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Framework for the string theory landscape

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It seems likely that string theory has a landscape of vacua that includes very many metastable de Sitter spaces. However, as emphasized by Banks, Dine, and Gorbatov, no current framework exists for examining these metastable vacua in string theory. In this paper we attempt to correct this situation by introducing an eternally inflating background in which the entire collection of accelerating cosmologies is present as intermediate states. The background is a classical solution which consists of a bubble of zero cosmological constant inside de Sitter space, separated by a domain wall. At early and late times the flat space region becomes infinitely big, so an S-matrix can be defined. Quantum mechanically, the system can tunnel to an intermediate state which is pure de Sitter space. We present evidence that a string theory S-matrix makes sense in this background, and that it contains metastable de Sitter space as an intermediate state.

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I. INTRODUCTION

For string theorists, the importance of making connections with cosmology is self evident. It would be disappointing to find that a consistent quantum theory of gravity has nothing to say about the quantum origin of the universe. Over the last decade the lessons from both string theory and black hole physics have dramatically changed the way we think about space and time *without* seriously changing the way we think about the universe.

However, because of the recent awareness of a large and diverse Landscape of metastable de Sitter vacua [1], the situation may be changing. A cosmology combining the string-theoretic Landscape with the ideas of eternal inflation may hold the key to a number of cosmological puzzles such as the peculiar fine tuning of the cosmological constant.

In order to give a reasonably rigorous basis to these ideas it is important to find a framework for studying eternal inflation which is capable of being adapted to string theory. Thus far, string theory requires the existence of an asymptotic boundary on which some kind of S-matrix data can be defined. The S-matrix formulation in flat space is familiar. Anti-de Sitter space provides another space with an asymptotic boundary description. In both cases the asymptotic boundary is infinite in area and allows particles to separate and propagate freely.

By contrast, de Sitter space does not allow this kind of asymptotic description. The space-like asymptotic boundaries of de Sitter space are problematic. This is particularly so because the de Sitter vacua of string theory are at best metastable [2]. This means that if we follow any time-like path through the geometry we eventually will find the de Sitter vacuum decaying. The future time-like boundary is not well described by classical eternal de Sitter space. In particular, tunneling transitions can occur, creating expanding bubbles of other vacua. Typically the bubble nucleation rate is very small and the inflation rate in the surrounding vacuum is very large. The result is that the space between bubbles inflates so fast that each bubble remains isolated beyond the horizon of any other bubble. This is the phenomenon of *eternal inflation* [3] (see [4] for a review and references).

If the interior of a bubble is a vacuum with a positive vacuum energy, the space inside the bubble will also inflate, albeit more slowly than the parent space. The expected result is that the bubble interiors are themselves eternally inflating regions spawning more bubbles. Following a given time-like curve in this bubble bath, an observer will see a sequence of vacua which can only end when there is no longer a positive cosmological constant. This can happen in two ways. The first is that the observer is swallowed by a region of negative cosmological constant. This inevitably leads to a big crunch. Singularities of this kind may ultimately be resolved or avoided in quantum gravity, but regardless there is another kind of endpoint with a more optimistic outlook: a bubble of supersymmetric vacuum with exactly vanishing cosmological constant can end the sequence. In fact some observers will exit onto the supersymmetric moduli space while others crash into singularities.

Let us suppose a bubble forms in which the cosmological constant is positive but smaller than in the background. The bubble nucleation is quantum mechanical and cannot be entirely represented by a classical history, but once the bubble forms it evolves in a classical way.

The process can be illustrated diagrammatically. Let us begin with the Penrose diagram for pure de Sitter space in Fig. 1(a). We find it helpful to visualize the $1 + 1$ dimensional case and to draw the entire space-time as a conformal diagram as in Fig. 1(b).

The bubble evolution is shown in Fig. 2(a). The straight horizontal lower boundary of the bubble represents the quantum tunneling event. The space-like upper edge of the bubble is the asymptotic future boundary of the inflating space in the bubble. If, however, the space in the bubble has negative cosmological constant then there will be a future big crunch singularity. Most interesting

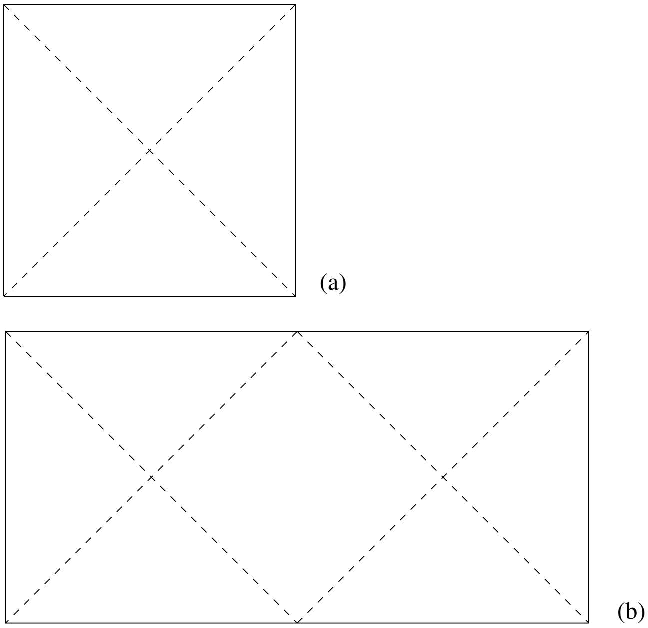

FIG. 1. On the left, the Penrose diagram for de Sitter space. On the right, the conformal diagram for $1 + 1$ dimensional de Sitter space.

for us is the case where the interior of the bubble is a supersymmetric vacuum with vanishing cosmological constant. Then the diagram looks like Fig. 2(b). The future boundary is a portion of the light-like future infinity of Minkowski space. We will call it a "hat." Only in this case can we describe the future in terms of well separated asymptotically free particles. That is key to adapting the method to string theory.

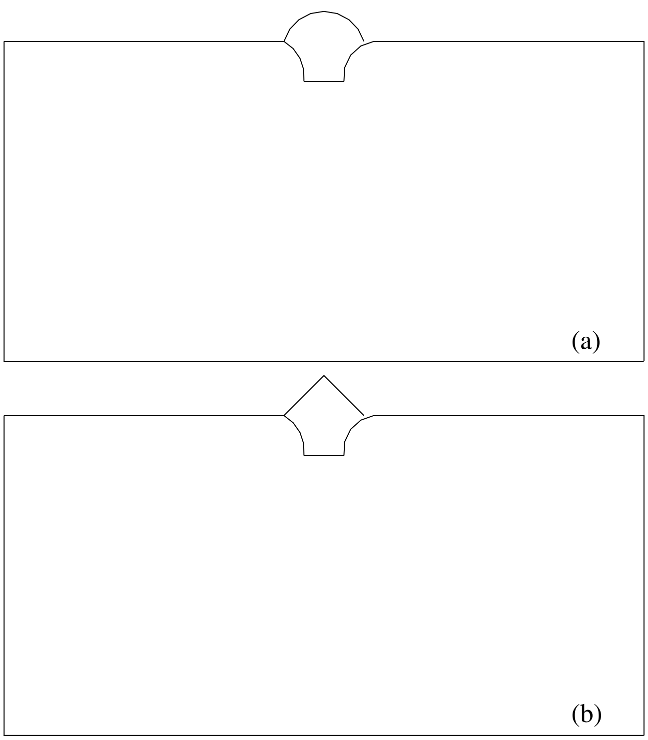

FIG. 2. On the left, a bubble of smaller cosmological constant forming inside de Sitter space. On the right, a bubble of flat space forming inside de Sitter space.

The true asymptotic future will have an infinity of bubbles forming a fractal set including singularities and hats. The later the bubble forms, the smaller it will appear on the diagram. The region near the upper edge of the Penrose diagram is populated with an infinity of bubbles of all different kinds, separated from their parent bubble by domain walls. We will refer to the domain walls as branes. In the case of four space-time dimensions the branes are 2-branes or membranes. A curious property of the fractal future is that the coordinate volume is completely swallowed up by bubbles although the proper volume of space continues to be dominated by undecayed de Sitter space. To be more precise, the expected coordinate volume left in de Sitter space is zero.

At present there is no string theory framework or holographic framework in which the ideas can be tested. String theory as we now know it relies very heavily on the existence of an asymptotic boundary of space-time. The nature of this boundary dictates the nature of the holographic degrees of freedom as well as the physical observables. In asymptotically flat space the boundary at infinity defines the scattering states of string theory. In anti-de Sitter space the time-like boundary also permits well-defined boundary data.

By contrast, de Sitter space does not have a clear boundary description. Most likely eternal de Sitter space is not possible and in any case there are good reasons to believe that string-theoretic de Sitter vacua are at best metastable, with a lifetime shorter than the recurrence time. This means that the past and future boundaries of de Sitter space have to be replaced by a quantum fractal of hats and crunches. How exactly string theory can accommodate this is far from obvious.

In this paper we suggest a framework for this purpose. We will explore the existence of a background geometry, with two key properties.

Property one is that there should be asymptotic past and future boundaries on which the geometry tends to infinite flat space. This allows us to formulate asymptotic states and an S-matrix.

Property two is that the metastable de Sitter vacua should occur as intermediate resonant states in the S-matrix.

II. SOLUTIONS INTERPOLATING BETWEEN $\Lambda = 0$ AND $\Lambda > 0$

Let us begin with a simplified model including gravity and a single scalar field ϕ. The scalar potential is assumed to have two minima, one at $\phi = 0$ with a positive value of the vacuum energy and one with vanishing energy density. The zero energy minimum can be at finite

ϕ or infinite ϕ. For definiteness we will take the case of infinite ϕ, so the potential looks like Fig. 3.

If we ignore the effects of gravity then there are spatially homogeneous solutions of the scalar field equations that closely resemble the "S-brane" solutions of open string field theory. The geometric background is flat space-time and the solutions begin and end at $\phi = \infty$ where the potential vanishes. The solutions climb part way up the potential and then roll back down in a time-symmetric way.

Suppose that the initial kinetic energy of the scalar was close to the value of the potential at $\phi = 0$. In that case one might wonder if the field can tunnel through to the minimum with positive energy. If so we would have just the situation that we are looking for: an asymptotic vacuum with a vanishing cosmological constant making a transition to a de Sitter space. If the tunneling rate is finite the vacuum should eventually tunnel back out of the potential well and roll back to $\Lambda = 0$.

Unfortunately this is not possible. Tunneling cannot happen in a homogeneous way. If space is infinite, the zero mode of the field has infinite "mass." It behaves like a completely classical coordinate.

A possible way out of this no-go situation is to replace the minimum at $\phi = 0$ by a broad flat region and eliminate the barrier. The flat region could support a de Sitter space, if not forever than at least for a long period. Eternal inflation is also possible. In this case the de Sitter-like state would decay locally into regions that would indefinitely grow, tending toward the vacuum at $\phi = \infty$. However if the inflation rate in the undecayed regions is larger than the decay rate, the region in false vacuum expands faster than it gets eaten up. The true vacuum regions remain isolated islands in an inflating sea. Trying to formulate string theory in this background has all of the same problems of string theory in de Sitter space. One *could* imagine formulating string theory in the true vacuum regions, but they are produced in a chaotic, unpredictable way. There is no simple classical solution in this case which contains true vacuum regions.

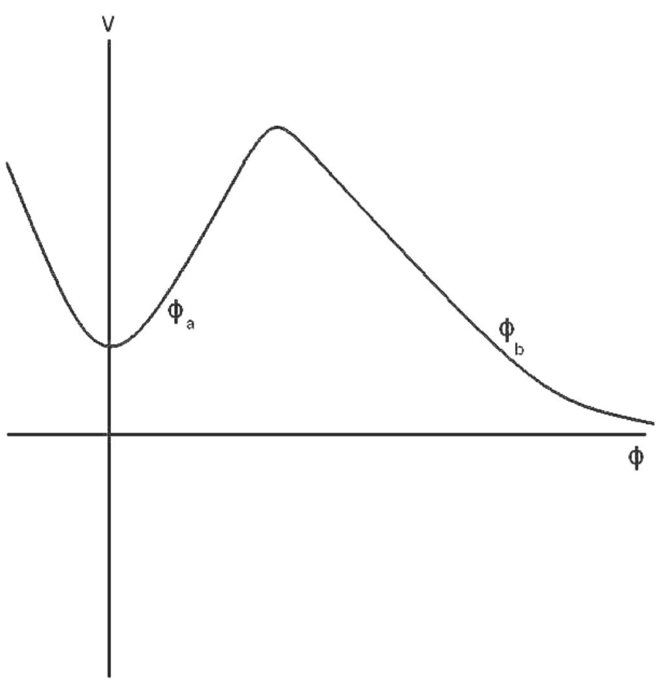

FIG. 3. A potential which has a metastable minimum with positive energy and a stable minimum at infinity with zero energy.

A. S-Branes

An interesting possibility is that space-filling unstable D-branes or brane-antibrane systems might serve as a starting point for eternal inflation. We believe this is an interesting avenue to explore but we will find that non-perturbative methods are required. As an illustration, consider a system of N unstable D9-branes in type IIa string theory. The basic requirement for eternal inflation is that the rate for the open string tachyon to fall off the top of its potential should be smaller than the inflation rate at the top of the potential.

Assuming we can ignore corrections, the rate for the tachyon to fall, call it γ, is order one in string units.

$$\gamma \sim \frac{1}{l_s} \tag{2.1}$$

where l_s is the string length scale.

