Path integral approach to residual gauge fixing

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In this paper we study the question of residual gauge fixing in the path integral approach for a general class of axial-type gauges including the light-cone gauge. We show that the two cases—axial-type gauges and the light-cone gauge—lead to very different structures for the explicit forms of the propagator. In the case of the axial-type gauges, fixing the residual symmetry determines the propagator of the theory completely. On the other hand, in the light-cone gauge there is still a prescription dependence even after fixing the residual gauge symmetry, which is related to the existence of an underlying global as well as local symmetry.

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I. INTRODUCTION

Light-front field theories [1] have been studied vigorously in the past. Quantization on the light front (as opposed to equal-time quantization) leads to a larger number of kinematical generators of the Poincaré algebra resulting in a trivial vacuum state of the theory [2]. Therefore, nonperturbative calculations can, in principle, be carried out in a simpler manner in such theories.

More recently, it has been observed both in the conventional light-front frame as well as in a generalized light-front frame [3–5] that canonical quantization (within the Hamiltonian formalism) of a gauge theory in the light-cone gauge

$$n \cdot A = 0, \qquad n^2 = 0, \tag{1}$$

where A_μ can be thought of as a matrix in the adjoint representation of the gauge group in the case of a non-Abelian theory, leads to a doubly transverse propagator [6,7]. For example, in this case the propagator has the explicit form (with the identity matrix neglected in the case of a non-Abelian theory)

$$D^{(\mathrm{DT})}_{\mu\nu}(n,p) = -\frac{1}{p^2}\left[g_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{(n\cdot p)} + \frac{p^2}{(n\cdot p)^2} n_\mu n_\nu\right], \tag{2}$$

and satisfies

$$n^\mu D^{(\mathrm{DT})}_{\mu\nu}(n,p) = 0 = D^{(\mathrm{DT})}_{\mu\nu}(n,p) n^\nu = p^\mu D^{(\mathrm{DT})}_{\mu\nu}(n,p) = D^{(\mathrm{DT})}_{\mu\nu}(n,p) p^\nu. \tag{3}$$

On the other hand, when calculated in the path integral formalism using the naive Faddeev-Popov procedure, the inverse of the two point function in the light-cone gauge (1) takes the form (once again, we neglect the identity matrix in the case of a non-Abelian theory)

$$(\Gamma^{(\mathrm{PI})})^{-1}_{\mu\nu}(n,p) = -\frac{1}{p^2}\left[g_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{(n\cdot p)}\right]. \tag{4}$$

While this is transverse with respect to n^μ, it is not transverse with respect to the momentum, namely,

$$\begin{aligned} n^\mu(\Gamma^{(\mathrm{PI})})^{-1}_{\mu\nu}(n,p) &= 0 = (\Gamma^{(\mathrm{PI})})^{-1}_{\mu\nu}(n,p) n^\nu,\\ p^\mu(\Gamma^{(\mathrm{PI})})^{-1}_{\mu\nu}(n,p) &= \frac{n_\nu}{(n\cdot p)},\\ (\Gamma^{(\mathrm{PI})})^{-1}_{\mu\nu}(n,p)p^\nu &= \frac{n_\mu}{(n\cdot p)}. \end{aligned} \tag{5}$$

It is, of course, not clear *a priori* whether, in the light-cone gauge, the inverse of the two point function corresponds exactly to the propagator of the theory. We will show explicitly (in an appendix) that such an identification can, in fact, be made in the light-cone gauge. Therefore, there is a manifest difference between the two structures in (2) and (4) in addition to the fact that one has to further specify a prescription for handling the unphysical poles at $n \cdot p = 0$. This difference has led to several papers [8] where the Lagrangian density of the (Abelian) theory is modified arbitrarily by hand in order to reproduce the propagator (2).

We note that such a difference is not restricted to light-front theories alone. In fact, even in a gauge theory quantized at equal-time in the light-cone gauge, such a difference does appear. Furthermore, even in the temporal gauge in a theory quantized at equal-time, such a phenomenon arises. Normally, one ascribes this to a residual gauge invariance in the path integral approach. More specifically, in the canonical analysis of a gauge theory, two first class constraints arise which necessitate two gauge fixing conditions [6,7], while in the naive path integral approach one only imposes a single gauge fixing condition such as in (1) which leaves behind a residual gauge symmetry. However, a residual gauge invariance has a very different effect in the sense that while the original gauge invariance constrains the structure of the theory strongly enough to make the two point function noninvertible, this is not the case when there is a residual gauge invariance. Rather, in the case of the temporal gauge, we know that residual gauge fixing removes the arbitrary prescription dependence of the unphysical poles in the propagator [9,10]. It is for this reason that we would first like to understand the path integral formulation of the theory in the light-cone gauge as much as is possible without fixing the residual gauge invariance before going into a systematic analysis of residual gauge fixing. The paper is organized as follows. In Sec. II, we study systematically various properties of the theory in the naive gauge fixing of the light-cone gauge in the path integral formalism. We show, in particular, that the free theory in this gauge has in general a global symmetry which allows for an arbitrary term involving the tensor structure $n_\mu n_\nu$ in the propagator. However, in this case, there arises an additional Becchi-Rouet-Stora (BRS) symmetry in the free action which restricts the path integral propagator of the naively gauge fixed theory to have the form (4). The same arbitrariness of the tensor structure in the propagator can also be understood as merely arising as a result of a field redefinition which, in fact, is more along the lines of a residual gauge fixing. In Sec. III, we study the question of fixing the residual gauge symmetry for the gauge $n \cdot A = 0$ in a Yang-Mills theory for both axial-type gauges as well as the light-cone gauge. The two cases have quite different features and so we discuss them separately. We derive the forms of the completely gauge fixed propagator in both the cases. In the case of axial-type gauges the propagator has no further prescription dependence while the unphysical poles in the propagator in the light-cone gauge do require a prescription even after fixing the residual gauge symmetry. We trace the origin of such a behavior in the light-cone gauge to an underlying global as well as local invariance of the free theory. We present a brief conclusion in Sec. IV. In Appendix A, we show that the inverse of the two point function in the naive light-cone gauge indeed corresponds to the propagator, while in Appendix B, we compile some useful formulas for transformations into light-cone variables.

