

**Microscopic entropy of nondilatonic branes: A 2D approach**

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We investigate nondilatonic  $p$ -branes in the near-extremal, near-horizon regime. A two-dimensional gravity model, obtained from dimensional reduction, gives an effective description of the brane. We show that the  $\text{AdS}_{p+2}/\text{CFT}_{p+1}$  correspondence at finite temperature admits an effective description in terms of a  $\text{AdS}_2/\text{CFT}_1$  duality endowed with a scalar field, which breaks the conformal symmetry and generates a nonvanishing central charge. The entropy of the  $\text{CFT}_1$  is computed using Cardy formula. Fixing in a natural way a free, dimensionless, parameter introduced in the model by a renormalization procedure, we find exact agreement between the  $\text{CFT}_1$  entropy and the Bekenstein-Hawking entropy of the brane.

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**I. INTRODUCTION**

The brane solutions of string and M-theory are important for several reasons. The nondilatonic  $p$ -brane solutions play a crucial role in the formulation of the antide Sitter/Conformal Field theory (AdS/CFT) correspondence [1–3]. For instance, in the 3-brane case, the low energy limit of string theory is found to have two different descriptions, each of them splitting into two decoupled pieces. The first is free bulk supergravity and the near-horizon geometry of the extremal 3-brane ( $\text{AdS}_5 \times S^5$ ) and the second is free bulk supergravity and  $\mathcal{N} = 4$ ,  $U(N)$  super Yang-Mills theory (SYM) in four dimensions. This led Maldacena to identify string theory on  $\text{AdS}_5 \times S^5$  and the SYM theory as duals. Similar arguments led Maldacena to propose a duality between string theory on the near-horizon geometry of extremal nondilatonic  $p$ -branes in  $D$ -dimensions ( $\text{AdS}_{p+2} \times S^{D-p-2}$ ) and a conformal field theory in  $p + 1$  dimensions.

Brane solutions are also interesting from a slightly different, albeit related, point of view.  $p$ -branes are classical solutions of supergravity (SUGRA) theories in  $D$  dimensions. Being gravitational configurations, they may be endowed with an event horizon and become black  $p$ -branes. From this point of view they can be considered as a generalization of charged black hole solutions of general relativity. In particular, one can associate to them a thermodynamical entropy using Bekenstein-Hawking area law. Similarly to the black hole case, one has to face the problem of giving a microscopical interpretation of the Bekenstein-Hawking entropy of the brane.

In view of the AdS/CFT correspondence, one is tempted to use the  $\text{CFT}_{p+1}$  dual theory to compute the microscopical entropy of the near-horizon, near-extremal  $p$ -brane. However, this is not so easy. The AdS/CFT duality is assumed to hold at zero temperature, corresponding to the extremal brane, which has also zero entropy. Near-

extremal branes have nonvanishing temperature and entropy, but finite temperature effects break conformal invariance. If the AdS/CFT duality survives finite temperature effects, the near-extremal brane should be described by a CFT at finite temperature. Indications that this could be the case come from calculations for the 3-brane.

Klebanov *et al.* compared the entropy of the 3-branes with that of finite temperature, weak-coupled  $U(N)$  gauge theory. They found an agreement of the two results up to a numerical factor [4,5]. The origin of the discrepancy factor is qualitatively well understood. The gauge theory computation is performed at weak 't Hooft coupling, whereas the gravity description is assumed to be valid at strong 't Hooft coupling. Also the result for the 1-brane in  $D = 6$  indicates that near-extremal branes can be described by a finite temperature CFT. In this latter case, the near-extremal brane can be identified as a Bañados-Teitelboim-Zanelli (BTZ) black hole, whose entropy can be exactly reproduced using a two-dimensional (2D) CFT at finite temperature [6].

For M-branes (the 2,5-brane) the situation looks rather different. Here the AdS/CFT duality is of little help. The Klebanov *et al.* calculation, which uses a dual, weak-coupled, field theory, reproduces correctly the scaling behavior of the brane entropy with the temperature but not that with the number of branes  $N$ . Thus, for the 2,5-brane we have rather weak indications that the near-extremal brane can admit a description in terms of a finite temperature  $\text{CFT}_{p+1}$ .

Other attempts to explain the entropy of nonextremal  $p$ -branes use a generalization of the approach proposed by Strominger and Vafa to compute the entropy of extremal BPS black holes [7]. One tries to explain the entropy of the brane in terms of states of the string living on the brane [8–12].

Considering this situation it is worth exploring other possibilities to describe near-extremal nondilatonic  $p$ -branes, which can be used to give a microscopic interpretation of the brane entropy. In Ref. [13] has been

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proposed an effective description of the near-extremal 3-brane in terms of a AdS<sub>2</sub>/CFT<sub>1</sub> duality endowed with a scalar field which breaks the conformal symmetry and generates a nonvanishing central charge. The Bekenstein-Hawking entropy of the 3-brane could be matched by CFT<sub>1</sub> calculations up to a numerical factor.

In this paper we improve the method used in Ref. [13] and generalize it to all the relevant nondilatonic branes. In particular, we will be able to reproduce exactly the thermodynamical entropy of the near-extremal nondilatonic  $p$ -branes fixing in a natural way a dimensionless, renormalization parameter, which appears as free parameter in our calculations.

In Sec. II we briefly review some basic facts about black  $p$ -branes. Later on (Sec. III) we perform a dimensional reduction of the brane to obtain a 2D gravity model, which gives an effective description of the near-horizon, near-extremal brane. In Sec. IV we study the group of asymptotical symmetries (ASG) of the 2D solutions. A one-dimensional CFT will emerge as dual description of the 2D bulk gravity theory. The central charge of the associated Virasoro algebra is computed in section V. Making use of a renormalization procedure, we will be able to find finite charges associated with the ASG, at the price of

$$ds^2 = [H(r)]^{-2(d-2)/\delta}[-f(r)dt^2 + \sum_{i=1}^p dx_i dx^i] + [H(r)]^{2(p+1)/\delta}[f^{-1}(r)dr^2 + r^2 d\Omega_{d-1}^2] \quad H(r) = 1 + \left(\frac{h_p}{r}\right)^{d-2}, \quad (2)$$

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{d-2} \quad D = d + p + 1, \quad \delta = (p + 1)(d - 2) + \frac{a^2(D - 2)}{2}, \quad h_p^{2(d-2)} + r_0^{d-2} h_p^{d-2} = \frac{\delta Q^2}{2(d - 2)^2(D - 2)},$$

where  $Q$  is the RR charge,  $r_0$  and  $h_p$  are integration constants related to the mass and charge of the brane. The solution (2) can be regarded as a generalization of the Reissner-Nordstrom charged black hole solution, for this reason it is called black  $p$ -brane.

