$ with the coordinate x:

$$ds^2 = k\left[\frac{dx^2}{x^2 - 1} - \frac{x^2 - 1}{(x + \delta)^2}(dt - \lambda\cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2\right], \tag{7}$$

with

$$\Phi - \Phi_0 = -\tfrac{1}{2}\ln(x + \delta), \tag{8}$$

where now $-\infty \le x \le +\infty$. The three ranges of interest are $1 \le x \le +\infty$, ($x = \cosh\sigma$) which is the first NUT region above, $-\infty \le x \le -1$ ($x = -\cosh\sigma$) which is a second NUT region, and $-1 \le x \le +1$ ($x = -\cos\tau$), which is a Taub region with a big bang at $\tau = 0$ and a big crunch at $\tau = \pi$. We shall see shortly that this embedding is very natural from the point of view of string theory, since x is a natural coordinate on the group $SL(2, \mathbb{R})$, which plays a crucial role in defining the complete theory. It is interesting to sketch the behavior of the function $G_{tt} = F(x) = (1 - x^2)/(x + \delta)^2$. This is done in Fig. 1. Note that $F(x)$ vanishes at $x = \pm 1$ and so for $x = 1 - \tau$ where τ is small, the metric of the (τ, ξ) space is

$$ds^2 = k\left[-(2\tau)^{-1}d\tau^2 + \frac{2\tau}{(1 + \delta)^2}d\xi^2\right], \tag{9}$$

which is of Misner form, and so the essential features of the Taub-NUT space-time persist in this stringy version of the space-time. Note that, unlike general relativity's Taub-NUT solution, there is a genuine curvature singularity in the metric, and it is located at $x = -\delta$. The dilaton diverges there, and hence the string theory is strongly coupled at this place, but it is arbitrarily far from the regions of Misner space connecting the Taub and NUT regions, so we will not need to worry about this locus for the questions of interest in this paper.

Note that the (x, t) plane is fibered over a family of S^2s which have *constant* radius, as opposed to a radius vary-

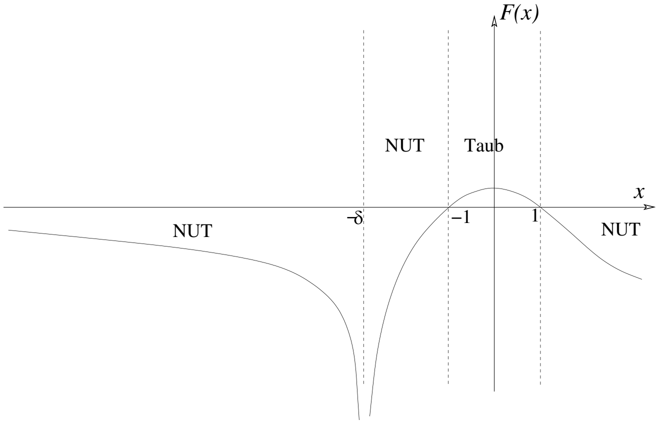

FIG. 1. The various regions in the stringy Taub-NUT geometry. There are two NUT regions, containing CTCs, and a Taub region, which is a cosmology. Note that there is a curvature singularity in the second NUT region, when $x = -\delta$.

ing with x. This does not mean that we lose key features of the geometry, since, e.g., in the Taub region, we still have a cosmology in which the universe has S^3 topology, but its volume is controlled entirely by the size of the circle fiber $(dt - \lambda\cos\theta d\phi)$, which ensures that the universe's volume vanishes at the beginning and the end of the cosmology.

The constancy of the S^2s is in fact a feature, not a bug. It allows the geometry to be captured in an exact conformal field theory, as we shall recall in the next section. This geometry is the ''near-horizon'' limit of a spacetime constructed as confirmation of the statement in Ref. [19] that the metric in question is indeed obtainable from the original Taub-NUT metric in a series of steps using the symmetries of the heterotic string theory action [63–65]. This geometry is, in string frame

$$ds^2 = (a^2 + f_2^2)\left\{-\frac{f_1}{f_2^2}[dt + (\rho + 1)l\cos\theta d\phi]^2 + f_1^{-1}dr^2 + (r^2 + l^2)(d\theta^2 + \sin^2\theta d\phi^2)\right\}, \tag{10}$$

where f_1 is as before, $\rho^2 \geq 1$ and

$$f_2 = 1 + (\rho - 1)\frac{Mr + l^2}{r^2 + l^2}, \quad \text{and} \quad a = (\rho - 1)l\frac{r - M}{r^2 + l^2}. \tag{11}$$

This metric has the full asymptotically flat part of the geometry and connects smoothly onto the throat region, which develops in an ''extremal'' limit (analogous to that taken for charged black holes). Figure 2 shows a cartoon of this. The metric (5) is obtained from it in the extremal limit $\rho \to \infty$, $M \to 0$, $l \to 0$, where $m = \rho M$ and $\ell = \rho l$ are held finite. The limit is taken in the neighborhood of $f_1 = 0$, and σ is the scaled coordinate parameterizing r in that region. The coordinate t has to be rescaled as well

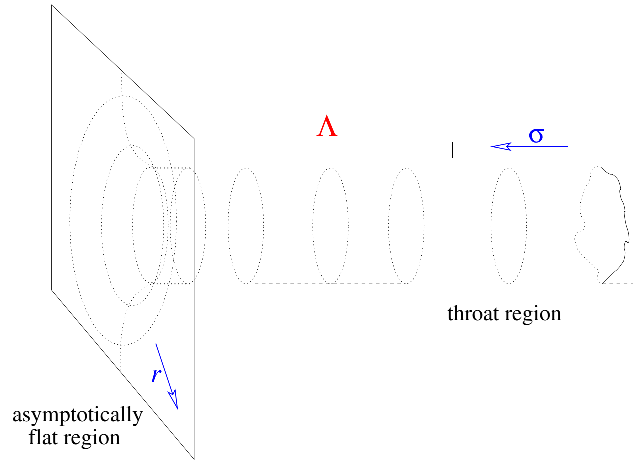

FIG. 2 (color online). A schematic showing the asymptotically flat region connected to the throat region located near the horizon at extremality. In the extremal limit, the typical measure, Λ of the distance from a point on the outside to a point near the horizon region diverges logarithmically, and the throat region is infinitely long. The coordinate σ is used for the exact throat region in low-energy metric (5), while r is the coordinate for the general low-energy metric (10).

to get matching expressions. The parameters of metric (5) are recovered as $\lambda = l/m$ and $\delta^2 = 1 + l^2/M^2$.

The stringy embedding giving rise to the metric (10) (we have not displayed the other fields of the solution here) is carried out starting from the metric (1) as follows: (The details are in Refs. [63–65]) First, an O(1, 1) boost (a subgroup of the large group of perturbative noncompact symmetries possessed by the heterotic theory) is used to generate a new solution, mixing the t direction with a U(1) gauge direction. This generates a gauge field A_t, a nontrivial dilaton, and since there is a coupling of t to ϕ in the original metric, a gauge field A_ϕ and an antisymmetric tensor background $B_{t\phi}$. So the solution has electric and magnetic charges under a U(1) of the heterotic string, and nontrivial axion and dilaton charge. We will not need the forms of the fields here. It turns out that the dilaton has a behavior which is ''electric'' in its behavior in a sense inherited from the behavior of charged dilaton black holes; it decreases as one approaches the horizon. Such holes do not support the development of throats in the string frame metric, but their ''magnetic'' cousins, where the dilaton has the opposite behavior, do support throats[5]. Using the $SL(2,\mathbb{R})$ S-duality of the four dimensional effective action of the heterotic string, which combines an electric-magnetic duality with an inversion of the axi-dilaton field $\tau = a + ie^{-\Phi}$, a solution with magnetic character can be made

[5]In fact, an exact conformal field theory can be written for pure magnetic dilaton black holes in four dimensions [69], and it can be realized as a heterotic coset model as well [19].

[63,64], which supports a throat in the string frame metric. This is the solution whose metric we have displayed in Eq. (10).

So in summary, there is an embedding of general relativity's celebrated Taub-NUT solution into heterotic string theory which preserves all of the interesting features: the NUT regions containing CTCs, and the Taub region with its big bang and big crunch cosmology, and (crucially) the Misner regions connecting them. There is a throat part of the geometry which decouples from the asymptotically flat region in an extremal limit, but which captures all of the features of the Taub-NUT geometry of interest to us here.