II. NAIVE LIGHT-CONE GAUGE FIXING

In this section, we will study various properties of a light-cone gauge fixed theory in the path integral formalism. Although the entire analysis in this paper will be carried out in the usual Minkowski spacetime for simplicity, all of our discussions hold for a theory quantized either on an equal-time surface or on the light-front, both in the conventional as well as generalized light-front frames (which are related by a change of frame) [2–5,7]. Furthermore, since our interest lies in studying the structure of the propagator, we will restrict ourselves to analyzing the free Maxwell theory in this section and comment on possible differences which may arise in a fully interacting non-Abelian theory. In the next section, our systematic analysis of residual gauge fixing will be carried out within the context of a fully interacting non-Abelian gauge theory.

The gauge invariant action for the Maxwell theory has the form

$$S_{\rm inv} = \int d^4x \mathcal{L}_{\rm inv} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}. \tag{6}$$

The path integral for the theory with a naive light-cone gauge fixing (1) has the form [11]

$$Z = N\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\delta(n\cdot A)e^{iS_{\rm inv}}, \tag{7}$$

where N is a normalization constant and $\Delta_{\rm FP}[A]$ represents the Faddeev-Popov determinant which can, in general, be field dependent in a non-Abelian theory. However, in a general axial-type gauge (including the light-cone gauge that we are analyzing here), it is known that the Faddeev-Popov determinant is independent of the field variables even in a non-Abelian theory [12].

It is clear from the structure of the generating functional in (7) that any n-point Green's function involving the gauge fields will be transverse with respect to the lightlike vector n^μ, namely,

$$\begin{aligned} n^{\mu_1}\langle 0|T(A_{\mu_1}\cdots A_{\mu_n})|0\rangle &= N\int \mathcal{D}A_\mu \Delta_{\rm FP}[A] \\ &\quad\times \delta(n\cdot A) n\cdot A A_{\mu_2}\cdots A_{\mu_n} e^{iS_{\rm inv}} \\ &= 0. \end{aligned} \tag{8}$$

In particular, this would imply that the propagator would satisfy

$$n^\mu D_{\mu\nu}(n,p) = 0, \tag{9}$$

which is satisfied by both (2) and (4).

We note that when $n^2 = 0$, the gauge fixing condition as well as the invariant action $S_{\rm inv}$ in (6) are invariant under the infinitesimal global transformation

$$\delta A_\mu = \zeta n_\mu \partial\cdot A, \tag{10}$$

where ζ represents the constant infinitesimal parameter of transformation. The change in the path integral measure under such a field redefinition is easily seen to be a field independent constant. Conversely, if we incorporate the gauge fixing condition as well as the Faddeev-Popov determinant into the action, we can write the generating functional in the form

$$Z = N\int \mathcal{D}A_\mu \mathcal{D}F\mathcal{D}\bar{c}\mathcal{D}c e^{iS}, \tag{11}$$

where

$$S = S_{\rm inv} - \int d^4x(Fn\cdot A - \bar{c}n\cdot\partial c), \tag{12}$$

and F represents an auxiliary field implementing the gauge condition. It is easy to verify that the action as well as the generating functional in this case are invariant under the infinitesimal global transformations

$$\delta A_\mu = \zeta n_\mu \partial \cdot A, \qquad \delta F = \zeta \Box \partial \cdot A, \qquad \delta c = 0 = \delta \bar{c}. \tag{13}$$

If we add sources into the path integral as

$$Z[J^\mu, J, \eta, \bar{\eta}] = e^{iW[J^\mu, J, \eta, \bar{\eta}]} = N \int \mathcal{D}A_\mu \mathcal{D}F \mathcal{D}\bar{c} \mathcal{D}c e^{i(S+S_{\text{source}})}, \tag{14}$$
$$S_{\text{source}} = \int d^4x [J^\mu A_\mu + JF + i(\bar{\eta} c - \bar{c}\eta)],$$

then, the Ward identity for the global invariance of (13) can be easily derived to be

$$\int d^4x \left[n \cdot J(x) \partial_\mu \frac{\delta W}{\delta J_\mu(x)} + J(x) \Box \partial_\mu \frac{\delta W}{\delta J_\mu(x)} \right] = 0. \tag{15}$$