Let us compare (2.1) with the inflation rate at the top of the tachyon potential. The energy density of a D9-brane is $1/g$ in string units. Thus for a stack of N branes the energy density is

$$\epsilon = \frac{N}{g l_s^{10}}, \tag{2.2}$$

where g is the closed string coupling constant. The Hubble expansion rate is

$$H \sim \sqrt{\Lambda} \sim \sqrt{G\epsilon}.$$

The 10-dimensional Newton constant G is given by $G = g^2 l_s^8$ giving

$$H = \frac{1}{l_s}\sqrt{Ng}. \tag{2.3}$$

Thus the figure of merit, H/γ, is of order

$$\frac{H}{\gamma} \sim \sqrt{Ng}. \tag{2.4}$$

Evidently the ratio of expansion rate to tachyon decay rate is proportional to the 't Hooft coupling constant for the open string theory on the branes. It is clear that to push the system into the regime of eternal inflation the 't Hooft coupling must be at least of order 1.

Things are not as bad as they could be. The open string theory must be nonperturbative to exhibit eternal inflation but the closed string coupling can be arbitrarily small. In other words eternal inflation may occur in the 't Hooft limit

$$N \to \infty, \qquad g \to 0, \qquad Ng \to 1, \tag{2.5}$$

without any need for closed string quantum corrections.

Although we do not know how to track the system to strong coupling it is interesting to extrapolate some formulas from weak 't Hooft coupling to $gN = 1$.

First consider the Hawking temperature at the top of the potential. In de Sitter space the Hawking temperature is of order the Hubble constant. From (2.3) we see that the temperature at $gN = 1$ is

$$T \sim \frac{1}{l_s}.$$

In other words the Hawking temperature at the onset of eternal inflation is the Hagedorn temperature. Perhaps this was to be expected.

The entropy of a de Sitter horizon is the usual $A/4G$, where A is the eight-dimensional area. The area is of order

$$A \sim \frac{1}{H^8}.$$

Thus the entropy is

$$S \sim \frac{1}{g^6 N^4} = \frac{N^2}{(gN)^6}. \tag{2.6}$$

For $gN = 1$ we get the interesting result that the de Sitter entropy is just the square of the number of branes,

$$S \sim N^2. \tag{2.7}$$

This is very suggestive since the fields of the open string theory are in the adjoint representation of SU(N). Perhaps there is a matrix description of a single causal patch. However it is not likely that the excited open string states decouple in the 't Hooft limit. Unlike the case of a system of BPS branes, there is no reason to expect a decreasing energy scale as N grows. One reasonable possibility is that the causal patch is approximately described by the lowest string modes while it is inflating but as it decays the higher string modes become increasingly important. Unfortunately we do not have the tools to pursue this further but it is obviously a worthwhile direction for the future.

B. The Bounce Background

The discussion of the previous section does not in itself help us find a string theory background with the properties discussed in section I, namely, that it permits a description of asymptotic states composed of freely moving, well-separated particles, and that it includes metastable de Sitter vacua as intermediate states. The homogeneous classical solutions of the coupled gravity-scalar field equations generally have space-like singularities either in the past or the future.

Our strategy for finding solutions will be to begin with Euclidean solutions of the coupled gravity-scalar equations and to continue them to the Minkowski signature.

The Euclidean solutions we will consider are Euclidean versions of Friedmann-Robertson-Walker (FRW) geometries. They have the form

$$ds^2 = c^2 dy^2 + a(y)^2 d\Omega_{D-1}^2, \qquad \phi = \phi(y), \tag{2.8}$$

where c is a constant and the number of space-time dimensions is D.

The variable y runs over a finite range,

$$0 \le y \le \pi. \tag{2.9}$$

The equations of motion are the usual FRW equations apart from some changes of sign. In four dimensions they are:

$$\begin{aligned} \dot a^2/a^2 = H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot\phi^2 - V(\phi)\right] + \frac{1}{a^2}, \\ \ddot\phi = -3H\dot\phi + \partial_\phi V(\phi). \end{aligned} \tag{2.10}$$

The topology of such solutions is that of the D-sphere but the symmetry is only that of the $D-1$ sphere, $O(D)$. We are interested in solutions where the scalar ϕ is inside the well at the north pole of the sphere, $y = 0$, and outside the well at the south pole, $y = \pi$. Smoothness at the poles $y = 0, \pi$ requires boundary conditions,

$$\begin{aligned} a \to cy \quad (y = 0), a \to c(\pi - y) \\ (y = \pi), \partial_y\phi = 0 \quad (y = 0, \pi). \end{aligned} \tag{2.11}$$

The Eqs. (2.10) and (2.11) are the Coleman-de Luccia equations [5] for the instanton that governs the decay of a metastable de Sitter space. However at the moment we are interested in these equations for another purpose. We will analytically continue the solution to Minkowski signature to give a classical background.

In order to facilitate the continuation we will write the metric of the $D-1$ sphere in a form that emphasizes the U(1) rotational symmetry under a particular U(1) in $O(D-1)$. As an illustration we work with the case $D = 4$.

$$d\Omega_3^2 = d\alpha^2 + (\sin^2\alpha)d\beta^2 + (\sin^2\alpha\sin^2\beta)d\theta^2, \tag{2.12}$$

where α and β have range $(0, \pi)$ and θ has range $(0, 2\pi)$. The U(1) symmetry of interest is $\theta \to \theta + \text{const}$.

The solution (2.8) will be continued by letting $\theta \to$ it:

$$\begin{aligned} ds^2 = c^2dy^2 + a(y)^2[d\alpha^2 + (\sin^2\alpha)d\beta^2 \\ - (\sin^2\alpha\sin^2\beta)dt^2]. \end{aligned} \tag{2.13}$$

The angular variable β now runs from 0 to 2π while the time-like variable t runs from $-\infty$ to $+\infty$. As in the Euclidean solution, the field ϕ depends only on y.

We can rewrite (2.13) in the form

$$ds^2 = c^2 dy^2 + a(y)^2 [d\Omega_{D-2}^2 - (\sin^2\alpha \sin^2\beta) dt^2]. \tag{2.14}$$

To get a feel for the geometry we can fix α and β and study the geometry in the y, t plane. For example, set $\alpha = \beta = \pi$,

$$ds^2 = c^2 dy^2 - a(y)^2 dt^2. \tag{2.15}$$

Now define the conformal coordinate z by

$$cdy/a(y) = dz, \tag{2.16}$$

$$ds^2 = a(y)^2[-dt^2 + dz^2]. \tag{2.17}$$

Since $a(y)$ tends to zero linearly at $y = 0, \pi$ the range of z is infinite, from $z = -\infty$ to $z = +\infty$. The two-dimensional geometry is conformal to the entire z, t plane. Note however that there is no symmetry under $z \to -z$.

We can also draw a Penrose diagram consisting of a diamond as in Fig. 4. In the figure we have superimposed surfaces of constant t. The original rotation symmetry $\theta \to \theta + c$ is now realized as time translation symmetry.

The geometry however is not geodesically complete. The points $z = \pm\infty$ are at a finite distance and the light-like boundaries of the diamond are also not at light-like infinity. Indeed time-like geodesics cross these light-like surfaces at a finite proper time.

To complete the geometry let us look at the vicinity of the point $y = 0$. Letting $\rho = cy$ the metric has the familiar Rindler form,

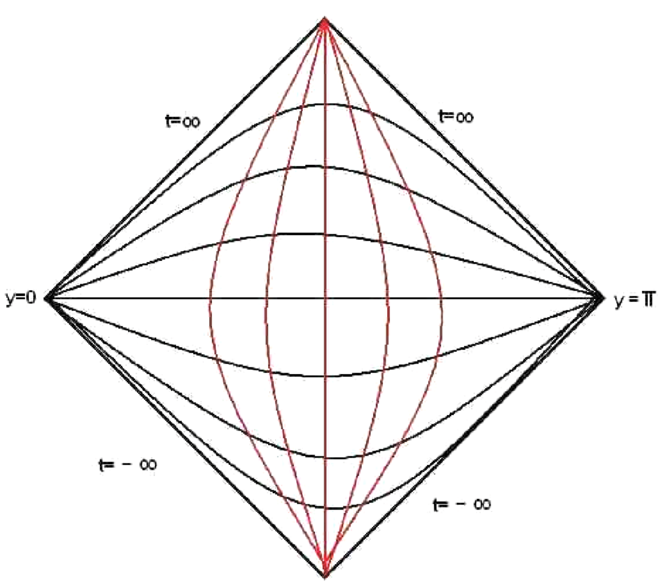

FIG. 4 (color online). Wick rotation of the Euclidean geometry yields a Lorentzian geometry with this Penrose diagram. The geometry is not geodesically complete and can be analytically continued to yield the full Lorentzian solution.

$$ds^2 = -\rho^2 dt^2 + d\rho^2. \tag{2.18}$$

As in the Rindler case, the geometry can be continued past $\rho^2 = 0$, into the region where ρ becomes a time-like coordinate—behind the horizon. We can continue ρ^2 from positive values (region I) to negative values in two ways: either toward the future (into region IV) or the past (region V). In region I the original θ shift symmetry became a time-like translation symmetry but in regions IV and V the symmetry reverts back to space-like transformations that act on surfaces of constant (time-like) ρ.

The surfaces of constant ρ on the original Euclidean geometry (2.8) are obviously $D - 1$ spheres. After continuing to the Minkowski signature they become hyperboloids. In region I the hyperboloids have a time-like direction and are of a single sheet. In regions IV and V they are also hyperboloids but they are space-like. The symmetry ensures that the geometry on each space-like hyperboloid is homogeneous with uniform negative curvature. In fact the geometry in these regions has the form of a conventional Minkowski signature FRW cosmological geometry with negatively curved open spatial sections. However in region V the solution is time reversed relative to region IV.

The equation of motion in region IV is FRW but with a reidentification of ρ as FRW time and t as one of the coordinates in the space-like hyperboloids. Recall from (2.11) that at $\rho = 0$,

$$da/d\rho = 1, \qquad d\phi/d\rho = 0. \tag{2.19}$$

These now serve as initial conditions for the FRW evolution.

It is clear that the field in region IV will roll to the point $\phi = 0$ as the FRW geometry evolves. This implies that the FRW cosmology in region IV is asymptotically de Sitter and terminates on a space-like infinitely inflated boundary. Moreover region V is the exact time reverse of region IV.

Everything we have said about the vicinity of $y = 0$ is also true near $y = \pi$. Continuing past this point is facilitated by redefining $\rho = c(\pi - y)$. Continuation now produces two new regions, II (future) and III (past) which evolve as open FRW geometries—region II in the usual sense and region III in the time reversed sense. However the initial conditions for regions II and III are not the same as for regions IV and V. The field starts at ϕ_b on the other side of the barrier. Instead of rolling to $\phi = 0$ it rolls to vanishing cosmological constant. The FRW geometry will be conventional and end with a time-like and light-like infinity. In other words the future boundary of the geometry contains a hat. In Fig. 5(a) we put all these elements together into a single conventional Penrose diagram. Figure 5(b) indicates the values of the field ϕ at various locations.

The interesting thing is that the geometry contains not only a future hat but also a past inverted hat where well

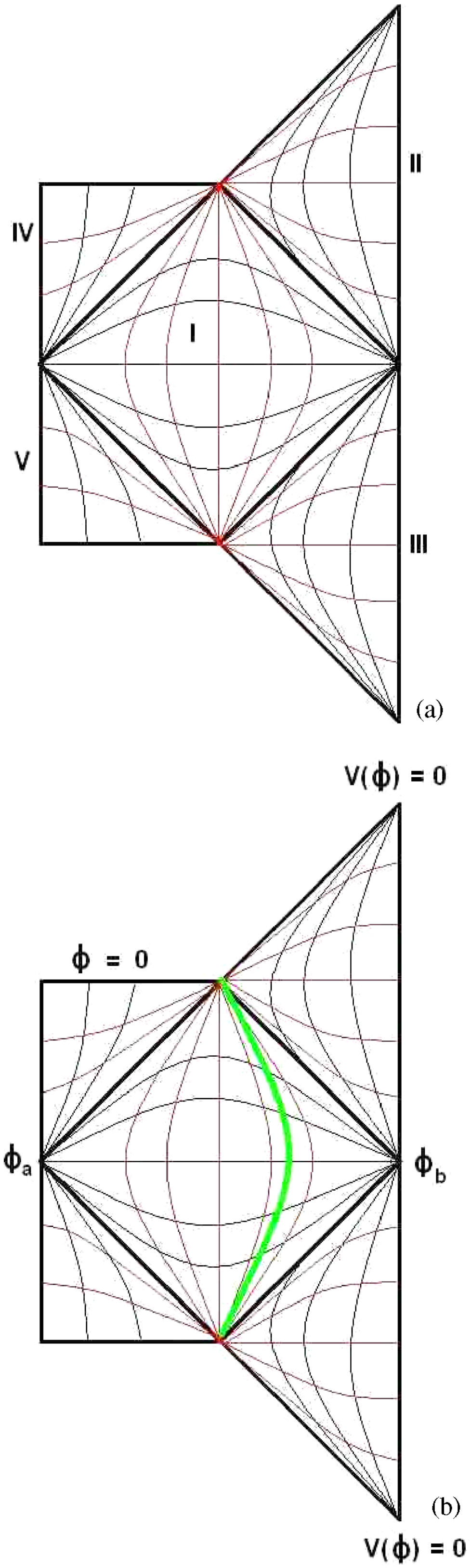

FIG. 5 (color online). On the left, the Penrose diagram for the full Lorentzian geometry. On the right, the value of the scalar field is shown. At the top right of the diagram, the scalar field rolls to its true minimum giving $\Lambda = 0$; at the top left the field is in the false vacuum and $\Lambda > 0$.

separated particles can be injected into the system. The past and future hats will be denoted h^- and h^+.