The next thing we need to recall is that this throat geometry arises as the low-energy limit of a complete description in terms of a conformal field theory, as presented in Ref. [19].

III. EXACT CONFORMAL FIELD THEORY

A. The Definition

In Ref. [19], the "heterotic coset model" technique was presented, and one of the examples of the application of the method was the model in question, from which the low-energy metric in Eq. (7) was derived, for $x = \cosh\sigma$. The other regions that have been presented here (making up $-\infty \leq x \leq 0$) are easily obtained from the same conformal field theory by choosing different coordinate patches in the parent model, as we shall see.

Actions can be written for a large class of conformal field theories obtained as coset models [70–74], by using gauged WZNW models [75–80]. The ungauged model [81,82] has some global symmetry group G which defines a conformal field theory [83–85] with an underlying current algebra, and coupling it to gauge fields charged under a subgroup $H \subset G$ gives the coset. Such models have been used to generate conformal field theories for many studies in string theory, including cosmological contexts (see the introduction for some references). It is important to note that the vast majority of these models use a particular sort of gauging. The basic world-sheet field is group valued, and we shall denote it as $g(z, \bar{z})$. The full global invariance is $G_L \times G_R$, realized as $g(z, \bar{z}) \to g_L g(z, \bar{z}) g_R^{-1}$, for $g_L, g_R \in G$. The sorts of group actions gauged in most studies are $g \to h_L g h_R^{-1}$, for $h_L, h_R \in H$, and it is only a restricted set of choices of the action of h_L and h_R which allow for the writing of a gauge-invariant action. These are the "anomaly-free" subgroups, and the typical choice that is made is to correlate the left and right actions so that the choice is essentially left-right symmetric. This also gives a symmetric structure on the world-sheet, as appropriate to bosonic strings and to superstrings if one considers supersymmetric WZNW models. For these anomaly-free subgroups, a gauge extension of the basic WZNW action can be written which is H-invariant, and the resulting conformal field theory is well-defined. The supersymmetric models can of course be turned into heterotic string theories too, by simply tensoring with the remaining conformal field theory structures needed to make a left-right asymmetric model.

The general heterotic coset model goes beyond this, and exploits the basic fact that the heterotic string is asymmetric in how it is built. The idea is to allow oneself the freedom to choose to gauge far more general subgroups. This might well produce anomalies, but permits one to choose to retain certain global symmetries which might be of interest (such as space-time rotations) and/or use in the conformal field theory. Introducing right-moving fermions to achieve a right-moving supersymmetry is easy to do, and they contribute extra terms to the anomaly, making matters worse in general Their couplings (the effective charges they carry under H) are completely determined by supersymmetry, so one has no choice. Of course, one does not have a well-defined model if there are anomalies, so ultimately they must be eliminated. This is achieved as follows [19]. Note that the left-moving fermions can be introduced with *arbitrary* couplings (charges under H), since there is no requirement of left-moving supersymmetry in the heterotic string. The anomaly they contribute comes with the opposite sign to that of the others, since they have the opposite chirality. The requirement that the anomaly cancels can be satisfied, since it just gives a set of algebraic equations to solve for the charges. The resulting model is a conformal field theory with (0,1) world-sheet supersymmetry, (enhanced to $(0, 2)$ when G/H is Kähler [86–88]) naturally adapted to the heterotic string.

It is important to note that the types of heterotic models obtained by this method are very different from the types of models obtained by gaugings that do not cancel the anomalies against those of the gauge fermions. One way to see the difference is to note that since the anomaly is proportional to k, the cancellation equation puts the gauge charge at the same order as the metric. This means that there is a nontrivial modification of the geometry one would read off from the WZNW action, traceable to the left-moving fermions. We will explain this more shortly.

By way of example, we simply present the model relevant to our study here [19]. The group in question is $SL(2, \mathbb{R}) \times SU(2)$, and the group elements are denoted g_1 and g_2 respectively. Let the levels of the models be denoted k_1 and k_2, respectively. We are interested in a $U(1)_A \times U(1)_B$ subgroup (A and B are just means of distinguishing them) which acts as follows:

$$U(1)_A \times U(1)_B\colon \begin{cases} g_1 \to & e^{\epsilon_A \sigma_3/2} g_1 e^{(\delta\epsilon_A + \lambda\epsilon_B)\sigma_3/2} \\ g_2 \to & g_2 e^{i\epsilon_B \sigma_3/2} \end{cases}. \quad (12)$$

Notice that there is a whole global $SU(2)_L$ of the original $SU(2)_L \times SU(2)_R$ untouched. This is a deliberate choice to give a model with space-time SU(2) invariance (rotations) in the end. With that, and the other asymmetry introduced by the presence of λ and δ, the gauging is very anomalous. Once right-moving supersymmetry fermions are introduced, the anomalies are proportional to $-k_1(1-\delta^2)+2\delta^2$ from the AA sector, $k_1\delta\lambda+2\delta\lambda$ from the AB sector, and $k_2+k_1\lambda^2+2(1+\lambda^2)$ from the BB sector. The k-independent parts come from the fermions. Next, four left-moving fermions are introduced. Two are given charges $Q_{A,B}$ under $U(1)_{A,B}$ and the other two are given charges $P_{A,B}$. Their anomalies are $-2(Q_A^2+P_A^2)$, $-2(Q_AQ_B+P_AP_B)$, and $-2(Q_B^2+P_B^2)$, respectively, from the various sectors AA, AB, BB. So we can achieve an anomaly-free model by asking that:

$$\begin{aligned}-k_1(1-\delta^2)&=2(Q_A^2+P_A^2-\delta^2),\\ k_1\delta\lambda&=2(Q_AQ_B+P_AP_B-\delta\lambda),\\ k_2+k_1\lambda^2&=2[Q_B^2+P_B^2-(1+\lambda^2)].\end{aligned}\qquad(13)$$

It is a highly nontrivial check on the consistency of the model to note that in the solution-generating techniques used to verify the observation made in Ref. [19] that our stringy solution (5) can be obtained from the basic Taub-NUT solution (1), the charges in the resulting throat metric turn out to be given in terms of the parameters M, l and ρ in such a way that they satisfy the anomaly equations above, in the large k limit (which is appropriate to low-energy). See Ref. [63].

The central charge of this four dimensional model is

$$c=\frac{3k_1}{k_1-2}+\frac{3k_2}{k_2+2},\qquad(14)$$

where the -2 from gauging is cancelled by the $+2$ from four bosons on the left and right. We can ask that this be equal to 6, as is appropriate for a four dimensional model, tensoring with another conformal field theory to make up the internal sector, as desired[6]. The result is that $k_1=k_2+4$.

In Ref. [19], the metric for the throat region was discovered by working in the low-energy limit where k_1 and k_2 are large, and denoted simply as k. In this paper, we study the case of going beyond this large k (low-energy) approximation and derive the geometry which is correct to all orders in the $\alpha'\sim 1/k$ expansion.

B. Writing The Full Action

The $G=\mathrm{SL}(2,\mathbb{R})\times\mathrm{SU}(2)$ WZNW model is given by

$$S(g_1,g_2)=-k_1I(g_1)+k_2I(g_2),\qquad(15)$$

where

$$I(g)=-\frac{1}{4\pi}\int_\Sigma d^2z\mathrm{Tr}(g^{-1}\partial_zgg^{-1}\partial_{\bar z}g)-i\Gamma(g),\qquad(16)$$

with

$$\Gamma(g)=\frac{1}{12\pi}\int_{\mathcal{B}}d^3\sigma\epsilon^{abc}\mathrm{Tr}(g^{-1}\partial_agg^{-1}\partial_bgg^{-1}\partial_cg).\qquad(17)$$

The group valued fields $g_1(z,\bar z)\in\mathrm{SL}(2,\mathbb{R})$ and $g_2(z,\bar z)\in\mathrm{SU}(2)$ map the world-sheet Σ with coordinates $(z,\bar z)$ into the group $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SU}(2)$. Part of the model is defined by reference to an auxiliary space-time $\mathcal{B}$, whose boundary is Σ, with coordinates σ^a. The action $\Gamma(g)$ is simply the pullback of the $G_L\times G_R$ invariant threeform on G.