This leads to a constraint on the form of the gauge propagator of the form

$$n^\nu \partial_\mu^{(x)} \frac{\delta^2 W}{\delta J_\mu(x) \delta J_\lambda(y)} + n^\lambda \partial_\mu^{(y)} \frac{\delta^2 W}{\delta J_\mu(y) \delta J_\nu(x)} = 0. \tag{16}$$

Together with (9), this determines the general form of the propagator for the gauge field in the light-cone gauge (1) to be

$$D_{\mu\nu}(n, p) = -\frac{1}{p^2} \left[\eta_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{(n \cdot p)} + \alpha \frac{p^2}{(n \cdot p)^2} n_\mu n_\nu \right], \tag{17}$$

where α is arbitrary. We note that when $n^2 = 0$, this satisfies $n^\mu D_{\mu\nu}(n, p) = 0$ as is expected from (9). For $\alpha = 1$, this corresponds to the doubly transverse propagator of (2) while for $\alpha = 0$, this coincides with the path integral propagator of (4). It is worth noting here that if we identify the tensor structure in (17) with the sum over polarization vectors,

$$\sum_\lambda \epsilon_\mu(p, \lambda) \epsilon_\nu(p, \lambda) = -\eta_{\mu\nu} + \frac{n_\mu p_\nu + n_\nu p_\mu}{(n \cdot p)} - \alpha \frac{p^2}{(n \cdot p)^2} n_\mu n_\nu, \tag{18}$$

where we have chosen the polarization vector $\epsilon_\mu(p, \lambda)$ to be real, then, when $n^2 = 0$, we see that for any value of α, we have

$$\sum_\lambda \epsilon^\mu(p, \lambda) \epsilon_\mu(p, \lambda) = -2. \tag{19}$$

This does not, however, imply that the polarization vectors are summed over only the physical ones in (18). In fact, the arbitrariness in the $n_\mu n_\nu$ term in (18) signifies that the polarization sum does contain polarization vectors proportional to the lightlike vector, $\epsilon_\mu(p, \lambda) \sim n_\mu$. On the other hand, such polarization vectors would lead to zero norm states and, therefore, cannot represent physical polarizations.

This issue can be further understood by noting that while

$$P_{\mu\nu}^{(\mathrm{T})} = \eta_{\mu\nu} - \frac{n_\mu n_\nu}{n^2}, \qquad P_{\mu\nu}^{(\mathrm{L})} = \frac{n_\mu n_\nu}{n^2}, \tag{20}$$

define transverse and longitudinal projection operators with respect to a vector n^μ when $n^2 \neq 0$, they are not defined for a lightlike vector. In fact, one can define transverse and longitudinal projection operators for a lightlike vector only in conjunction with another vector with a nonzero inner product. Thus, for example, if we take the second vector as the gradient operator, then for a lightlike vector n^μ we can define [7]

$$P_{\mu\nu}^{(\mathrm{T})}(n, \partial) = \eta_{\mu\nu} - \frac{n_\mu \partial_\nu + n_\nu \partial_\mu}{(n \cdot \partial)} + \frac{\partial^2}{(n \cdot \partial)^2} n_\mu n_\nu,$$
$$P_{\mu\nu}^{(\mathrm{L})} = \frac{n_\mu \partial_\nu + n_\nu \partial_\mu}{(n \cdot \partial)} - \frac{\partial^2}{(n \cdot \partial)^2} n_\mu n_\nu. \tag{21}$$

It can be checked that these define orthogonal projection operators and satisfy

$$n^\mu P_{\mu\nu}^{(\mathrm{T})} = 0 = P_{\mu\nu}^{(\mathrm{T})} n^\nu = \partial^\mu P_{\mu\nu}^{(\mathrm{T})} = P_{\mu\nu}^{(\mathrm{T})} \partial^\nu,$$
$$n^\mu P_{\mu\nu}^{(\mathrm{L})} = n_\nu, \qquad P_{\mu\nu}^{(\mathrm{L})} n^\nu = n_\mu, \qquad \partial^\mu P_{\mu\nu}^{(\mathrm{L})} = \partial_\nu, \tag{22}$$
$$P_{\mu\nu}^{(\mathrm{L})} \partial^\nu = \partial_\mu, \qquad P_{\mu\nu}^{(\mathrm{T})} + P_{\mu\nu}^{(\mathrm{L})} = \eta_{\mu\nu}.$$

This allows us to decompose any vector and, in particular, the gauge field as

$$A_\mu = A_\mu^{(\mathrm{T})} + A_\mu^{(\mathrm{L})}, \qquad A_\mu^{(\mathrm{T})} = P_{\mu\nu}^{(\mathrm{T})} A^\nu,$$
$$A_\mu^{(\mathrm{L})} = P_{\mu\nu}^{(\mathrm{L})} A^\nu. \tag{23}$$

We note that by construction,

$$n \cdot A^{(\mathrm{T})} = 0 = \partial \cdot A^{(\mathrm{T})}, \qquad n \cdot A^{(\mathrm{L})} = n \cdot A,$$
$$\partial \cdot A^{(\mathrm{L})} = \partial \cdot A. \tag{24}$$