We are interested in nondilatonic branes: M-branes (the 2-brane and 5-brane in 11 dimensions) and dyonic branes with equal magnetic and electric charges in  $D = 2p + 4$ , with  $p$  odd (1-brane in  $D = 6$  and 3-brane in  $D = 10$ ). M-branes are nondilatonic simply because there is no dilaton in 11 dimensions. Dyonic branes with equal charges in  $D = 2p + 4$  and  $p$  odd are self-dual solutions of theories with self-dual  $(p + 2)$ -field strengths. They are characterized by a constant dilaton and can be thought of as intrinsically nondilatonic. In what follows we will consider only nondilatonic branes.

In the extremal limit ( $r_0 = 0$ ) the metric (2) becomes:

$$ds^2 = [H(r)]^{-2/p+1} \left(-dt^2 + \sum_{i=1}^p dx_i dx^i\right) + [H(r)]^{2/d-2} \times (dr^2 + r^2 d\Omega_{d-1}^2), \quad h_p^{d-2} = \frac{Q}{\sqrt{\alpha}(d-2)},$$

$$\alpha = \frac{2(D-2)}{(d-2)(p+1)}. \quad (3)$$

introducing a dimensionless free parameter  $\beta$ . The entropy of the  $p$ -brane is then calculated via the Cardy formula. Fixing the parameter  $\beta$  we reproduce exactly the thermodynamical entropy of the brane.

## II. NON-DILATONIC BLACK $p$ -BRANES

In this section we briefly review some well known facts about  $p$ -branes. Black  $p$ -branes are classical Ramond-Ramond (RR) charged solutions of SUGRA theory in  $D$  dimensions [14–19]. They can be also considered as the low energy limit of string and M-theory.

In the Einstein frame the bosonic part of the action reads:

$$A = \frac{1}{2k_D^2} \int d^D x \sqrt{-g} \left[ R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2n!} F_n^2 e^{a\phi} \right], \quad (1)$$

where  $\phi$  is the dilaton,  $F_n$  is the field strength of an  $(n - 1)$ -form potential  $F_n = dA_{n-1}$ , and  $a$  is a constant depending on the dimensional reduction that produces the action in  $D$  dimensions. The metric part of the electric solution of the action (1) is [20–22] (the magnetic solution is obtained using Hodge duality)

The brane tension  $T_p$  and the  $D$ -dimensional Newton constant  $G_D$ , can be written as follows,

$$T_p = \frac{2\pi}{(2\pi l_D)^{p+1} g_s}, \quad 2k_D^2 = 16\pi G_D = \frac{(2\pi l_D)^{D-2} g_s^2}{2\pi}, \quad (4)$$

where for  $D = 10$   $l_D$  is the string length and  $g_s$  is the string coupling constant, whereas for  $D = 11$   $l_D$  is the Planck length and  $g_s = 1$ . We can also define,

$$e_p = \frac{1}{\sqrt{2}k_D} \int_{S^{d-1}} *F_{d-1} = \frac{Q\Omega_{d-1}}{\sqrt{2}k_D}. \quad (5)$$

For a single  $p$ -brane the flux of the RR field is  $\sqrt{2}k_D T_p$ . If we consider  $N$  coincident  $p$ -branes we have

$$N = \frac{e_p}{\sqrt{2}k_D T_p} = \frac{\Omega_{d-1} Q}{2k_D^2 T_p}, \quad h_p^{d-2} = \frac{(2\pi l_D)^{d-2} N g_s}{\sqrt{\alpha}(d-2)\Omega_{d-1}}. \quad (6)$$

The AdS/CFT duality arises considering the near-horizon limit,

$$(r, l_D) \rightarrow 0 \quad \text{keeping fixed } u^{(p+1)/(d-2)} = r l_D^{-\alpha(p+1)/2} \quad (7)$$

of the brane solution. In this limit the extremal brane (3) becomes [1,21]:

$$ds^2 = l_D^4 \lambda_p^2 u^2 \left( -dt^2 + \sum_{i=1}^p dx_i dx^i \right) + \frac{du^2}{\lambda_p^2 u^2} + R_{0p}^2 d\Omega_{d-1}^2, \quad (8)$$

where  $R_{0p} = h_p$  and  $\lambda_p = (d-2)/[(p+1)R_{0p}]$ . The near-horizon region has the  $AdS_{p+2} \times S^{d-1}$  geometry. This was the starting point of the Maldacena conjecture about a duality between string theory in this background and conformal field theory in  $(p+1)$  dimensions. The isometry group of  $AdS_{p+2}$  is identified with the conformal symmetry of  $(p+1)$ -dimensional Minkowski space.

Excitations above extremality break the conformal symmetry and the brane acquires finite temperature and entropy. The near-horizon, near-extremal form of the solution can be easily obtained by taking in Eq. (2) the near-horizon limit (7) and the energy above extremality finite. One finds

$$ds^2 = l_D^4 \lambda_p^2 u^2 \left\{ - \left[ 1 - \left( \frac{u_0}{u} \right)^{p+1} \right] dt^2 + \sum_{i=1}^p dx_i dx^i \right\} + \frac{du^2}{\lambda_p^2 u^2 \left[ 1 - \left( \frac{u_0}{u} \right)^{p+1} \right]} + R_{0p}^2 d\Omega_{d-1}^2, \quad (9)$$

where  $u$  and  $u_0$  are defined as in Eq. (7).