With reference to the $\mathrm{U}(1)_A\times\mathrm{U}(1)_B$ action chosen in Eq. (12), the gauge fields are introduced with the action

$$\begin{aligned}S(g_1,g_2,A)=\frac{k_1}{8\pi}\int d^2z\{&-2(\delta A_{\bar z}^A+\lambda A_{\bar z}^B)\mathrm{Tr}[\sigma_3g_1^{-1}\partial_zg_1]-2A_z^A\mathrm{Tr}[\sigma_3\partial\bar z g_1g_1^{-1}]+A_z^AA_{\bar z}^A(1+\delta^2+\delta\mathrm{Tr}[\sigma_3g_1\sigma_3g_1^{-1}])\\&+\lambda^2A_z^BA_{\bar z}^B+\lambda\delta A_z^AA_{\bar z}^B+A_z^BA_{\bar z}^A(\lambda\delta+\lambda\mathrm{Tr}[\sigma_3g_1\sigma_3g_1^{-1}])\}+\frac{k_2}{8\pi}\int d^2a\{2iA\bar z^B\mathrm{Tr}[\sigma_3g_2^{-1}\partial_zg_2]+A_z^BA_{\bar z}^B\},\end{aligned}\qquad(18)$$

and we note that we have written the generators as

$$t_{A,R}^{(1)}=-\delta\frac{\sigma_3}{2},\quad t_{A,L}^{(1)}=\frac{\sigma_3}{2},\quad t_{B,R}^{(1)}=-\lambda\frac{\sigma_3}{2},\quad t_{B,R}^{(2)}=-i\frac{\sigma_3}{2}.\qquad(19)$$

The anomaly under variation $\delta A_a^{A(B)}=\partial_a\epsilon_{A(B)}$ can be written as

$$\mathcal{A}_{ab}=\frac{1}{4\pi}\mathrm{Tr}[t_{a,L}t_{b,L}-t_{a,R}t_{b,R}]\epsilon_a\int d^2zF_{z\bar z}^b,\qquad(20)$$

(no sum on a,b) and we have defined $\mathrm{Tr}=-k_1\mathrm{Tr}_1+k_2\mathrm{Tr}_2$. The right-moving fermions have an action

$$I_R^F=\frac{i}{4\pi}\int d^2z\mathrm{Tr}(\Psi_R\mathcal{D}_z\Psi_R),\qquad(21)$$

where Ψ_R takes values in the orthogonal complement of the Lie algebra of $\mathrm{U}(1)_A\times\mathrm{U}(1)_B$, (so there are four right-movers, in fact) and

$$\mathcal{D}_z\Psi_R=\partial_z\Psi_R-\sum_aA^a\bar z[t_{a,R},\Psi_R],\qquad(22)$$

The four left-moving fermions have action

[6]Actually, we can also choose other values of c, and adjust the internal theory appropriately.

$$I_L^F = -\frac{ik_1}{4\pi}\int d^2z\{\lambda_L^1[\partial_z + Q_A A_z^A + Q_B A_z^B]\lambda_L^2\} + \frac{ik_2}{4\pi}\int d^2z\{\lambda_L^3[\partial_z + P_A A_z^A + P_B A_z^B]\lambda_L^4\}. \quad (23)$$

Under the gauge transformation $\delta A_a^{A(B)} = \partial_a \epsilon_{A(B)}$, these two sets of fermion actions yield the anomalies discussed earlier, but at one-loop, while the WZNW model displays its anomalies classically. It is therefore hard to work with the model in computing a number of properties. In particular, in working out the effective space-time fields it is useful to integrate out the gauge fields. It is hard to take into account the effects of the successful anomaly cancellation if part of them are quantum and part classical. The way around this awkward state of affairs [19] is to bosonize the fermions. The anomalies of the fermions then appear as classical anomalies of the action. The bosonized action is

$$\begin{aligned} I_B = \frac{1}{4\pi}\int d^2z\{&|\partial_z\Phi_2 - P_A A_z^A - (P_B+1)A_z^B|^2 + |\partial_z\Phi_1 \\ &- (Q_B+\lambda)A_z^B - (Q_A+\delta)A_z^A|^2 - \Phi_1[(Q_B-\lambda)F_{z\bar z}^B \\ &+ (Q_A-\delta)F_{z\bar z}^A] - \Phi_2[(P_B-1)F_{z\bar z}^B + P_A F_{z\bar z}^A] \\ &+ [A\bar z^A A_z^B - A_z^A A\bar z^B][\delta Q_B - \lambda Q_A - P_A]\}, \end{aligned} \quad (24)$$

which under variations

$$\delta A_a^{A(B)} = \partial_a\epsilon_{A(B)}, \quad \delta\Phi_1 = (Q_A+\delta)\epsilon_A + (Q_B+\lambda)\epsilon_B, \quad \delta\Phi_2 = P_A\epsilon_A + (P_B+1)\epsilon_B, \quad (25)$$

manifestly reproduces the anomalies presented earlier.

C. Extracting the Low-Energy Metric

At this stage, it is possible to proceed to derive the background fields at leading order by starting with the Lagrangian definition given in the previous section and integrating out the gauge fields, exploiting the fact that they appear quadratically in the action. As these fields are fully quantum fields, this procedure is only going to produce a result which is correct at leading order in the $1/k$ expansion, where k is large. This is because we are using their equations of motion to replace them in the action, and neglecting their quantum fluctuations. Before turning to how to go beyond that, let us note that there is an important subtlety even in the derivation of the leading order metric. This is not an issue for coset models that are not built in this particularly heterotic manner, and so is a novelty that cannot be ignored.

The coordinates we use for SL(2, $\mathbb{R}$) and SU(2) are

$$g_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{t_+/2}(x+1)^{1/2} & e^{t_-/2}(x-1)^{1/2} \\ e^{-t_-/2}(x-1)^{1/2} & e^{-t_+/2}(x+1)^{1/2}\end{pmatrix}, \quad (26)$$

where $t_\pm = t_L \pm t_R$, and $-\infty \le t_R, t_L, x \le \infty$, and the Euler angles

$$g_2 = \begin{pmatrix} e^{i\phi_+/2}\cos\frac{\theta}{2} & e^{i\phi_-/2}\sin\frac{\theta}{2} \\ -e^{-i\phi_-/2}\sin\frac{\theta}{2} & e^{-i\phi_+/2}\cos\frac{\theta}{2}\end{pmatrix}, \quad (27)$$

where $\phi_\pm = \phi \pm \psi$, $0 \le \theta \le \pi$, $0 \le \psi \le 4\pi$, and $0 \le \phi \le 2\pi$. Note that the full range of x is available here, while remaining in SL(2, $\mathbb{R}$). In Ref. [19], the range $x = \cosh\sigma \ge 1$ was used. The larger range reveals the connection to the Taub and the other NUT region. This extension is very naturally inherited from the SL(2, $\mathbb{R}$) embedding[7].

The gauge we fix to before integrating out the gauge fields is

$$t_L = 0, \quad \psi = \pm\phi, \quad (28)$$

where the sign choice depends on which coordinate patch we investigate, such that + refers to the North Pole on the S^2 parameterized by (θ, ϕ) and − refers to the South Pole, and we write $t_R = t$. One can then read off various space-time fields from the resulting σ-model, by examining terms of the form $C_{ij}\partial_z\chi^i\partial\bar z\chi^j$, where here χ^j, is a place holder for any world-sheet field, and j denoted which field is present. When i, j are such that $\chi^i\chi^j$ run over the set of fields t, x, θ, ϕ, then the symmetric parts of C_{ij} give a metric we shall call $G^0_{\mu\nu}$, and the antisymmetric parts give the antisymmetric tensor potential $B_{\mu\nu}$. When i, j are such that χ^i is one of the bosonized fermions and χ^j is one of t, x, θ, ϕ, the C_{ij} is a space-time gauge potential, either from the (1) or the (2) sector: $A_\mu^{(1,2)}$.

Note that $G^0_{\mu\nu}$ is *not* the correct space-time metric at this order. This is a crucial point [19]. The anomaly cancellation requirement means that the contribution from the left-movers has a significant modification to the naive metric. The most efficient way of seeing how it is modified is to refermionize the bosons, using as many symmetries as one can to help in deducing the normalization of the precise couplings. After some work [19], it transpires that the correct metric (to leading order) is

$$G_{\mu\nu} = G^0_{\mu\nu} - \frac{1}{2k}[A^1_\mu A^1_\nu + A^2_\mu A^2_\nu], \quad (29)$$

where it can be seen that because $A \sim Q$ and from the anomaly Eqs. (13) we have $Q \sim \sqrt{k}$, this gives a nontrivial correction to the metric one reads off naively. This is the clearest sign that these heterotic coset models are quite different from coset models that have commonly been used to make heterotic string backgrounds by tensoring together ordinary cosets. In those cases, typically $A \sim Q \sim 1$ and so at large k, the correction is negligible.