Therefore, each of the four vectors $A_\mu^{(\mathrm{T})}, A_\mu^{(\mathrm{L})}$ carries only 2 degrees of freedom. Furthermore, under a gauge transformation,

$$\delta A_\mu = \partial_\mu \theta(x), \tag{25}$$

it follows, using (22), that

$$\delta A_\mu^{(\mathrm{T})} = 0, \qquad \delta A_\mu^{(\mathrm{L})} = \partial_\mu \theta(x). \tag{26}$$

Consequently, we see that $A_\mu^{(\mathrm{T})}$ is gauge invariant and carries only the physical degrees of freedom while $A_\mu^{(\mathrm{L})}$

consists of the two unphysical (gauge) degrees of freedom. From the definition of $A_\mu^{(L)}$ in (23) and (21), we see that it is completely determined from a knowledge of $n \cdot A$ and $\partial \cdot A$. While the gauge fixing condition (1) specifies one of the components, $\partial \cdot A$ remains arbitrary and the transformation (10) [or (13)] merely reflects the arbitrariness in this component.

So far we have discussed the consequences on the structure of the propagator following from the global invariance in (10). Such a global invariance will be quite important in the analysis of the form of the propagator after residual gauge fixing to be discussed in the next section. The Ward identity (15) is quite general and does not depend on the particular structure of the theory. For example, if the action had an additional term of the form

$$S_{\text{additional}} = -\frac{1}{2\xi}\int d^4x (\partial \cdot A)^2, \tag{27}$$

it would still be invariant under the global transformation (10) and the Ward identity (15) would continue to hold. Thus, it is quite curious as to why the path integral propagator has a unique form corresponding to $\alpha = 0$. This constraint, in fact, comes from an additional BRS invariance that the free theory develops. In fact, it can be checked that in addition to the usual BRS transformations [13] under which the action (14) is invariant, it is also invariant under a new BRS transformation of the form

$$\begin{aligned} \tilde{\delta} A_\mu &= \tilde{\omega} n_\mu c, \qquad \tilde{\delta} c = 0, \qquad \tilde{\delta}\bar{c} = \tilde{\omega}\partial \cdot A, \\ \tilde{\delta} F &= \tilde{\omega}\Box c, \end{aligned} \tag{28}$$

where $\tilde{\omega}$ represents an anticommuting global parameter. This transformation anticommutes with the usual BRS transformation and is also nilpotent (like the conventional BRS transformation), but only on-shell (when the ghost equations of motion are used) which is a reflection of the absence of some auxiliary field in the theory. This new BRS invariance leads to a Ward identity of the form

$$\int d^4x \left[\partial_\mu \frac{\delta W}{\delta J_\mu(x)}\eta(x) + n \cdot J(x)\frac{\delta W}{\delta \bar{\eta}(x)} + J(x)\Box \frac{\delta W}{\delta \bar{\eta}(x)}\right] = 0. \tag{29}$$

It is clear that unlike the global transformation (10), the BRS invariance is very specific to a specific theory and, correspondingly, the resulting Ward identity is also. From (29), it can be easily derived that

$$\partial_\mu^{(x)} \frac{\delta^2 W}{\delta J_\mu(x)\delta J_\nu(y)} - n^\nu \frac{\delta^2 W}{\delta \bar{\eta}(y)\delta \eta(x)} = 0. \tag{30}$$

This relates the divergence of the gauge propagator to the ghost propagator (it does not say that the gauge propagator is transverse) and thereby determines $\alpha = 0$.

It is worth pointing out here that the global invariance of (10) or the new BRS invariance of (28) cannot be incorporated into a fully interacting theory which would include a non-Abelian theory. However, the violation of the invariance will occur only at the higher order terms in the number of fields. Namely, the violation of the Ward identities in (15) or (29) will manifest only in the structure of the higher point functions. As far as the structure of the propagator is concerned, all of our discussions will hold in a fully interacting theory as well.

There is yet another interesting and suggestive way to see the arbitrariness in the $n_\mu n_\nu$ term in the propagator. For example suppose we start with the path integral propagator in (4) in the momentum space, namely,

$$\langle 0|T[A_\mu(x)A_\nu(y)]|0\rangle \to -\frac{1}{p^2}\left[\eta_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{(n \cdot p)}\right]. \tag{31}$$

Then, under a field redefinition

$$A'_\mu(x) = A_\mu(x) - \frac{1}{2\beta} n_\mu \frac{1}{n \cdot \partial}\partial \cdot A, \tag{32}$$

which preserves the gauge fixing condition, the propagator would change in the momentum space to (β is a constant)

$$\langle 0|T[A'_\mu(x)A'_\nu(y)]|0\rangle \to -\frac{1}{p^2}\left[\eta_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{(n \cdot p)} + \alpha \frac{p^2}{(n \cdot p)^2} n_\mu n_\nu\right], \tag{33}$$

where we have identified $\alpha = \frac{4\beta - 1}{4\beta^2}$. This has the same structure as (17). This derivation, however, is quite suggestive for the following reason. Let us consider a field redefinition of the form (32) in the path integral (7). While the gauge fixing condition is invariant under such a redefinition, the action is not. In fact, under this redefinition