Using the Bekenstein-Hawking formula, we can easily calculate the entropy of the brane (9). Working in the canonical ensemble, we can express the entropy  $S_p$  as a function of the temperature  $T$  and volume  $V$  of the brane,

$$S_p = a_p VT^p, \quad (10)$$

where  $a_p$  depends on the number  $N$  of coincident branes and is given for the 1, 2, 3, 5-brane under consideration as follows,

$$\begin{aligned} a_1 &= \pi N^2, & a_2 &= \frac{2^{7/2} \pi^2}{27} N^{3/2}, \\ a_3 &= \frac{\pi^2}{2} N^2, & a_5 &= \frac{2^7}{3^6} \pi^3 N^3. \end{aligned} \quad (11)$$

Klebanov *et al.* tried to give a microscopic interpretation of the thermodynamical entropy of the brane using a system of weak interacting brane excitations [4,5]. They could reproduce the scaling behavior (10), but they found an expression for the coefficients  $a_p$  depending on a parameter  $\tilde{n}$ , which characterizes the field content of the model:

$$a_1 = \frac{\pi}{2} \tilde{n}, \quad a_2 = \frac{7}{8\pi} \zeta(3) \tilde{n}, \quad a_3 = \frac{\pi^2}{12} \tilde{n}, \quad a_5 = \frac{\pi^3}{40} \tilde{n}. \quad (12)$$

For the 3- and 1-brane the AdS/CFT correspondence allows an easy identification of parameter  $\tilde{n}$ . The CFT<sub>4</sub> dual to AdS<sub>5</sub> is well known, it is  $\mathcal{N} = 4$ ,  $U(N)$  SYM. This fact enables us to identify  $\tilde{n} = 8N^2$  [4]. In this way the statistical entropy is in agreement with the thermodynamical one up to a 3/4 factor. The origin of the discrepancy

factor is well understood. The gauge theory computation is performed at zero 't Hooft coupling, whereas the gravity description is assumed to be valid in the strong coupling regime [23,24]. For the 1-brane the AdS/CFT correspondence allows us to reproduce exactly the thermodynamical result (11). In this case, neglecting the 3-sphere of constant radius, the brane solution (9) is nothing but the Bañados-Teitelboim-Zanelli (BTZ) black hole, whose microscopic entropy has been calculated by Strominger [6], leading to the identification  $\tilde{n} = 2N^2$

For the 2 M-branes the situation is more involved. The AdS/CFT correspondence is here of little help, because the dualities AdS<sub>4</sub>/CFT<sub>3</sub> and AdS<sub>7</sub>/CFT<sub>6</sub> are poorly understood. Moreover, in order to explain the dependence on  $N$  in Eq. (11) we need a behavior  $\tilde{n} \sim N^{3/2}$  and  $\tilde{n} \sim N^3$ , respectively, for the 2- and five brane, which is very hard to achieve using a field theory. In spite of some progress, achieved considering a D-brane-D-antibrane system [25–29], this still remains a puzzling point, which is related with our lack of knowledge about M-theory.

In this paper we will use a 2D approach to the problem of giving a microscopical interpretation for the entropy of nondilatonic branes. The first step in this direction is to perform a dimensional reduction in order to obtain a 2D effective description of the brane. This will be the subject of the next section.

### III. DIMENSIONAL REDUCTION

The near-horizon, near-extremal nondilatonic brane solution (9) factorizes as direct product of a  $(p+2)$ -dimensional spacetime, which is asymptotically AdS <sub>$p+2$</sub> , and a  $(d-1)$ -sphere  $S^{d-1}$  of constant radius. This fact allow us to derive, by dimensional reduction, a 2D effective gravity model, which describes the spherically symmetric excitations of the brane above extremality. We can perform the dimensional reduction from  $D$  to two dimensions using the ansatz:

$$ds_D^2 = ds_2^2 + \phi^{2/p} \sum_{i=1}^p dx_i dx^i + R_{0p}^2 d\Omega_{d-1}^2, \quad (13)$$

where  $\phi$  is a scalar field, which parametrizes the volume  $\mathcal{V}$  of the brane embedded in the  $(p+2)$ -dimensional spacetime,

$$\mathcal{V} = \phi V. \quad (14)$$

For the RR field strength we have  $F_n^2/n! = Q^2/h_p^{2(d-1)}$ . Performing the dimensional reduction in the  $D$ -dimensional action (1) we get the 2D effective model,

$$A_{2D} = k \int d^2x \sqrt{-g} \phi \left\{ R + \left( \frac{p-1}{p} \right) \frac{(\nabla\phi)^2}{\phi^2} + \Lambda \right\}, \quad (15)$$

where the cosmological constant  $\Lambda = R_{(d-1)} - (F_n^2/2n!)$ ,  $R_{(d-1)}$  being the scalar curvature of  $S^{d-1}$  and the constant  $k$

is

$$k = \frac{\Omega_{d-1} R_{0p}^{d-1} V}{2k_D^2} = \frac{2\pi N V R_{0p}}{\sqrt{\alpha}(d-2)(2\pi l_D)^{p+1} g_s}. \quad (16)$$

The class of 2D gravity models described by the action (15) has been already investigated in the literature [30–32]. In particular, they admit the, asymptotically AdS, 2D black hole solutions,

$$ds^2 = -[b^2 r^2 - A^2 (br)^{1-p}] dt^2 + \frac{dr^2}{b^2 r^2 - A^2 (br)^{1-p}}, \quad (17)$$

$$\phi = \phi_0 (br)^p,$$

where  $b^2 = \Lambda/[p(1+p)]$  and  $\phi_0, A$  are integration constants. The thermodynamical behavior of the 2D black hole is characterized by mass  $m_{bh}$ , temperature  $T_{bh}$  and entropy  $S_{bh}$ ,

$$m_{bh} = \frac{p}{2} \phi_0 A^2 b, \quad T_{bh} = \frac{b(p+1)}{4\pi} A^{2/p+1}, \quad (18)$$

$$S_{bh} = 2\pi \Phi_0 A^{2p/p+1}.$$

The 2D black hole solution (17) gives an effective description of the  $D$ -dimensional brane solution (9). The 2D integrations constant  $A$  and  $\phi_0$  can be identified in terms of the physical parameters of the brane. Comparing Eq. (9) with Eq. (17), we can easily see that  $A$  is related to energy of the brane excitations above extremality:

$$A^2 = \lambda_p^{p+1} u_0^{p+1}. \quad (19)$$

We can fix the value of  $\phi_0$  using a (classical) scale symmetry of the 2D action (15). Rescaling the scalar field  $\phi \rightarrow \mu \phi$  the action changes as  $A_{2D} \rightarrow \mu A_{2D}$ . In this way we can change the normalization factor in front of the action. Choosing the normalization of Ref. [30] ( $k = 1/2$ ) we have  $\phi_0 = 2k$ . One can easily check that the thermodynamical behavior of the 2D black hole solution reproduces exactly the thermodynamics of the near-extremal brane (9). In fact, we have  $S_{\text{brane}} = S_{bh}$ ,  $E_{\text{brane}} = m_{bh}$  and  $T_{\text{brane}} = T_{bh}$ , where  $E_{\text{brane}}$  is the energy of a brane excitation above extremality. This fact has a natural interpretation. The thermodynamics of the brane is determined by the behavior on the horizon of the 2D  $(r, t)$  section of metric (9), which is exactly given by the 2D black hole.

