PHYSICAL REVIEW D **70,** 126009 (2004)

Heterotic string compactified on half-flat manifolds

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(Received 7 September 2004; published 9 December 2004)

We study the effective action of the heterotic string compactified on particular half-flat manifolds which arise in the context of mirror symmetry with Neveu-Schwarz–Neveu-Schwarz flux. We explicitly derive the superpotential and Kähler potential at lowest order in α' by a reduction of the bosonic action. The superpotential contains new terms depending on the Kähler moduli which originate from the intrinsic geometrical flux of the half-flat manifolds. A generalized Gukov formula, valid for all manifolds with SU(3) structure, is derived from the gravitino mass term. For the half-flat manifolds it leads to a superpotential in agreement with our explicit bosonic calculation. We also discuss the inclusion of gauge fields.

DOI: 10.1103/PhysRevD.70.126009 PACS numbers: 11.25.Mj

I. INTRODUCTION

The stabilization of moduli remains one of the central problems when trying to relate string theory to low-energy particle physics. Recently, flux compactifications were intensively studied as a method to tackle this problem, mostly in the context of type II strings or M-theory [1–39]. The analysis is particularly straightforward within the context of type IIB strings on Calabi-Yau spaces where a combination of Neveu-Schwarz–Neveu-Schwarz (NS-NS) and Ramond-Ramond (RR) flux can be used to fix all complex structure moduli as well as the axion dilaton [8]. If all moduli are successfully stabilized in such models [40] the radius of the internal space is usually not much larger than the string scale. This excludes very large additional dimensions and a low string scale and means that low-energy supersymmetry remains as the only known option to stabilize the electroweak scale. Explicit examples for type II brane models can be found, for example, in Refs. [41–49]. The construction of phenomenologically attractive *supersymmetric* type II brane models has so far proven difficult, however, see Ref. [50].

The situation is somewhat reversed in the context of heterotic string models. It has been known for a long time that supersymmetric models with broadly the right phenomenological properties can be obtained easily and in large numbers [51,52]. NS-NS flux in heterotic compactifications has been introduced some time ago [53–57] and there are also a number of more recent discussions [58–62] of the subject. However, discarding the $E_8 \times E_8$ or SO(32) gauge fields whose vacuum expectation values are tied to the curvature via the Bianchi identity, the NS-NS three-form field strength is the only antisymmetric tensor field in heterotic theories which implies an apparent lesser degree of flexibility in fixing moduli through flux, as compared with type II theories. In particular, no even-degree form field strength is available whose flux could fix the Kähler moduli.

In this paper, we are going to address this problem by considering the heterotic string on particular six-dimensional manifolds with SU(3) structure and nonvanishing (intrinsic) torsion. We will see that these manifolds encode even-degree flux "geometrically" and we will compute the resulting Kähler moduli superpotential explicitly. The existence of these manifolds is suggested by type II mirror symmetry with NS-NS flux, as has been argued in Refs. [17,20]. This conjecture was generalized and further evidence was provided for it in Refs. [63,64]. Explicit noncompact examples were constructed in Refs. [65,66]. Also, consistency of embedding such backgrounds in string/M-theory was discussed in Ref. [67].

In Ref. [17], it was proposed that type IIB (IIA) on a Calabi-Yau three-fold with NS-NS flux is mirror-symmetric to IIA (IIB) on a particular class of six-dimensional half-flat manifolds with SU(3) structure. In the following, we will refer to these manifolds as half-flat mirror manifolds. Under the mirror map, the original odd-degree NS-NS flux, which generates a superpotential for the complex structure moduli, is mapped to even-degree geometrical flux of the half-flat mirror manifolds, which generates a superpotential for the Kähler moduli. In this paper, we are not interested in type II mirror symmetry by itself but merely as a means of "defining" the half-flat mirror manifolds. Our goal is to consider the heterotic string on the so-defined manifolds with torsion.

The heterotic string on non-Kähler manifolds was already discussed in a number of papers [68–74] and the supersymmetric solutions were classified in terms of the five torsion classes of manifolds with SU(3) structure. However, the lack of knowledge of internal properties (such as the moduli space) of general manifolds with SU(3) structure makes it difficult to derive the effective

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action for such theories and, so far, general properties of the superpotentials have been discussed [69–71].

In comparison, we can see a number of advantages in our approach. First of all, type II mirror symmetry strongly suggests the existence of the half-flat mirror manifolds and it imposes very strong constraints on them. In fact, mirror symmetry provides us with a concrete set of relations describing half-flat mirror manifolds, which allows the calculation of much of the low-energy effective action. Their mirror symmetry origin implies that a half-flat mirror manifold should exist for each Calabi-Yau three-fold with a mirror and for each set of NS flux parameters. Hence, we are dealing with a large class of manifolds which is closely linked to Calabi-Yau three-folds. This will hopefully lead to models preserving many of the attractive features of heterotic Calabi-Yau compactifications while at the same time enhancing the flexibility for moduli stabilization through flux.

In this paper, we will mainly focus on zeroth order in α', that is, on the gravitational sector of the heterotic string and we will discuss only some aspects of including gauge fields. The full gauge field sector will be included in a forthcoming publication [75]. Our main aim is to derive the effective four-dimensional $N = 1$ supergravity for the heterotic string on half-flat mirror manifolds to this order in α'. In particular, we will compute the superpotential which will be done in two largely independent ways, namely, from the bosonic action and the gravitino mass term. We also obtain a general Gukov-type formula for the superpotential which we expect to hold for all heterotic compactifications on manifolds with SU(3) structure and to first order in α'.

The outline of the paper is as follows. In the next section, we present a brief review of the ten-dimensional action of the heterotic string and of the half-flat mirror manifolds on which we are going to carry out the dimensional reduction. Section III reduces the bosonic part of the action on half-flat mirror manifolds at zeroth order in α', at first without and eventually including NS-NS flux. In Sec. IV, we present an alternative derivation of the superpotential from fermionic terms in the action. Based on the gravitino mass term, we first derive a Gukov-type formula for the heterotic string on SU(3) structure manifolds and then show that, specialized to half-flat mirror manifolds, it reproduces the previous result for the superpotential. Section V discusses some steps necessary to include gauge fields and we conclude in Sec. VI. Two appendices present some relevant results in special geometry and the calculation of the potential from the superpotential in the general case.

II. REVIEW OF BACKGROUND MATERIAL

In this section, we present some background material in order to set up our notation and conventions. First, we review the ten-dimensional effective action of the heterotic string [52] which is the action we would like to compactify to four dimensions. Then we describe the half-flat mirror manifolds [17] on which we are going to carry out the dimensional reduction.

A. Ten-dimensional effective action for the heterotic string

The ten-dimensional effective action for the heterotic string is given, to leading order in α', by ten-dimensional $N = 1$ supergravity coupled to ten-dimensional super-Yang-Mills theory with gauge group $E_8 \times E_8$ or SO(32). In this paper, we will focus on the $E_8 \times E_8$ case for definiteness but most of our considerations will directly apply to the SO(32) case as well. Ten-dimensional coordinates are denoted by (x^M), labeled by curved indices $M, N, \ldots = 0, \ldots, 9$.

The ten-dimensional $N = 1$ supergravity multiplet consists of the metric $\hat{g}_{MN}$, the dilaton $\hat{\phi}$, the NS-NS two-form $\hat{B}_{MN}$ and their fermionic partners, the gravitino $\hat{\Psi}_M$ and the dilatino $\hat{\lambda}$, both ten-dimensional Majorana-Weyl spinors which we take to be of positive chirality. Here and in the following a hat denotes a ten-dimensional quantity. To lowest (zeroth) order in the α' expansion the bosonic part of the effective action is given by [52]

$$S_{0,\text{bosonic}} = -\frac{1}{2\kappa_{10}^2}\int_{M_{10}} e^{-2\hat{\phi}}\Big[\hat{R}\star\mathbf{1} - 4d\hat{\phi}\wedge\star d\hat{\phi} + \frac{1}{2}\hat{H}\wedge\star\hat{H}\Big], \tag{2.1}$$

where $\hat{H} = d\hat{B}$ is the three-form field strength of $\hat{B}$ and $\hat{R}$ is the Riemann curvature scalar. We will later find it useful to consider some of the fermionic terms. To zeroth order in α' they read

$$S_{0,\text{fermionic}} = -\frac{1}{2}\int_{M_{10}} d^{10}x\sqrt{-\hat{g}}e^{-2\hat{\phi}}\Big\{\bar{\hat{\Psi}}_M\Gamma^{MNP}D_N\hat{\Psi}_P - \frac{1}{24}(\bar{\hat{\Psi}}_M\Gamma^{MNPQR}\hat{\Psi}_R + 6e^{-\hat{\phi}}\bar{\hat{\Psi}}^N\Gamma^P\hat{\Psi}^Q)\hat{H}_{NPQ} + \ldots\Big\}, \tag{2.2}$$

where the dots stand for additional four-fermion terms and terms which involve the dilatino. Here, Γ_M are the ten-dimensional gamma matrices which are taken to be real, conjugation is defined as $\bar{\psi} = \psi^\dagger\Gamma_0$ for a spinor ψ, and multi-indexed Γ symbols denote antisymmetrized products of gamma matrices with unit norm, as usual. For convenience we have chosen the overall dilaton factor to be the same as in the bosonic part of the action by appropriately rescaling the gravitino field.

The ten-dimensional Yang-Mills multiplet consists of the gauge field $\hat{A}_M$, with field strength $\hat{F}_{MN}$, and its superpartner, the gaugino, both in the adjoint $E_8 \times E_8$ [or SO(32)]. The kinetic terms for these fields along with

$\hat{R}^2$ terms and additional four-fermion terms involving the gauginos arise at order α'. The bosonic among those terms are given by

$$S_{1,\mathrm{bosonic}} = -\frac{\alpha'}{16\kappa_{10}^2}\int_{M_{10}} d^{10}x\sqrt{-\hat{g}}e^{-2\hat{\phi}}\{\mathrm{Tr}(\hat{F}^2) - \mathrm{tr}(\tilde{R}^2)\} \tag{2.3}$$

where $\mathrm{tr}(\tilde{R}^2)$ really stands for the Gauss-Bonnet combination. The curvature two-form $\tilde{R}$ is computed in terms of the modified connection

$$\tilde{\omega}_{IJ}{}^K = \omega_{MN}{}^P + \frac{1}{2}\hat{H}_{MN}{}^P, \tag{2.4}$$

where ω is the Levi-Civita connection. The other modification to the action at this order appears in the definition of the field strength $\hat{H}$ which now becomes

$$\hat{H} = d\hat{B} + \frac{\alpha'}{4}(\omega_{\mathrm{L}} - \omega_{\mathrm{YM}}). \tag{2.5}$$

Here, ω_{L} and ω_{YM} are the usual Lorentz and Yang-Mills Chern-Simons three-forms defined by

$$\omega_{\mathrm{L}} = \mathrm{tr}\Big(\tilde{R}\wedge\tilde{\omega} - \frac{1}{3}\tilde{\omega}\wedge\tilde{\omega}\wedge\tilde{\omega}\Big) \tag{2.6}$$

$$\omega_{\mathrm{YM}} = \mathrm{Tr}\Big(\hat{F}\wedge\hat{A} - \frac{1}{3}\hat{A}\wedge\hat{A}\wedge\hat{A}\Big). \tag{2.7}$$

The trace Tr denotes $1/30$ of the trace in the adjoint for $E_8\times E_8$ or the trace in the fundamental for SO(32), as usual. These are the only corrections to the action at order α'. Further terms appear at order α'^2 which, however, will not concern us here. In fact, throughout most of the paper we will focus on the leading, zeroth order in α' for which we present a complete analysis. In addition, we discuss some aspects related to the gauge fields.