$$S_{\text{inv}}[A] = S_{\text{inv}}[A'] - \frac{1}{2\xi}\int d^4x (\partial \cdot A')^2, \tag{34}$$

where we have identified $\xi = \frac{(2\beta-1)^2}{1-4\beta}$. This shows that the field redefinition induces an additional term in the action which is reminiscent of a covariant gauge fixing term. In fact, it can be written in the path integral in the form of a delta function $\delta(\partial \cdot A' - \sqrt{\xi} f)$. While this is suggestive and seems to imply that the field redefinition must somehow correspond to fixing the residual gauge invariance of the theory, it is not quite complete for a variety of reasons. First, the field redefinition is meaningful only over a limited range of the parameters ξ. In fact, the Jacobian of the field transformation which has the form

$$J = \det \left|\frac{\partial A_\mu}{\partial A'_\nu}\right| = \frac{2\beta}{2\beta - 1}, \tag{35}$$

becomes singular for $\beta = \frac{1}{2}$ exactly at the point where

$\xi = 0$. Furthermore, we do not quite see the Faddeev-Popov determinant arising from such a field redefinition. Thus, we conclude from all this analysis that a systematic understanding of the residual gauge fixing is necessary in order to fully appreciate the structure of the propagator in the path integral formalism and we will do this in the next section.

III. RESIDUAL GAUGE FIXING

The question of residual gauge fixing within the path integral approach for the temporal gauge has previously been studied in some detail in [9,14]. In this section, we will systematically study the question of gauge fixing for the residual gauge invariance in a Yang-Mills theory in a general class of axial-type gauges including the light-cone gauge. However, we will divide the study into two cases—axial-type gauges and the light-cone gauge—because as we will see the two cases lead to quite different results.

A. Axial-type gauges

Let us consider a Yang-Mills theory described by the gauge invariant action

$$S_{\rm inv} = -\frac{1}{4}\int d^4x {\rm Tr} F_{\mu\nu}F^{\mu\nu}, \tag{36}$$

where the gauge fields are assumed to belong to the adjoint representation of the gauge group and the field strength is defined to be

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]. \tag{37}$$

For simplicity, we have scaled the coupling constant to unity. The action in (36) is invariant under the infinitesimal gauge transformation

$$\delta A_\mu = D_\mu \theta(x) = \partial_\mu \theta(x) - i[A_\mu(x), \theta(x)], \tag{38}$$

where $\theta(x)$ is the infinitesimal parameter of transformation.

Let us consider an axial-type gauge of the form

$$n\cdot A = 0, \qquad |n^2| = 1. \tag{39}$$

This can, therefore, describe the temporal gauge or the axial gauge depending on the choice of the vector n^μ. Furthermore, since the gauge condition (39) involves a Lorentz scalar, it can hold in any frame including the general light-front frame (which is not related to the Minkowski frame through a Lorentz transformation, for details see [7]). The naive generating functional

$$Z = N\int \mathcal{D}A_\mu e^{iS_{\rm inv}}, \tag{40}$$

does not exist because of the gauge invariance of the theory. We would like to impose the general axial gauge (39) and following Faddeev-Popov [11], we introduce the identity

$$\Delta_{\rm FP}^{-1}[A] = \int \mathcal{D}\theta\,\delta(n\cdot A^{(\theta)}), \tag{41}$$

where $A_\mu^{(\theta)}$ represents a gauge transformed potential [see (38)]

$$A_\mu^{(\theta)}(x) = A_\mu(x) + D_\mu\theta(x). \tag{42}$$

The Faddeev-Popov determinant in (41) is manifestly gauge invariant and the standard procedure of Faddeev-Popov can be followed to separate the volume of gauge orbits as

$$\begin{aligned} Z &= N\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\int \mathcal{D}\theta\,\delta(n\cdot A^{(\theta)})e^{iS_{\rm inv}} \\ &= N\Big(\int\mathcal{D}\theta\Big)\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\delta(n\cdot A)e^{iS_{\rm inv}}. \end{aligned} \tag{43}$$

In the derivation above, we have inserted the identity from (41) in the first step while we have made an inverse gauge transformation and used the gauge invariance of the Faddeev-Popov determinant in the second step. This would, therefore, seem to have separated out the infinite gauge volume element from the path integral. However, this is not entirely true.

In fact, let us note that the delta function constraint in (43) is invariant under gauge transformations of the form (the Faddeev-Popov determinant is gauge invariant)

$$A_\mu(x) \to A_\mu(x) + D_\mu\bar\theta(x), \qquad n\cdot\partial\bar\theta(x) = 0, \tag{44}$$

where $\bar\theta(x)$ is an arbitrary function independent of $n\cdot x$. Namely, in the case of axial-type gauges, the gauge transformation parameters can be grouped into two classes—ones that do not depend on the coordinate $n\cdot x$ and others that do—and the gauge fixing condition (39) cannot determine the transformation parameters which do not depend on the coordinate $n\cdot x$. This is the reason why the separation of the infinite gauge volume is incomplete in (43) and manifests in a residual gauge symmetry of the generating functional since each factor in the path integral is invariant under gauge transformations of the form (44).