This sets the scene for what we will have to do when we have constructed the exact effective σ-model. We will again need to correct the naive metric in a way which

[7]See Ref. [32] for a discussion of how an SL(2, $\mathbb{R}$) structure also provides a natural extension for the discussion of wavefunctions in related spacetimes.

generalizes Eq. (29), in order to get the right space-time metric.

D. The Exact Effective Action

In the previous section, we treated the gauge fields as classical fields, substituting their on-shell behavior into the action to derive the effective σ-model action for the rest of the fields and ignoring the effects of quantum fluctuations arising at subleading order in the large k expansion. To include all of the physics and derive a result valid at any order in k, we need to do better than this. For ordinary coset models, this sort of thing has been achieved before, using a number of methods. To our knowledge, this was first done in Ref. [89] in the context of the $SL(2,\mathbb{R})/U(1)$ coset model studied as a model of a two-dimensional black hole [90]. The exact metric and dilaton were written down by appealing to a group theoretic argument, writing the exact expressions for the quadratic Casimirs for G and for H, in terms of the target space (G/H) fields, and then equating their difference to the Laplacian for the propagation of a massless field (the tachyon) in the background. The proposed metric and dilaton were verified at higher orders by explicit calculation in Ref. [91,92], and the argument was generalized and applied to a number of other models in a series of papers [93,94]. An elegant alternative method was developed in Refs. [95,96], and is the one we adapt for use here. We must extend it to work for the heterotic coset models, since although heterotic backgrounds are considered in some of those works, they are of the mildly heterotic type which are essentially similar to the superstring models; an asymmetric arrangement of fermions is merely tensored in as dressing.

Since there will be a fair amount of messy computation in what follows, we state the key ideas in what follows: It is known [95,97,98] that the exact effective action for the WZNW model defined in Ref. [16] is extremely simple to write down. One takes the form of the basic action at level k, $kI(g)$, where g is a quantum field, and one writes for the full quantum effective action $(k - c_G)I(g)$, where now g should be taken as a classical field, and c_G is the dual Coxeter number of the group G. This is particularly simple since k only enters the action as an overall multiplicative factor, which then gets shifted. The key observation of Refs. [95,96] is that this can be applied to a gauged WZNW model as well, by exploiting the fact that if one writes $A_z = \partial_z h_z h_z^{-1}$ and $A\bar{z} = \partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1}$, the action can be written as the sum of two formally decoupled WZNW models, one for the field $g' = h_{\bar{z}}^{-1} g h_z$ at level k and the other for the field $h' = h_{\bar{z}}^{-1} h_z$ at level $2c_H - k$. To write the exact effective action, one shifts the levels in each action: $k \to k - c_G$ and $2c_H - k \to 2c_H - k - c_H = c_H - k$, and treats the fields as classical. Transforming back to the original variables, one gets the original gauged WZNW model with its level shifted according to $k \to k - c_G$, together with a set of new terms for $A_z, A_{\bar{z}}$ which are proportional to $c_H - c_G$, and have no k dependence. Because there is no multiplicative factor of k in these new terms, it is easy to see that the large k contribution to the result of integrating out the gauge fields will be the same as before. For results exact in k, there will be a family of new contributions to the σ-model couplings upon integrating out the gauge fields. In this effective action, they are to be treated as classical fields now and so once the integration is done, there are no further contributions from quantum fluctuations to take into account. The metrics derived using this method are the same as those constructed using the algebraic approach, which is a useful consistency check [95,96].

Note that the new pieces in the effective action are nonlocal in the fields $A_z, A_{\bar{z}}$ (although local in the $h_z, h_{\bar{z}}$). This difficulty does not present a problem for the purposes of reading off the space-time fields, since it is enough to work in the zero-mode sector of the string to capture this information. This amounts to dropping all derivatives with respect to σ on the world-sheet and working with the reduced "point-particle" Lagrangian for that aspect of the computation [96].

Let us turn to the model in question. Here, we exploit the fact [19,66,67] that our heterotic coset model, in its bosonized form (where all the anomalies are classical) can be thought of as an asymmetrically gauged WZNW model for G/H supplemented by another asymmetrically gauged WZNW model for $SO(\dim G - \dim H)/H$, representing the fermions. We should be able to carry out a similar set of changes of variables to write the whole model as a set of decoupled WZNW models, transform to the effective action, and then rewrite it back in the original variables to see what new terms the effective action supplies us with. Then we have to integrate out the gauge fields and—crucially—correctly refermionize the bosons to read off the space-time fields. This is the subject of the next subsection. The reader wishing to skip to the result can pick up the story again at the beginning of subsection III G.

E. Computation of the Exact Effective Action

As noted above, the fermions can also be represented as a gauged WZNW model based on the coset $SO(D)/H$, with $D = \dim G - \dim H = 6 - 2 = 4$. Doing this, the complete classical action can be written as

$$S = -k_1 I(g_1) + k_2 I(g_2) + I(g_f), \tag{30}$$

with $g_1 \in SL(2,\mathbb{R})$, $g_2 \in SU(2)$, and $g_f \in SO(4)$. It is convenient to write

$$g = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_f \end{pmatrix} \in SL(2,\mathbb{R}) \times SU(2) \times SO(4). \tag{31}$$

To gauge the subgroup $H = \mathrm{U}(1)_A \times \mathrm{U}(1)_B$ we introduce the covariant derivative

$$\mathcal{D}_\mu g = \partial_\mu g + A^a_{\mu,L} g - g A^a_{\mu,R}, \tag{32}$$

where $A_{\mu,L} = A^a_\mu t_{a,L}$ and $A_{\mu,R} = A^a_\mu t_{a,R}$. These are the gauge fields, which take values in the Lie algebra of H. With $f_L \in H_L$, $f_R \in H_R$, the gauge transformation is written

$$g \to f_L g f_R^{-1}. \tag{33}$$

The $t_{a,L}$ are left generators, and $t_{a,R}$ are right generators of H. Using the block diagonal notation above, we can write

$$A = A^a \begin{pmatrix} t_a^{(1)} & 0 & 0 \\ 0 & t_a^{(2)} & 0 \\ 0 & 0 & t_a^{(f)} \end{pmatrix} \in \mathrm{Lie}(H), \tag{34}$$

where $t_a^{(1)}$ and $t_a^{(2)}$ are 2×2 matrices, and $t_a^{(f)}$ are 4×4 matrices.

The gauged WZNW model is

$$S_{gWZNW} = -k_1[I(g_1) + S_1(g_1, A)] + k_2[I(g_2) + S_1(g_2, A)] + [I(g_f) + S_1(g_f, A)], \tag{35}$$

where

$$S_1(g, A) = \frac{2}{4\pi}\int d^2z \mathrm{Tr}\{A_{\bar{z},L}\partial_z g g^{-1} - A_{z,R} g^{-1}\partial_{\bar{z}} g - A_{\bar{z},L} g A_{z,R} g^{-1} + \frac{1}{2}(A_{z,L}A_{\bar{z},L} + A_{z,R}A_{\bar{z},R})\}. \tag{36}$$

Since there is no gauge-invariant extension for the Wess-Zumino term $\Gamma(g)$ for general subgroup H, this action has (in general) classical anomalies. However, there is a unique extension such that the anomalies do not depend on g, but only on gauge fields [99]. This extension has been used in the expression above.