B. Half-flat manifolds

We will now briefly describe the particular six-dimensional manifolds on which we are going to carry out the reduction of the ten-dimensional heterotic effective action. In general terms, these manifolds arise as the mirrors of Calabi-Yau manifolds with (a particular type of) NS-NS flux, as constructed in Ref. [17]. Before we get to this specific definition in terms of mirror symmetry it is useful to review the main properties of the general manifolds with SU(3) structure and their classification in terms of torsion classes following [76] and then specialize to the case of half-flat mirror manifolds.

A six-dimensional manifold is said to have SU(3) structure if it admits a globally defined spinor[1] which we denote η. From a physical point of view this is the most practical definition as this globally defined spinor ensures that the action obtained by compactifying on such manifolds preserves some supersymmetry.

The geometric properties of manifolds with SU(3) structure are better described in terms of two invariant forms J and Ω which can be defined as bilinears in the spinor η as follows

$$J_{mn} = -i\eta^\dagger\gamma_{mn}\eta, \qquad \Omega_{mnp} = -\frac{i||\Omega||}{\sqrt{8}}\eta^\dagger\gamma_{mnp}\eta^*. \tag{2.8}$$

Here, γ_m, with indices $m, n\ldots = 5,\ldots,9$ are six-dimensional Euclidean gamma matrices which are chosen to be purely imaginary. As before, multiple indices denote antisymmetrization. Note that the normalization of Ω is different from what can be found in the literature and was chosen in order to agree with the usual moduli space conventions. Indeed, it is easy to check using gamma matrix algebra and Fierz identities that

$$\Omega_{mnp}\bar{\Omega}^{mnp} = 3!||\Omega||^2, \tag{2.9}$$

provided the spinor η satisfies $\eta^\dagger\eta = 1$.

Manifolds with SU(3) structure can be classified by their intrinsic torsion and it will be useful to briefly review this. For a more complete account see, for example, Ref. [76]. It is well known that the SU(3) structure induces a metric on the manifold [77]. The Levi-Civita connection associated to this metric violates in general the structure, but there always exists a connection which we denote $\nabla^{(\mathrm{T})}$, which does preserve it. In other words, denoting any of the invariant objects η, J, or Ω by ξ we have

$$\nabla^{(\mathrm{T})}\xi = 0. \tag{2.10}$$

Any connection, and in particular $\nabla^{(\mathrm{T})}$ defined above, can be expressed in terms of the Levi-Civita connection ∇ as

$$\nabla^{(\mathrm{T})}_m = \nabla_m + \kappa_m, \tag{2.11}$$

where κ_m are matrices whose entries constitute the contorsion tensor κ_{mnp}. Unlike the Levi-Civita connection, this connection has a torsion $T_{mnp} = \kappa_{[mn]p}$. Note that the contorsion tensor is antisymmetric in its last two indices and can be thought of as a one-form taking values in so(6), the Lie algebra of SO(6). Thus, we can decompose it under the SU(3) structure group as

$$\kappa_m = \kappa_m^0 + \kappa_m^{\mathrm{su}(3)}, \tag{2.12}$$

where $\kappa_m^{\mathrm{su}(3)}$ takes values in $\mathrm{su}(3) = \mathbf{8}$, the Lie algebra of SU(3), and κ_m^0 takes values in the complement $\mathrm{su}(3)^\perp = \mathbf{1}\oplus\mathbf{3}\oplus\bar{\mathbf{3}}$ of su(3) within so(6). The action of $\kappa^{\mathrm{su}(3)}$ on the SU(3) invariant tensors ξ vanishes and, hence, the left-hand side of the compatibility condition (2.10) only depends on κ^0 which is called the "intrinsic contorsion." This intrinsic contorsion can be used to classify SU(3)

[1]For definiteness we will take this spinor to be Weyl, but one can as well work with Majorana spinors.

structures and it is useful, in this context, to analyze its SU(3) representation content. From what has been said above, the intrinsic contorsion κ^0 is an element of the SU(3) representation

$$(\mathbf{3}\oplus\bar{\mathbf{3}})\otimes(\mathbf{1}\oplus\mathbf{3}\oplus\bar{\mathbf{3}}) = (\mathbf{1}\oplus\mathbf{1})\oplus(\mathbf{8}\oplus\mathbf{8})\oplus(\mathbf{6}\oplus\bar{\mathbf{6}}) \oplus(\mathbf{3}\oplus\bar{\mathbf{3}})\oplus(\mathbf{3}\oplus\bar{\mathbf{3}})'. \tag{2.13}$$

The five terms on the right-hand side of this relation correspond to the five torsion classes [76], denoted by $\mathcal{W}_1, \ldots, \mathcal{W}_5$, of six-dimensional manifolds with SU(3) structure. These classes are a useful tool to characterize the intrinsic torsion and the associated SU(3) structure. The intrinsic torsion can also be read off from the exterior derivatives dJ and $d\Omega$ since Eq. (2.10) implies that

$$(dJ)_{mnp} = 6\kappa^0_{[mn}{}^r J_{r|p]} \tag{2.14}$$

$$(d\Omega)_{mnpq} = 12\kappa^0_{[mn}{}^r \Omega_{r|pq]}. \tag{2.15}$$

Therefore, a practical way to specify the intrinsic torsion of an SU(3) structure is to explicitly write down expressions for dJ and $d\Omega$. As can be seen from Eqs. (2.14) and (2.15), these expressions contain information about various of the five torsion classes, namely,

$$dJ \in \mathcal{W}_1 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4, \qquad d\Omega \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_5. \tag{2.16}$$

It will turn out that the first torsion class $\mathcal{W}_1$ plays a special role in the case we address in this paper. Thus we define the corresponding contorsion to be

$$\kappa_{mnp}|_{\mathcal{W}_1} = \kappa_1\Omega_{mnp} + \bar{\kappa}_1\bar{\Omega}_{mnp}, \tag{2.17}$$

where κ_1 is given by

$$\kappa_1 = \frac{i\int\sqrt{g}(dJ)_{mnp}\bar{\Omega}^{mnp}}{6\int\sqrt{g}\Omega_{mnp}\bar{\Omega}^{mnp}}, \tag{2.18}$$

and we have used Eq. (2.14).

In this paper, we are interested in a more special class of manifolds with SU(3) structure, namely, half-flat manifolds. They are defined as six-dimensional SU(3) structure manifolds with the invariant forms J and Ω satisfying

$$d\Omega_- = 0, \qquad dJ\wedge J = 0, \tag{2.19}$$

where Ω_- is the imaginary part of Ω. Comparison with Eq. (2.16) reveals [76] that these conditions are equivalent to vanishing torsion classes $\mathcal{W}_1^-$, $\mathcal{W}_2^-$ (these being the imaginary parts of the classes $\mathcal{W}_1$ and $\mathcal{W}_2$), $\mathcal{W}_4$, and $\mathcal{W}_5$.

The specific half-flat manifolds considered in this paper arise in the context of mirror symmetry with NS-NS flux [17]. Let us briefly review how this comes about. Consider a mirror pair X and Y of Calabi-Yau manifolds and introduce a standard symplectic basis $(\tilde{\alpha}_I, \tilde{\beta}^I)$, where $I = 0, \ldots h^{2,1}(X)$, of the third cohomology on X. We start with, say, type IIB on X in the presence of NS-NS flux $\tilde{H} = e_i\tilde{\beta}^i$, where $i = 1, \ldots, h^{2,1}(X)$ and $\zeta = (e_i)$ are real flux parameters. Is there any compactification on the IIA side which is mirror-symmetric to this configuration? Evidence for this has been presented in Refs. [17,20] and it has been shown that the mirror configuration is given by IIA on a half-flat manifold $\hat{Y}_\zeta$, closely related to the original mirror Calabi-Yau Y, but without NS-NS flux. Moreover, it has been argued that the moduli spaces of metrics on Y and $\hat{Y}_\zeta$ are identical for all values of the flux ζ.

Let us now describe the structure of these half-flat mirror manifolds $\hat{Y}_\zeta$ in more detail. Matching of the moduli spaces of metrics, together with the correspondence between metrics and SU(3) structures implies that the forms J and Ω have expansions

$$J = v^i\omega_i \tag{2.20}$$

$$\Omega = z^A\alpha_A - \mathcal{G}_A\beta^A, \tag{2.21}$$

similar to the ones on the associated Calabi-Yau manifold Y. Here (ω_i), where $i, j, \ldots = 1, \ldots, h^{1,1}(Y)$, are (1, 1)-forms and (α^A, β_A), where $A, B, \ldots = 0, \ldots, h^{2,1}(Y)$, are three-forms, suitable for the expansion of J and Ω while the coefficients v^i and z^A are the analog of Kähler and complex structure moduli. For simplicity, we will continue to use Calabi-Yau terminology and refer to Kähler and complex structure moduli and moduli spaces, although the manifolds $\hat{Y}_\zeta$ are generally neither Kähler nor complex. The three-forms (α^A, β_A) satisfy the standard normalizations

$$\int_{\hat{Y}_\zeta}\alpha_A\wedge\beta^B = \delta_A^B, \qquad \int_{\hat{Y}_\zeta}\alpha_A\wedge\alpha_B = \int_{\hat{Y}_\zeta}\beta^A\wedge\beta^B = 0, \tag{2.22}$$

and we also introduce dual four-forms $\tilde{\omega}^i$ such that

$$\int_{\hat{Y}_\zeta}\omega_i\wedge\tilde{\omega}^j = \delta_i^j. \tag{2.23}$$

So far, all relations are identical to the corresponding Calabi-Yau ones. However, unlike in the Calabi-Yau case the forms ω_i and (α_A, β^B) are not all closed and, in particular, do not form a basis of the second and third cohomology. Rather, as shown in Ref. [17], mirror symmetry requires them to satisfy the differential relations

$$d\alpha_0 = e_i\tilde{\omega}^i, \qquad d\alpha_a = 0, \qquad d\beta^A = 0, \qquad d\omega_i = e_i\beta^0, \qquad d\tilde{\omega}^i = 0, \tag{2.24}$$

where we have introduced indices $a, b, \ldots = 1, \ldots, h^{2,1}(Y)$. The real parameters e_i are precisely the NS-NS flux parameters on the mirror side mentioned earlier and they encode the degree to which the half-flat mirror manifold $\hat{Y}_\zeta$ "deviates" from the associated Calabi-Yau manifold Y. Using the above relations together with the expansions (2.20) and (2.21) for J and Ω it is easy to show that

$$dJ = v^i e_i \beta^0 \tag{2.25}$$

$$d\Omega = e_i \tilde{\omega}^i. \tag{2.26}$$

As discussed, the right-hand sides of these relations specifiy the intrinsic torsion and the SU(3) structure of the manifolds $\hat{Y}_\zeta$. Comparison with the conditions (2.19) shows that they are indeed half-flat manifolds.