To separate out the volume associated with the residual (restricted) gauge transformations, we will follow again the method of Faddeev-Popov and fix a gauge. We note that a covariant gauge condition such as

$$\partial\cdot A = \sqrt{\xi}f(x), \tag{45}$$

where ξ is an arbitrary constant and $f(x)$ is an arbitrary function, can be implemented through a gauge transformation of the type (44) provided

$$\bar\theta(x) = \frac{1}{\partial\cdot D}[-\partial\cdot A(x) + \sqrt{\xi}f(x)]. \tag{46}$$

However, since $\bar\theta(x)$ does not depend on $n\cdot x$, such a condition (46) makes sense only at a given value of $n \cdot x = \tau$ where τ is an arbitrary fixed constant. Thus, the residual gauge fixing condition (45) can be implemented only at a fixed $n \cdot x = \tau$. In this case, we can use the identity,

$$\bar{\Delta}^{-1}_{\rm FP}[A]|_{n\cdot x=\tau} = \int \mathcal{D}\bar{\theta}\delta[\partial \cdot A^{(\bar{\theta})} - \sqrt{\xi}f(x)]_{n\cdot x=\tau}. \quad (47)$$

This second Faddeev-Popov determinant in (47) is manifestly invariant under a restricted gauge transformation (44) and is defined on the space of functions annihilated by $n \cdot \partial$ [see (44)]. Following the earlier derivation in (43), we can now write the generating function as

$$\begin{aligned} Z &= N\left(\int \mathcal{D}\theta\right)\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\delta(n\cdot A)\bar{\Delta}_{\rm FP}[A]|_{n\cdot x=\tau} \\ &\quad \times \int \mathcal{D}\bar{\theta}\delta[\partial\cdot A^{(\bar{\theta})} - \sqrt{\xi}f(x)]_{n\cdot x=\tau}e^{iS_{\rm inv}} \\ &= N\left(\int \mathcal{D}\theta\right)\left(\int \mathcal{D}\bar{\theta}\right)\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\bar{\Delta}_{\rm FP}[A]|_{n\cdot x=\tau} \\ &\quad \times \delta(n\cdot A)\delta(\partial\cdot A - \sqrt{\xi}f)_{n\cdot x=\tau}e^{iS_{\rm inv}}. \end{aligned} \quad (48)$$

The gauge volume is now completely extracted and can be absorbed into the normalization factor N.

The second delta function can be exponentiated using the 't Hooft trick of using a Gaussian weight factor [15] leading to

$$S = S_{\rm inv} - \frac{1}{2\xi}\int d^4x\delta(n\cdot x - \tau)(\partial\cdot A)^2, \quad (49)$$

which resembles (34) except for the fact that it is defined only for a fixed value of $n \cdot x$. The generating functional takes the form

$$Z = N\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\bar{\Delta}_{\rm FP}[A]|_{n\cdot x=\tau}\delta(n\cdot A)e^{iS}, \quad (50)$$

where we have absorbed the gauge volume elements into the normalization constant. As is well known, in an axial-type gauge, the Faddeev-Popov determinant $\Delta_{\rm FP}[A]$ is trivial [12]. However, the determinant coming from the second gauge fixing is not and can be written in the form of an action of the form

$$\begin{aligned} \bar{\Delta}_{\rm FP}[A]|_{n\cdot x=\tau} &= \int \mathcal{D}\bar{c}\mathcal{D}c\, e^{iS_{\rm ghost}}, \\ S_{\rm ghost} &= -\int d^4x\delta(n\cdot x-\tau)\bar{c}\partial^\mu D_\mu c. \end{aligned} \quad (51)$$

To determine the propagator, we note that in the axial-type gauges (39) with the second gauge fixing term (49) in the action, the Green's function for the theory has to satisfy the equation

$$\begin{aligned} &\left(\delta^\mu_\sigma - \frac{n^\mu n_\sigma}{n^2}\right)\left[\eta^{\sigma\lambda}\Box - \partial^\sigma\partial^\lambda + \frac{1}{\xi}\delta(n\cdot x-\tau)\partial^\sigma\partial^\lambda\right] \\ &\quad \times D_{\lambda\nu}(x,y) \\ &\quad = \left(\delta^\mu_\nu - \frac{n^\mu n_\nu}{n^2}\right)\delta^4(x-y), \\ &n^\mu D_{\mu\nu} = 0 = D_{\mu\nu}n^\nu. \end{aligned} \quad (52)$$

We note that with the available tensor structures, we can construct two linearly independent, orthogonal second rank symmetric projection operators which will be transverse to the vector n^μ. Namely,

$$\begin{aligned} P_{\mu\nu} &= \eta_{\mu\nu} - \epsilon(n^2)n_\mu n_\nu \\ &\quad - \frac{[\partial_\mu - \epsilon(n^2)n_\mu n\cdot\partial][\partial_\nu - \epsilon(n^2)n_\nu n\cdot\partial]}{[\Box - \epsilon(n^2)(n\cdot\partial)^2]} \\ Q_{\mu\nu} &= \frac{[\partial_\mu - \epsilon(n^2)n_\mu n\cdot\partial][\partial_\nu - \epsilon(n^2)n_\nu n\cdot\partial]}{[\Box - \epsilon(n^2)(n\cdot\partial)^2]}, \end{aligned} \quad (53)$$

where $\epsilon(n^2)$ represents the sign of n^2. Each of these two structures satisfies

$$n^\mu P_{\mu\nu} = 0 = n^\mu Q_{\mu\nu} = P_{\mu\nu}n^\nu = Q_{\mu\nu}n^\nu. \quad (54)$$