1. A Change of Variables

By the change of variables

$$A_{z,L} = -\partial_z h_z h_z^{-1}, \quad A_{z,R} = -\partial_z \tilde{h}_z \tilde{h}_z^{-1}, \quad h, \tilde{h} \in H, \quad A_{\bar{z},L} = -\partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1}, \quad A_{\bar{z},R} = -\partial_{\bar{z}} \tilde{h}_{\bar{z}} \tilde{h}_{\bar{z}}^{-1}, \tag{37}$$

we find

$$S_1(g, h) = \frac{2}{4\pi}\int d^2z \mathrm{Tr}\{-\partial_z g g^{-1}\partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1} + g^{-1}\partial_{\bar{z}} g \partial_z \tilde{h}_z \tilde{h}_z^{-1} - \partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1} g \partial_z \tilde{h}_z \tilde{h}_z^{-1} g^{-1} + \tfrac{1}{2}(\partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1}\partial_z h_z h_z^{-1} + \partial_{\bar{z}} \tilde{h}_{\bar{z}} \tilde{h}_{\bar{z}}^{-1}\partial_z \tilde{h}_z \tilde{h}_z^{-1})\}. \tag{38}$$

The Polyakov-Wiegmann identity [82] leads to the identities

$$\begin{aligned} I(h_{\bar{z}}^{-1} g \tilde{h}_z) &= I(g) + I(h_{\bar{z}}^{-1}) + I(\tilde{h}_z) + \frac{2}{4\pi} \times \int d^2z \mathrm{Tr}[-\partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1}\partial_z g g^{-1} - \partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1} g \partial_z \tilde{h}_z \tilde{h}_z^{-1} g^{-1} + g^{-1}\partial_{\bar{z}} g \partial_z \tilde{h}_z \tilde{h}_z^{-1}], \\ I(h_{\bar{z}}^{-1} h_z) &= I(h_{\bar{z}}^{-1}) + I(h_z) + \frac{2}{4\pi} \times \int d^2z \mathrm{Tr}[-\partial_{\bar{z}} h_{\bar{z}} h_{\bar{z}}^{-1}\partial_z h_z h_z^{-1}], \\ I(\tilde{h}_{\bar{z}}^{-1} \tilde{h}_z) &= I(\tilde{h}_{\bar{z}}^{-1}) + I(\tilde{h}_z) + \frac{2}{4\pi} \times \int d^2z \mathrm{Tr}[-\partial_{\bar{z}} \tilde{h}_{\bar{z}} \tilde{h}_{\bar{z}}^{-1}\partial_z \tilde{h}_z \tilde{h}_z^{-1}]. \end{aligned} \tag{39}$$

Using these, the classical action can be written as[8]

$$S_1 = -I(g) + I(h_{\bar{z}}^{-1} g \tilde{h}_z) - \tfrac{1}{2}[I(h_{\bar{z}}^{-1} h_z) + I(\tilde{h}_{\bar{z}}^{-1} \tilde{h}_z)] - \tfrac{1}{2}C],$$
$$\text{where} \quad C \equiv I(h_{\bar{z}}^{-1}) - I(\tilde{h}_{\bar{z}}^{-1}) - I(h_z) + I(\tilde{h}_z).$$

The term C is not manifestly gauge-invariant, but the others are. Note that if $A_L = A_R$, then $C = 0$, in which case the gauging is classically anomaly-free. Otherwise, the anomalous terms C_i may look disturbing, but in fact they cancel, $\sum k_{(i)} C_i = 0$, as will follow from the anomaly cancellation Eqs. (13).

Taking all this into account, we can write the action as

$$S = -\sum_{i=1,2,f}\{k_{(i)} I(h_{\bar{z}}^{-1} g_i \tilde{h}_z) - (k_{(i)} - 2c_H)\frac{1}{2}[I(h_{\bar{z}}^{-1} h_z) + I(\tilde{h}_{\bar{z}}^{-1} \tilde{h}_z)]\}, \tag{40}$$

with $k_{(1)} = k_1$, $k_{(2)} = -k_2$ and $k_{(f)} = -1$ and we note that $h_{\bar{z}}^{-1} g h_z \in G$, $h_{\bar{z}}^{-1} h_z \in H$, and $\tilde{h}_{\bar{z}}^{-1}\tilde{h}_z \in H$. Now, as promised in the previous section, we have achieved the rewriting of the full action in the form of a sum of WZNW actions, which allows us to write down the quantum effective action in a very simple way.

2. Effective Action

Using the simple prescription given above,

$$\begin{aligned} &\text{for} \quad \mathrm{G}: k_{(i)} \to k_{(i)} - c_{G_i}, \\ &\text{while for } H: \ -k_{(i)} + 2c_H \to (-k_{(i)} + 2c_H) - c_H = -(k_{(i)} - c_H), \end{aligned} \tag{41}$$

we find the effective action

[8] In this case of Abelian H, the Jacobian for the change of variables vanishes.

$$S^{eff} = -\sum_{i=1,2,f}\{(k_{(i)} - c_{G_i})I(h_{\bar{z}}^{-1}g_i\tilde{h}_z) - (k_{(i)} - c_H)\frac{1}{2}[I(h_{\bar{z}}^{-1}h_z) + I(\tilde{h}_{\bar{z}}^{-1}\tilde{h}_z)]\}, \quad (42)$$

where $G_1 = \mathrm{SL}(2,\mathbb{R})$, $G_2 = \mathrm{SU}(2)$, $G_f = \mathrm{SO}(4)$, $H = \mathrm{U}(1)\times\mathrm{U}(1)$. Again, the action is manifestly gauge-invariant. It is important to note here that the level constant for the fermionic sector $k_{(f)} = 1$ is *not* shifted.

3. Return to the Original Variables

We now change variables back to the original ones, using the identities given above. We find

$$S^{\mathrm{eff}} = -\sum_{i=1,2,f}\left\{(k_{(i)} - c_{G_i})\left[I(g) + S_1(g,A) + \frac{1}{2}[I_2(A_L) + I_2(A_R)] + \frac{1}{2}C_i\right] - (k_{(i)} - c_H)\frac{1}{2}[I_2(A_L) + I_2(A_R)]\right\}, \quad (43)$$

where $I_2(A_L) \equiv I(h_{\bar{z}}^{-1}h_z)$, $I_2(A_R) \equiv I(\tilde{h}_{\bar{z}}^{-1}\tilde{h}_z)$. Observe that the C_i's have come back into the action. Rewritten, this is

$$S^{\mathrm{eff}} = -\sum_{i=1,2,f}(k_{(i)} - c_{G_i})\left[I(g) + S_1(g,A) - \frac{\lambda_i}{2}[I_2(A_L) + I_2(A_R)]\right], \quad (44)$$

where $\lambda_i = \frac{c_{G_i} - c_H}{k_{(i)} - c_{G_i}}$.

F. Extracting the Exact Geometry

As we stated earlier, a problem with working with this action is that it has terms which are nonlocal in the gauge fields. Since we are going to integrate these out, this is inconvenient. To avoid this complication, we shall reduce to the zero-mode sector [96], which is enough to extract the information we want. The zero-mode sector is obtained by letting fields depend on world-sheet time only. So ∂_z and $\partial\bar{z} \to \partial_\tau$. We also denote A by a in this limit. This leads to the desired simplifications. Note the additional simplification that the WZ part of the WZNW action vanishes in this sector, i.e., $\Gamma(g) \to 0$.

The resulting action is

$$S_0^{\mathrm{eff}} = -\sum\frac{(k_{(i)} - c_{G_i})}{4\pi}\int d\tau\Big\{\mathrm{Tr}(g^{-1}\partial g g^{-1}\partial g) + 2\mathrm{Tr}[a_{\bar{z}}, L\partial g g^{-1} - a_{z,R}g^{-1}\partial g - a_{\bar{z}}, Lga_{z,R}g^{-1} + \tfrac{1}{2}(a_{z,L}a_{\bar{z}}, L + a_{z,R}a_{\bar{z}}, R)] - \lambda_i\tfrac{1}{2}\mathrm{Tr}[(a_{\bar{z}}, L - a_{z,L})^2 + (a_{\bar{z}}, R - a_{z,R})^2] + \frac{1}{2}\mathrm{Tr}[a_{z,R}a_{z,R} - a_{\bar{z}}, Ra_{\bar{z}}, R + a_{\bar{z}}, La_{\bar{z}}, L - a_{z,L}a_{z,L}]\Big\}. \quad (45)$$

This is a local action quadratic in a. It is going to be useful to simplify the notation, so let us define

$$\begin{aligned}
L^a = L^a_M\partial X^M &= \sum(k_{(i)} - c_{G_i})\mathrm{Tr}(t_{a,R}g^{-1}\partial g),\\
-R^a = -R^a_M\partial X^M &= \sum(k_{(i)} - c_{G_i})\mathrm{Tr}(t_{a,L}\partial g g^{-1}),\\
M_{ab} &= \sum(k_{(i)} - c_{G_i})\mathrm{Tr}(t_{a,L}gt_{b,R}g^{-1} - t_{a,L}t_{b,L}),\\
\tilde{M}_{ab} &= \sum(k_{(i)} - c_{G_i})\mathrm{Tr}(t_{b,L}gt_{a,R}g^{-1} - t_{a,R}t_{b,R}) = M_{ba} + 2H_{ab},\\
&= \sum(c_{G_i} - c_H)\frac{1}{2}\mathrm{Tr}(t_{a,L}t_{b,L} + t_{a,R}t_{b,R}),\\
H_{ab} &= \sum(k_{(i)} - c_{G_i})\frac{1}{2}\mathrm{Tr}(t_{a,L}t_{b,L} - t_{a,R}t_{b,R}),\\
\mathrm{g} = g_{MN}\partial X^M\partial X^N &= \sum(k_{(i)} - c_{G_i})\mathrm{Tr}(g^{-1}\partial g g^{-1}\partial g).
\end{aligned} \quad (46)$$