The point of view taken in this paper is that mirror symmetry with NS-NS flux provides us with a practical "definition" of the half-flat manifolds $\hat{Y}_\zeta$ as well as with a set of relations which allows us to deal with them. The evidence for mirror symmetry with NS-NS flux was obtained in the context of IIA and IIB supergravity [17] and one should, hence, expect the above relations to be valid only in the large complex structure limit. We will, therefore, work in this limit, in addition to the large radius limit in Kähler moduli space which is mandatory whenever supergravity theories are considered. In this paper we are not interested primarily in mirror symmetry itself but in using the so-defined manifolds in the context of the heterotic string. We can see a number of advantages in this method compared to, for example, working with the heterotic string on general manifolds of SU(3) structure or even general half-flat manifolds. First, mirror symmetry strongly suggests that the manifolds $\hat{Y}_\zeta$ actually exist although we are not aware that examples of these manifolds have been explicitly constructed except for the noncompact cases considered in Ref. [65]. Second, we have a relatively simple and explicit set of differential relations, describing these half-flat mirror manifolds, which facilitates concrete calculations. And finally, from mirror symmetry one expects a half-flat mirror manifold $\hat{Y}_\zeta$ for each Calabi-Yau space X with a mirror Y and each set of flux parameters ζ. This means we are dealing with a large class of manifolds closely related to Calabi-Yau manifolds. Hopefully this allows one to keep some of the phenomenologically attractive features of heterotic Calabi-Yau compactifications [51] while gaining additional benefits, for example, in terms of moduli stabilization through flux.

III. HETEROTIC ON HALF-FLAT: THE BOSONIC ACTION TO LOWEST ORDER IN α'

We will now carry out the dimensional reduction of the heterotic string on the half-flat mirror manifolds[2] $\hat{Y}$ described in the previous section. We only consider the reduction of the bosonic part of the action which should be sufficient to obtain all the relevant information about the four-dimensional effective action. However, the reduction of some of the fermionic terms provides some additional insights and confirmation of the bosonic results and we will come back to this in the following section. For now, we restrict the calculation to lowest (zeroth) order in α' which, in particular, means we will not deal with gauge fields at this stage. We will discuss the inclusion of gauge fields later.

A. The reduction

We would now like to compactify the zeroth order bosonic action (2.1) on a half-flat mirror manifold $\hat{Y}$. As usual in flux compactifications, the collective modes are taken to be the same as for the corresponding case without flux, that is, as for the reduction on the associated Calabi-Yau manifold Y, in our case. This approach is in line with the earlier statement that the moduli spaces of the half-flat mirror manifolds $\hat{Y}$ and the associated Calabi-Yau manifolds Y are identical. Of course, one expects the flux to induce a low-energy potential and, potentially, masses for some of the previously massless fields. The idea will be that this "flux" scale is sufficiently lower than the string and Kaluza-Klein scales. Only then can heavy string/Kaluza-Klein modes be neglected while modes acquiring masses from flux effects can be kept. This can be achieved by sufficiently small flux parameters e_i and/or large radii of the internal manifold. At any rate, this separation of scales can be consistently checked once the low-energy potential has been computed. Although one expects the flux parameters e_i to be quantized (since the NS-NS flux of the mirror is quantized) we will here work in a supergravity approximation and view them as continuous parameters. We also adopt the general principle that our low-energy effective theory should reduce to the standard one, obtained from the reduction on the associated Calabi-Yau manifold Y, in the limit of vanishing flux parameters, $e_i \to 0$.

We split ten-dimensional coordinates as $(x^M) = (x^\mu, x^m)$ with external indices $\mu, \nu, \ldots = 0, 1, 2, 3$ and internal indices $m, n, \ldots = 4, \ldots 9$. The ten-dimensional metric for our reduction then takes the form

$$ds_{10}^2 = e^{2\phi} g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n, \tag{3.1}$$

where g_{mn} is the metric on the half-flat mirror manifold $\hat{Y}$ induced by the SU(3) structure and $g_{\mu\nu}$ is the four-dimensional metric. We have also introduced the zero mode

$$\phi = \hat{\phi} - \frac{1}{2} \ln \mathcal{V}, \tag{3.2}$$

of the dilaton where $\mathcal{V}$ is the volume

[2] For convenience, we will drop the index ζ on $\hat{Y}_\zeta$ from here on.

$$\mathcal{V} = \frac{1}{v}\int_{\hat{Y}} d^6x\sqrt{g} \tag{3.3}$$

of the internal space $\hat{Y}$, measured relative to a fixed reference volume v. The dilaton factor in front of the four-dimensional part of the metric (3.1) has been chosen so that we arrive at a canonically normalized Einstein-Hilbert term in four dimensions. As we have already explained, the moduli space of internal metrics g_{mn} on $\hat{Y}$ is parametrized by Kähler moduli v^i, where $i, j, \ldots = 1, \ldots, h^{1,1}(Y)$ and complex structure moduli z^a, where $a, b, \ldots = 1, \ldots, h^{2,1}(Y)$. More specifically, we can write the following standard equations for the deformations of the metric

$$\begin{aligned} \delta g_{\alpha\bar{\beta}} &= -i\omega_{i\alpha\bar{\beta}}\delta v^i \\ \delta g_{\bar{\alpha}\bar{\beta}} &= -\frac{1}{||\Omega||^2}\bar{\Omega}_{\bar{\alpha}}{}^{\gamma\delta}(\chi_a)_{\gamma\delta\bar{\beta}}\delta z^a, \end{aligned} \tag{3.4}$$

where we have introduced a set of (2, 1)-forms χ_a and holomorphic (antiholomorphic) indices $\alpha, \beta, \ldots$ $(\bar{\alpha}, \bar{\beta}, \ldots)$ on the internal space. Finally, we have the following zero mode expansion for the NS-NS two-form

$$\hat{B} = B + b^i\omega_i \tag{3.5}$$

$$\hat{H} = H + db^i \wedge \omega_i + (b^ie_i)\beta^0, \tag{3.6}$$

where B is a four-dimensional two-form with field strength $H = dB$ and b^i are $h^{1,1}(Y)$ real scalar fields. Note that the last term in the Ansatz (3.6) for the field strength $\hat{H}$ is new compared to the Calabi-Yau case and results, via Eq. (2.24), from the fact that the (1, 1)-forms ω_i are no longer closed. This term does have the form of (a particular type of) H flux, although it should be kept in mind that it originates from the intrinsic flux encoded in the half-flat mirror manifolds. For now we will not include genuine H flux into the calculation but defer this until later in the section.

Inserting the Ansätze (3.1), (3.2), (3.3), (3.4), (3.5), and (3.6) into the ten-dimensional bosonic action (2.1) one finds, after integrating over the internal space

$$\begin{aligned} S_4 = -\frac{1}{2\kappa_4^2}\int\Big\{&R\star 1 + 2d\phi\wedge\star d\phi + \frac{1}{2}da\wedge\star da \\ &+ 2h^{(1)}_{i\bar{j}}dt^i\wedge\star d\bar{t}^{\bar{j}} + 2h^{(2)}_{a\bar{b}}dz^a\wedge\star d\bar{z}^{\bar{b}} + 2\kappa_4^2 V * 1\Big\}, \end{aligned} \tag{3.7}$$

with the four-dimensional Newton constant $\kappa_4^2 = \kappa_{10}^2/v$ and the scalar potential

$$V = 4\kappa_4^{-2}e^{2\phi+K^{(1)}+K^{(2)}}[e_ie_jh^{ij}_{(1)} + 4(e_ib^i)^2]. \tag{3.8}$$

The complex Kähler moduli t^i are defined by

$$t^i = b^i + iv^i, \tag{3.9}$$

and the four-dimensional two-form B has been dualized to the scalar a. The Kähler and complex structure moduli space metrics are defined as usual by

$$h^{(1)}_{ij} = \frac{1}{4v\mathcal{V}}\int_{\hat{Y}}\omega_i\wedge\star\omega_j \tag{3.10}$$

$$h^{(2)}_{a\bar{b}} = -\frac{\int_{\hat{Y}}\chi_a\wedge\bar{\chi}_{\bar{b}}}{\int_{\hat{Y}}\Omega\wedge\bar{\Omega}}, \tag{3.11}$$

with inverse metrics $h^{ij}_{(1)}$ and $h^{a\bar{b}}_{(2)}$ and associated Kähler potentials

$$K^{(1)} = -\ln(8\mathcal{V}) \qquad K^{(2)} = -\ln\Big(i\int_{\hat{Y}}\Omega\wedge\bar{\Omega}\Big). \tag{3.12}$$

In this calculation, we have used the following result for the integrated scalar curvature of half-flat mirror manifolds

$$\int_{\hat{Y}}\sqrt{g}R_{\rm hf} = v\exp(K^{(2)})e_ie_jh^{ij}_{(1)}, \tag{3.13}$$

which was proven in Ref. [17], as well as the special geometry relations (A11) and (A20) in order to evaluate the integral $\int_{\hat{Y}}\beta^0\wedge\star\beta^0$. The two contributions to the four-dimensional potential (3.8) originate from this nonvanishing scalar curvature and the additional term in the Ansatz for the NS-NS three-form field strength $\hat{H}$ in Eq. (3.6), respectively.

B. Four-dimensional supergravity

The four-dimensional action derived in the previous subsection should be the bosonic part of an $N = 1$ supergravity theory. We would now like to make this explicit comparing it to the standard $N = 1$ supergravity action [78].

The kinetic terms in (3.7) are easy to deal with since they are identical to the ones arising in standard Calabi-Yau compactifications. We introduce chiral superfields S, T^i, and Z^a satisfying

$$S| = a + ie^{-2\phi} \tag{3.14}$$

$$T^i| = t^i \tag{3.15}$$

$$Z^a| = z^a, \tag{3.16}$$

where the bar denotes the lowest component of the multiplet. Then the Kähler potential reproducing the kinetic terms in Eq. (3.7) can be written as

$$K = \kappa_4^{-2}(K^{(S)} + K^{(1)} + K^{(2)}), \tag{3.17}$$

where

$$K^{(S)} = -\ln[i(\bar{S} - S)], \tag{3.18}$$

and $K^{(1)}$ and $K^{(2)}$ are given in (3.12). In order to perform a concrete calculation one needs to express these Kähler potentials in terms of the low-energy fields. This is done via holomorphic prepotentials $\mathcal{F}$ and $\mathcal{G}$ and the respective equations are given in (A4) and (A10).

Having fixed the Kähler potential and the superfields in terms of component fields via Eqs. (3.14), (3.15), and (3.16) we now have to check whether the potential (3.8), obtained from dimensional reduction, can be reproduced from the standard supergravity expression

$$V = \kappa_4^{-4} e^{\kappa_4^2 K}(K^{X\bar{Y}} D_X W D_{\bar{Y}} \bar{W} - 3\kappa_4^2 |W|^2), \tag{3.19}$$

for a suitable choice of superpotential W. In this expression, we have used indices $X, Y, \ldots$ to label all chiral superfields $(\Phi^X) = (S, T^i, Z^a)$ and D_X denotes the Kähler-covariant derivative defined by

$$D_X W = \partial_X W + \kappa_4^2 K_X W. \tag{3.20}$$

Further $K^{X\bar{Y}}$ is the inverse of the Kähler metric $K_{X\bar{Y}}$.