However, it is easy to check that the first structure, in addition, is transverse to ∂^μ,

$$\partial^\mu P_{\mu\nu} = 0 = P_{\mu\nu}\partial^\nu. \quad (55)$$

Since the propagator has to be transverse to n^μ, we can expand it as

$$D_{\mu\nu}(x,y) = P_{\mu\nu}a(x,y) + Q_{\mu\nu}b(x,y), \quad (56)$$

where we can think of $a(x,y)$, $b(x,y)$ respectively as the transverse and the longitudinal components of the propagator. Substituting (56) into (52), it is easily determined that $a(x,y)$ and $b(x,y)$ satisfy

$$\begin{aligned} &\Box a(x,y) = \delta^4(x-y), \\ &\left[\epsilon(n^2)(n\cdot\partial)^2 + \frac{1}{\xi}\delta(n\cdot x-\tau)[\Box - \epsilon(n^2)\times \right. \\ &\quad \left.(n\cdot\partial)^2]\right]b(x,y) = \delta^4(x-y). \end{aligned} \quad (57)$$

The first equation is straightforward to solve and gives (in four dimensions)

$$a(x,y) = -\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot(x-y)}}{k^2}. \quad (58)$$

The second equation in (57) is a bit more involved because of the delta function, but leads to a solution of the form

$$b(x, y) = \int \frac{d^3k_T}{(2\pi)^3} e^{-ik_T\cdot(x_T - y_T)}\left[-\frac{\xi}{k_T^2} + \epsilon(n^2) \times (|n\cdot x - n\cdot y| - |n\cdot x - \tau| - |n\cdot y - \tau|)\right]. \tag{59}$$

Here we have defined the transverse coordinates and momenta as

$$x_T^\mu = x^\mu - \epsilon(n^2)n^\mu(n\cdot x),$$
$$k_\mu^T = k_\mu - \epsilon(n^2)n_\mu(n\cdot k). \tag{60}$$

This defines the completely gauge fixed propagator which is well behaved without any unphysical pole for any finite value of ξ. In particular, for $\xi = 0$, we note that the longitudinal part of the propagator (59) vanishes for $n\cdot x = \tau$ or $n\cdot y = \tau$ as we would expect from the residual gauge fixing. In the temporal gauge, for example, $n^\mu = (1, 0, 0, 0)$ and with the transverse and the longitudinal parts given in (58) and (59) respectively, the propagator (56) takes the form

$$D_{ij}(x, y) = -\int \frac{d^4k}{(2\pi)^4}\left(\eta_{ij} + \frac{k_ik_j}{\vec{k}^2}\right)\frac{e^{ik\cdot(x-y)}}{k^2} - \int \frac{d^3k}{(2\pi)^3}\frac{k_ik_j}{\vec{k}^2} e^{-i\vec{k}\cdot(\vec{x}-\vec{y})} \times \left(\frac{\xi}{\vec{k}^2} + |x^0 - y^0| - |x^0 - \tau| - |y^0 - \tau|\right). \tag{61}$$

For $\xi = 0$, the form of the propagator in the temporal gauge in (61) had already been obtained in [9] where it has also been argued that the longitudinal part of the propagator is quite crucial in obtaining the correct value for the Wilson line. It is worth recalling that the propagator for the theory in the path integral in the temporal gauge (without the residual gauge fixing) has the form

$$D_{ij}(x, y) = -\int \frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot(x-y)}}{k^2}\left(\eta_{ij} + \frac{k_ik_j}{k_0^2}\right) = -\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)}\left[\frac{1}{k^2}\left(\eta_{ij} + \frac{k_ik_j}{\vec{k}^2}\right) - \frac{1}{k_0^2}\frac{k_ik_j}{\vec{k}^2}\right]. \tag{62}$$

We note from the above expression that while the transverse part of the propagator is well defined without any unphysical pole and coincides with that in (61), the longitudinal part depends on the prescription for handling the pole at $k_0 = 0$. The residual gauge fixing has removed this arbitrariness in the longitudinal part in (61) for any finite value of ξ. This is, in fact, a very general result. As we have argued in the last section, any residual invariance of the quadratic part of the action—local or global—does lead to an arbitrariness in the definition of the propagator which reflects in some form of prescription dependence. In the case of axial-type gauges, this is completely fixed by the residual gauge fixing. However, as we will show next, the behavior is quite different in the light-cone gauge.

B. Light-cone gauge

Let us next study the action (36) in the light-cone gauge

$$n\cdot A = 0, \qquad n^2 = 0, \tag{63}$$

which has been studied by various groups from different points of view [10,16]. The discussion for the gauge fixing in the path integral approach, in this case, follows exactly as discussed for the axial-type gauges and we obtain the generating functional in (43) as a result of the naive light-cone gauge fixing. The analysis of the residual symmetry, however, differs from the earlier case.