The region of space where the field is near the top of the barrier defines a domain wall separating the phases where the vacuum energy is near V(0) or near 0. Depending of the details of the potential the domain wall may be thin or thick. In Fig. 5(b) the domain wall is the thick line.

Figure 5 is a conventional Penrose diagram. At each point there is a local $D - 2$ sphere whose radius is a function of y. The radius goes to zero at the two vertical boundaries of the diagram. We will be particularly interested in the observer located on the right boundary. Let us consider the history of such an observer. In the remote past she finds herself in a contracting FRW geometry with the field ϕ rolling up the potential. The potential reaches its maximum value at ϕ_b. At that point the field turns around and the potential starts to decrease toward 0. The entire history of this point closely resembles the behavior of S-branes in open string field theory. In the remote past and future FRW regions the field is homogeneous, but on negatively curved slices and not flat space.

In the frame of the observer, a spherical domain wall initially contracts to a minimum radius and then bounces to become an expanding spherical wall. For this reason we call this solution a "bounce." As we will demonstrate, solutions of this type provide backgrounds that satisfy the two criteria mentioned in the introduction: there are initial and final boundaries where incoming and outgoing free particles can propagate, and de Sitter space is an intermediate resonance in scattering amplitudes.

The bounce geometry is particularly simple in the thin wall approximation. The thin wall geometry was discussed for solutions including the bounce background in [6]. Similar solutions were discussed in [7]. The thin wall limit is applicable when the vacuum energy at $\phi = 0$ is very small. In that case a sharp domain wall separates the geometry into two domains. For $y < y_{dw}$ the field is constant and equal to 0. The space in this region is exactly de Sitter space. For $y > y_{dw}$ the field is at the minimum where the vacuum energy vanishes. The space is flat in this domain. The de Sitter domain includes part of region I and all of regions IV and V. The flat domain includes the remaining part of region I as well as regions II and III. This is illustrated in Fig. 6.

The Euclidean version of the thin wall geometry can be visualized by starting with a D-sphere embedded in $D + 1$ dimensions. The sphere is the Euclidean version of de Sitter space. The flat portion of the space is a D-plane, also embedded in $D + 1$ dimensions. Let the plane intersect the sphere as in Fig. 7(a). The result is the thin wall geometry.

Similarly the Minkowski version begins with a hyperboloid embedded in $D + 1$ dimensional Minkowski space. Intersecting the hyperboloid with a plane as in Fig. 7(b) yields the thin wall bounce geometry.

In the above discussion, we have assumed that both the flat region and the de Sitter region are four-dimensional. The same framework applies if the tunneling to flat space entails the decompactification of some dimensions, so the flat region could be 10- or 11-dimensional.

III. THE CAUSAL PATCH

We begin with the concept of a causal patch of pure classical de Sitter space. Pick a point **F** on the future boundary and a point **P** on the past boundary. Such a pair of points defines a causal patch. Now construct the future light cone of **P** and the past light cone of **F**. The interiors of these light cones define the causal future and causal past of **P** and **F**. The intersection of the causal future of **P** and the causal past of **F** define the causal patch of **F**, **P**. The intersection of the two light cones is the bifurcate horizon of the causal patch and the light cones themselves are the future and past event horizons.

Classical de Sitter space has the symmetry $O(D, 1)$, part of which acts to move the points **P** and **F** to new points. In fact any causal patch of de Sitter Space can be transformed to any other causal patch by means of this symmetry. Evidently the choice of a particular causal patch is a *gauge symmetry* [8]. We will return to this point.

Now consider the quantum version of de Sitter space in which the boundaries are replaced by quantum fractals of crunches and hats in both the past and future. Let us pick a point on the past fractal but not entirely arbitrarily. We choose the point **P** to be the tip of a past hat and the point **F** to be the tip of a future hat. Otherwise the points are arbitrary. We believe that even in this quantum case, the choice of points **P**, **F** should be viewed as a gauge choice. One could in principle choose an observer who ends or begins in a singularity. Such observers should be gauge equivalent to observers who begin and end in flat space, so

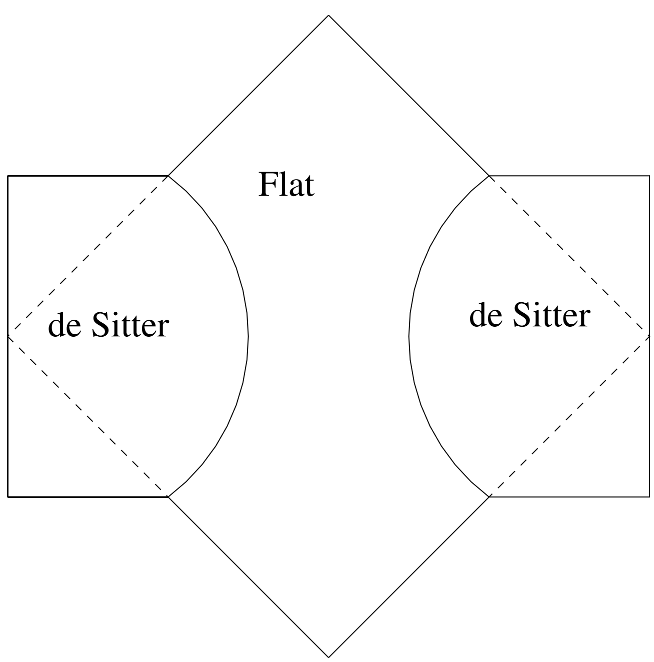

FIG. 6. The thin-wall limit of the geometry consists of flat space separated by a domain wall from de Sitter space. The true Penrose diagram is half of the figure. The diamond-shaped region is the causal patch of an observer at the center.

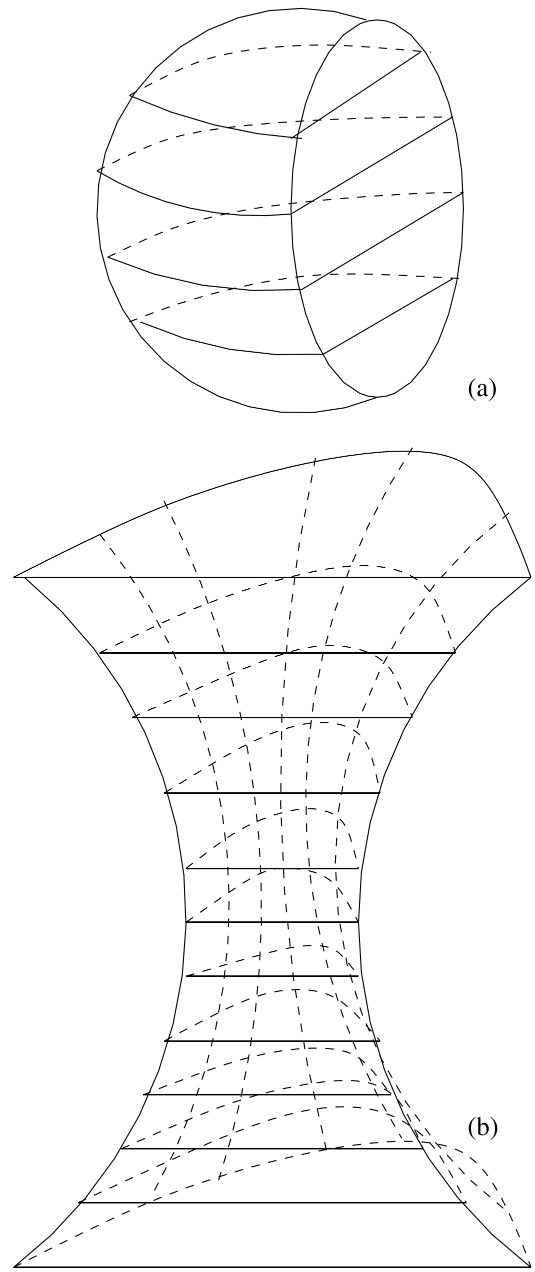

FIG. 7. Both the Euclidean and Lorentzian geometries can be embedded in higher dimensional flat space. On the left, the Euclidean version is the surface of a sphere intersected by a plane. On the right, the Lorentzian version is a hyperboloid intersected by a plane. In both cases, the domain wall is at the boundary between the curved part and the flat part. The cosmological constant jumps across the brane.

we choose to work with the flat space observers because they are simpler.

A special case of this construction is the causal patch of the bounce geometry. The regions II and III each have a point at time-like infinity, as well as light-like infinities. The points at time-like infinity are the tips of the future and past hats. We choose the tips of these hats to be the points **P** and **F**. The causal patch is the diamond-shaped region shown in Fig. 6. Note that the causal patch is partly in the flat domain and partly in the de Sitter domain.

The bifurcate horizon in this case lies at $y = 0$ where the local sphere has vanishing area. The implication of this fact is that the quantum description of the patch should be in terms of pure states rather than the entangled states that characterize de Sitter space or the Schwarzschild black hole. The causal patch can be foliated with space-like surfaces that cover the whole spatial geometry, from $y = 0$ to $y = \pi$, as in Fig. 8.

It may seem surprising that an initial contracting FRW solution does not lead to a singularity. The singularity, if it existed, would be right at the center of the diagram in Fig. 6. The "corners" where the branes end could easily be interpreted as naked singularities which could create infinitely energetic light-like shock waves that collide at the center. But this is not the case. This is made clear by the Euclidean version of the theory. The Minkowski and Euclidean geometries agree along the space-like static surface that divides the diagram in two. Indeed it is possible to define a Hartle-Hawking state on this surface. The origin, where the shock waves would collide in the Minkowski case is a completely ordinary nonsingular point in the Euclidean continuation. It is the center of the flat region of Fig. 7(a). It follows that there is no singularity in the Minkowski case.

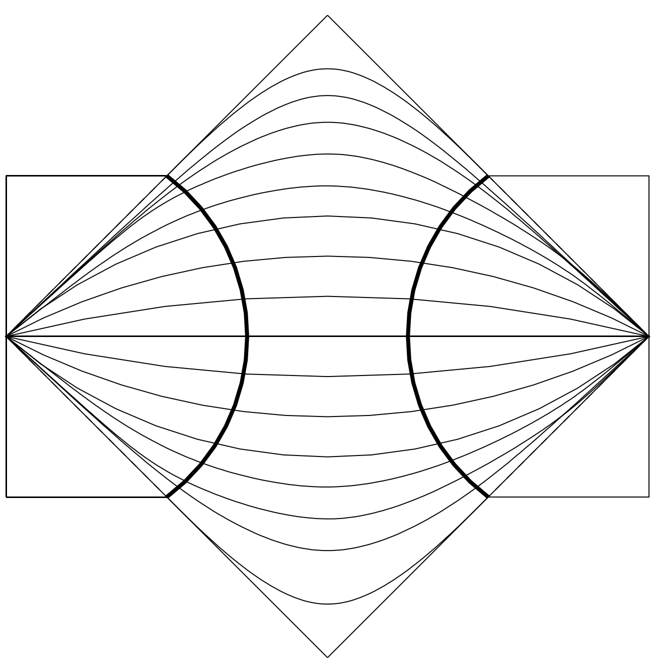

FIG. 8. The causal patch of our observer can be foliated by time slices as shown. The true Penrose diagram is half of the figure.

In ordinary de Sitter space the causal patch has a static geometry. In fact a causal patch is often referred to as a static patch. But in the bounce geometry, the causal patch is by no means static. An observer in this region sees a spherical brane that contracts, bounces, and expands. In general the time dependence will lead to particle creation. In particular as the field rolls from its initial value at ϕ_b to the minimum at zero energy, particles will be produced.

Particle Distribution

The distribution of particles in the FRW regions (regions II and III of Fig. 5(a)] is determined by the symmetry of the solution, namely $O(D-1,1)$ which acts in the FRW regions on space-like hyperboloids. In the FRW coordinates, the implication is that the particle distribution is uniform on the spatial slices. In the embedding coordinates, the same symmetry is described as boost invariance, so the implication is that the particle distribution is boost invariant. Such a distribution will have an infinite number of particles concentrated along the light-like directions or, equivalently, far out on the hyperboloid. Since the hyperboloid expands with time, the particle density tends to zero.

The asymptotic states on the hats have $O(D-1,1)$ symmetry and are uniquely specified by continuation from the Euclidean theory. One way to think of it is to use the Euclidean theory to define a Hartle-Hawking state on the symmetry plane of the solution and then evolve that state forward and backward to the hats.

Mathematically, a state of this kind should satisfy cluster decomposition. The way to guarantee this is to begin with the flat space Fock vacua on the hats. Call these states $|0\rangle_p$ and $|0\rangle_f$. The correct asymptotic states (i.e., the states obtained from time-evolving the Hartle-Hawking state) are obtained by exponentiating an $O(D-1,1)$-symmetric "cluster" operator and applying it to the Fock vacuum. Such cluster operators can be built by starting with an arbitrary rotationally symmetric connected operator on a space-like hyperboloid. Integrating the position of the operator over the hyperboloid will project out the $O(D-1,1)$-invariant part of the operator. Note that the invariance under $O(D-1,1)$ does not determine a unique cluster operator. For the case of a noninteracting field theory the cluster operator is quadratic and the result is a squeezed state. More generally the clusters are superpositions of any number of field operators.