In this notation the action can be written as

$$S_0^{\text{eff}} = -\frac{1}{4\pi}\int d\tau\{g - 2a_{\bar{z}}^a R_a - 2a_z^a L_a - 2a_{\bar{z}}^a a_z^b (M_{ab} - G_{ab} + H_{ab}) - a_z^a a_z^b (G_{ab} + H_{ab}) - a_{\bar{z}}^a a_{\bar{z}}^b (G_{ab} - H_{ab})\}. \quad (47)$$

Defining

$$z^i = \begin{pmatrix} a_{\bar{z}}^a \\ a_z^b \end{pmatrix}, \qquad B_i = \begin{pmatrix} R_a \\ L_b \end{pmatrix}^T,$$
$$A_{ij} = \begin{pmatrix} G - H & M - (G - H) \\ M^T - (G - H)^T & G + H \end{pmatrix} = \begin{pmatrix} G_- & M - G_- \\ \tilde{M} - G_+ & G_+ \end{pmatrix}, \quad (48)$$

where $G_+ = G + H$ and $G_- = G - H$, the action can be further simplified to

$$S_0^{\text{eff}} = -\frac{1}{4\pi}\int d\tau\{g - 2B_i z^i - z^i A_{ij} z^j\}. \quad (49)$$

Now we can complete the square, and get

$$S_0^{\text{eff}} = -\frac{1}{4\pi}\int d\tau\{g - A_{ij}(z + A^{-1}B)^i (z + A^{-1}B)^j + A^{kl}B_k B_l\}, \quad (50)$$

where $A^{kl} \equiv (A^{-1})_{kl}$.

The equations of motion for z (i.e., the equations of motion for the gauge fields a_z and $a\bar{z}$) are now easily read off,

$$\delta z \Rightarrow \qquad z^i = -A^{ik}B_k. \quad (51)$$

Inserting this into the action, we end up with

$$S_{\min}^{\text{eff}} = -\frac{1}{4\pi}\int d\tau[g + B_k A^{kl} B_l]. \quad (52)$$

To write out this explicitly we need to invert the matrix A_{ij}. If we write this inverted matrix as

$$A^{-1} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}, \quad (53)$$

then we can write

$$a_{\bar{z}}^a = -p_{ab}R_b - q_{ab}L_b, \quad (54)$$

$$a_z^a = -r_{ab}R_b - s_{ab}L_b, \quad (55)$$

and

$$S_{\min}^{\text{eff}} = -\frac{1}{4\pi}\int d\tau[g + R^a p_{ab} R^b + R^a(q_{ab} + r_{ba})L^b + L^a s_{ab} L^b] = -\frac{1}{4\pi}\int d\tau[g_{MN} + R_M^a p_{ab} R_N^b + R_M^a (q_{ab} + r_{ba}) L_N^b + L_M^a s_{ab} L_N^b]\partial X^M \partial X^N = -\frac{1}{4\pi}\int d\tau \frac{1}{2} C_{MN} \partial X^M \partial X^N. \quad (56)$$

So, finding the coefficients C_{MN} means finding the matrices p, q, r, s. Explicitly,

$$C_{MN} = 2[g_{MN} + R_M^a p_{ab} R_N^b + R_M^a (q_{ab} + r_{ba}) L_N^b + L_M^a s_{ab} L_N^b]. \quad (57)$$

Note that C_{MN} is not automatically symmetric.

Now let us recall the parameterization of the gauge groups. The generators of the gauge group $H = U(1)_A \times U(1)_B$, when acting on the $H \subset SL(2, \mathbb{R})$ part are

$$t_{A,L}^{(1)} = \frac{1}{2}\sigma_3, \qquad t_{B,L}^{(1)} = 0, \qquad t_{A,R}^{(1)} = -\frac{\delta}{2}\sigma_3, \qquad t_{B,R}^{(1)} = -\frac{\lambda}{2}\sigma_3. \quad (58)$$

The generators of H when acting on the $H \subset SU(2)$ part are

$$t_{A,L}^{(2)} = 0, \qquad t_{B,L}^{(2)} = 0, \qquad t_{A,R}^{(2)} = 0, \qquad t_{B,R}^{(2)} = -\frac{i}{2}\sigma_3. \quad (59)$$

We note once more that this gauging leaves the global $SU(2)_L$ symmetry untouched, and so it will survive as a global symmetry of the final model; the SU(2) invariance of Taub-NUT. Finally, introduce the generators of H when acting on the fermionic part, $H \subset SO(4)$:

$$t_{A,L}^{(f)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -Q_A & & \\ Q_A & 0 & & \\ & & 0 & P_A \\ & & -P_A & 0 \end{pmatrix}, \quad t_{A,R}^{(f)} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -\delta & & \\ \delta & 0 & & \\ & & 0 & 0 \\ & & 0 & 0 \end{pmatrix}, \quad t_{B,L}^{(f)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -Q_B & & \\ Q_B & 0 & & \\ & & 0 & P_B \\ & & -P_B & 0 \end{pmatrix},$$
$$t_{B,R}^{(f)} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -\lambda & & \\ \lambda & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{pmatrix}. \quad (60)$$

Note that the t_R are fixed by (0, 1) world-sheet supersymmetry, while in the t_L, the $Q_{A,B}$ and $P_{A,B}$ are chosen to cancel the anomaly *via* Eq. (13). The group elements are chosen as

$$g_1 == \frac{1}{\sqrt{2}}\begin{pmatrix} e^{\frac{t_+}{2}}(x^2+1)^{1/2} & e^{\frac{t_-}{2}}(x^2-1)^{1/2} \\ e^{-\frac{t_-}{2}}(x^2-1)^{1/2} & e^{-\frac{t_+}{2}}(x^2+1)^{1/2} \end{pmatrix} \in \mathrm{SL}(2,\mathbb{R}), \tag{61}$$

$$g_2 = e^{\frac{i\phi}{2}\sigma_3} e^{\frac{i\theta}{2}\sigma_2} e^{\frac{i\psi}{2}\sigma_3} \tag{62}$$

$$= \begin{pmatrix} e^{\frac{i\phi_+}{2}}\cos\frac{\theta}{2} & e^{\frac{i\phi_-}{2}}\sin\frac{\theta}{2} \\ -e^{-\frac{i\phi_-}{2}}\sin\frac{\theta}{2} & e^{-\frac{i\phi_+}{2}}\cos\frac{\theta}{2} \end{pmatrix} \in \mathrm{SU}(2), \tag{63}$$

$$g_f = \exp\left\{\begin{pmatrix} \Phi_1 \frac{i\sigma_2}{\sqrt{2}} & \\ & -\Phi_2 \frac{i\sigma_2}{\sqrt{2}} \end{pmatrix}\right\}$$

$$= \begin{pmatrix} \cos\frac{\Phi_1}{\sqrt{2}} & \sin\frac{\Phi_1}{\sqrt{2}} & & \\ -\sin\frac{\Phi_1}{\sqrt{2}} & \cos\frac{\Phi_1}{\sqrt{2}} & & \\ & & \cos\frac{\Phi_2}{\sqrt{2}} & -\sin\frac{\Phi_2}{\sqrt{2}} \\ & & \sin\frac{\Phi_2}{\sqrt{2}} & \cos\frac{\Phi_2}{\sqrt{2}} \end{pmatrix} \in \mathrm{SO}(4), \tag{64}$$

where $t_L, t_R, x \in \mathbb{R}$, $\theta \in (0, \pi)$, $\phi \in (0, 2\pi)$, $\psi \in (0, 4\pi)$, and Φ_1, and Φ_2 are 2π periodic. Also, $\phi_\pm = \phi \pm \psi$ and $t_\pm = t_L \pm t_R$. We have already gauge-fixed the fermionic sector.