The potential (3.8) is quadratic in the axionic fields b^i which are part of the chiral multiplets T^i. This suggest that the superpotential may be a linear function in the fields T^i. In fact, we claim that W is given by

$$W = \sqrt{8} e_i T^i. \tag{3.21}$$

Let us now verify this claim. We first note that, using the expression (3.17) for the Kähler potential, the prefactor in the reduction potential (3.8) can be rewritten as

$$4\exp(2\phi + K^{(1)} + K^{(2)}) = 8 e^{\kappa_4^2 K}. \tag{3.22}$$

This correctly matches the $e^{\kappa_4^2 K}$ prefactor of the supergravity potential (3.19). With the superpotential (3.21), the various Kähler-covariant derivatives are given by

$$D_S W = -\frac{1}{2} e^{2\phi} W \tag{3.23}$$

$$D_i W = \sqrt{8} e_i + K_i^{(1)} W \tag{3.24}$$

$$D_a W = K_a^{(2)} W. \tag{3.25}$$

For the nonvanishing components of the Kähler metric we have $K_{S\bar{S}} = e^{2\phi}/4$, $K_{i\bar{j}} = h^{(1)}_{i\bar{j}}$, and $K_{a\bar{b}} = h^{(2)}_{a\bar{b}}$. Using these, and Eq. (A4) we find

$$D_S W D_{\bar{S}} \bar{W} K^{S\bar{S}} = |W|^2 \tag{3.26}$$

$$D_i W D_{\bar{j}} \bar{W} K^{i\bar{j}} = 8 e_i e_j h^{ij}_{(1)} - 32(e_i v^i)^2 + 3|W|^2 \tag{3.27}$$

$$D_a W D_{\bar{b}} \bar{W} K^{a\bar{b}} = 3|W|^2. \tag{3.28}$$

In the second line, we have used (A7) which holds for special geometries with a cubic prepotential. The result in the third line can be proved using a similar cubic prepotential (A18) for the complex structure moduli which is justified in the large complex structure limit. Inserting the relations (3.22) and (3.26), (3.27), and (3.28) into the supergravity potential (3.19), using the explicit form (3.21) of W we indeed correctly reproduce the potential (3.8) obtained from the reduction.

To summarize our results so far, we have derived, to lowest order in α', the bosonic part of the four-dimensional effective action of the heterotic string on half-flat mirror manifolds $\hat{Y}$. We have shown that this action is indeed the bosonic part of a four-dimensional $N = 1$ supergravity theory with Kähler potential (3.17) and superpotential (3.21). This latter statement has been proved for large complex structure since we have used the relation (3.28) which, as far as we know, only holds in this limit. Given that the relations which define the half-flat mirror manifolds can only be expected to hold for large complex structure this is perhaps not surprising. However, our result indicates that the definition of the half-flat mirror manifolds indeed has to be modified away from the large complex structure limit.

C. Including H flux

Our previous calculation can be generalized by adding an arbitrary three-form $H_{\rm flux}$, harmonic on the internal space, to the Ansatz (3.6) for the NS-NS field strength $\hat{H}$. In the analogous Calabi-Yau case, the forms (α_A, β^B) constitute a basis of harmonic three-forms and the most general NS-NS flux is simply given by an arbitrary linear combination of these forms. Here, we have to be more careful. From Eq. (2.24) we know that α_0 is not even closed which means it does not define a cohomology class. All other forms (α_a, β^b) are closed but not necessarily co-closed. However, we know that

$$d \star \alpha_a = \mathcal{O}(e_i), \qquad d \star \beta^b = \mathcal{O}(e_i), \tag{3.29}$$

since these forms are harmonic in the Calabi-Yau limit $e_i \to 0$. Hence, the forms (α_a, β^b) define cohomology classes and they differ from the harmonic representative by exact forms of the order e_i. This understood, we write the following Ansatz for the NS-NS flux

$$H_{\rm flux} = \epsilon_A \beta^A + \mu^A \alpha_A, \tag{3.30}$$

where

$$\epsilon_A = (0, \epsilon_a), \qquad \mu^A = (0, \mu^a). \tag{3.31}$$

We have allowed indices in (3.30) to run over all values to keep expressions covariant but we have set $\mu^0 = 0$ in accordance with the above discussion. Also we note that dealing with the flux parameter ϵ_0 is a bit more subtle as it was argued in [17] that it reproduces the mirror of the zero-NS flux. For this reason, we have also set $\epsilon_0 = 0$. However, all other flux parameters (ϵ_a, μ^a) are kept arbitrary. The so-defined NS-NS flux satisfies

$$dH_{\text{flux}} = 0, \qquad d \star H_{\text{flux}} = (\text{second order in flux}). \tag{3.32}$$

For the second relation, we have used Eq. (3.29) and, here and in the following, "nth order in flux" refers to a quantity proportional to a product of n of the flux parameters e_i, ϵ_a, or μ^a.

We would now like to repeat our reduction of the lowest order bosonic action (2.1), using the Ansätze (3.1), (3.2), (3.3), (3.4), (3.5), and (3.6), but modifying the expression for $\hat{H}$ by adding to it the NS-NS flux (3.30). The kinetic terms are, of course, unmodified by the additional NS-NS flux and the four-dimensional effective action is still of the form (3.7), where only the potential V has a different form. Combining our earlier expression (3.6) for the field strength $\hat{H}$ with the H flux (3.30) we have

$$\hat{H} = H + db^i \wedge \omega_i + (b^i e_i)\beta^0 + \epsilon_a \beta^a + \mu^a \alpha_a. \tag{3.33}$$

The contribution to the potential which originates from the nonvanishing scalar curvature (3.13) of the half-flat mirror manifolds remains the same. However, we have to consider the additional terms which arise from this new form of $\hat{H}$ when inserted into the form field kinetic term. To do this, we note, the term proportional to β^0 in the above expression looks like an ordinary H flux and can be treated on the same footing. To this end, we define the modified flux parameters $\tilde{\epsilon}_A = (e_i b^i, \epsilon_a)$. With these, the potential takes the form

$$V = 4e^{2\phi + K^{(1)} + K^{(2)}} e_i e_j h^{ij}_{(1)} - 2e^{2\phi + K^{(1)}} (\tilde{\epsilon}_A + \mu^c \mathcal{M}_{cA}) \times (\text{Im}\mathcal{M})^{-1AB} (\tilde{\epsilon}_B + \mu^d \bar{\mathcal{M}}_{dB}). \tag{3.34}$$

To obtain the last term we have used (A11) and (A12) and the matrix $\mathcal{M}$ is defined in Eq. (A19). Since we have neglected second order flux terms in H_{flux} this potential is correct up to quadratic terms in the flux and there are possible corrections of cubic and higher order in flux which we have not calculated. Let us also note that despite the explicit minus sign which appears in the above formula, this potential is manifestly positive definite as the matrix $[\text{Im}(\mathcal{M})]^{-1}$ is negative definite. When deriving the potential from the superpotential, this feature will arise from the no-scale structure which annihilates the negative contribution in (3.19). We note that Eq. (3.34) reduces to the previous formula (3.8) for the potential in the absence of H flux by setting $\epsilon_a = 0$ and $\mu^a = 0$, remembering that $\tilde{\epsilon}_0 = e_i b^i$.

As before, it has to be checked that the above result can be embedded into four-dimensional $N = 1$ supergravity. Since the kinetic terms are unmodified, the definition of superfields is still given by (3.14), (3.15), and (3.16) and the Kähler potential is the standard one, Eq. (3.17). Given these results, is the modified potential (3.34) of the supergravity form (3.19) for a suitable superpotential W? It is shown in Appendix B that this is indeed the case, provided one is working in the large complex structure limit. The superpotential then reads

$$W = \sqrt{8}(e_i T^i + \epsilon_a Z^a + \mu^a \mathcal{G}_a), \tag{3.35}$$

with arbitrary flux parameters e_i, ϵ_a, and μ^a.

IV. GRAVITINO MASS AND THE SUPERPOTENTIAL

In this section we propose another approach to compute the superpotential which will turn out to be more suitable for further generalizations and for obtaining some more insight when α' corrections are taken into account. Previously, we have derived the moduli superpotential by dimensional reduction of the bosonic action and by comparing the result with the standard form of four-dimensional $N = 1$ supergravity. However, there is also a more direct method using Gukov's formula [79,80] which, in the appropriate form, has led to the correct result for a number of different compactifications. In this section, we are going to explore this second approach and its relation to the results of the previous section, for the case of heterotic string on half-flat mirror manifolds.

We will proceed in two steps. First, we will derive the appropriate version of Gukov's formula from the four-dimensional gravitino mass term which we obtain as a dimensional reduction of the appropriate terms in the ten-dimensional action, an approach also considered in Refs. [59,69]. As we will see, the resulting Gukov-type formula applies to the heterotic string on all manifolds of SU(3) structure and is valid to first order in α'. As a second step, we then apply this general formula to our particular half-flat mirror manifolds and show that it specializes to the superpotential (3.35), derived in the previous section.

A. Gukov formula from the gravitino mass term

The mass term for the gravitino Ψ in four dimensions is given by

$$S_{\Psi,\text{mass}} = -\frac{1}{2}\int_{M_4} d^4x\sqrt{-g}\{M_{3/2}\Psi^\dagger_\mu \gamma^0 \gamma^{\mu\nu}\Psi^*_\nu + \text{h.c.}\}, \tag{4.1}$$

where the four-dimensional gamma matrices γ_μ are chosen to be real and the chirality matrix

$$\gamma = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_{\mu\nu\rho\sigma} \tag{4.2}$$

is purely imaginary. In the context of $N = 1$ four-dimensional supergravity the gravitino mass can be written as

$$M_{3/2} = \exp(\kappa_4^2 K/2)W, \tag{4.3}$$

and the invariant function $G = K + \ln|W|^2$ can be computed from the gravitino mass using the relation

$$e^G = |M_{3/2}|^2. \tag{4.4}$$

If the Kähler potential has been computed independently or the holomorphic part of $M_{3/2}$ can be identified then the superpotential can be obtained directly from $M_{3/2}$. We are now going to apply these facts to the gravitino mass term which descends from the ten-dimensional theory.

A quick inspection of the ten-dimensional action (2.2) reveals which parts potentially contribute to the gravitino mass terms in four dimensions. The most obvious one is the flux term $\Psi_M\Gamma^{MNPQR}\Psi_N H_{PQR}$. This term was also considered in Refs. [59,62,69] and, as we will show, it gives rise to the well-known superpotential $W \sim \int H \wedge \Omega$ which was proposed in Refs. [79,80]. This result for W is definitely correct for Calabi-Yau manifolds, but if the internal manifold has only SU(3) structure there will be a further contribution from the gravitino kinetic term in ten dimensions. This additional contribution will turn out to be proportional to the first torsion class, $\mathcal{W}_1$, of the SU(3) structure manifold. The reason this term appears in four dimensions is that on such manifolds the globally defined spinor η is no longer covariantly constant with respect to the Levi-Civita connection.