On the other hand both the nondilatonic brane and the 2D black hole seem to have a dual descriptions in terms of a conformal field theory. For the brane the dual theory is a  $CFT_{p+1}$ , whereas for the 2D black hole it is a  $CFT_1$  [30,33–38]. This gives a strong indication that the  $CFT_1$  can be used both to give an effective description of the  $CFT_{p+1}$  and to obtain a microscopic derivation of the entropy of the brane, in agreement with the philosophy of the holographic principle. To achieve this goal we first need to investigate the asymptotical symmetries of the 2D

solutions, which will be the topic discussed in the next section.

#### IV. ASYMPTOTICAL SYMMETRIES

The group of asymptotical symmetry (ASG) of the metric (17) is the group of transformations which leaves the asymptotic,  $r \rightarrow \infty$ , behavior of the metric invariant. The case  $p = 1$  is well known; it corresponds to 2D antide Sitter spacetime ( $\text{AdS}_2$ ). Its ASG was investigated in various papers and the problem of the microscopical explanation of the entropy of the corresponding 2D black hole has been completely solved [33–37]. In this paper we focus our attention on the other three cases ( $p = 2, 3, 5$ ). The asymptotical symmetries of the metric (15) were investigated in Ref. [30]. In that paper the ASG was identified with the group of reparametrizations of the 1-dimensional,  $r \rightarrow \infty$ , timelike boundary of the  $\text{AdS}_2$  spacetime (the  $\text{diff}_1$  group). It was shown that the generators of the group satisfy a Virasoro algebra. Unfortunately, the charges associated with the generators and the central extension of the Virasoro algebra were found to be divergent. A renormalization procedure was not applicable directly because it erases identically the charges. The divergence is due to the power behavior  $\phi \sim r^p$  of the scalar field for  $r \rightarrow \infty$ . In this paper we propose a general method to renormalize the charges in a consistent way. Our method is a generalization of the renormalization procedure proposed in Ref. [13].

We can separate a finite from a divergent part in the charges performing the change of coordinate:

$$(br)^{p-1} \rightarrow (br)^{p-1} + C. \quad (20)$$

The parameter  $C$  is a free dimensionless parameter. It must be related to  $A$ , the only physical dimensionless parameter appearing in the 2D metric (17). In general we will therefore have  $C = \beta f(A)$ , where  $\beta$  is a dimensionless parameter not depending on  $A$  and  $f(A)$  is an arbitrary function of the parameter  $A$ . The form of the function  $f(A)$  can be determined using scale symmetry arguments. The metric part of the 2D black hole vacuum (the metric ((17) with  $A = 0$ ) is invariant under the scale transformation  $r \rightarrow \mu r$  and  $t \rightarrow \mu^{-1} t$ , which is a subgroup of the full isometry group of 2D AdS spacetime. The scale transformation changes the solution for the dilaton in Eq. (17) but this change is just a scale factor, which can be absorbed in the arbitrary integration constant  $\phi_0$ . The scale transformation maps one into the other the classical vacuum solutions (17). This is a consequence of the on-shell scale symmetry of the 2D action (15) discussed at the end of the previous section. The symmetry of the vacuum solution can be maintained for the massive black hole states if the parameter  $A$  transforms as  $A \rightarrow \mu^{p+1/2} A$ . After the change of coordinates (20), the black hole vacuum will be invariant under the scale transformation only if

$$C = \beta A^{2(p-1)/p+1}. \quad (21)$$

Notice that our scale symmetry argument fixes  $C$  up to an arbitrary dimensionless parameter  $\beta$ , whose value cannot be fixed by the renormalization procedure

A uniform treatment of both even and odd branes is not possible. In the following we will distinguish the two cases. Performing the change of coordinate (21) and expanding the solution (17) near  $r \rightarrow \infty$  we have for  $p = 2$ ,

$$\begin{aligned} g_{tt} &= -b^2 r^2 - \frac{2}{(p-1)} \beta A^{2(p-1)/p+1} (br)^{3-p} + A^2 (br)^{1-p} - \frac{3-p}{(p-1)^2} \beta^2 A^{4(p-1)/p+1} (br)^{-(p+1)} + O[r^{2(1-p)}], \\ g_{rr} &= \frac{1}{b^2 r^2} - \frac{2\beta A^{2(p-1)/p+1}}{(br)^{p+1}} + \frac{A^2}{(br)^{p+3}} + \frac{3\beta^2 A^{4(p-1)/p+1}}{(br)^{2p}} + O[r^{-2(p+1)}], \\ \phi &= \phi_0 \left[ b^p r^p + \frac{p}{p-1} \beta A^{2(p-1)/p+1} (br) + \frac{p}{2(p-1)^2} \beta^2 A^{4(p-1)/p+1} (br)^{2-p} \right] + O[r^{3-2p}]. \end{aligned} \quad (23)$$