The states defined in this way are guaranteed to be nonsingular, particularly at the origin. But if cluster operators are tampered with the result will generally lead to a FRW singularity at the origin. The condition of no singularity is enough to uniquely determine the $O(D-1,1)$-invariant to be the state that evolves from the Hartle-Hawking state. We call these states $|V\rangle_{\rm in,out}$.

The full space of states includes many noninvariant states. In general perturbing the asymptotic states can lead to singularities. The potential singularities are due to the infinity of particles moving out along the light cone. If we trace them back they all appear to intersect at the origin, i.e., the center point of Fig. 6. This is obviously a potential source of trouble. But as long as the boundary conditions far out on the hyperboloid are unmodified, the particles will be absorbed by the time dependent field before focusing at the origin. Any localized perturbation which is made by operators in a bounded region of the asymptotic hyperboloids should be nonsingular. In other words, localized perturbations of the Hartle-Hawking state will not lead to singularities. Asymptotic states which are not obtained by localized perturbations acting on the "vacuum" state will lead to singularities in general. Thus there is an incoming free particle Hilbert space of states and a similar outgoing space.

Later we will see that in the quantum version of the bounce there are nonperturbative processes in which metastable de Sitter space appears as an intermediate state. In fact it is likely that every de Sitter vacuum appears as an intermediate state.

IV. HORIZON COMPLEMENTARITY

The classical geometry of the bounce solution will be modified by nonperturbative quantum corrections. Among those corrections are the bubble nucleation events that turn the de Sitter boundaries into fractal populations of bubble universes. These bubble universes are on the far side of the event horizon. According to classical general relativity, events behind the horizon are completely decoupled from the causal patch and cannot influence, in any way, observations in the patch. It has been argued that this decoupling of the bubbles from our universe makes these other universes more metaphysical than physical.

We believe that the complete decoupling is a feature of classical physics, that does not survive in a complete quantum theory of gravity. The basis for this belief is the last decade of experience in understanding black hole horizons, particularly in the context of string theory. That experience can be summarized by two principles: The Principle of Black Hole Complementarity [8,9], or more generally Horizon Complementarity, and the Holographic Principle (see [10] for reviews and references). Let us review the Horizon Complementarity Principle.

The causal patch can be foliated with a set of space-like surfaces which all pass through the bifurcate horizon as in Fig. 9(a). Any such set of surfaces allow us to define a time variable t in the causal patch which runs from $-\infty$ to $+\infty$. Note that the time variable we defined in section II B is not suitable because it naturally foliates only region I. It is generally believed that a self-contained Hamiltonian description of physics in a causal patch is possible. Things coming in from the past horizon or going out through the future horizon are part of the initial or final conditions at infinite time.

The surface $t = \infty$ is comprised of two parts. One part is just the future hat itself which consists of time-like and light-like infinity. The other portion is not part of the boundary of the Penrose diagram but defines the future event horizon. The two regions can be distinguished as follows: Every point in the Penrose diagram represents a $D-2$ sphere. The area of the local $D-2$ sphere is finite everywhere in the horizon, but on the hat it is infinite. Similar things are true for the $t = -\infty$ surface.

Essentially identical things can be said about black hole geometries. In Fig. 9(b) a causal patch of the

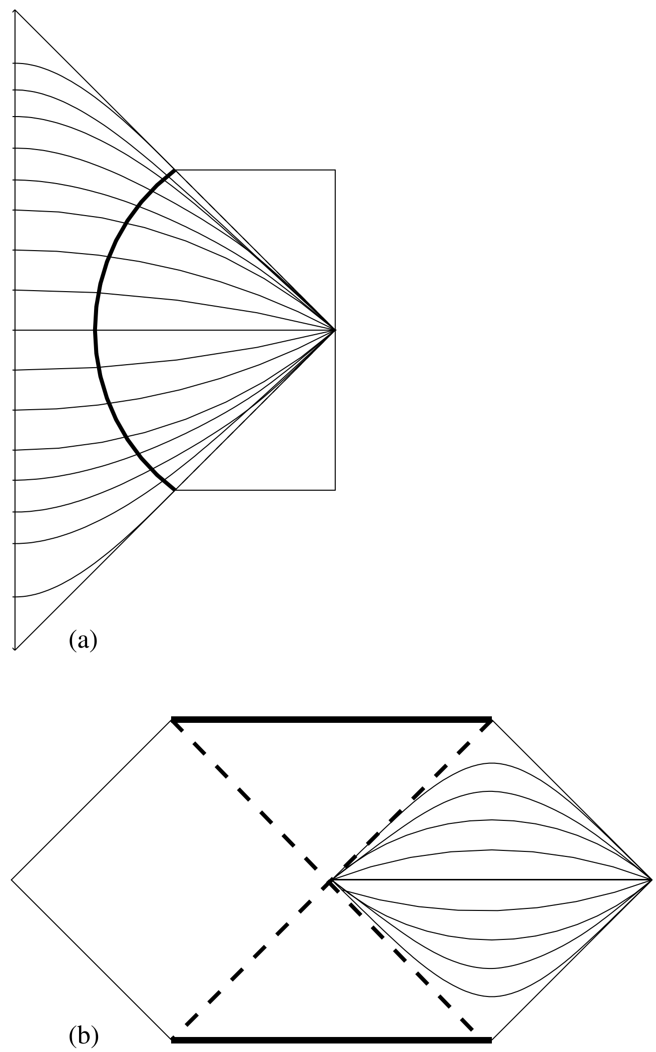

FIG. 9. There are similarities between the Penrose diagram for the bounce background (left) and the Schwarzschild black hole (right). Constant time surfaces are shown. In both cases, we believe that unitary evolution exists between *part* of the $t = -\infty$ surface and *part* of the $t = +\infty$ surface.

Schwarzschild geometry is shown foliated by Schwarzschild time slices. Again the asymptotic time slice $t = \infty$ consists of a horizon with finite area and the portion with infinite area, i.e., light-like and time-like infinity.

The claim that there is a self-contained physics in the causal patch is not particularly controversial. However there is a much stronger form of the claim, which for a black hole is called black hole complementarity. What it basically says is that from the vantage point of an observer at infinity, *no information is stored on the portion of the* $t = \infty$ *surface with finite area.* In the case of the formation and evaporation of a black hole, it says that there is an S-matrix connecting the states on the past asymptotic surface with states on the future surface. The evidence for this conjecture, in the case of a black hole is overwhelming. The bounce geometry is less familiar but we can see no reason why the same thing should not be true. Thus we postulate that there is an S-matrix connecting the past hat to the future hat.

There is an even stronger form of black hole complementarity that is the real reason for calling it complementarity. It is the assumption that the degrees of freedom in the Hawking radiation are redundant descriptions of the degrees of freedom behind the horizon. According to this formulation, not only do the degrees of freedom behind the horizon fail to commute with those in the Hawking radiation, but they can be expressed as functions of the Hawking radiation variables. The connection is of course extremely scrambled.

In many cases this strong form of complementarity can be proved from the weaker form. In these cases the existence of an S-matrix implies that the degrees of freedom behind the horizon are redundantly described in terms of the Hawking evaporation products. The geometry describing the formation and evaporation of a black hole is an example

Figure 10 is the standard Penrose diagram for the formation and evaporation of a black hole. Superimposed on the diagram is a scattering process in which particles **a** and **b** collide behind the horizon far from the singularity. We assume that the scattering process is a completely conventional low energy process such as low energy photon photon scattering.

Suppose we are interested in the outcome of the collision. For example we might want to know the angle of scattering. Since, in the in-falling frame, the process is at low energy, ordinary quantum electrodynamics can be used to describe it. Any observable can be described by a Hermitian operator including the angle of the outgoing particles.

Now using the conventional QED Hamiltonian, the observable in question can be run backwards and re-expressed as an operator in the Hilbert space of incoming asymptotic scattering states. In other words by solving the

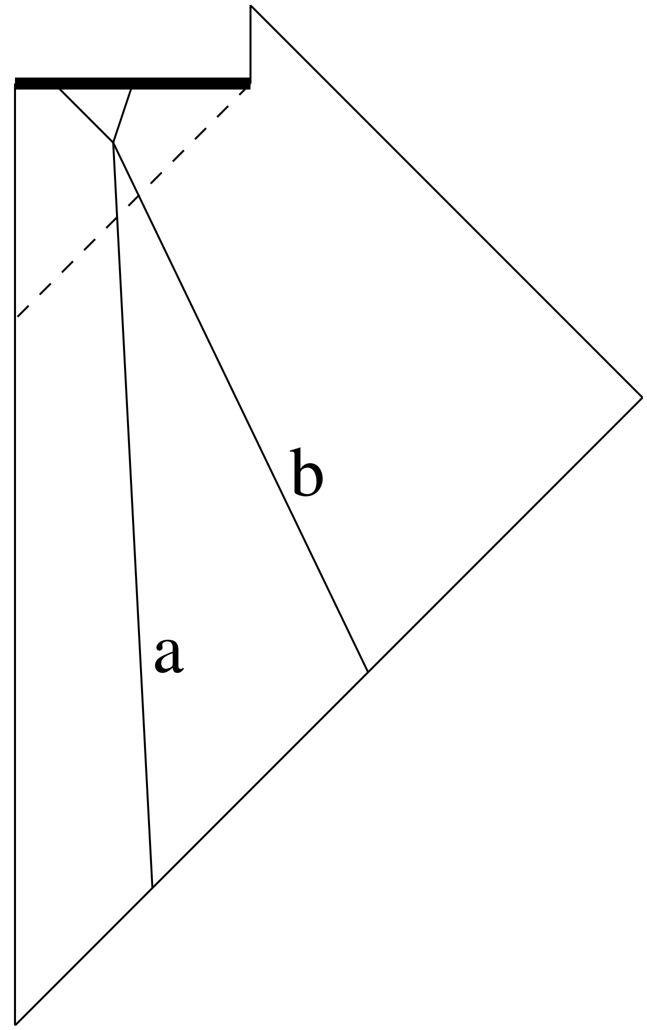

FIG. 10. Ordinary scattering behind the black hole horizon can be translated into a very complicated process occurring outside the horizon.

Heisenberg equations of motion, the observable can be written in terms of operators on past light-like infinity $\mathcal{I}^-$. Calling the observable Q,

$$Q_- = UQU^\dagger, \tag{4.1}$$

where Q_- is an operator on $\mathcal{I}^-$ and U is the unitary operator that connects the final state of the collision to the incoming states on $\mathcal{I}^-$. So far all of this can be done using only conventional low energy physics.

Next we assume the existence of an S-matrix S, that connects states on $\mathcal{I}^-$ to states on $\mathcal{I}^+$. This S-matrix is of course not something that we can compute by ordinary methods. Among other processes it describes the formation and evaporation of a black hole. It scrambles information and produces the Hawking radiation.

The operator Q_- can now be moved forward in time by conjugating it with the S-matrix

$$Q_+ = S^\dagger U Q U^\dagger S. \tag{4.2}$$

The operator Q_+ has exactly the same information as Q but it is an operator defined on I^+—the future null infinity–in the products of black hole evaporation.

This argument is very formal but it does show that the existence of an S-matrix implies that the degrees of freedom of the Hawking radiation are a complementary way of keeping track of events behind the horizon.

In the case of the bounce geometry, the Hawking radiation is replaced by the particle production in the FRW region due to the time dependence of the metric and the field ϕ. Assuming that the degrees of freedom beyond the horizon are redundantly described by the infinite sea of particles on the future hat brings us to a remarkable conclusion: The infinity of bubble universes are not at all out of contact with our universe. Their degrees of freedom are all around us in the very subtle many-particle correlations in the cosmic microwave background (CMB). Strictly speaking, this conclusion only applies to the final exit to zero cosmological constant.

The total number of bubbles in the multiverse is expected to be infinite. This raises the question of whether there are enough degrees of freedom in the causal patch to describe an infinite number of bubbles. This question can be answered by calculating the entropy bound on very late time slices. In Fig. 11 the Bousso-Penrose diagram for the bounce geometry is shown. The maximum entropy on the future hat is equal to the area at point **g** and that is infinite. Thus there is no bound on the amount of information that can be stored on the upper (or lower) hat [11].

The Conjecture

The conjecture that we would like to put forward is this:

(1) Asymptotic in and out "vacua," invariant under SO($D - 1$, 1) exist and lead to nonsingular behavior in the bulk. The vacua have an infinite number of particles when expanded in a flat space basis of states.
(2) A Hilbert Space of asymptotic in and out states consisting of localized perturbations on the respective vacua exist.
(3) The asymptotic in and out states are connected by an S-matrix.

The particle density in the asymptotic vacua vanishes because the FRW expansion dilutes the particles. This means that the asymptotic vacua are locally identical to some conventional flat space vacuum with zero cosmological constant. The only such vacua that we know of are on the moduli space of supersymmetric vacua of string theory. Thus we assume that the vacua on the hats are always supersymmetric, at least locally. Different metastable vacua may lead to different points on the supersymmetric moduli space.