Let us now see how this works in detail. We first have to decompose the ten-dimensional gamma matrices

$$(\gamma_M) = (\gamma_\mu \otimes \mathbf{1}, \gamma \otimes \gamma_m). \tag{4.5}$$

Note that we have chosen the four-dimensional gamma matrices, γ_μ, real and the six-dimensional ones, γ_m, imaginary so that the above decomposition leads to real ten-dimensional gamma matrices. Furthermore, we have to decompose the ten-dimensional gravitino $\hat{\Psi}_M$ in a way compatible with its Majorana-Weyl nature. The unique possibility, up to overall rescalings, for the case of a manifold with SU(3) structure is

$$\hat{\Psi}_M = e^{\phi/2}(\psi_M \otimes \eta + \psi_M^* \otimes \eta^*), \tag{4.6}$$

where ψ_M is a four-dimensional Weyl spinor of positive chirality. We recall that η is the six-dimensional globally defined Weyl spinor which exists on manifolds with SU(3) structure. The external components ψ_μ correspond to the four-dimensional gravitino while ψ_m represent spin 1/2 fields. In fact, in order not to have cross kinetic terms between the gravitino and the spin 1/2 fields one needs to redefine ψ_μ by some particular combination of ψ_m. However, this subtlety does not effect the gravitino mass which can be read off as the coefficient of the term $\frac{1}{2}\psi_\mu^\dagger\gamma^0\gamma^{\mu\nu}\psi_\nu^*$. On the other hand the normalization of the gravitino field is important since its kinetic term needs to be in canonical form in order to read off the correct gravitino mass. For this reason we have chosen the overall factor $e^{\phi/2}$ in Ansatz (4.6) and one can easily check that this leads to the correct kinetic term for the gravitino in four dimensions. Let us quickly sketch how this works. Inserting (4.6) and (4.5) into the ten-dimensional kinetic term from (2.2) and keeping only the terms involving the four-dimensional spacetime indices we obtain

$$\overline{\hat{\Psi}}_\mu\Gamma^{\mu\nu\rho}D_\nu\hat{\Psi}_\rho = e^\phi[\overline{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho(\eta^\dagger\eta) + \psi_\mu^T\gamma^{\mu\nu\rho}D_\nu\psi_\rho^*(\eta^T\eta^*)]. \tag{4.7}$$

Note that due to our conventions the above terms are the only combinations which survive as $\eta^T\eta = \eta^\dagger\eta^* \equiv 0$. Also recall that we have normalized the spinor η requiring $\eta^\dagger\eta = 1$ so that the terms above do not depend on the internal manifold. Consequently the integration over the six-dimensional space will only produce a volume factor which combines with the dilaton factor in Eq. (2.2) into the four-dimensional dilaton (3.2). Finally, taking into account the rescaling of the spacetime metric (3.1) we obtain for the four-dimensional gravitino kinetic term

$$S_{(3/2)\mathrm{k}in} = \frac{1}{2}\int_{M_4} d^4x\sqrt{-g}[\overline{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho + \psi_\mu^T\gamma^{\mu\nu\rho}D_\nu\psi_\rho^*], \tag{4.8}$$

which is indeed the correct kinetic term for the gravitino in four dimensions [78].

Having normalized the gravitino field correctly we can go ahead and derive the gravitino mass term. This can be done by inserting the decompositions (4.5) and (4.6), along with the Ansatz (3.1) for the ten-dimensional metric into the fermionic action (2.2) and keeping the terms with two four-dimensional gamma matrices and no spacetime derivatives. Let us consider the two relevant terms in (2.2) separately, starting with the kinetic term. We obtain

$$\overline{\hat{\Psi}}_\mu\Gamma^{\mu n\nu}D_n\hat{\Psi}_\nu = -(\psi_\mu \otimes \eta + \psi_\mu^* \otimes \eta^*)^T \times [\gamma^0\gamma^{\mu\nu} \otimes \gamma^n D_n](\psi_\nu \otimes \eta + \psi_\nu^* \otimes \eta^*). \tag{4.9}$$

From compatibility condition (2.10) we know that the spinor η is covariantly constant with respect to the connection with torsion. This implies

$$D_n\eta - \frac{1}{4}\kappa_{npq}\gamma^{pq}\eta = 0, \tag{4.10}$$

which, applied to Eq. (4.9), yields

$$\bar{\hat{\Psi}}_\mu \Gamma^{\mu n \nu} D_n \hat{\Psi}_\nu = \frac{1}{4}\psi_\mu^\dagger \gamma^0 \gamma^{\mu\nu} \psi_\nu^* (\eta^\dagger \gamma^n \gamma^{pq} \eta^*) \kappa_{npq} - \frac{1}{4}\psi_\mu^T \gamma^0 \gamma^{\mu\nu} \psi_\nu (\eta^T \gamma^n \gamma^{pq} \eta) \kappa_{npq}. \tag{4.11}$$

As before, we have discarded terms like $\eta^\dagger \gamma^n \gamma^{pq} \eta$ which vanish identically. Moreover, the properties of six-dimensional spinors and gamma matrices assure that only the totally antisymmetric part of $\eta^\dagger \gamma^n \gamma^{pq} \eta^*$ survives. Using (2.8) and taking care to include all the dilaton factors in Eqs. (3.1) and (4.6), we conclude that the torsion contribution to the gravitino mass term can be written as

$$M_{3/2}^{(T)} = \frac{e^\phi}{4\mathcal{V}} \int_{\hat{Y}} \sqrt{g}\, \eta^\dagger \gamma^{npq} \eta^* \kappa_{npq} = \frac{i e^\phi \sqrt{8}}{4\mathcal{V}||\Omega||} \int_{\hat{Y}} \sqrt{g}\, \Omega^{\bar{\alpha}\bar{\beta}\bar{\gamma}} \kappa_{\bar{\alpha}\bar{\beta}\bar{\gamma}}. \tag{4.12}$$

With Eq. (2.14) and the relation $J_m{}^n \Omega_{npq} = i\Omega_{mpq}$ one can also write the above expression in the following form

$$M_{3/2}^{(T)} = \frac{e^\phi \sqrt{8}}{24\mathcal{V}||\Omega||} \int_{\hat{Y}} \sqrt{g}\, \Omega^{\bar{\alpha}\bar{\beta}\bar{\gamma}} (dJ)_{\bar{\alpha}\bar{\beta}\bar{\gamma}} = \frac{i e^\phi \sqrt{8}}{4\mathcal{V}||\Omega||} \int_{\hat{Y}} \Omega \wedge dJ. \tag{4.13}$$

We recall from Eq. (2.13) that the torsion κ decomposes into five classes according to the various SU(3) representations it contains. Evidently, contracting with Ω in the above relation projects out the SU(3) singlet part which corresponds to the torsion class $\mathcal{W}_1$.

For the $\hat{H}$-dependent term in the fermionic action (2.2) the calculation is similar and was also discussed in Refs. [59,60,62,69]. One finds

$$\bar{\Psi}_\mu \Gamma^{\mu npq\nu} \Psi_\nu \hat{H}_{npq} = -(\psi_\mu \otimes \eta + \psi_\mu^* \otimes \eta^*)^T \times [\gamma^0 \gamma^{\mu\nu} \gamma^5 \otimes \gamma^{npq}] (\psi_\nu \otimes \eta + \psi_\nu^* \otimes \eta^*), \tag{4.14}$$

and comparison with Eq. (4.1) leads to the gravitino mass contribution

$$M_{3/2}^{(H)} = -\frac{i e^\phi \sqrt{8}}{24\mathcal{V}||\Omega||} \int_{\hat{Y}} \sqrt{g}\, \Omega^{\bar{\alpha}\bar{\beta}\bar{\gamma}} \hat{H}_{\bar{\alpha}\bar{\beta}\bar{\gamma}} = \frac{e^\phi \sqrt{8}}{4\mathcal{V}||\Omega||} \int_{\hat{Y}} \Omega \wedge H. \tag{4.15}$$

Adding up the two contributions (4.13) and (4.15) one finds for the gravitino mass

$$M_{3/2} = \frac{\sqrt{8} e^\phi}{4\mathcal{V}||\Omega||} \int_{\hat{Y}} \Omega \wedge (H + idJ). \tag{4.16}$$

From Eq. (4.4) this determines the supergravity function G.

Of course we do not know the Kähler potential for general manifolds with SU(3) structure. However, it is suggestive to identify the integral in Eq. (4.16) as the holomorphic part and, hence, the superpotential and the prefactor as the Kähler potential. Accepting this we find by comparison with Eq. (4.3) that[3]

$$W \sim \int_{\hat{Y}} \Omega \wedge (\hat{H} + idJ). \tag{4.17}$$

Note that $\hat{H} + idJ = d(\hat{B} + iJ)$ is precisely the holomorphic combination which determines the scalar components (3.15) of the superfields T^i. Equivalently, following the notation of Ref. [64] we can write the superpotential as

$$W \sim H_1 + \kappa_1, \tag{4.18}$$

where the SU(3) singlet component of the torsion, κ_1, was defined in (2.17). Likewise, H_1 is the SU(3) singlet component of $\hat{H}$ defined by

$$\hat{H}_{mnp}|_{\rm singlet} = -6H_1(\Omega + \bar{\Omega})_{mnp}. \tag{4.19}$$

Comparing again with (4.3) we can argue that the prefactor in Eq. (4.16) should determine the Kähler potential. Thus we can write

$$e^K \sim \frac{e^{2\phi}}{\mathcal{V}^2||\Omega||^2}. \tag{4.20}$$

We stress that one expects these results for G, W, and K to be valid for heterotic compactifications on all manifolds with SU(3) structure. In addition, they hold up to and including correction of order α' since the relevant ten-dimensional gravitino terms in Eq. (2.2) do not receive corrections at this order. This latter fact can be illustrated for standard Calabi-Yau compactifications. In this case it is straightforward to show that the formula (4.17) correctly reproduces the cubic gauge matter superpotential

[3]A similar formula for the superpotential was first proposed in Ref. [17] in the context of type II theories. In heterotic string compactifications it appears in Ref. [71], but to our knowledge it was never derived before in a systematic way as we do in this paper.

[52] which arises at order α'. We expect the relation (4.17) will be quite useful when computing the gauge matter superpotential in more general cases, such as for half-flat mirror manifolds.

B. Application to half-flat mirror manifolds

If the Kähler potential has been fixed by other means, the superpotential can be obtained from Eq. (4.16) exactly, including the prefactor. For example, using the Calabi-Yau Kähler potential (3.17) which, as we have seen, also applies to half-flat mirror manifolds one finds

$$W = \sqrt{8}\int_{\hat{Y}} \Omega \wedge (\hat{H} + idJ). \tag{4.21}$$

It is now just a simple exercise to obtain the expression of the superpotential in terms of the component fields in four dimensions. We recall from Eqs. (3.6) and (3.30) that the complete Ansatz for the NS-NS field strength, including NS-NS flux, is given by

$$\hat{H} = H + db^i\omega_i + (b^ie_i)\beta^0 + e_A\beta^A + m^a\alpha_a. \tag{4.22}$$

Using the expansion of the (3, 0)-form Ω in terms of the complex structure moduli (2.21), the particular expression for dJ, (2.25), and the integration rules (2.22) one immediately obtains

$$W = \sqrt{8}(e_iT^i + \epsilon_aZ^a + \mu^a\mathcal{G}_a), \tag{4.23}$$

which precisely coincides with (3.35). In summary, we have verified this result in two, largely independent ways, namely, by a reduction of the bosonic term and from a generalized Gukov-type formula which we have derived from the gravitino mass term.