In both cases the metric is asymptotically AdS. In view of Eqs. (22) and (23), we are led to impose the following boundary conditions,

$$\begin{aligned} g_{tt} &= -b^2 r^2 - 2\beta A^{2/3} br + \gamma_{tt} + \frac{\Gamma_{tt}}{br} + O[r^{-2}], \\ g_{rr} &= \frac{1}{b^2 r^2} - \frac{2\beta A^{2/3}}{b^3 r^3} + \frac{\gamma_{rr}}{b^4 r^4} + \frac{\Gamma_{rr}}{b^5 r^5} + O[r^{-6}], \\ g_{rt} &= \frac{\gamma_{rt}}{b^3 r^3} + O[r^{-4}], \\ \phi &= \phi_0 \left( \rho b^2 r^2 + 2\rho \beta A^{2/3} br + \gamma_{\phi\phi} + \frac{\Gamma_{\phi\phi}}{br} + O[r^{-2}] \right), \end{aligned} \quad (24)$$

for  $p = 2$ , whereas for  $p = 3, 5$  we have

$$\begin{aligned} g_{tt} &= -b^2 r^2 + \gamma_{tt} + \frac{\Gamma_{tt}}{b^2 r^2} + \frac{\theta_{tt}}{b^4 r^4} + O[r^{-6}], \\ g_{rr} &= \frac{1}{b^2 r^2} + \frac{\gamma_{rr}}{b^4 r^4} + \frac{\Gamma_{rr}}{b^6 r^6} + \frac{\theta_{rr}}{b^8 r^8} + O[r^{-10}], \\ g_{rt} &= \frac{\gamma_{rt}}{b^3 r^3} + \frac{\Gamma_{rt}}{b^5 r^5} + O[r^{-7}], \\ \phi &= \phi_0 [\rho (br)^p + \gamma_{\phi\phi} (br)^{p-2} + \Gamma_{\phi\phi} (br)^{p-4} \\ &\quad + \theta_{\phi\phi} (br)^{p-6} + O(r^{p-8})] \end{aligned} \quad (25)$$

In Eqs. (24) and (25),  $\rho, \gamma, \Gamma, \theta$  are boundary fields, which depend only on the coordinate  $t$  and describe deformations of the metric and of the scalar  $\phi$ .

The Killing vectors that preserve the boundary conditions (24) and define the ASG, are

$$\begin{aligned} \chi^r &= -\dot{\varepsilon}(t) \left( r + \frac{\beta A^{2/3}}{b} \right) + O[r^{-1}], \\ \chi^t &= \varepsilon(t) + \frac{\ddot{\varepsilon}(t)}{2b^4 r^2} \left( 1 - \frac{2\beta A^{2/3}}{br} \right) + O[r^{-4}], \end{aligned} \quad (26)$$

those that preserve the boundary conditions (25) are in-

$$\begin{aligned} g_{tt} &= -b^2 r^2 - 2\beta A^3 br - \beta^2 A^4 + \frac{A^2}{br} + O[r^{-2}], \\ g_{rr} &= \frac{1}{b^2 r^2} - \frac{2\beta A^3}{b^3 r^3} + \frac{3\beta^2 A^4}{b^4 r^4} + \frac{(1-4\beta^3)A^2}{b^5 r^5} + O[r^{-6}], \\ \phi &= \phi_0 [b^2 r^2 + 2\beta A^3 (br) + \beta^2 A^4], \end{aligned} \quad (22)$$

whereas for  $p = 3, 5$  we get

stead,

$$\chi^r = -\dot{\varepsilon}(t)r + O[r^{-2}], \quad \chi^t = \varepsilon(t) + \frac{\ddot{\varepsilon}(t)}{2b^4 r^2} + O[r^{-4}], \quad (27)$$

where  $\varepsilon(t)$  is an arbitrary function of time and the dot denotes derivative with respect to time. Notice that for  $p = 2$  both the boundary conditions (24) and the Killing vectors (26) depend on the parameters  $A, \beta$ . This dependence is a consequence of the coordinate transformation (21). For  $p$  odd, there are contributions to the unnormalized charge coming from terms of order higher than  $r^{-4}$  in Eq. (27). However, these terms do not contribute to the renormalized charges. We can consistently neglect them. The generators  $L_n$  of the ASG span a Virasoro Algebra:

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3-n)\delta_{n+m,0}, \quad (28)$$

where we allow for a nonvanishing central charge  $c$ . We can therefore identify the ASG as the  $diff_1$  group, the conformal group in one dimension. The  $diff_1$  group is an asymptotical symmetry only for the metric part of the solution (17). The ASG of the metric is broken by the nonconstant solution for the scalar  $\phi$ . However, it has been shown, for the  $p = 1$  case, that this breaking of the conformal symmetry just generates a nonvanishing central charge in the Virasoro algebra (28) [35]. The arguments of Ref. [35] can be straightforwardly extended to the generic 2D gravity model considered in this paper. Translated in terms of the one-dimensional CFT living on the boundary of our 2D spacetime this means that we have a ‘‘soft’’ breaking of the conformal symmetry generated by a conformal anomaly. We can still use (at least at the classical and semiclassical level) the centrally extended conformal algebra to constrain states and build representations for the one-dimensional boundary theory. In particular, at the semiclassical level one can show the validity of Cardy’s

formula [39], relating the degeneracy of states of the one-dimensional boundary CFT with its central charge.

The boundary fields  $\rho$ ,  $\gamma$ ,  $\Gamma$ ,  $\theta$  transform under the action of the  $diff_1$  group as conformal fields of definite weight. The only boundary field that contributes to the renormalized central charge is  $\rho$ , whose transformation law is:

$$\delta\rho = \varepsilon\dot{\rho} - p\dot{\varepsilon}\rho. \quad (29)$$

The extremal brane (8) has the  $AdS_{p+2} \times S^{d-1}$  geometry and is mapped by the dimensional reduction into the  $AdS_2$  spacetime, which is given by Eq. (17) with  $A = 0$ . Neglecting the scalar field  $\phi$ , we see that the isometry group of  $AdS_{p+2}$ , the group  $SO(2, p+1)$ , locally isomorphic to the conformal group in  $(p+1)$  dimensions, is mapped by the dimensional reduction into the ASG of  $AdS_2$ , namely, the  $diff_1$  conformal group. The nonconstant configuration for the scalar  $\phi$  breaks the conformal symmetry and generates a nonvanishing central charge in the Virasoro algebra. The dimensional reduction allows us to find an effective description of the  $AdS_{p+2}/CFT_{p+1}$  duality at finite temperature in terms of a  $AdS_2/CFT_1$

duality with the conformal symmetry broken by the scalar field  $\phi$ .