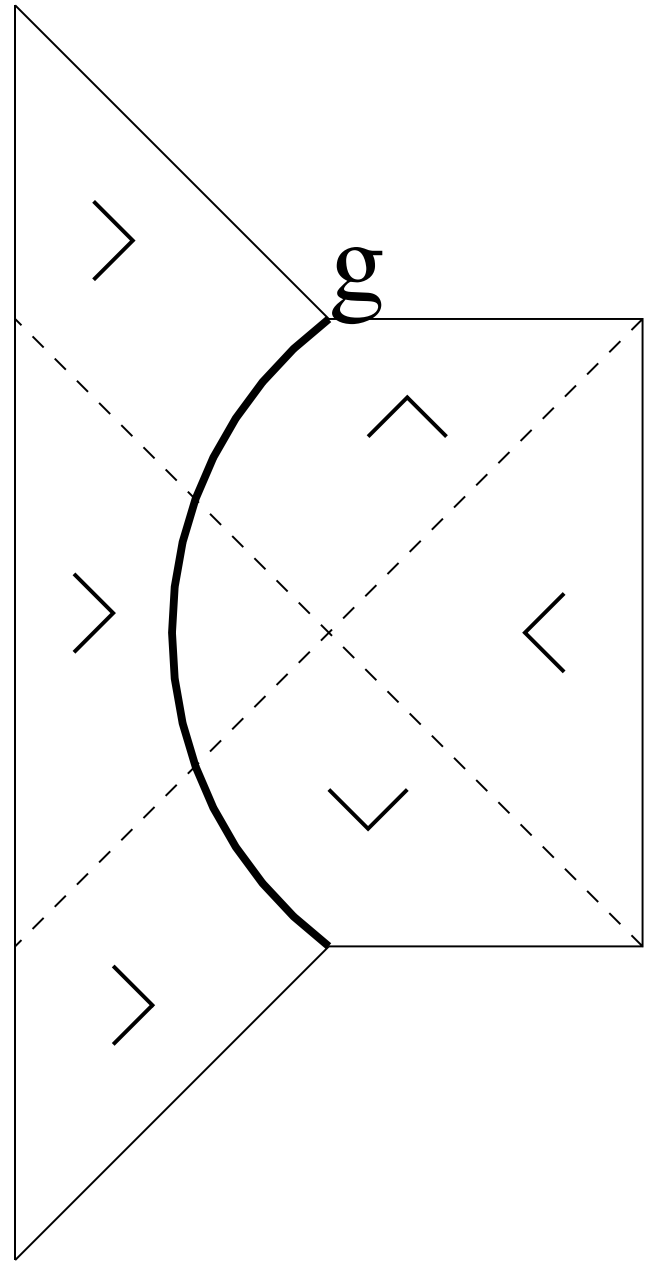

FIG. 11. There is no bound on the entropy contained in the upper "hat," as this Bousso-Penrose diagram shows.

V. PERTURBATIONS AROUND THE BOUNCE BACKGROUND

A. The equations of motion

Ultimately we would like to show that string theory in the bounce geometry exists and can be used to calculate the S-matrix. We are far from that goal but there are a number of interesting issues which are likely to come up. Let us for the moment forget string theory and think about quantum field theory in the bounce background. Presumably Feynman rules can be constructed for the perturbative processes. The obvious way to carry out a perturbation theory would be to start with the Green functions in the Euclidean version of the theory and continue them to the Minkowski signature. Then by means of the Lehmann-Symanzik-Zimmerman (LSZ) technique, an S-matrix can be constructed.

In Appendix A, we argue that quantum field theory is well-defined in the bounce background, explain some subtleties due to working on a compact space, and compute some correlation functions. In the main text, we focus on the nonperturbative processes because they are the most interesting for our purposes. We certainly do not have a systematic formulation but some idea of the nature of these processes can be gained by a kind of mini-superspace reduction of the theory. For simplicity we will work in the thin wall approximation, but the general case is a straightforward generalization.

Although the entire causal patch cannot be described in static coordinates with a time-like Killing vector, region I of the Penrose diagram does have a static description. For our present purpose this is sufficient. First consider the static metric for the pure bounce solution. The radial coordinate r is defined to be zero at the origin of the flat space region.

We want to describe a spherically symmetric bubble of $\Lambda = 0$ inside a region with positive Λ. We will closely follow [12]. In the simplest situation, the $\Lambda = 0$ region is pure Minkowski space and the $\Lambda > 0$ region is pure de Sitter space. If we use static coordinates on both sides, then the metric on each side has the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2, \tag{5.1}$$

with $f(r) = 1$ in the flat part and $f(r) = 1 - r^2/R^2$ in the de Sitter region, where R is the de Sitter radius.

We will study two spherically symmetric perturbations of this geometry. One possibility is to add a mass M at the center of the flat region, $r = 0$, so that the metric inside the brane is Schwarzschild and the metric outside remains pure de Sitter space. Another possibility is to add a mass M at the center of the de Sitter region, so that the metric is Schwarzschild-de Sitter on the $\Lambda > 0$ side and Minkowski space on the $\Lambda = 0$ side. To be clear, the time slices are spheres cut by planes just like Fig. 7(a) and we add a mass at the point on the sphere farthest from the plane. In both cases, the gravitational backreaction changes the motion of the brane, so we get a one-parameter family of brane motions. Let us describe the brane motion by $r(\tau)$ where τ denotes proper time along the domain wall trajectory and r is the Schwarzschild radial coordinate.

First we discuss adding a mass in the de Sitter region. In equations, the effect of adding a mass in the $\Lambda > 0$ region is that for $r < r(\tau)$ the metric is

$$ds^2 = -f_{\rm in}dt^2 + \frac{1}{f_{\rm in}}dr^2 + r^2 d\Omega^2, \tag{5.2}$$

with

$$f_{\rm in} = 1. \tag{5.3}$$

The metric for $r > r(\tau)$ is

$$ds^2 = -f_{\rm out}dt^2 + \frac{1}{f_{\rm out}}dr^2 + r^2 d\Omega^2, \tag{5.4}$$

with

$$f_{\rm out} = \left(1 - \frac{r^2}{R^2} - \frac{2GM}{r}\right). \tag{5.5}$$

To determine the domain wall position $r(\tau)$, we need to know what equation of motion it satisfies. The Israel junction condition gives the equation of motion for the brane. Given the Schwarzschild-like form of the metric on both sides, the condition is

$$\sqrt{f_{\rm in}(r) + \dot{r}^2} - \sqrt{f_{\rm out}(r) + \dot{r}^2} = 4\pi G\sigma r, \tag{5.6}$$

where $f_{\rm in}(r)$ is the metric function inside the bubble, $\dot{r}$ is the derivative of the Schwarzschild radial coordinate with respect to proper time, and σ is the tension of the domain wall.

The junction condition can be rearranged to look like an energy conservation equation for the brane motion,

$$4\pi\sigma r^2\sqrt{1 + \dot{r}^2} - \left(\frac{1}{2GR^2} + 8\pi^2 G\sigma^2\right)r^3 = M. \tag{5.7}$$

Each term has a physical interpretation. The square root term is the usual kinetic term for a membrane, and $-8\pi^2 G\sigma^2 r^3$ is the gravitational self-energy of a spherical membrane. The bubble of flat space replaces a region of positive vacuum energy with a region of zero vacuum energy, resulting in a change $-\frac{1}{2GR^2}r^3$ in the energy.

Why should the parameter M, which we thought of as a mass in the de Sitter region, have a nice interpretation as the energy of the bubble of flat space? Roughly, it is because nonsingular perturbations of de Sitter space involve adding equal masses on opposite sides of the spatial sphere. When we add a mass at the north pole of the spatial sphere, the gravitational backreaction adjusts the position of the brane so that the bubble of flat space effectively has positive energy.

Equation (5.7) is intuitive, but since we want to analyze the one-dimensional problem we will make it look like the energy conservation equation for a particle rather than a brane. Take the nonrelativistic limit $\dot{r} \ll 1$ and change variables to $u = r^2$. The equation becomes

$$\frac{1}{2}\pi\sigma\dot{u}^2 + 4\pi\sigma u - \left(\frac{1}{2GR^2} + 8\pi^2 G\sigma^2\right)u^{3/2} = M, \tag{5.8}$$

which has the form of an energy conservation equation for a nonrelativistic particle in a potential. The energy is given by M and the potential is

$$V(u) = 4\pi\sigma u - \left(\frac{1}{2GR^2} + 8\pi^2 G\sigma^2\right)u^{3/2}. \tag{5.9}$$

The potential is shown in Fig. 12. We will discuss the interpretation further in section V B.

A similar one-dimensional problem may be constructed for the family of solutions obtained by adding a mass at the center of the flat region. The boundary conditions are that outside the bubble, $r > r(\tau)$, the geometry is pure de Sitter while inside it is given by the metric of the massive particle in flat space. For $r < r(\tau)$ we take the metric to be Schwarzschild. We assume that the Schwarzschild radius of the mass M is much smaller than the other length scales in the problem. Now

$$f_{\rm in} = \left(1 - \frac{2MG}{r}\right) \tag{5.10}$$

and

$$f_{\rm out} = \left(1 - \frac{r^2}{R^2}\right). \tag{5.11}$$

The junction condition can be rearranged to the form

$$\frac{4\pi\sigma r^2\sqrt{1 - r^2/R^2 + \dot{r}^2}}{\sqrt{1 - r^2/R^2}} - \left(\frac{1}{2GR^2} - 8\pi^2 G\sigma^2\right)r^3 = -M. \tag{5.12}$$

To make this look like a conventional energy conservation equation for a particle, again let $u = r^2$ and take the limit where the minimum brane size is much smaller than the de Sitter radius, $G\sigma \ll 1/R$, so that the nonrelativistic motion occurs in the region $r \ll R$. The equation of motion becomes

$$\frac{1}{2}\pi\sigma\dot{u}^2 + 4\pi\sigma u - \frac{1}{2GR^2}u^{3/2} = -M. \tag{5.13}$$

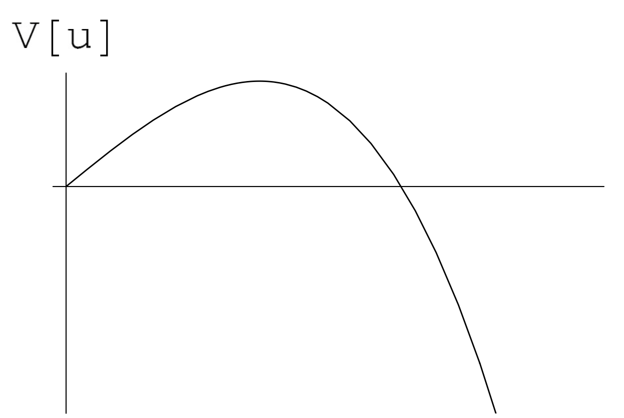

FIG. 12. The effective potential for the motion of the domain wall has a barrier. The classical evolution corresponds to coming in from infinity, bouncing off the barrier, and going back out. Tunneling in to the center means that the brane reaches 0 radius: it disappears, leaving pure de Sitter space.

The potential looks just like Fig. 12, but now positive mass corresponds to *negative* energy.

It is not obvious what happens to the causal structure for either of these perturbations. The question is important because the bounce solution is on the verge of having a horizon. In Appendix B, we show that adding a mass at the center of the flat region causes the brane to move out from the origin, as shown in Fig. 13, so no horizon forms. We believe that adding a mass at the center of the de Sitter region has exactly the opposite effect. The formation of a horizon indicates that at the field theory level the description in the causal patch will no longer be in terms of pure states. However, we expect that just as in the case of black holes the true description is in terms of pure states. An entire Cauchy surface is still in the backward lightcone of the observer.

B. de Sitter as a Resonance

For $M = 0$ there are two classical solutions. One of them is just the original bounce solution. The other is the static solution located at the minimum of the potential at $r = 0$. The interpretation of that solution is especially interesting. It represents the spherical brane of zero radius— in other words, no brane at all. This solution is just pure unmodified de Sitter space!

In the quantum theory the static solution with the degenerate vanishing domain wall is unstable; it can

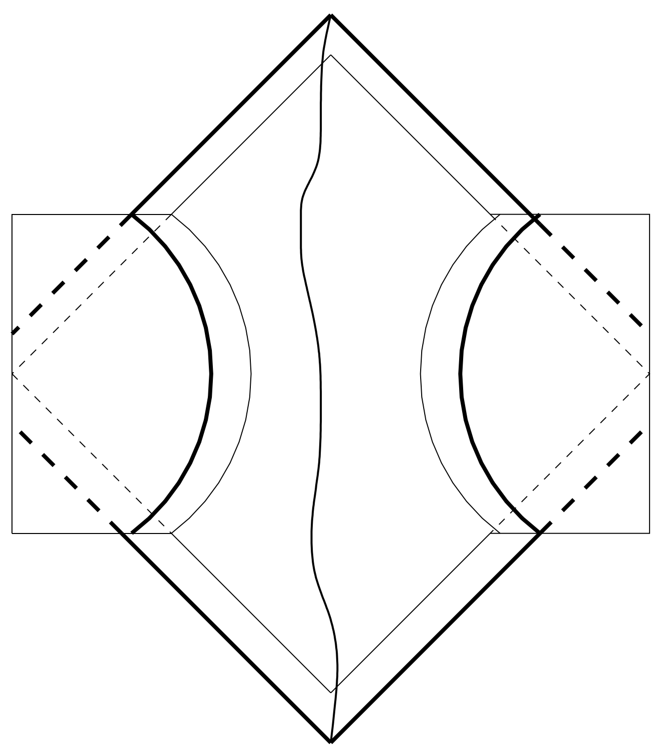

FIG. 13. Adding a mass inside the bubble causes the causal patch of the observer at the center to ‘‘expand’’ as shown in the figure. The new causal patch is shown in bold.

tunnel out through the barrier. This is just the instability of de Sitter space to bubble nucleation.