V. INCLUDING GAUGE FIELDS

Let us now discuss some properties of the heterotic $E_8 \times E_8$ string on half-flat mirror manifolds at first order in α'. At this order, the Bianchi identity (2.5) for $\hat{H}$ receives its gauge field and gravitational Chern-Simons correction and finding its solution becomes a nontrivial task. With

$$d\omega_{\rm YM} = \mathrm{Tr}(\hat{F}\wedge\hat{F}), \qquad d\omega_{\rm L} = \mathrm{tr}(\tilde{R}\wedge\tilde{R}), \tag{5.1}$$

the Bianchi identity leads to the well-known relation

$$d\hat{H} = \frac{\alpha'}{4}[\mathrm{tr}(\tilde{R}\wedge\tilde{R}) - \mathrm{Tr}(\hat{F}\wedge\hat{F})]. \tag{5.2}$$

It implies, as a condition for the Bianchi identity to be soluble, that the right-hand side has to be cohomologically trivial and, hence, that

$$[\mathrm{tr}(\tilde{R}\wedge\tilde{R})] = [\mathrm{Tr}(\hat{F}\wedge\hat{F})], \tag{5.3}$$

where the bracket $[\ldots]$ denotes the cohomology class. Traditionally, the way to satisfy this condition has been the standard embedding [51] although more general possibilities have been discussed in the literature [81–83].

Here, we will consider the simplest possibility, a generalization of the standard embedding to our compactifications. Let us first recall the standard Calabi-Yau case. The spin connection $\omega_m^{(\rm CY)}$ of the Calabi-Yau manifold Y takes values in SU(3) which means its nonvanishing components are of the form $\omega_m^{(\rm CY)\alpha\bar{\beta}}$. The standard embedding then amounts to setting the internal Yang-Mills connection equal to the Calabi-Yau spin connection, that is

$$A_m{}^{\alpha\bar{\beta}}|_{\rm background} = \omega_m^{(\rm CY)\alpha\bar{\beta}}. \tag{5.4}$$

Here, the indices $(\alpha, \bar{\beta})$ on A refer to an SU(3) subgroup of one of the E_8 factors of the gauge group. The trace of the square of such an SU(3) generator in the adjoint of E_8 is 30 times the trace of the square of an SU(3) generator in the fundamental. With the definition of Tr as $1/30$ of the trace in the adjoint of $E_8 \times E_8$, this means that the standard embedding indeed solves the cohomology constraint (5.3) and, even more strongly, leads to the right-hand side of Eq. (5.2) to vanish identically. Note that, at the level of background fields, the internal part of $\hat{H}$ is vanishing so that the modification (2.4) of the spin connection does not contribute for Calabi-Yau manifolds. The surviving low-energy gauge group is the maximal commutant of SU(3) within $E_8 \times E_8$ which is $E_6 \times E_8$. In addition, one obtains $h^{1,1}(Y)$ chiral multiplets in the $\overline{\mathbf{27}}$ of E_6 and $h^{2,1}(Y)$ chiral multiplets in the **27** of E_6.

Can this picture be adapted to half-flat mirror manifolds? There are two essential modifications. First of all, the spin connection $\omega_m^{(\rm hf)}$ of the half-flat manifold generally takes values in SO(6) rather than SU(3). Second, the internal background value of $\hat{H}$ is no longer vanishing due to the additional term in Eq. (3.6) and, if present, H flux in Eq. (3.30). Therefore, we have to work with the modified connection $\tilde{w}$ which is the correct object that enters the Bianchi identity. From Eqs. (2.4), (3.6), and (3.30) it is given by

$$\tilde{\omega}_m{}^{np} = \omega_m^{(\rm hf)np} + 1/2(b^ie_i\beta^0 + \epsilon_a\beta^a + \mu^a\alpha_a)_m{}^{np}. \tag{5.5}$$

This connection still generically takes values in SO(6). The generalization of the standard embedding to half-flat mirror manifolds is then characterized by

$$A_m{}^{np} = \tilde{\omega}_m{}^{np}, \tag{5.6}$$

where the index pair (np) on A refers to an SO(6) subgroup of one of the E_8 gauge factors. The trace of the square of an SO(6) generator in the adjoint of E_8 is still 30 times that of the trace in the fundamental of SO(6) and, hence, the above choice indeed provides a solution to the cohomology constraint (5.3). As for the Calabi-Yau case it sets the right-hand side of Eq. (5.2) identically zero.

However, the low-energy gauge group is now the commutant of SO(6) within $E_8 \times E_8$ which (modulo global issues) is given by $SO(10) \times E_8$. It is interesting to compare this to the standard Calabi-Yau case. Apparently, switching on flux has broken the gauge group from E_6 to SO(10). From the decomposition

$$\mathbf{78} \rightarrow \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1} \tag{5.7}$$

of the adjoint **78** of E_6 under SO(10) we conclude that the additional gauge bosons in the **16**, $\overline{\mathbf{16}}$, and **1** representations of SO(10) must have picked up a mass proportional to the flux parameters e_i, ϵ_a, and μ^a. For this to happen the additional gauge multiplets must pair up with chiral multiplets in the same SO(10) representations. To see how this works let us examine the decomposition of the fundamental of E_6 under SO(10) which is given by

$$\mathbf{27} \rightarrow \mathbf{16} + \mathbf{10} + \mathbf{1}. \tag{5.8}$$

In the standard Calabi-Yau case, we therefore have $h^{1,1}(Y)$ chiral multiplets in $\overline{\mathbf{16}}$ and $h^{2,1}(Y)$ chiral multiplets in **16**. One **16** and one $\overline{\mathbf{16}}$ (and one singlet) chiral multiplet have to be paired up with the additional gauge bosons, so they will pick up a mass proportional to flux parameters. It is reasonable to expect, therefore, that $h^{1,1}(Y) - 1$ antifamilies in $\overline{\mathbf{16}}$ and $h^{2,1}(Y) - 1$ families in **16** are left massless. This expectation should be confirmed by an explicit calculation of the four-dimensional effective theory including gauge matter. We remark that the general formula (4.17) for the superpotential should be valid including gauge matter and its evaluation should, hence, lead to the correct gauge matter superpotential. This will be discussed in detail in a forthcoming publication [75].

A final remark concerns the gauge kinetic function f of the low-energy gauge group. From a simple reduction of the ten-dimensional gauge field action (2.2) it is clear that, to order α', this function is given by the dilaton, as in the standard Calabi-Yau case. More precisely, fixing the normalization of the gauge field kinetic term by

$$-\frac{1}{4g_{\rm YM}^2}\int_{M_4} d^4x\sqrt{-g}{\rm Re}(f){\rm Tr}(F^2), \tag{5.9}$$

where F is the low-energy gauge field strength, and

$$g_{\rm YM}^2 = \frac{4\kappa_{10}^2}{\alpha' v}, \tag{5.10}$$

one finds that

$$f = S. \tag{5.11}$$

This result can be expected to receive threshold corrections at order $(\alpha')^2$ which result from terms at that order in the ten-dimensional effective action [84]. It would be interesting to calculate these corrections for half-flat mirror manifolds.

VI. CONCLUSION AND OUTLOOK

In this paper, we have considered the heterotic string on half-flat mirror manifolds which arise in the context of mirror symmetry with flux. More precisely, given a mirror pair (X, Y) of Calabi-Yau three-folds, the associated half-flat mirror manifolds $\hat{Y}_\zeta$ are the mirror duals of X with NS-NS flux $\zeta = (e_i)$.

Our main result is the complete derivation of the four-dimensional $N = 1$ effective action to lowest order in α' on such manifolds. We find that the Kähler potential for the dilaton S, the Kähler moduli T^i, and the complex structure moduli Z^a are the same as for the reduction on the associated Calabi-Yau manifolds Y while the superpotential is given by

$$W = \sqrt{8}(e_i T^i + \epsilon_a Z^a + \mu^a \mathcal{G}_a). \tag{6.1}$$

Here, the first term arises from the intrinsic, geometrical flux of the half-flat mirror manifold and the other two terms arise from NS-NS flux with electric and magnetic parameters ϵ_a and μ^a, respectively. The structure of this result certainly invites speculations about more general half-flat mirror manifolds which also contain intrinsic magnetic flux and generate the "missing" term $m^i \mathcal{F}_i$ in Eq. (6.1). Unfortunately, at present, there is no explicit description available for such manifolds.

We have confirmed the above result for W by two largely independent methods, namely, by a reduction of the bosonic action and via a reduction of some fermionic terms leading to the four-dimensional gravitino mass term. As a by-product, we have also obtained a Gukov-type formula for the superpotential which we expect to be valid for the heterotic string on all manifolds of SU(3) structure and includes order α' effects. It is given by

$$W \sim \int_{\hat{Y}} \Omega \wedge (\hat{H} + idJ), \tag{6.2}$$

where J is the two-form which, along with the three-form Ω, characterizes the SU(3) structure.

We have also argued that the standard embedding can be generalized to the heterotic string on half-flat mirror manifolds and leads to (in the case of $E_8 \times E_8$) a low-energy gauge group $SO(10) \times E_8$ rather than $E_6 \times E_8$. We also expect $h^{1,1}(Y) - 1$ antifamilies in the $\overline{\mathbf{16}}$ representation of SO(10) and $h^{2,1}(Y) - 1$ families in the **16** representation.

There certainly remains substantial work to be done concerning the inclusion of gauge and gauge matter fields. In particular, one would like to derive the four-dimensional effective theory for these fields, understand the way in which the flux parameters break E_6 to SO(10), and compute the gauge matter superpotential explicitly. For this latter task the general formula (6.2) will be quite useful. All these issues are currently under investigation [75].

An important application of our results concerns moduli stabilization in heterotic models. The superpotential (6.1) is independent of S so, as stands, at least the dilaton still represents a runaway direction. However, we have seen that the gauge kinetic function is still proportional to S and, hence, gaugino condensation would generate a nonperturbative superpotential [53]

$$W_{\text{gaugino}} \sim \exp(-cS) \tag{6.3}$$

for some appropriate constant c. Studying the combined effect of this gaugino superpotential and (6.1) is an interesting problem which we are currently investigating.

ACKNOWLEDGMENTS

S. G. is supported by JSPS. A. L. and A. M. are supported by PPARC. This work is supported in part by a Royal Society Japan-UK Joint Project, Reference No. NC/Japan/JP/15207. The authors would like to thank Mariana Graña, Yukiko Konishi, Taichiro Kugo, Jan Louis, and Dan Waldram for helpful discussions.

APPENDIX A: SOME USEFUL RESULTS ON SPECIAL GEOMETRY

In order to make the paper self-contained we add this appendix on special Kähler geometry and its particular realizations on Calabi-Yau manifolds. Our discussion will be carried out for a Calabi-Yau space Y with occasional reference to its mirror X. For an extensive cover of the subject see Ref. [85,86].