## V. CENTRAL CHARGE AND ENTROPY

Because the dual theory of the effective 2D gravity theory is an one-dimensional CFT, knowledge of the central charge in the Virasoro Algebra, allows us to calculate the entropy of the 2D black hole (hence of the near-extremal brane) via the Cardy formula. We can compute the central charge appearing in the Virasoro algebra (28) using a canonical realization of the ASG.

The gravitational Hamiltonian  $H$  is easily computed using the ADM parametrization of the metric :

$$ds^2 = -N^2 dt^2 + \sigma^2 (dr + N^r dt)^2, \quad (30)$$

where  $N$  and  $N^r$  are, respectively, the lapse and shift functions. According with the Regge-Teitelboim procedure [40–42] we must add surface terms  $J$  to  $H$ , needed to obtain well-defined variational derivatives. In the case under consideration we obtain:

$$\delta J = -\lim_{r \rightarrow \infty} \left\{ N \left( \sigma^{-1} \delta \phi' - \sigma^{-2} \phi' \delta \sigma - \frac{p-1}{p} \sigma^{-1} \phi^{-1} \phi' \delta \phi \right) - N^r \sigma^{-1} \delta \phi + N^r (\Pi_\phi \delta \phi - \sigma \delta \Pi_\sigma) \right\}, \quad (31)$$

where  $\Pi_\phi$  and  $\Pi_\sigma$  are, respectively, the momenta conjugate to  $\phi$  and  $\sigma$ . The ‘‘orthogonality problem’’ [43,44] typical of two dimensions, can be solved introducing the time-integrated charges [33]:

$$\hat{J} = \frac{b}{2\pi} \int_0^{2\pi/b} J dt. \quad (32)$$

The central charge  $c$  can be computed using the commutator

$$\delta_\omega \hat{J}(\varepsilon) = [\hat{J}(\varepsilon), \hat{J}(\omega)]. \quad (33)$$

However, in our case the time-integrated charges (32) are divergent and the final outcome of the calculation is a divergent central charge [30]. A renormalization procedure is needed in order to have finite charges.

After some manipulations we can write Eq. (31) in the following form,

$$\delta J = \delta J_I + \varepsilon \delta M, \quad (34)$$

where  $M$  is the charge associated with time translations ( $\varepsilon = 1$ ), while  $\delta J_I$  is a complicate function of the boundary fields and of  $\varepsilon$ , which for shortness we do not quote here. Equation (34) presents several divergences. The mass term  $\delta M$  is divergent because arbitrary excitations of the boundary fields have infinite energy [30]. We can eliminate this kind of divergences considering deformations of the boundary fields near the classical solution (on-shell deformations).

One could wonder why we limit ourself to consider only on-shell deformations. In the spirit of the AdS/CFT correspondence one expects the dual boundary theory to be quantum mechanical, and hence to involve arbitrary fluctuations of the boundary fields. The  $AdS_2/CFT_1$  correspondence discussed in this paper is based on the notion of asymptotic symmetries and is slightly different from the usual formulation of the AdS/CFT duality. In our case the conformal symmetry is generated by deformations that have a twofold interpretation. They can be seen both as deformations of the bulk metric and as deformations of the timelike boundary of AdS. As bulk degrees of freedom we can treat them classically. In this sense we can also define a classical boundary dynamics for the deformations. For the 1-brane it has been shown that this classical boundary theory has the form of a De Alfaro-Fubini-Furlan model coupled to an external source [36]. The quantum boundary theory can be obtained upon quantization of the classical boundary theory.

On-shell boundary deformations can be selected using appropriately the equations of motion induced on the boundary by the equation of motion for the bulk degrees of freedom. For  $p = 2$  we make use of the following boundary equation of motion

$$\frac{\dot{\rho}^2}{4b^2\rho} = 5\beta^2 A^{4/3} \rho - \gamma_{rr} \rho - 2\gamma_{\phi\phi}, \quad (35)$$

while for  $p$  odd we use,

$$\begin{aligned}
 & \frac{\dot{\rho}^2}{pb^2\rho} + p\rho\gamma_{rr} + 4\gamma_{\phi\phi} = 0, \\
 p\rho b^2\gamma_{rr}^2 - (p-3)\dot{\rho}b\gamma_{r\dot{t}} + \frac{p-1}{2p}\frac{\dot{\rho}^2}{\rho}\gamma_{\dot{t}\dot{t}} - \frac{p-1}{2p}\left(\frac{\dot{\rho}}{\rho}\right)^2\gamma_{\phi\phi} + \frac{2(p-1)}{p}b^2\frac{\gamma_{\phi\phi}^2}{\rho} + \frac{p-1}{2p}\frac{\gamma_{r\dot{t}}\dot{\rho}^2}{\rho} + \\
 & \frac{15(p-1)^2}{4p}b^2\gamma_{rr}\gamma_{\phi\phi} + \frac{(p-3)p}{2}b^2\rho\Gamma_{rr} + (p-3)(p-1)b^2\Gamma_{\phi\phi} + \frac{\dot{\rho}\dot{\gamma}_{rr}}{2} + \frac{p-1}{p}\frac{\rho\cdot\gamma_{\phi\phi}}{\rho} = 0.
 \end{aligned} \tag{36}$$

Equation (35) is obtained from the leading and Eqs. (36) from the leading and subleading, terms in the  $r = \infty$  expansion of the bulk field equations coming from the  $g_{\mu\nu}$  variation of the action (15). Taking the variation of Eqs. (35) and (36), evaluated on the classical solution (22) and (23), respectively, one finds that the divergent terms in the mass term of Eq. (34) vanish. Moreover, the finite part of  $M$  is equal to the mass  $m$  of the solution calculated using the prescription of Ref. [45].