To find the coefficients C_{MN} we now have to compute the group manifold metric g_{MN} and the vectors L_M and R_M. We also have to compute the matrix A_{ij} and find its inverse. This is all relatively straightforward and the details, involving a number of rather messy expressions, are left out. Having completed this task, we must worry about the effects of refermionization.

1. Refermionization and Back-Reaction on Metric

Assume that the local part of the action can be written (where we have reintroduced dependence on world-sheet space as well as time, which is necessary to deduce the B-field)

$$S = \frac{1}{2}\int d^2z C_{MN}\partial X^M \bar{\partial} X^N. \tag{65}$$

This expression can be rewritten as follows:

$$S = \frac{1}{2}\int d^2z C_{MN}\partial X^M \bar{\partial} X^N \tag{66}$$

$$= \frac{1}{2}\int d^2z[C_{\mu\nu}\partial X^\mu \bar{\partial} X^\nu + A^i_\mu(\partial X^\mu \bar{\partial}\Phi^i + \bar{\partial} X^\mu \partial\Phi^i) + B^i_\mu(\partial X^\mu \bar{\partial}\Phi^i - \bar{\partial} X^\mu \partial\Phi^i) + R_{ij}\frac{1}{2}(\partial\Phi^i \bar{\partial}\Phi^j + \bar{\partial}\Phi^i \partial\Phi^j) + F_{ij}\frac{1}{2}(\partial\Phi^i\bar{\partial}\Phi^j - \bar{\partial}\Phi^i\partial\Phi^j)], \tag{67}$$

$$= \frac{1}{2}\int d^2z[(C_{\mu\nu} - R^{ij}A^i_\mu A^j_\nu)\partial X^\mu \bar{\partial} X^\nu + R_{ij}(\partial\Phi^i + R^{ik}A^k_\mu \partial X^\mu)(\bar{\partial}\Phi^j + R^{jl}A^l_\nu \bar{\partial} X^\nu) B^i_\mu(\partial X^\mu \bar{\partial}\Phi^i - \bar{\partial} X^\mu \partial\Phi^i) + F_{ij}\frac{1}{2}(\partial\Phi^i\bar{\partial}\Phi^j - \bar{\partial}\Phi^i\partial\Phi^j)], \tag{68}$$

where

$$A^i_\mu = C_{\mu i} + C_{i\mu}, \quad B^i_\mu = C_{\mu i} - C_{i\mu}, \quad R_{ij} = C_{(ij)}, \quad F_{ij} = C_{[ij]}. \tag{69}$$

Note that in the zero-mode sector where we keep only symmetric terms, which means $F_{ij} = 0$ and $B^i_\mu = 0$. This is (almost) the form required for refermionization, and we can read off the metric from the first term. Before refermionisation, we must rescale the Φs in the action (68) that the term $R_{ij}\partial\Phi^i\bar{\partial}\Phi^j$ becomes $\delta_{ij}\partial\tilde{\Phi}^i\bar{\partial}\tilde{\Phi}^j$. This is done by

$$\Phi^i = U^i_j\tilde{\Phi}^j, \tag{70}$$

with $R_{ij}U^i_k U^j_l = \delta_{kl}$. This corrects the A^i_μ to $\mathcal{A}^i_\mu = R^{ij}A^j_\mu$, where $R^{ij} = (R^{-1})_{ij}$. The space-time metric is then

$$G_{\mu\nu} = C_{(\mu\nu)} - R^{ij}A^i_{(\mu}A^j_{\nu)} = G^0_{\mu\nu} - \mathcal{A}^i_\mu\mathcal{A}^j_\nu. \tag{71}$$

Carrying out the computation, we find that the final expression for the exact metric simplifies in a remarkable way to the following [using Eq. (14) we write $k_1 = k$, $k_2 = k - 4$]:

$$ds^2 = G_{\mu\nu}dX^\mu dX^\nu = (k-2)\left\{\frac{dx^2}{x^2-1} - \frac{x^2-1}{D(x)}(dt + 2\lambda A^M_\phi d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2\right\}, \tag{72}$$

where

$$D(x) = (x+\delta)^2 - \frac{4}{k+2}(x^2-1), \tag{73}$$

and $2A^M_\phi = \pm 1 - \cos\theta$ is a Dirac monopole connection where $\pm$ refers to the N(S) Pole on the S^2. The ± 1 can be gauged away by, e.g., a shift of t to match the form given in Sec. I. The dilaton is generated by the effects of two Jacobians. One comes from the determinant, $\det A$, arising from integrating out the gauge fields, but there is another contribution coming from the change of variables from Φ to $\tilde{\Phi}$. That Jacobian is

$$\left|\frac{\partial\Phi}{\partial\tilde{\Phi}}\right| = \det U = (\det R)^{-1/2}. \tag{74}$$

This results in [100]:

$$e^{2\Phi} = (\det A)^{-1/2}(\det R)^{-1/2}, \tag{75}$$

where the determinants can be written as follows. Define

$$p = k - 2 + 2P_B, \quad q = (k+2)\delta + 2Q_A, \quad r = (k+2)\lambda + 2Q_B. \tag{76}$$

Then

$$\det A = \Delta(x) = [(k-2)px - (2P_A r - pq)]^2 + 4(r^2 - p^2), \tag{77}$$

and

$$\det R = 4(k+2)(k-2)^3 \frac{D(x)}{\Delta(x)}. \tag{78}$$

The result is that the exact dilaton is

$$\Phi - \Phi_0 = -\tfrac{1}{4}\ln[D(x)], \tag{79}$$

where we have absorbed a nonessential constant into the definition of Φ_0. The expressions for the exact fields $B_{\mu\nu}, A^i_\mu$ are somewhat involved, but straightforward to read off. We will not list them here, as we will not need them in what follows.

As a useful check on our procedure, it is worth noting that the large k limit gives the expressions originally written in Ref. [19]. In this limit, we get $D \to (\delta + x)^2$, and the metric becomes that given in Eq. (7), and the dilaton becomes

$$\Phi - \Phi_0 \to -\tfrac{1}{2}\ln(x + \delta). \tag{80}$$

G. Properties of the Exact Metric

As already stated in the previous section, the final result for the exact space-time metric is (after a trivial shift in t)

$$ds^2 = (k-2)\left[\frac{dx^2}{x^2-1} + F(x)(dt - \lambda\cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2\right],$$

where $$F(x) = -\frac{x^2-1}{D(x)} = -\left(\frac{(x+\delta)^2}{x^2-1} - \frac{4}{k+2}\right)^{-1}. \tag{81}$$

This is a pleasingly simple form to result from such an involved computation. In fact, its relation to the leading order result is reminiscent in form to the relation between the leading order and exact results for the black hole SL(2, ℝ)/U(1) model [89,90].

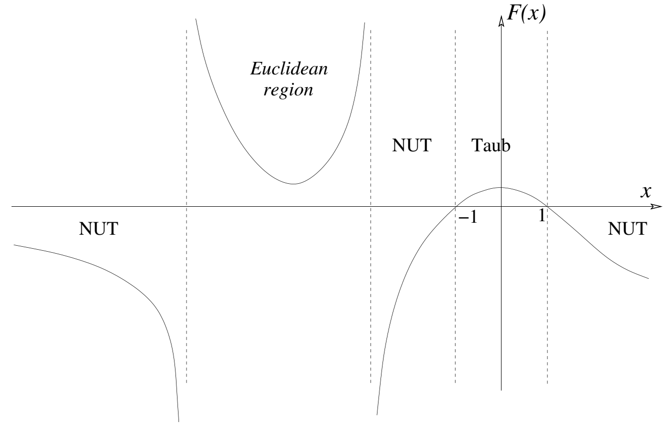

FIG. 3. The various regions in the stringy Taub-NUT geometry for arbitrary k, with all $1/k$ corrections included. Compare to the leading order result in Fig. 1. Note that the singularity splits in order to incorporate a finite sized region of Euclidean signature in the second NUT region.

It is interesting to sketch the behavior of $G_{tt} = F(x)$, as it contains the answer to our original questions about the fate of the Taub and NUT regions of the space-time once the contributions of the stringy physics are included. This result is plotted in Fig. 3, and it should be contrasted with Fig. 1.