For the program of defining and studying an S-matrix, the state at $r = 0$—the pure de Sitter space—is an intermediate resonance, a singularity as a function of energy. The singularity is not exactly at zero energy because the finite lifetime of the state shifts the singularity into the complex plane.

In a semiclassical analysis of the S-matrix we can distinguish two types of histories. The first consist of small fluctuations around the bounce solution. These can be studied using conventional perturbation theory in the bounce background. The second type of history involves the formation and decay of the resonant pure de Sitter intermediate states. These histories begin with an incoming solution identical to the bounce solution. The domain wall moves inward until the point where it comes to rest. This occurs at the point

$$r = \frac{8\pi G\sigma}{16\pi^2 G^2\sigma^2 + R^{-2}}. \tag{5.14}$$

At this point the fictitious particle tunnels to the origin where it remains for a time t. It then tunnels back to the point (5.14) and continues on its original course with a time delay t. The process can be thought of as an interrupted bounce. It can be illustrated by the conformal diagram shown in Fig. 14.

The amplitude for this process has the form

$$A = \gamma \int_0^\infty dt, \tag{5.15}$$

where γ is the Coleman-de Luccia tunneling rate given by

$$\gamma \sim e^{-S}. \tag{5.16}$$

Here S is the action for the Euclidean bounce solution. From (5.15) the amplitude appears to diverge. To see the meaning of this, let us shift M away from zero. In that case the integrand picks up an additional factor e^{iMt} and the result of the integration is

$$A = \gamma \frac{1}{M}. \tag{5.17}$$

The pole at $M = 0$ is the standard pole in the energy indicative of a sharp intermediate state. However in a more precise calculation including rescattering corrections the pole associated with an unstable state gets shifted into the complex plane. Thus the amplitude will have the approximate form

$$A = \gamma \frac{1}{M + i\gamma}. \tag{5.18}$$

In Appendix C, we give a more careful argument for the same result which does not rely on using the nonrelativistic approximation or defining the energy via the mass of the added particle, but the result is exactly the same.

In fact the pure de Sitter space is not a single resonant state. It consists of a large number of nearby states implied by the thermal density matrix describing the quantum mechanics of de Sitter space. Although, like any unstable state, the de Sitter states are not well-defined quantum states, they should have precise meaning as resonances in the complex plane. In this respect they are like black holes.

Although our goal is the description of nonsupersymmetric metastable de Sitter vacua, supersymmetry plays a central role. The entire possibility of an S-matrix description depends on the existence of vacua with exactly zero cosmological constant. There is no reason to expect such vacua in the absence of supersymmetry. Indeed the S-matrix elements described in this paper are a sector of the S-matrix of a theory whose asymptotic states are classified by supersymmetry.

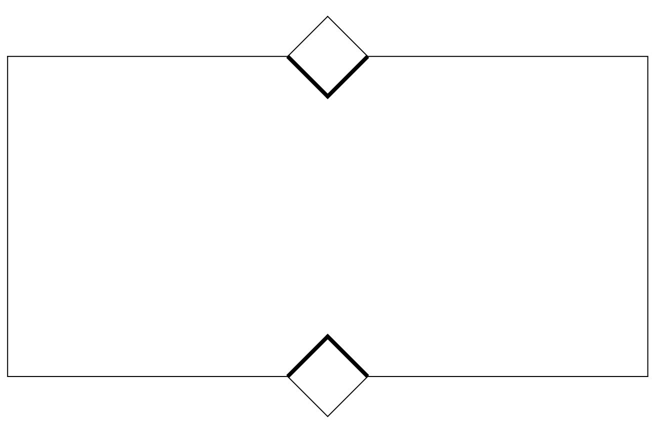

FIG. 14. A "solution" in which the brane tunnels into the minimum at zero size for a time can be illustrated by this conformal diagram. The evolution is classical except at the points where the brane disappears and is nucleated. It is not really nucleated at zero size unless the tension is zero.

C. Bubble Collisions

Up until now we have ignored the possibility of collisions between bubbles. These are not only possible but are inevitable. To see this recall that along any time-like trajectory, an observer eventually is swallowed by a bubble nucleation event. Consider a trajectory that eventually ends at a point where the domain wall meets the future hat, so that it remains always outside the bubble but approaches the bubble wall at infinite time on the conformal diagram. Classically an observer following such a trajectory sees herself in de Sitter space. Eventually she will encounter a bubble nucleation. Obviously the expanding bubble, which she is inside of, will collide with the original bubble. This process will occur an infinite number of times.

One might think that the bubble collisions would have the effect of connecting all the bubbles together into one big bubble that covers the whole space. But this is not correct. Guth and Weinberg analyzed this situation in [13]. They find that the bubbles cluster into disconnected structures that maintain their island-like character.

Bubble collisions certainly make nonperturbative corrections to the final state of the hat. But because the entire cluster forms in a background with $O(D-1,1)$ symmetry, the final state must have this invariance. This means that the perturbations caused by such collisions are uniformly distributed over the hyperbolic plane representing a spatial slice through the FRW region. In Fig. 15 we show one of Escher's drawings of the two-dimensional hyperbolic plane tessellated by "Angels and Devils." The different bubbles that coalesce to form the cluster should be distributed like the devils in the figure, although much more sparsely.

Since the final states are expected to lie on the supersymmetric moduli space, the different patches will eventually settle down to give an inhomogeneous state with varying massless scalar fields distributed symmetrically over the negatively curved geometry. The background field is given by the average field and the variations are part of the distribution of massless particles in the final state. If this picture is correct then bubble collisions do not destroy the overall picture but they do give nonperturbative corrections to the particle content on the hat. We hope to return to this point in the near future.

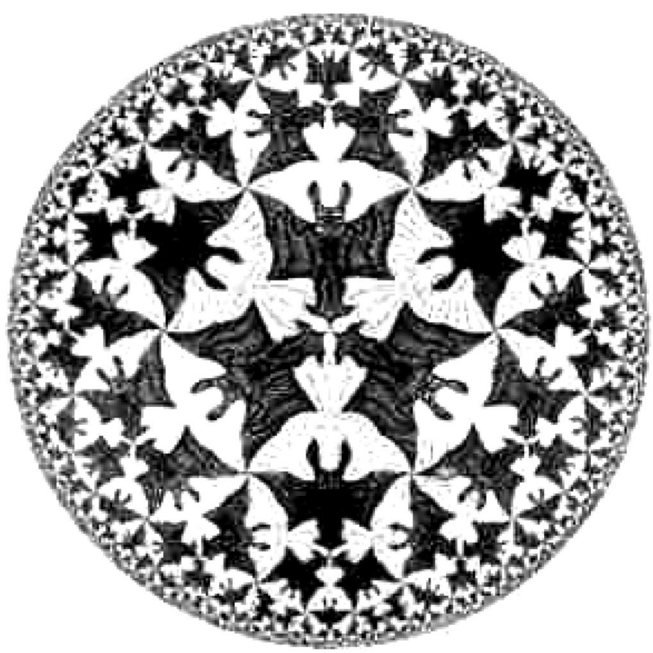

FIG. 15. Escher's famous figure illustrates properties of the hyperbolic plane, which is the spatial geometry seen by the FRW observer.

VI. CONCLUSION

In this paper we have suggested a framework for studying eternal inflation within the context of an S-matrix description that may be adaptable to string theory. The metastable de Sitter vacua appear as resonant intermediate states in the scattering matrix elements.

This framework, when combined with horizon complementarity, leads to a conceptually far-reaching conclusion: The infinity of bubble universes out beyond an observer's horizon is not truly decoupled. Radiation analogous to Hawking radiation exists and encodes the degrees of freedom of these other bubbles in a scrambled way. This radiation is essentially the cosmic microwave background, but because our universe still has a nonvanishing cosmological constant there cannot presently be enough information to encode all the other bubbles. It is interesting to quantify this a little more. Given the current value of the cosmological constant, the entropy of our horizon is about 10^{120}. Most of this is in the horizon degrees of freedom. The amount of nonhorizon entropy in ordinary stuff is about 10^{100}. This means that the horizon degrees of freedom contain enough information to describe the features of 10^{20} universes like our own.[1]

But if we wait long enough our universe will tunnel to an open FRW bubble with an infinite number of particles. Those observers who survive the transition will have enough available information to reconstruct the rest of the universe.

The reason we emphasize this point is not to suggest that there is a practical way of testing the hypothesis of eternal inflation. Even if we could wait long enough to enter the FRW era and collect enough quanta, the information would be in a hopelessly scrambled form. Our motivation is to dispel the idea that discussing the portion of the universe beyond our classical horizon is pure metaphysics. We would argue that the rest of the universe will become imprinted in the causal patch and is physically meaningful.

Another important question is the unitarity of the S-matrix. The initial and final vacuum states are determined by the background de Sitter vacuum that the bouncing bubble is embedded in. If the de Sitter vacuum is replaced by another minimum in the Landscape, the boundary conditions in the FRW spatial infinity will change. Moreover it seems possible that transitions can occur

[1]W. Fischler has pointed out to us that it is misleading to say this information is stored in the CMB. The characteristic wavelength of the Hawking radiation is horizon size, so it is the far infrared of the CMB. Additionally, the time scale to extract information from the Hawking radiation is extremely long, just as it is for black holes. For a black hole, the time to extract one bit of information is r^3/l_p^2, where r is the Schwarzshild radius. Assuming that for de Sitter space we should replace the Schwarzshild radius by the de Sitter radius, the time is 10^{120} times the age of the universe.

between these asymptotic vacua. Consider a history involving a tunneling to a particular de Sitter intermediate state. The system can then tunnel back to the original state or it might tunnel to some other nearby de Sitter minimum. If it does the latter it can subsequently tunnel to a different final state. Thus it seems that transitions between asymptotic vacua are possible. If this is the right idea then the S-matrix would only be unitary after summing over the different sectors with different boundary conditions.

We think that this may not be the right idea. To see why, consider pure de Sitter space. In this case the choice of points **P** and **F** is obviously a gauge choice. Now consider the real situation in which the initial and final boundaries are replaced by fractals containing an infinite number of hats. In fact one can expect an infinite number of hats with every possible boundary condition. But if we are right about the implications of horizon complementarity, then each hat contains all the degrees of freedom of the other hats. That suggests that the choice of hat in the initial and final state is a *gauge choice*. One may pick a gauge in which the boundary conditions are some specific point on the moduli space or a different gauge with a different asymptotic behavior. Some choices of gauge may be more convenient than others, making manifest the physics with the particular asymptotic behavior. The physics of other gauges will only exist in a scrambled form.

If this latter view is right then the S-matrix with a given asymptotic vacuum may be unitary by itself.

Finally let us address the issue of formulating string theory in bounce backgrounds. As an example, the KKLT model [14] relies on nonperturbative instanton effects to stabilize the Kahler moduli, which means that the domain wall solution is not really a classical string theory solution because the domain wall is a nonperturbative object. This does not necessarily mean that string theory cannot be formulated in this kind of background, but it does mean that some nonperturbative version of the Fischler-Susskind mechanism will be needed to define the string action. On the other hand, perturbative constructions of de Sitter minima in supercritical string theory [15] do not present the same difficulty.

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APPENDIX A: PERTURBATION THEORY IN THE BOUNCE BACKGROUND

Perturbation theory in the bounce background is well-defined, but it has several unique features compared to ordinary flat space perturbation theory. One difference is that the spatial slices are compact, which leads to interesting restrictions on the allowed perturbations. Another difference is that an infinite number of particles are produced. These complications make perturbation theory more confusing, but no less well-defined. We first compute some simple correlators and discuss the particle production. Then we deal with subtleties due to the unusual geometry of the background.

1. Correlation Functions in the Bounce Background: Overview and computation of particle production

To begin, we will compute the correlator of a conformally coupled scalar field in the thin-wall approximation. In spite of the time dependence of the background, the correlators in the $\Lambda = 0$ region are exactly the same as they would be in ordinary flat space!

The reason for this is that we can conformally map the Euclidean version of the bounce, shown in Fig. 7(a), to the flat plane using a conformal mapping which is trivial in the flat region.

It may be surprising that the complicated geometry has no effect on correlators in the flat region. A helpful analogy is the uniformly accelerated mirror in flat space. The correlators of a massless field in flat space are unaffected by the presence of the mirror *if* the acceleration is uniform [16].

The Conformal Mapping. The Euclidean space consists of a sphere sliced by a plane. For simplicity, we consider the special case where the plane slices the sphere exactly in half. We coordinatize the flat part in the standard way. The half sphere we coordinatize by stereographic projection from the opposite pole onto the plane. The metric is

$$\begin{aligned} ds^2 &= dr^2 + r^2 d\Omega^2, \quad \text{for } r < 1, \\ ds^2 &= \frac{1}{(r^2+1)^2}(dr^2 + r^2 d\Omega^2), \quad \text{for } r > 1. \end{aligned} \tag{A1}$$

In these coordinates, it is clear that the space is conformally flat both inside and outside the domain wall. The conformal factor is continuous at the domain wall, so the space is conformally flat everywhere. A subtlety is that the correlator of a conformally coupled field is not well-defined. This is easiest to see in two dimensions, where a massless field is conformally coupled. The shift symmetry $\phi \to \phi + c$ ensures that the correlator is undefined. Correlators of derivatives *are* well-defined.