This is not the end of the story. The term  $\delta J_I$  in Eq. (34) contains also divergent parts. The presence of these divergences can be traced back to the large  $r$  behavior of the scalar field,  $\phi \sim r^p$ . Because this behavior is shared by all the classical solutions of the 2D bulk theory, the most natural way to remove the divergences is to subtract the contribution of the massless background solution ( $A = 0$  in Eqs. (22) and (23)),

$$ds^2 = -b^2r^2dt^2 + \frac{dr^2}{b^2r^2}, \quad \phi = \phi_0 b^p r^p. \tag{37}$$

Indicating with  $J_{bg}$  the charges obtained evaluating Eq. (31) on the massless background and defining the renormalized charges  $J_R = J - J_{bg}$ , we get, respectively, for  $p$  even and odd,

$$\begin{aligned}
 \delta J_R &= \frac{\phi_0\beta A^{2/3}}{b}(\dot{\varepsilon}\delta\dot{\rho} - \dot{\varepsilon}\delta\rho) + \varepsilon\delta m, \delta J_R \\
 &= -\frac{\phi_0\beta A^{2(p-1)/p+1}}{b}p\left(\frac{p-2}{2^{p-3}}\dot{\varepsilon}\delta\rho + \frac{p-1}{2^{p-1}}\dot{\varepsilon}\delta\dot{\rho}\right) \\
 &+ \varepsilon\delta m.
 \end{aligned} \tag{38}$$

Notice that we use here a renormalization prescription that is slightly different from that used in Ref. [13]. In that paper the charges have been renormalized subtracting only their divergent part. Here, we have chosen a more natural procedure, which in general gives a different finite result for the charges. We can recover the results of Ref. [13] for the entropy of the 3-brane, by fixing appropriately the value of the renormalization parameter  $\beta$ .

Taking into account that the time-integrated charges are defined only up to a total time derivative, we can integrate the variations  $\delta J_R$  in Eqs. (38) to obtain,

$$J_R[\varepsilon] = -\frac{p\phi_0\beta A^{2(p-1)/p+1}}{2^n b}\varepsilon\dot{\rho}, \tag{39}$$

where  $n = (0, 1)$  respectively, for  $p = (\text{even, odd})$ . The term proportional to  $m$  in Eq. (38) has been canceled by choosing appropriately the integration constant.  $J_R(\varepsilon)$  in Eq. (39) is related to the energy-momentum tensor  $T_{\dot{t}\dot{t}}$  of the one-dimensional CFT,  $J_R(\varepsilon) = \varepsilon T_{\dot{t}\dot{t}}$ . Using the conformal transformation of the field  $\rho$  given in Eq. (29), expanding in Fourier modes and using Eqs. (28) and (33), near the classical  $\rho = 1$  solutions, we find the value of the central charge in the virasoro algebra:

$$\frac{c}{12} = \frac{\phi_0\beta A^{2(p-1)/p+1}}{2^n}. \tag{40}$$

For  $p \neq 1$ , the result for central charge we have obtained involves the parameter  $A$ . Because  $A^2$  is essentially the 2D black hole mass, this implies that there is a different CFT (characterized by a different value of the central charge) for every state in the black hole spectrum (characterized by the black hole mass). This is completely different from Strominger's AdS<sub>3</sub>/CFT<sub>2</sub> calculation of the entropy of the BTZ black hole [6] (Corresponding to the  $p = 1$  case of Eq. (40)). In this latter case the central charge of the CFT<sub>2</sub> depends only on the 3D Newton constant  $G_N$  and cosmological constant  $\lambda$ , the black hole mass enters only in the eigenvalue of the Virasoro operator  $L_0$ . The BTZ black hole is described by a single CFT whose excitation are in correspondence with states in the black hole spectrum.

The dependence of the central charge (40) from the state-dependent parameter  $A$  seems a general (and probably unavoidable) consequence of using a dimensionally reduced AdS<sub>2</sub>/CFT<sub>1</sub> correspondence to describe higher dimensional AdS/CFT dualities. More in general, it is likely that any attempt to use Cardy-like formulas to compute the entropy of a CFT <sub>$d$</sub>  with  $d \geq 3$  (see for instance Ref. [46]) will lead to the dependence of the central charge from some state-dependent parameter. The dimensional reduction (13) introduces in the 2D gravity theory a scalar field  $\phi$ . Owing to its scalar character the nonconstant background solution for  $\phi$  breaks the conformal symmetry of the AdS background and produces a nonvanishing central charge. Obviously the central charge can only depend on the parameters appearing in the background solution for  $\phi$ . In the most favorable situation, the 1-brane when renormalization is not necessary,  $c$  depends only on the dimensionless constant  $\phi_0$  entering in the solution (17) for  $\phi$ . In the general case, owing to the coordinate transformation (20), the background solution for  $\phi$ , hence also

the central charge  $c$ , will also depend on the parameter  $A$ . The dependence of the central charge from the black hole mass in the  $\text{AdS}_2/\text{CFT}_1$  correspondence can be also understood just by power-counting. In two dimensions  $G_N$  is dimensionless, if we want to produce a dimensionless quantity such as the central charge using  $\lambda$  we must also use the black hole mass. This is not the case for the BTZ black hole, in three dimensions  $G_N$  has the dimensions of a length, so that one can construct  $c$  out of  $G_N$  and  $\lambda$ .

The central charge (40) depends on the renormalization parameter  $\beta$ . The presence of this arbitrary dimensionless constant is a consequence of our renormalizations procedure. From the point of view of the 2D gravity theory,  $\beta$  is just a free parameter. However its value can be constrained using arguments stemming from the  $\text{AdS}/\text{CFT}$  duality. The central charge in the Virasoro algebra is a rational function of the conformal weights of the boundary fields, so that we can expect  $\beta$  to be a rational number. Moreover, all the information about physical parameters of the  $p$ -brane is contained in the 2D parameters  $A$  and  $\phi_0$ . The dimensionless parameter in Eq. (40) must encode the information about the degrees of freedom of the  $\text{CFT}_{p+1}$  living on the brane, leading again to a rational value for  $\beta$ . We fix  $\beta$ , choosing the value

$$\beta = \frac{2^n}{p^2}. \quad (41)$$

The central charge takes the simple form,

$$c = \frac{12}{p} \phi_0 A^{2(p-1)/p+1}. \quad (42)$$

The entropy associated with the boundary  $\text{CFT}_1$  characterized by eigenvalue  $l_0$  of the operator  $L_0$  and central charge  $c$ , is given by the Cardy formula  $S = 2\pi\sqrt{cl_0/6}$ , [39]. The eigenvalue of  $L_0$  is given in terms of the mass of the 2D black hole,  $l_0 = m_{bh}/b$ , whereas  $c$  can be read from Eq. (42). We get for the entropy,

$$S = 2\pi\phi_0 A^{2p/p+1}. \quad (43)$$

Equation(43) holds for all the nondilatonic branes discussed in this paper. Using Eqs. (16) and (18) to express  $\phi_0$  and  $A$  in terms of the brane temperature  $T$  and brane parameters  $N, V$ , we reproduce exactly the thermodynamical entropy (10),  $S_p = a_p VT^p$ , with coefficients  $a_p$  given by Eq. (11). By fixing appropriately the value of the renormalization parameter  $\beta$  our microscopical calculation of the brane entropy, which uses an effective  $\text{AdS}_2/\text{CFT}_1$  duality, is in perfect agreement with the thermodynamical result.