A Kähler manifold of complex dimension n is called special Kähler if its geometry is completely determined in terms of a holomorphic function $\mathcal{H}$, called the prepotential. When written in terms of projective coordinates, which we denote by X^P, where $P, Q, \ldots = 0, \ldots, n$ the prepotential is a homogeneous function of degree two which implies that $X^P \mathcal{H}_P = 2\mathcal{H}$ with derivatives $\mathcal{H}_P = \frac{\partial \mathcal{H}}{\partial X^P}$. In terms of the prepotential, the Kähler potential has the form

$$K = -\ln i(\bar{X}^P \mathcal{H}_P - X^P \bar{\mathcal{H}}_P). \tag{A1}$$

It is also useful to introduce a $(n+1) \times (n+1)$ matrix $\mathcal{Q}$

$$\mathcal{Q}_{PQ} = \bar{\mathcal{H}}_{PQ} + 2i \frac{\mathrm{Im}(\mathcal{H}_{PR})\mathrm{Im}(\mathcal{H}_{QS})X^Q X^S}{\mathrm{Im}(\mathcal{H}_{RS})X^R X^S}, \tag{A2}$$

which plays the role of gauge coupling matrix in type II compactifications and which satisfies $\mathcal{H}_P = \mathcal{Q}_{PQ} X^Q$.

It is well known that the moduli space of Calabi-Yau manifolds is governed by two such special Kähler geometries: one for the complexified Kähler moduli and one for the complex structure moduli. Let us now describe these two moduli spaces in turn.

We start with the Kähler moduli space of the Calabi-Yau manifold Y which has dimension $n = h^{1,1}(Y)$. We denote its projective coordinates by T^I with indices $I, J, \ldots = 0, \ldots, h^{1,1}(Y)$. It is also useful to introduce indices $i, j, \ldots = 1, \ldots, h^{1,1}(Y)$.

In the large radius limit of the Calabi-Yau space, the prepotential, which we call $\mathcal{F}$, is known explicitly and given by

$$\mathcal{F} = -\frac{1}{6} \frac{d^{(Y)}_{ijk} T^i T^j T^k}{T^0}, \tag{A3}$$

where $d^{(Y)}_{ijk}$ are the triple intersection numbers of the manifold Y. Introducing affine coordinates $t^i = T^i/T^0$, one finds from Eq. (A1) for the associated Kähler potential

$$K^{(1)} = -\ln[i d^Y_{ijk}(t^i - \bar{t}^i)(t^j - \bar{t}^j)(t^k - \bar{t}^k)] \equiv -\ln 8\mathcal{V}, \tag{A4}$$

where $\mathcal{V}$ can be interpreted as the volume of the Calabi-Yau space. It is useful to describe the moduli space in terms of the Kähler form J which can be expanded as

$$J = v^i \omega_i, \tag{A5}$$

where $v^i = \mathrm{Im}(t^i)$ and (ω_i) is a basis of the second cohomology of Y. Then, the metric $h^{(1)}_{ij}$ on the Kähler moduli space can be written as

$$h^{(1)}_{ij} = \partial_i \bar{\partial}_j K^{(1)} = \frac{1}{4\mathcal{V}} \int_Y \omega_i \wedge \star \omega_j. \tag{A6}$$

A useful relation which can be derived from the explicit Kähler potential (A4) is

$$h^{ij}_{(1)} K^{(1)}_j = (t^i - \bar{t}^i) = 2iv^i, \tag{A7}$$

where $K^{(1)}_i$ denote the derivatives of the Kähler potential (A4) with respect to the fields t^i and $h^{ij}_{(1)}$ is the inverse of the Kähler metric.

One can also explicitly compute the coupling matrix defined in (A2) which we denote by $\mathcal{N}$. The components of $\mathcal{N}$ together with the ones of $[\mathrm{Im}(\mathcal{N})]^{-1}$ are given by

$$\mathrm{Re}(\mathcal{N}_{ij}) = -d^{(Y)}_{ijk} b^k, \qquad \mathrm{Im}(\mathcal{N}_{ij}) = -4\mathcal{V} h^{(1)}_{ij}, \qquad [\mathrm{Im}(\mathcal{N})]^{-1ij} = -\frac{h^{ij}_{(1)}}{4\mathcal{V}} - \frac{b^i b^j}{\mathcal{V}}, \qquad \mathrm{Re}(\mathcal{N}_{i0}) = \frac{1}{2} d^{(Y)}_{ijk} b^j b^k,$$
$$\mathrm{Im}(\mathcal{N}_{i0}) = 4\mathcal{V} h^{(1)}_{ij} b^j, \qquad [\mathrm{Im}(\mathcal{N})]^{-1i0} = -\frac{b^i}{\mathcal{V}}, \qquad \mathrm{Re}(\mathcal{N}_{00}) = -\frac{1}{3} d^{(Y)}_{ijk} b^i b^j b^k, \tag{A8}$$
$$\mathrm{Im}(\mathcal{N}_{00}) = -\mathcal{V} - 4\mathcal{V} h^{(1)}_{ij} b^i b^j, \qquad [\mathrm{Im}(\mathcal{N})]^{-100} = -\frac{1}{\mathcal{V}}.$$

Let us now pass to the complex structure moduli space of the same Calabi-Yau manifold Y which has dimension $n = h^{2,1}(Y)$. We denote the projective coordinates on this moduli space by Z^A, where $A, B, \ldots = 0, \ldots, h^{2,1}(Y)$ and also introduce lowercase indices $a, b, \ldots = 1, \ldots, h^{2,1}(Y)$. The prepotential is called $\mathcal{G}$. In general, an explicit expression for this prepotential cannot be written down. However, one can still derive some useful formulas when working with a generic $\mathcal{G}$. Most of the properties of this space can be described in terms of the holomorphic (3, 0)-form Ω. Recall that in a real, symplectic basis (α_A, β^B) of three-forms it can be expanded as

$$\Omega = Z^A \alpha_A - \mathcal{G}_A \beta^A. \tag{A9}$$

It follows immediately that the Kähler potential can be written as

$$K^{(2)} = -\ln[i(\bar{Z}^A \mathcal{G}_A - Z^A \bar{\mathcal{G}}_A)] = -\ln\left(i \int_Y \Omega \wedge \bar{\Omega}\right). \tag{A10}$$

Let us here denote the coupling matrix (A2) by $\mathcal{M}$. It turns out that, for the complex structure moduli space, this matrix has a proper geometric interpretation in terms of the integrals

$$B_{AB} = \int_{Y_3} \alpha_A \wedge *\alpha_B = \int_{Y_3} \alpha_B \wedge *\alpha_A = B_{BA},$$
$$C^{AB} = -\int_{Y_3} \beta^A \wedge *\beta^B = -\int_{Y_3} \beta^B \wedge *\beta^A = C^{BA}, \tag{A11}$$
$$A_A{}^B = -\int_{Y_3} \beta^B \wedge *\alpha_A = -\int_{Y_3} \alpha_A \wedge *\beta^B,$$

which can be expressed as [87,88]

$$A = \mathrm{Re}(\mathcal{M})[\mathrm{Im}(\mathcal{M})]^{-1},$$
$$B = -\mathrm{Im}(\mathcal{M}) - \mathrm{Re}(\mathcal{M})[\mathrm{Im}(\mathcal{M})]^{-1}\mathrm{Re}(\mathcal{M}), \tag{A12}$$
$$C = [\mathrm{Im}(\mathcal{M})]^{-1}.$$

A particularly useful insight can be obtained by choosing a different basis for the third cohomology of Y. One can define complex (2, 1)-forms χ_a via Kodaira's formula [89]

$$\frac{\partial \Omega}{\partial z^a} = -K^{(2)}_a \Omega + \chi_a, \tag{A13}$$

where $z^a = Z^a/Z^0$ are the affine coordinates and $K^{(2)}_a$ denote the derivatives of the complex structure Kähler potential (A10) with respect to z^a. Then the forms $(\Omega, \chi_a, \bar{\chi}_a, \bar{\Omega})$ form a basis for the third cohomology of Y. In this new basis, the metric $h^{(2)}_{ab}$ on the complex structure moduli space has the simple form

$$h^{(2)}_{a\bar{b}} \equiv \partial_a \partial_{\bar{b}} K^{(2)} = -\frac{\int_{\hat{Y}} \chi_a \wedge \bar{\chi}_{\bar{b}}}{\int_{\hat{Y}} \Omega \wedge \bar{\Omega}}. \tag{A14}$$

The transformation from the symplectic basis (α_A, β^A) to the complex basis defined above can be summarized as

$$\beta^A = \tilde{f}^A \Omega + \tilde{f}^{Aa} \chi_a + \mathrm{h.c.},$$
$$\alpha^A = f_A \Omega + f_A{}^a \chi_a + \mathrm{h.c.}, \tag{A15}$$

where

$$\tilde{f}^A = -\frac{\bar{Z}^A}{\int_Y \Omega \wedge \bar{\Omega}},$$
$$\tilde{f}^{Aa} = \frac{h^{a\bar{b}}_{(2)}}{\int_Y \Omega \wedge \bar{\Omega}} \bar{D}_{\bar{b}} \bar{Z}^A,$$
$$f_A = -\frac{\bar{\mathcal{G}}_A}{\int_Y \Omega \wedge \bar{\Omega}}, \tag{A16}$$
$$f_A{}^a = \frac{h^{a\bar{b}}_{(2)}}{\int_Y \Omega \wedge \bar{\Omega}} \bar{D}_{\bar{b}} \bar{\mathcal{G}}_A,$$

and by $h^{a\bar{b}}_{(2)}$ we denote the inverse of the metric (A14). The Kähler-covariant derivatives D are defined by

$$\bar{D}_{\bar{b}} \bar{Z}^A = \partial_{\bar{b}} \bar{Z}^A + \overline{K}^{(2)}_{\bar{b}} \bar{Z}^A, \qquad \bar{D}_{\bar{b}} \bar{\mathcal{G}}_A = \partial_{\bar{b}} \bar{\mathcal{G}}_A + \overline{K}^{(2)}_{\bar{b}} \bar{\mathcal{G}}_A. \tag{A17}$$

Until now all the formulas for the complex structure moduli space were generic and can be applied to any Calabi-Yau manifold. However, in the limit of *large complex structures* one can be somewhat more explicit. For this we rely on mirror symmetry which relates the complex structure deformations of the Calabi-Yau manifold Y to Kähler deformations on the mirror X. As a result, the prepotential $\mathcal{G}$ is now given by a cubic formula similar to Eq. (A3), that is,

$$\mathcal{G} = -\frac{1}{6} \frac{d^{(X)}_{abc} Z^a Z^b Z^c}{Z^0}. \tag{A18}$$

Here, $d^{(X)}_{abc}$ are the triple intersection numbers of the mirror Calabi-Yau manifold X. The matrix $\mathcal{M}$ can be computed explicitly in this limit and is given by

$$\mathrm{Re}(\mathcal{M}_{00}) = -\frac{1}{24} d^{(X)}_{abc}(Z^a + \bar{Z}^a)(Z^b + \bar{Z}^b)(Z^c + \bar{Z}^c), \qquad \mathrm{Im}(\mathcal{M}_{00}) = -\frac{e^{-K^{(2)}}}{8}[1 + h^{(2)}_{ab}(Z^a + \bar{Z}^a)(Z^b + \bar{Z}^b)],$$
$$\mathrm{Re}(\mathcal{M}_{a0}) = \frac{1}{8} d^{(X)}_{abc}(Z^b + \bar{Z}^b)(Z^c + \bar{Z}^c), \qquad \mathrm{Im}(\mathcal{M}_{a0}) = \frac{e^{-K^{(2)}}}{4} h^{(2)}_{ab}(Z^b + \bar{Z}^b), \tag{A19}$$
$$\mathrm{Re}(\mathcal{M}_{ab}) = -\frac{1}{2} d^{(X)}_{abc}(Z^c + \bar{Z}^c), \qquad \mathrm{Im}(\mathcal{M}_{ab}) = -\frac{e^{-K^{(2)}}}{2} h^{(2)}_{ab}.$$