A hidden assumption in this section is that the whole effect of the domain wall on the field ϕ is through the geometry. In other words, we implicitly imposed continuity of the field and its derivative at the domain wall rather than a more complicated boundary condition.

2. Subtleties due to compactness and symmetry of the background

It is well known that the total charge on a compact space must be zero, roughly because there is no place for electric field lines to end except on charges. Since the bounce solution has compact spatial slices, we must restrict to electrically neutral perturbations. There is a less familiar gravitational analog of the restriction on total charge which we will describe.

First, consider a familiar example from string theory. When quantizing the closed bosonic string in lightcone gauge, one imposes various gauge conditions on the worldsheet metric. These conditions completely fix the worldsheet reparameterization invariance except for the transformations

$$\sigma \rightarrow \sigma + \text{const}, \tag{A2}$$

where σ is the periodic coordinate on the string worldsheet. The unfixed gauge freedom means we must impose a condition on the physical states, restricting to states which are invariant under the residual gauge transformation. Since the gauge freedom is translations along the string, we restrict to states with zero worldsheet momentum. This restriction is the familiar level matching condition.

To state the condition in a form which will generalize, the residual coordinate transformation is generated by the Killing vector

$$\xi = \frac{\partial}{\partial \sigma} = \frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}. \tag{A3}$$

This leads to the constraint

$$0 = \int d\Sigma T_{ab} \xi^a \hat{n}^b, \tag{A4}$$

where the integral is taken over a spatial slice, T_{ab} is the worldsheet stress tensor, ξ^a is the Killing vector, and $\hat{n}^b$ is the unit normal vector to the spatial slice. In traditional notation, the constraint is

$$0 = \int d\sigma (T_{zz} - T_{\bar{z}\bar{z}}), \tag{A5}$$

which is the level matching constraint.

Analogous constraints appear in the bounce solution. Recall that the gauge invariance for graviton fluctuations is

$$h_{ab} \rightarrow h_{ab} + D_a \xi_b + D_b \xi_a. \tag{A6}$$

If ξ is a Killing vector field, then by definition it satisfies

$$D_a \xi_b + D_b \xi_a = 0 \tag{A7}$$

so it does not generate any gauge transformation. As a result, the graviton gauge fixing leaves unfixed the coordinate transformations generated by the Killing vectors of the background. It is easiest to think about in the Euclidean geometry, Fig. 7(a). The geometry is invariant under rotations of the $(D - 1)$-sphere. In the Lorentzian version, some of these rotations become boosts. If we use coordinates so that the bounce is a hyperboloid cut by the plane $x = \text{const}$, then the Killing vectors are rotations and boosts which leave x fixed. The corresponding quantities which are set to zero are once again

$$0 = \int d\Sigma T_{ab} \xi^a \hat{n}^b, \tag{A8}$$

where the integral is again over a spatial slice. The constraints coming from the rotation generators are, for example,

$$0 = \int d\Sigma (y T_{0z} - z T_{0y}), \tag{A9}$$

which sets the angular momentum in the yz plane equal to zero. (We continue to use the embedding coordinates because the symmetries are clearest there.) The constraints coming from boost generators are

$$0 = \int d\Sigma y T_{00} \tag{A10}$$

if we choose to integrate over the spatial slice $x_0 = 0$. These constraints are less familiar, setting the dipole moment of the energy distribution equal to zero in the symmetry directions. Just as angular momentum is the charge associated to rotations, the dipole moment of the mass distribution is the charge associated to boosts and it should be set to zero.

Why are not there analogous restrictions in flat space, which after all has many Killing vectors? The difference is spatial compactness. In flat space, the modes which are unfixed by the gauge conditions are not normalizable, so we do not need to impose their equations of motion as constraints on the physical states.

The analogy with the level matching condition is incomplete because four-dimensional gravity has propagating degrees of freedom while two-dimensional gravity does not. At higher order in perturbation theory, there cannot be a restriction on the matter stress-energy tensor, roughly because there is also stress energy in gravitational waves. Another way to see the same thing is to note that the quantity we are setting to zero, $\int d\Sigma T_{ab} \xi^a \hat{n}^b$, is not gauge invariant.

Once we allow propagating gravitons rather than just computing the backreaction, we may as well discuss pure gravitational perturbation theory with no matter since no new issues arise when adding matter. In pure gravitational perturbation theory, the combination of compact spatial slices and symmetries leads to a *linearization instability*. We thank Vincent Moncrief for explaining the situation to us and refer the reader to his review article [17], which we follow here, for a more thorough description and references to the literature. The bottom line is that the linearized equations of motion have spurious solutions which cannot be continued to higher order; they are not approximations to any solution of the full equations of motion. To be clear, there is no instability of the background. What is unstable is the linear approximation to the equations of motion for small fluctuations about the background.

A simple example of a situation where some solutions to the linearized equations cannot be continued to higher order is a cone defined by $x^2 + y^2 - z^2 = 0$. Say we perturb around the point (0, 0, 0) and ask which points are on the cone. The point $\epsilon(a, b, c)$ satisfies the cone equation to first order in ϵ for any (a, b, c), so it appears that we can move in three directions and stay on the cone. We know this is a wrong result; the linearized equations do not give an accurate picture of the space of solutions. If we blindly computed in perturbation theory, at second order we would find a quadratic constraint on the first order fluctuations, namely $a^2 + b^2 - c^2 = 0$. The true linear approximation to the theory near such a conical point is the linearized equations of motion *plus* an extra constraint quadratic in the fluctuations.

In the gravity problem, the basic result is that solutions with compact spatial slices and Killing vectors are conical points in the space of solutions. Just as in the cone example, there are solutions of the linearized equations of motion which are not approximations to any full solution. The true linear approximation consists of the linearized equations of motion plus a second order condition [17]

$$\int d\Sigma G_{ab} \xi^a \hat{n}^b, \tag{A11}$$

where G_{ab} is the Einstein tensor to second order in the metric fluctuations. There is one such condition for each Killing vector.

To summarize, there are some extra subtleties in doing gravitational perturbation theory in the bounce background, but these subtleties are well understood by relativists and constitute an inconvenience rather than a disease. We have not yet determined the most convenient way to compute in such a background.[2] At the moment our main goal is simply to establish that perturbation theory makes sense in the bounce background.

[2]Pure de Sitter space is the simplest case of a background with linearization instabilities. While there is no conceptual problem, it is unclear what is the most convenient way to compute in such a background. Higuchi and Weeks [18] have computed the graviton propagator in de Sitter space without worrying about the linearization instability. As suggested by Moncrief, one could possibly use their propagator and impose the extra constraints on the states. We have not yet pursued this possibility.

APPENDIX B: MOTION OF DOMAIN WALL WITH MASS INSIDE

Here is the computation to support the claim of section VA that adding a mass inside the bubble causes the domain wall to move out so that there is no horizon at all. Begin with the junction condition

$$\sqrt{1 - 2GM/r + \dot{r}^2} - \sqrt{1 - r^2/R^2 + \dot{r}^2} = 4\pi G\sigma r. \tag{B1}$$

We will deal with small M. At the time when $\dot{r} = 0$, the radius of the bubble is given by r_0, where r_0 satisfies

$$\sqrt{1 - 2GM/r_0} - \sqrt{1 - r_0^2/R^2} = 4\pi G\sigma r_0. \tag{B2}$$

We will expand around a solution with $M = 0$ which has the same minimum size. We think of this solution as having a different tension σ_1:

$$1 - \sqrt{1 - r_0^2/R^2} = 4\pi G\sigma_1 r_0. \tag{B3}$$

For small M, subtracting the two equations shows that

$$4\pi G r_0(\sigma_1 - \sigma) = GM/r_0. \tag{B4}$$

It is more convenient to think of τ as a function of r rather than vice versa. We call the unperturbed solution $\tau(r)$ and the perturbed solution $\tilde{\tau}(r)$. Derivatives with respect to r are denoted by primes. Then $\tilde{\tau}(r)$ and $\tau(r)$ solve

$$\sqrt{1 - 2GM/r + 1/\tilde{\tau}'^2} - \sqrt{1 - r^2/R^2 + 1/\tilde{\tau}'^2} = 4\pi G\sigma r, \tag{B5}$$

$$\sqrt{1 + 1/\tau'^2} - \sqrt{1 - r^2/R^2 + 1/\tau'^2} = 4\pi G\sigma_1 r. \tag{B6}$$

Expanding $\tilde{\tau}(r) = \tau(r) + \epsilon(r)$, these equations become

$$\sqrt{1 - 2GM/r + 1/\tau'^2 - 2\epsilon'/\tau'^3} - \sqrt{1 - r^2/R^2 + 1/\tau'^2 - 2\epsilon'/\tau'^3} = 4\pi G\sigma r, \tag{B7}$$

$$\sqrt{1 + 1/\tau'^2} - \sqrt{1 - r^2/R^2 + 1/\tau'^2} = 4\pi G\sigma_1 r. \tag{B8}$$

Subtracting the two equations, expanding the square roots for small ϵ and small M, and using the formula (B4) to eliminate the tensions, we get

$$-\frac{GM}{r\sqrt{1 + 1/\tau'^2}} - \frac{\epsilon'}{\tau'^3\sqrt{1 + 1/\tau'^2}} + \frac{\epsilon'}{\tau'^3\sqrt{1 - r^2/R^2 + 1/\tau'^2}} = -\frac{GMr}{r_0^2}. \tag{B9}$$

Simplify this by using the equation satisfied by $\tau(r)$, (B6), which is more useful in the form

$$1 + 1/\tau'^2 = r^2/r_0^2, \tag{B10}$$

to get

$$GMr_0/r^2 + \epsilon'(r^2/r_0^2 - 1)^{3/2}\left(\frac{r_0}{r} - \frac{r_0}{r\sqrt{1 - r_0^2/R^2}}\right) = GMr/r_0^2, \tag{B11}$$

or

$$\epsilon' = \frac{GM(r^3/r_0^3 - 1)}{r\left(1 - 1/\sqrt{1 - r_0^2/R^2}\right)(r^2/r_0^2 - 1)^{3/2}}. \tag{B12}$$

Our interest is not really in the function $\tilde{\tau}(r)$ because we know that the proper time approaches infinity as the size of the bubble approaches infinity, so $\tau \to \infty$ as $r \to \infty$. We want to know where on the Penrose diagram the brane ends up, or equivalently what light ray it approaches asymptotically. For this purpose we introduce the coordinate x^+ defined by

$$dx^+ = dt - \frac{dr}{1 - r^2/R^2}, \tag{B13}$$

$$x^+ = 0 \quad \text{at} \quad t = r = 0. \tag{B14}$$

Recall that

$$d\tau^2 = (1 - r^2/R^2)dt^2 - \frac{dr^2}{1 - r^2/R^2}, \tag{B15}$$

so

$$dt^2 = dr^2\left(\frac{1}{(1 - r^2/R^2)^2} + \frac{\tau'^2}{1 - r^2/R^2}\right). \tag{B16}$$

The unperturbed solution asymptotically approaches the light ray $x^+ = 0$ as one can see from the Penrose diagram; the perturbed solution is

$$\Delta x^+ = \int_{r_0}^{\infty} \frac{dr}{1 - r^2/R^2}\left(\sqrt{1 + \tilde{\tau}'^2(1 - r^2/R^2)} - 1\right). \tag{B17}$$

Since $x^+(\tau = \infty) = 0$ for the unperturbed solution, we expand $\tilde{\tau}$ as above and keep only the first order piece. Thus

$$x^+(\tau = \infty) = \int_{r_0}^{\infty} dr \frac{\epsilon'}{\sqrt{1 - r^2/R^2 + 1/\tau'^2}}. \tag{B18}$$

Using (B10) and (B12), this becomes

$$x^+(\tau = \infty) = \frac{GMr_0}{\sqrt{1 - r_0^2/R^2} - 1} \int_{r_0}^{\infty} dr \frac{r^3/r_0^3 - 1}{r^2(r^2/r_0^2 - 1)^{3/2}}. \tag{B19}$$

The integral is clearly convergent, and up to a constant can be done by dimensional analysis. The final answer is

$$x^+(\tau = \infty) = -(\text{const})\frac{GM}{1 - \sqrt{1 - r_0^2/R^2}}. \tag{B20}$$

Looking back at the definition of x^+, Eq. (B13), we see that negative x^+ means that now the brane is asymptotic to a light ray which has positive r at $t = 0$. The brane has moved out. The conformal diagram is shown in the main text, Fig. 13.

APPENDIX C: DE SITTER POLES IN THE S-MATRIX

We are in an unusual situation where we understand the semiclassical solutions, but we do not know how to pick out a time variable and define an energy. This is a problem because we want to look for poles in the S-matrix as a function of energy. We do know how to compute the action for semiclassical configurations. Our strategy will be to compute the action semiclassically, and then get at the energy through the back door by using the semiclassical formula $\frac{\partial S}{\partial t} = -E$.