Notice that our general formula for the brane entropy (43) holds also for  $p = 1$ , although in this case no renormalization procedure, hence no fixing of the parameter  $\beta$  is needed. In our 2D approach, the 1-brane (the BTZ black hole) becomes after dimensional reduction the  $\text{AdS}_2$  black

hole, whose microscopical entropy has been already calculated in Ref. [33–37].

It is important to stress that even if we leave the parameter  $\beta$  unfixed, our method allows to reproduce the right dependence of the thermodynamical entropy (10) from  $T$  and  $N$ . From this point of view our  $\text{CFT}_1$  results represent an improvement with respect to those obtained in Ref. [4] using the  $\text{CFT}_{p+1}$ . In fact the  $\text{CFT}_{p+1}$  calculations do not have the predictive power to determine, for the 2- and 5-brane, the right dependence of the entropy from the parameter  $N$ .

The weak point in our derivation is the fact that we do not have any compelling reason to fix  $\beta$  as in Eq. (41). However, we can argue that Eq. (41) may not be a simple coincidence. First, this value of  $\beta$  seems to be rather special. With this choice the central charge (42) takes a simple form for all branes and the dimensionless factor in the entropy (43) becomes  $p$ -independent. Second, the factor  $12/p$  appearing in the central charge (42) seems related to the number of degrees of freedom of the  $\text{CFT}_{p+1}$  living on the brane. The way how the information about  $\text{CFT}_{p+1}$  degrees of freedom is encoded in the central charge of the  $\text{CFT}_1$  may be extremely non trivial. However, our result seems to support recent attempts to find generalization of the Cardy formula for CFTs in  $d > 2$  [46].

If we do not fix the renormalization parameter  $\beta$ , the entropy (43) will depend on it. The dependence of the entropy from a dimensionless parameter can be also understood in terms of the classical scale symmetry of the 2D action (15) mentioned in Sec. III. Rescaling the scalar field  $\phi$ , the 2D action changes by an overall factor. This scale symmetry appears as a subgroup of the isometry group of  $\text{AdS}_2$ . In fact, the metric (37) is invariant under the transformations

$$r \rightarrow \mu r, \quad t \rightarrow \mu^{-1} t. \quad (44)$$

This scale symmetry is broken by the scalar field, which encodes the information about the embedding of the brane in the  $D$ -dimensional space-time. In fact  $\phi$  transforms as  $\phi \rightarrow \mu^p \phi$ . If we want to preserve the scale symmetry, the parameter  $\phi_0$  must scale as  $\phi_0 \rightarrow \mu^{-p} \phi_0$ . Using this transformation law into Eq. (43) we see that also the entropy scales in a similar way. This explains the dependence of the entropy from a dimensionless parameter, which is undetermined, at least at the classical level. Conversely, for the  $\text{AdS}_{p+1}$  spacetime in Eq. (8) the scale transformation (44) can be promoted to an exact isometry transforming the brane coordinates  $x^i \rightarrow \mu^{-1} x^i$ .

## VI. CONCLUSION

In this paper we have used a 2D approach to study the microscopic entropy of near-extremal nondilatonic  $p$ -branes and, more in general, to investigate the  $\text{AdS}/\text{CFT}$  correspondence at finite temperature. Performing a



dimensional reduction, we have found a 2D gravity model that gives an effective description of the  $p$ -brane in the near-horizon, near-extremal regime. The AdS/CFT duality survives the dimensional reduction. An AdS<sub>2</sub>/CFT<sub>1</sub> duality gives an effective description of the AdS <sub>$p+2$</sub> /CFT <sub>$p+1$</sub>  correspondence at finite temperature. Finite temperature effects are taken into account in the 2D model as a breaking of the conformal symmetry, which generates a nonvanishing central charge in the Virasoro algebra. Using this procedure, we have calculated the entropy of the boundary CFT<sub>1</sub>. Fixing in a natural way a dimensionless free renormalization parameter, we have reproduced exactly the Bekenstein-Hawking entropy of all relevant nondilatonic  $p$ -branes in the near-extremal, near-horizon regime.

Our results represent an important improvement, in particular, for what concerns the 2- and 5-brane. For these branes methods based on the AdS <sub>$p+2$</sub> /CFT <sub>$p+1$</sub>  duality cannot explain the dependence of the entropy from the number of branes  $N$ . This is probably due to our lack of knowledge about M-theory and about the AdS <sub>$p+2$</sub> /CFT <sub>$p+1$</sub>  duality for  $p = 2, 5$ . Our 2D approach is more successful simply because it is almost completely based on 2D gravitational

physics, therefore largely independent from the details of the fundamental theory in 11 dimensions.

On the other hand, the fact that a 2D model can be used as a unifying framework to describe all the relevant nondilatonic branes, indicates that the 2D description could be more general than it could seem at first sight. The reason behind this generality can be easily recognized. Similarly to what happens for black holes, also for black branes the thermodynamical behavior is essentially determined by the 2D  $(r, t)$  sections of the spacetime and largely independent from the transverse dimensions.

The weakness of our 2D approach is that it is not fully predictive. The microscopic entropy of the brane is determined up to a dimensionless renormalization constant, which from the 2D point of view is a free parameter. However, the values of this parameter that lead to agreement between statistical and thermodynamical entropy are natural from the point of view of the brane and seem to have an universal character. This may be the consequence of the existence of a general and deep relationship between the central charge of the one-dimensional CFT and the number of the degrees of freedom of the brane.

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