The components of $[\mathrm{Im}(\mathcal{M})]^{-1}$ read

$$[\mathrm{Im}(\mathcal{M})]^{-1ab} = -2e^{K^{(2)}}[h^{ab}_{(2)} + (Z^a + \bar{Z}^a)(Z^b + \bar{Z}^b)],$$
$$[\mathrm{Im}(\mathcal{M})]^{-1a0} = -4e^{K^{(2)}}(Z^a + \bar{Z}^a), \tag{A20}$$
$$[\mathrm{Im}(\mathcal{M})]^{-100} = -8e^{K^{(2)}}.$$

As a simple application of the above formulas and as a warm-up for the next section we can rewrite the potential [5], obtained by turning on H fluxes in Calabi-Yau compactifications, in a more suggestive way which makes it easier to read off the superpotential. As in Eq. (3.30), the H flux[4]

$$H = \epsilon_A \beta^A + \mu^A \alpha_A. \tag{A21}$$

can be expanded in terms of the symplectic basis (α_A, β^B). With Eqs. (A11), this potential can be written as

$$e^{-K}V_H = 4e^{-K^{(2)}}\int H \wedge *H$$
$$= -4e^{-K^{(2)}}(\epsilon_A + \mu^C\mathcal{M}_{AC})\mathrm{Im}\mathcal{M}^{-1AB} \times (\epsilon_B + \mu^D\bar{\mathcal{M}}_{BD}). \tag{A22}$$

On the other hand, writing the H flux in the complex basis defined in (A15) the above formula reads

$$e^{-K}V_H = 8e^{-K^{(2)}}(\epsilon_A\tilde{f}^{Aa} + \mu^A f_A{}^a)(\epsilon_B\bar{\tilde{f}}^{Bb} + \mu^B\bar{f}_B{}^b) \times \int \chi_a \wedge *\bar{\chi}_b + 8e^{-K^{(2)}}|\epsilon_A\tilde{f}^A + \mu^A f_A|^2 \times \int \Omega \wedge *\bar{\Omega}. \tag{A23}$$

Inserting the relations (A16) and using (A14) we obtain

$$e^{-K}V_H = 8h^{ab}_{(2)}(\epsilon_A D_a Z^A + \mu^A D_a\mathcal{G}_A)(\epsilon_B\bar{D}_b\bar{Z}^B + \mu^B\bar{D}_b\bar{\mathcal{G}}_B) + 8|\epsilon_A Z^A + \mu^A\mathcal{G}_A|^2. \tag{A24}$$

Thus we can write

$$V_H = h^{a\bar{b}}_{(2)}(D_a W_H)(\overline{D_b W_H}) + |W_H|^2, \tag{A25}$$

where we have defined

$$W_H = \sqrt{8}(\epsilon_A Z^A + \mu^A\mathcal{G}_A). \tag{A26}$$

Let us stress that, at this level, Eq. (A25) does not yet have the structure of the usual supergravity relation (3.19) between the potential and the superpotential since the term $-3|W|^2$ is not correctly reproduced. However, for heterotic strings compactified on Calabi-Yau manifolds in the presence of H fluxes also the dilaton and the Kähler moduli have to be included in calculating the potential. Their contribution is precisely $4|W_H|^2$ which cancels against $-3|W|^2$ leaving behind precisely the factor $|W_H|^2$ present in (A25).

APPENDIX B: SUPERPOTENTIAL INCLUDING NS-NS FLUX

Having defined all the technical tools in the previous section, we are now ready to show that the scalar potential in Eq. (3.34) can be indeed obtained from the superpotential (3.35) using the general supergravity formula (3.19). To do this it will be useful to replace $\tilde{\epsilon}_0 = e_i b^i$ in the potential (3.34) and pull apart the contributions to the potential coming from the torsion of the half-flat mirror manifold and the one coming from the H flux, writing the potential as

$$V = V_T + V_H + V_{\mathrm{mix}}. \tag{B1}$$

Here, V_T arises from the torsion of the internal manifold, V_H is due to H flux, and V_{mix} is the mixed term which is present when both are taken into account simultaneously. Explicitly, these parts are given by

$$V_T = -2e^{2\phi+K^{(2)}}e_i[\mathrm{Im}(\mathcal{N})]^{-1ij}e_j,$$
$$V_H = -2e^{2\phi+K^{(1)}}(\epsilon_A + \mu^C\mathcal{M}_{AC})[\mathrm{Im}(\mathcal{M})]^{-1AB} \times (\epsilon_B + \mu^D\bar{\mathcal{M}}_{BD}),$$
$$V_{\mathrm{mix}} = -4(e_i b^i)e^{2\phi+K^{(1)}}\{\epsilon_a[\mathrm{Im}(\mathcal{M})]^{-10a} + \mu^c[\mathrm{Im}(\mathcal{M})]^{-10A}\mathrm{Re}\mathcal{M}_{Ac}\}, \tag{B2}$$

where, in the second equation, we have used the convention

$$\epsilon_A = (0, \epsilon_a), \qquad \mu_A = (0, \mu_a). \tag{B3}$$

Taking into account Eq. (A8) one finds that V_T defined above is precisely the potential obtained in Eq. (3.8) while V_H is the potential we have discussed in Eq. (A22).

[4]Unlike in the main part of the paper, ϵ_A and μ^A denote arbitrary flux parameters, that is, we allow $\epsilon_0 \neq 0$ and $\mu^0 \neq 0$.

Let us split the superpotential in its two main pieces

$$W = W_T + W_H, \tag{B4}$$

where W_T was defined in Eq. (3.21) and W_H is taken from Eq. (A26) with the specific flux parameters (B3) inserted. Note that W_T depends only on the Kähler moduli T^i while W_H depends only on the complex structure moduli Z^a. We would like to reproduce the potential (B1) by inserting this superpotential, as well as the standard Kähler potential (3.17), into the general supergravity formula (3.19). We start by evaluating the Kähler-covariant derivatives which now read

$$\begin{aligned} D_S W &= -\frac{1}{2} e^{\phi} W, \\ D_i W &= \sqrt{8} e_i + K_i^{(1)} W, \\ D_a W &= D_a W_H + K_a^{(2)} W_T. \end{aligned} \tag{B5}$$

Using the Kähler metric $h_{ij}^{(1)}$ in terms of the fields t^i, to be derived from the Kähler potential (A4), we obtain

$$\begin{aligned} &D_S W (\overline{D_S W}) K^{\bar{S}S} = |W|^2 \qquad D_i W (\overline{D_j W}) K^{\bar{j}i} = 8 e_i e_j h_{(1)}^{ij} - 2i\sqrt{8}(e_i v^i)(W - \bar{W}) + 3|W|^2 \\ &D_a W (\overline{D_b W}) K^{\bar{b}a} = D_a W_H (\overline{D_b W_H}) h_{(2)}^{\bar{b}a} + (K_a^{(2)} \overline{K}_b^{(2)} h_{(2)}^{\bar{b}a}) |W_T|^2 + [D_a W_H (\overline{K_b^{(2)} W_T}) h_{(2)}^{\bar{b}a} + \text{h.c.}], \end{aligned} \tag{B6}$$

where we have used

$$K_i^{(1)} \overline{K}_j^{(1)} h_{(1)}^{ij} = 3, \tag{B7}$$

which follows for the cubic prepotential (A3).

In the large complex structure limit, the prepotential for the complex structure moduli space is given by (A18) and thus, in analogy with the Kähler moduli space, we have $K_a^{(2)} (\overline{K}_b^{(2)}) h_{(2)}^{\bar{b}a} = 3$. Using this relation and further splitting W as in Eq. (B4) we obtain from the supergravity formula (3.19)

$$\begin{aligned} e^{-K} V = {}& 8 e_i e_j h_{(1)}^{ij} - 32 (e_i v^i)^2 + 4|W_T|^2 \\ &+ D_a W_H (\overline{D_b W_H}) h_{(2)}^{\bar{b}a} + |W_H|^2 \\ &+ (W_T - \bar{W}_T)(W_H - \bar{W}_H) + 4(W_T \bar{W}_H + \bar{W}_T W_H) \\ &+ [\partial_a W_H (\overline{K}_b^{(2)}) h_{(2)}^{\bar{b}a} \bar{W}_T + \text{h.c.}]. \end{aligned} \tag{B8}$$

It was shown in the main part of the paper that the first line in the above equation reproduces the potential V_T from Eq. (B2) while we have proved in Eq. (A25) that the second line gives rise to the potential V_H.

In order to evaluate the mixed terms in the last line which contain half-flat as well as H flux, we need to compute the expressions $\partial_a W_H (\overline{K}_b^{(2)}) h_{(2)}^{\bar{b}a}$. In general this is a complicated task, but in our case, as we work in the large complex structure limit, this computation is fairly easy. First of all note that in this case we can derive a formula similar to Eq. (A7), namely,

$$h_{(2)}^{a\bar{b}} (\overline{K}_b^{(2)}) = -(z^a - \bar{z}^a). \tag{B9}$$

Then, making explicit use of the cubic formula for the prepotential $\mathcal{G}$, Eq. (A18), one can rewrite the potential (B8) into the following form

$$\begin{aligned} V = {}& V_T + V_H + 32(e_i b^i) e^K \epsilon_a (Z^a + \bar{Z}^a) \\ &+ 8 e^K \{ e_i \bar{T}^i \mu^a [\bar{\mathcal{G}}_a + 3\mathcal{G}_a + (\bar{Z}^b - Z^b) \mathcal{G}_{ba}] + h.c. \}. \end{aligned} \tag{B10}$$

Working from the other end, we now rewrite the mixed part of the potential V_{mix} in Eq. (B2), obtained from the reduction, by using the explicit form of the matrix $\mathcal{M}$ given in Eq. (A19). One can easily show that

$$\begin{aligned} [\text{Im}(\mathcal{M})]^{-10A} \text{Re}(\mathcal{M})_{Ac} = {}& -2 e^{K^{(2)}} [\bar{\mathcal{G}}_a + 3\mathcal{G}_a \\ &+ (\bar{Z}^b - Z^b) \mathcal{G}_{ba}]. \end{aligned} \tag{B11}$$

Inserting this relation into Eq. (B2) and using the expression for $[\text{Im}(\mathcal{M})]^{-10a}$ from (A20) one obtains

$$\begin{aligned} V_{\text{mix}} = {}& 32(e_i b^i) e^K \epsilon_a (Z^a + \bar{Z}^a) \\ &+ 8 e^K \{ e_i \bar{T}^i \mu^a [\bar{\mathcal{G}}_a + 3\mathcal{G}_a + (\bar{Z}^b - Z^b) \mathcal{G}_{ba}] + h.c. \}. \end{aligned} \tag{B12}$$

Comparing the result (B10) from the supergravity side with the reduction result (B1), where V_{mix} is given by the formula above, we see that the two potentials are indeed the same. This proves that the potential obtained by compactifying the heterotic string on half-flat mirror manifolds with H flux (3.34), can be obtained from the $N = 1$ supergravity formula (3.19) with the superpotential given by (3.35).

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