We are also able to extract information about S-matrix elements in this way. In a one-dimensional quantum mechanics problem, the S-matrix consists merely of phases. It is precisely true that the phase shift, which is naturally thought of as a function of energy, is the Fourier transform of the action, which is naturally thought of as a function of the time:

$$e^{i\delta(E)} = \int dt e^{iEt} e^{iS(t)}. \tag{C1}$$

Here $\delta(E)$ is the phase shift, and $S(t)$ is the action, as will be defined more carefully below. The meaning of the formula is that the S-matrix can be thought of in the usual energy representation or in the time representation, and as always they are related by Fourier transformation. In the energy representation, one computes energy eigenfunctions and extracts the phase relationship between the leftmoving and rightmoving waves. There is a meaningless constant in deciding the zero of the phase shift. In the time representation, one computes the amplitude to start from an arbitrarily chosen point far to the right and come back to that point in a time t. The amplitude is given by an integral over paths as usual in quantum mechanics. We call this amplitude $e^{iS(t)}$. Again, there will be a meaningless constant which depends on the choice of point. With these definitions, Eq. (C1) is exact, but in practice we will estimate $S(t)$ using semiclassical techniques.

Semiclassically, we do the Fourier transform by saddle points. The saddle point condition is

$$\frac{\partial S}{\partial t} = -E, \tag{C2}$$

which is a standard equation in classical mechanics.

Evaluating the function gives

$$e^{i\delta(E)} = e^{i[Et+S(t)]}, \tag{C3}$$

or

$$\delta(E) = Et + S(t) = -t\frac{\partial S}{\partial t} + S(t), \tag{C4}$$

which identifies $\delta(E)$ and $S(t)$ as Legendre transforms of each other. This is the standard relationship: the Legendre transform is the semiclassical approximation to the Fourier transform. The end result is that Eq. (C4) enables us to turn semiclassical computations of the action into semiclassical computations of the S-matrix.

To illustrate the technique, we begin with a simple quantum mechanics problem. Consider a particle moving in the potential shown in Fig. 16. We will approximate $S(t)$ by semiclassical methods and show that our resulting S-matrix has the right behavior. For short times, the only important trajectories are those which bounce off the wall. The action is simply

$$S(t) = \int dt \frac{1}{2} m\dot{x}^2 = \frac{1}{2} m\dot{x}^2 t = \frac{1}{2} m \frac{d^2}{t}. \tag{C5}$$

Here we have chosen to start and end our paths at $x = d$.

For long times, the only important trajectories are those which tunnel into the well and stay there for a long time. The action for these trajectories can be estimated as a sum of three contributions: the action outside the well, the action inside the well, and the action associated with tunneling through the barrier. We will calculate each of these in a crude way. We could do better for this problem, but the point is that crude estimates provide information about the S-matrix.

The action associated with tunneling is an imaginary factor $S_{\text{tunneling}} = i\gamma$. While the particle is inside the well, it is described by a harmonic oscillator for which the minimum potential energy is nonzero:

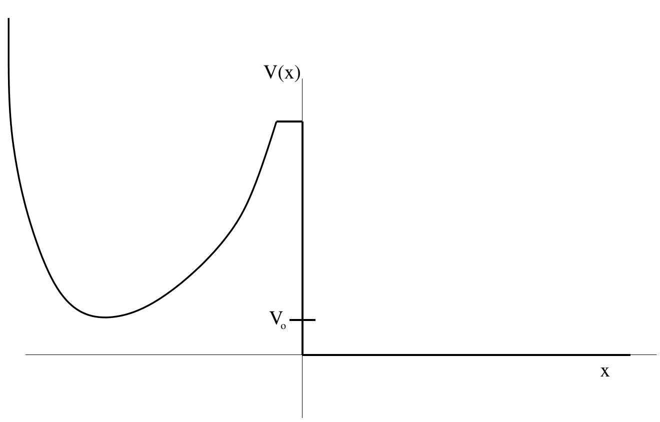

FIG. 16. The S-matrix of a quantum mechanical particle moving in this potential exhibits a pole at the energy of the bound state.

$$S_{\text{H.O.}} = \int dt\left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - V_0\right). \tag{C6}$$

Over one period,

$$\left\langle\frac{1}{2}m\dot{x}^2\right\rangle = \left\langle\frac{1}{2}kx^2\right\rangle \tag{C7}$$

so the only contribution is from V_0. If the particle is in the well for many oscillations, then the dominant contribution to the action is simply

$$S_{\text{H.O.}} = -V_0 t. \tag{C8}$$

There will also be an oscillatory piece which we choose not to compute. Finally, for these paths the particle spends almost all of its time inside the well, so we simply

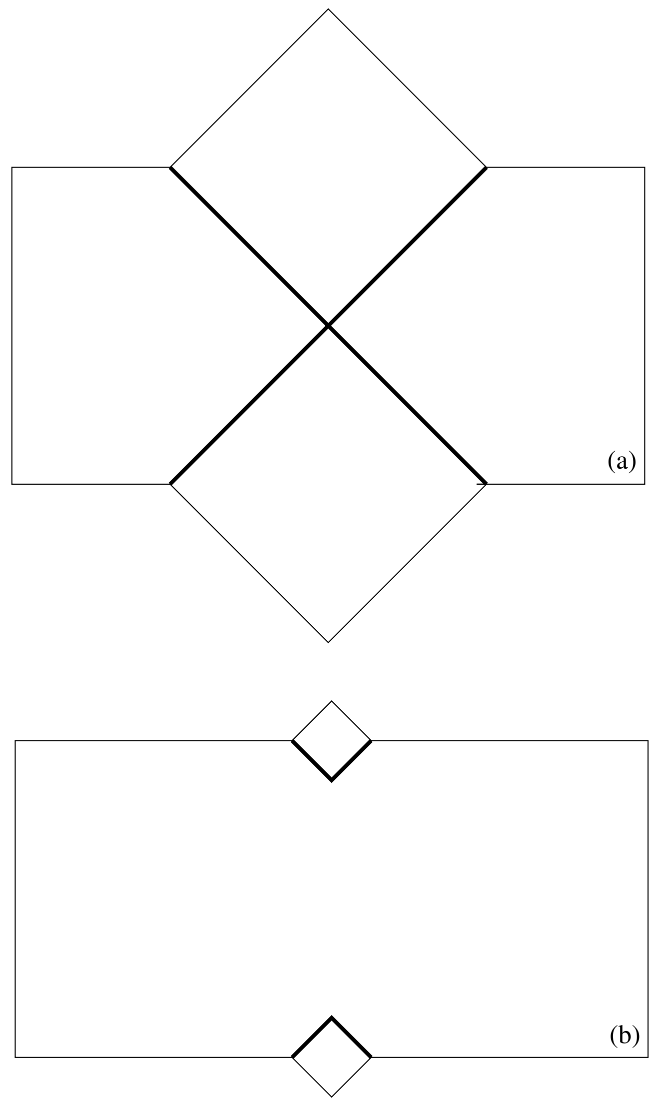

FIG. 17. In the limit of zero brane tension, the domain wall always moves at the speed of light. Conformal diagrams for the classical solution (left) and the tunneling solution (right) are shown.

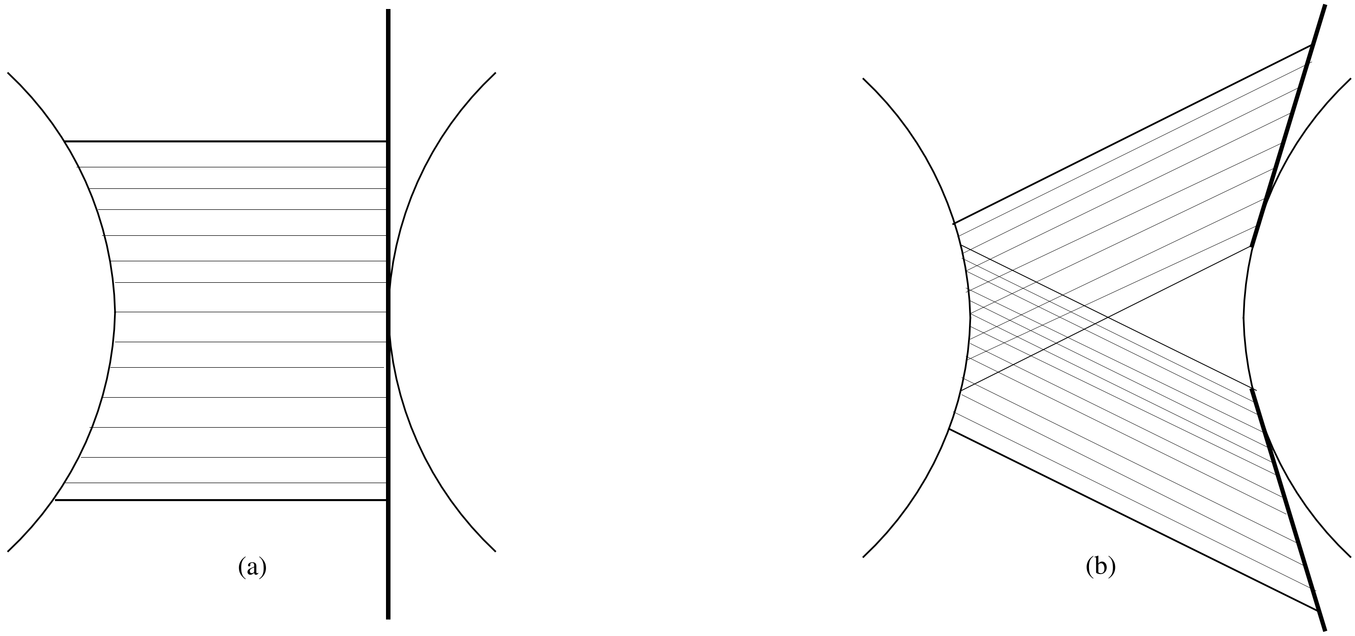

FIG. 18. At left, a slice through the embedding shows the classical solution in the limit of zero brane tension. The hyperboloid is sliced through by a plane (thick vertical line). At right, a tunneling solution. The hyperboloid is sliced by two semi-infinite planes (thick rays). In both cases, time slices in the rest frame of the bubble are shown. They are horizontal lines on the left; on the right they are boosted so they are not horizontal anymore. One can see that the area between the top and bottom horizontal lines on the left is the same as the area between the top and bottom boosted horizontal lines on the right, so the action is equal for the two configurations aside from the tunneling factor.

ignore the action due to the time it spends outside the well. So we arrive at the formula for long times

$$S(t) = -V_0 t + 2i\gamma. \tag{C9}$$

Now let us see whether we get a pole in the S-matrix. We Fourier transform to get

$$e^{i\delta(E)} = \int_0^\infty dt e^{i(Et - V_0 t + 2i\gamma)} = \frac{ie^{-2\gamma}}{E - V_0}, \tag{C10}$$

which has a pole at the resonance! In order to get the imaginary part of the pole, we would have to do a little better, but this example makes it clear that a crude semiclassical understanding of the action can yield important information about the S-matrix. If we adjust our energy scale so that $V_0 = 0$, then $S(t)$ does not depend on t for long times, as one can see from (C9). This corresponds to a pole at $E = 0$.

Now for the real problem. For simplicity, we take the limit where the tension of the brane is extremely small, so that it is essentially always moving at the speed of light. However, our results will be general. In the zero tension limit, the conformal diagram for the bounce solution looks like Fig. 17(a). A tunneling solution is shown in Fig. 17(b).

The corresponding embedding is the hyperboloid cut by the plane $x = 1$, as shown in Fig. 18(a). We know the gravitational action is simply

$$S = \frac{1}{G} \int d^4x \sqrt{g}(R + \Lambda). \tag{C11}$$

Since there is no time-like Killing vector, it is not obvious which time variable to use in defining $S(t)$. We will simply using the embedding time since it is easy to work with. In addition, we are interested in an observer in the flat part of the bubble and the embedding time *is* a Killing vector in the flat part of the geometry, so we might expect that at very late times when the branes have gone most of the way to infinity this time is the right one to use.

The action is infinite, because the space at the top and bottom of the diagram becomes infinitely big. The action all comes from the de Sitter part of the geometry, and is proportional to

$$S \sim \frac{\Lambda}{G} \int d^4x \sqrt{g}. \tag{C12}$$

We choose to cut it off when the branes get out to a distance t_0, which is also the time from the origin.

Now consider a tunneling solution. In the limit of zero brane tension, the bubbles will nucleate at zero size. Consider for a moment the two points from which the bubbles nucleate. There are enough symmetries in de Sitter space that only the invariant distance between the two points matters, so I can place them time symmetrically and along the same spatial axis. Each individual bubble nucleation is a boost of one which starts at $t = 0$.

They each are defined by planes which are tangent to the hyperboloid at the time of bubble nucleation. A slice through the geometry is shown in Fig. 18(b). It is natural to cut off the integral along a time slice in the rest frame of the bubble. Then a geometric argument (Fig. 18) shows that the action for the tunneling solution is exactly the same as the action for the classical solution aside from the tunneling factor.

The time delay can be arbitrarily long for tunneling solutions, so for long times the action is NOT a function of the time delay. This is exactly what happened in the quantum mechanics example when we chose $V_0 = 0$. Just like in that example, the fact that the action is independent of the time delay indicates a pole at $E = 0$. Since the action was not a function of the time delay, our confusion about which time to use was unimportant. A miracle has occurred: We have been able to conclude not only that all of these solutions have the same energy, but also that they all have zero energy without having to precisely define what we mean by time.

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