Several remarks are in order. The first is that the Taub and NUT regions, although modified somewhat, survive to all orders. The second is that the local structure of the chronology horizons separating these regions is completely unaffected by the stringy corrections! $F(x)$ still vanishes at $x = \pm 1$ and furthermore for $x = 1 - \tau$ where τ is small, the metric of the (τ, ξ) space (the space over each point of the S^2) becomes

$$ds^2 = (k-2)\left[-(2\tau)^{-1}d\tau^2 + \frac{2\tau}{(1+\delta)^2}d\xi^2\right], \tag{82}$$

which is again of Misner form.

Notice that the singularity we observed in $F(x)$ (and the space-time) has now split into two. Recalling the definition of $D(x)$ given in Eq. (73), we can write the Ricci scalar as

$$R = -\frac{1}{2(k-2)D^2}[2D(x^2-1)D'' - 3(x^2-1)(D')^2 + \lambda^2(x^2-1)D + 6xDD'], \tag{83}$$

(where a prime means d/dx). R diverges if and only if $D(x) = 0$. These singularities are located at

$$x_\pm = \frac{-\delta \pm \sqrt{a^2 + a(\delta^2 - 1)}}{(1-a)}, \quad a = \frac{4}{k+2}, \tag{84}$$

and the region in between them has Euclidean signature. Such a region was noticed in Ref. [101] in the context of the exact metric for the SL(2, ℝ)/U(1) coset giving the

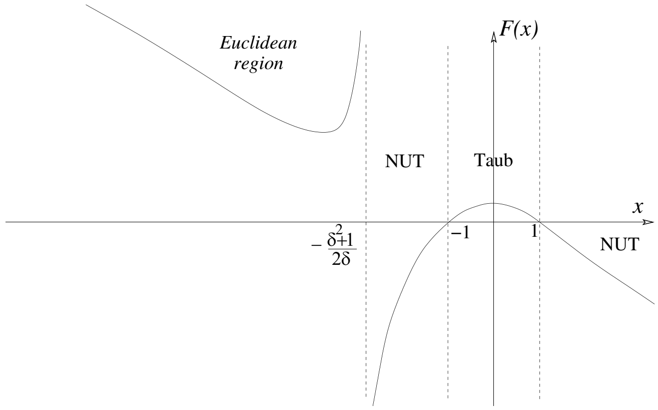

FIG. 4. The various regions in the stringy Taub-NUT geometry for the smallest value of k possible. This is the "most stringy" geometry. Compare to the leading order result in Fig. 1 and the intermediate k result in Fig. 3. The Euclidean region has grown and occupied the entire region to the left, making the second nut region of finite extent.

two-dimensional black hole. This region remains entirely within the second NUT region, however, and never approaches the Misner horizons. Its size goes as $1/(k-2)$. The model only seems to make sense for $k > 2$, of course, and it interesting to note that the limiting behavior of this metric as $k \to 2^+$ is that the Euclidean region grows until it fills the entire left hand side of the sketch (see Fig. 4), with one singularity at $x = -(\delta^2 + 1)/(2\delta)$, and the other, when last seen, was moving off to $x = -\infty$.

IV. DISCUSSION

Our goal was to identify a stringy laboratory for the study of a number of issues of interest, which allows a controlled study of various physical phenomena. Closed timelike curves are very common in general relativity, but the theory is silent about their physical role in a complete theory of gravity. They can appear after a cosmology passes through a certain type of spacelike "big crunch" singularity, and it is natural to wonder if the full theory somehow modifies the geometry in a way which obstructs this process of formation, realizing the so-called chronology protection conjecture [6]. The model upon which a great deal of the study within general relativity has been focused is the Taub-NUT space-time (or local parts of it). Quite satisfyingly, this is precisely the model that we study here, furthering earlier work which showed how to embed it into string theory in a way which allows a complete definition in terms of conformal field theory.

The study of the model we performed here was to go beyond the low-energy truncation and compute the all orders in α' geometry, thereby including the effects of the entire string spectrum on the background. Our embedding (into heterotic string theory) was chosen so as to permit such corrections to occur, at least in principle. Somewhat surprisingly (perhaps) we found that the key features of the Taub-NUT geometry persist to all orders. This includes the fact that the volume of the universe in the Taub cosmology vanishes as a circle shrinks to zero size, at the junction (described by Misner space) where the CTCs first appear. There is no disconnection of the Taub region from the NUT regions containing the CTCs, to all orders in α'. Note that the strength of the string coupling near the junctions is not particularly remarkable, and so an appeal to severe corrections purely due to string loops may not help modify the geometry further.

We have therefore ruled out a large class of possible modification to the geometry which could have destroyed the chronology horizons and prevented the formation of the CTC regions (from the point of view of someone starting in the cosmological Taub region). As remarked upon in the introduction, there is still the possibility that there is an instability of the *full* geometry to back-reaction by probe particles or strings. A large class of such effects are likely missed by our all orders computation of the metric. There are studies of Misner space in various dimensions (in its orbifold representations) that signal such an instability [21,23,24], and the fate of the chronology horizons embedded in our geometry should be examined in the light of those studies. The nature of the space-time in which they are embedded is important, however, and so it seems that the relevant geometry to study such back-reaction effects is the fully corrected geometry we have derived here, since it takes into account the full α' effects.

Quantum effects may well be important even though the string coupling is not strong at the chronology horizons, and even if there are no (as we have seen here) modifications due to α' corrections. Radically new physics can happen if there are the right sort of special (for example, massless) states arising in the theory there together with (crucially) certain types of new physics. Strings wrapped on the t-circle are candidate such states. Following these states could shed new light on the validity of the geometry if they are accompanied by the appropriate physics, such as in the mechanism of Ref. [102]. Such probe heterotic strings are hard to study in the sigma model approach, but it would be interesting to undergo such an investigation. The study of probes directly in the full conformal field theory (i.e., without direct reference to the geometry) may well be the most efficient way to proceed.

Another (less often considered) possibility is that the result of this paper is a sign that the theory is telling us that it is perfectly well-defined in this geometry. The conformal field theory is (at face value) well-defined, and there are no obvious signs of a pathology. Perhaps string theory is able to make sense of all of the features of Taub-NUT. For example, the shrinking of the spatial circle away to zero size at the big bang or big crunch might not produce a pathology of the conformal field theory even though there might be massless states appearing from wrapped heterotic strings. They might simply be incorporated into the physics in a way that does not invalidate the geometry: The physics, as defined by the world-sheet model, would then carry on perfectly sensibly through that region. This would mean that would be another geometry that a dual heterotic string sees which is perfectly smooth through this region. It would be interesting to construct this geometry[9].

In this scenario, if we accept that the conformal field theory is telling us that the stringy physics is well behaved as it goes through from the Taub region to the NUT region, we have to face the possibility that the CTCs contained in the NUT regions might well be acceptable, and part of the full physics as well.

While it is perhaps too early to conclude this with certainty, it is worth noting that most objections that are raised about physics with CTCs are usually ones based on paradoxes arrived at using macroscopic and manifestly classical reasoning, or reasoning based on our very limited understanding of quantum theory outside of situations where there is an asymptotic space-time region to which we make reference. Some CTCs fall outside of those realms, opening up new possibilities. We must recall that time, just like space, is supposed to arrive in our physics as an approximate object, having a more fundamental quantum mechanical description in our theory of quantum gravity. The ubiquity of CTCs in theories of gravity might be a sign that (appropriately attended to) they are no more harmful than closed spatial circles. Rather than try to discard CTCs, we might also keep in mind the possibility that they might play a natural role in the full theory, when we properly include quantum mechanics. Here, we saw them remain naturally adjoined to a toy cosmology, surviving all α' corrections. This is just the sort of scenario where CTCs might play a role in nature; a natural way to render meaningless the usual questions about the lifetime of the universe prior to the ''big bang'' is to have the big bang phase adjoined to a region with CTCs[10]. This is an amusing alternative to the usual scenarios, and may be naturally realized within string theory, or its fully nonperturbative successor.

[9]The right-handed world-sheet parity flip which generates a dual geometry is no longer achievable by axial-vector duality as in simpler cases such as the $SL(2,\mathbb{R})/U(1)$ black hole [89,103]. It only works for $\delta = \pm 1$, $\lambda = 0$. Here, it is natural to explore whether $\delta \rightarrow -\delta$ combined with other actions might generate it, but a fiberwise duality rather like that which relates [104,105] an NS5-brane to an ALE space might be more appropriate.

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[10]Although it is in the very different context of eternal inflation, the role of CTCs in cosmology has been speculated about before [106].

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