Let us note that given a lightlike vector n^μ, one can define a dual lightlike vector $\tilde{n}^\mu$ such that

$$\tilde{n}^2 = 0, \qquad n\cdot\tilde{n} \neq 0. \tag{64}$$

For example, with $n^\mu = (1, 0, 0, -1)$, we can define $\tilde{n}^\mu = (1, 0, 0, 1)$. Correspondingly, one can label the coordinates as $x^\mu = (n\cdot x, \tilde{n}\cdot x, x_T^\mu)$ where x_T^μ is transverse to both n^μ and $\tilde{n}^\mu$. In such a case, the delta function constraint in (43) can again be seen to be invariant under a residual gauge transformation (44)

$$A_\mu(x) \to A_\mu(x) + D_\mu\bar{\theta}(x), \qquad n\cdot\partial\bar{\theta}(x) = 0. \tag{65}$$

However, because of the lightlike nature of n^μ, the implications of (65) in this case are different and, in particular, it implies that the parameter of gauge transformation must be independent of $\tilde{n}\cdot x$. This difference from the earlier case leads to the essential difference in the structure of the propagator in the light-cone gauge.

Once again, as in the axial-type gauges, if we would like to impose a covariant gauge fixing, for the residual gauge symmetry, of the form

$$\partial\cdot A(x) = \sqrt{\xi}f(x), \tag{66}$$

we can implement it only at a fixed value of $\tilde{n}\cdot x = \tau$ since the parameter of the residual gauge transformation does not depend on $\tilde{n}\cdot x$. Therefore, incorporating the second covariant gauge fixing term into the action, we can write the generating functional in the form (50)

$$Z = N\int \mathcal{D}A_\mu \Delta_{\rm FP}[A]\bar{\Delta}_{\rm FP}[A]|_{\tilde{n}\cdot x=\tau}\delta(n\cdot A)e^{iS}, \tag{67}$$

where

$$S = S_{\rm inv} - \frac{1}{2\xi}\int d^4x\,\delta(\tilde{n}\cdot x - \tau)(\partial\cdot A)^2. \qquad (68)$$

We note once again that $\Delta_{\rm FP}[A]$ is trivial in the light-cone gauge, but the second Faddeev-Popov determinant leads to a ghost action much like (51) where the ghost action is defined only for $\tilde{n}\cdot x = \tau$.

To define the propagator, we note that in the light-cone gauge (63), the Green's function of the theory described by (68) would satisfy

$$\begin{aligned}\left(\delta^{\mu}_{\sigma} - \frac{n^{\mu}\tilde{n}_{\sigma}}{n\cdot\tilde{n}}\right)\Big[\eta^{\sigma\lambda}\Box - \partial^{\sigma}\partial^{\lambda} + \frac{1}{\xi}\partial^{\sigma} \\ \times\,\delta(\tilde{n}\cdot x - \tau)\partial^{\lambda}\Big]D_{\lambda\nu}(x,y) \\ = \left(\delta^{\mu}_{\nu} - \frac{n^{\mu}\tilde{n}_{\nu}}{n\cdot\tilde{n}}\right)\delta^4(x-y), \\ n^{\mu}D_{\mu\nu} = 0 = D_{\mu\nu}n^{\nu}.\end{aligned} \qquad (69)$$

There are two linearly independent second rank symmetric projection operators which vanish when contracted with either n^{μ} or n^{ν} [one was already given in (21)],

$$\begin{aligned}P_{\mu\nu} &= \eta_{\mu\nu} - \frac{n_{\mu}\partial_{\nu} + n_{\nu}\partial_{\mu}}{(n\cdot\partial)} + \frac{\partial^2}{(n\cdot\partial)^2}n_{\mu}n_{\nu} \\ Q_{\mu\nu} &= \eta_{\mu\nu} - \frac{n_{\mu}\partial_{\nu} + n_{\nu}\partial_{\mu}}{(n\cdot\partial)}.\end{aligned} \qquad (70)$$

It is easy to check that while both vanish when contracted with n^{μ} or n^{ν}, the first structure is in addition transverse to ∂^{μ}, namely,

$$\partial^{\mu}P_{\mu\nu} = 0 = P_{\mu\nu}\partial^{\nu}. \qquad (71)$$

Thus, much like the case of the axial-type gauges, we can decompose the propagator as

$$\begin{aligned}D_{\mu\nu}(x,y) &= P_{\mu\nu}a(x,y) + (P_{\mu\nu} - Q_{\mu\nu})c(x,y) \\ &= P_{\mu\nu}a(x,y) + n_{\mu}n_{\nu}b(x,y),\end{aligned} \qquad (72)$$

where we can think of $a(x, y)$, $b(x, y)$ respectively as the transverse and the longitudinal components of the propagator. Substituting (72) into (69), we can derive the equations for the coefficient functions to be

$$\begin{aligned}\Box a(x,y) = \delta^4(x-y), \\ n\cdot\partial\left[1 - \frac{1}{\xi}\delta(\tilde{n}\cdot x - \tau)\right]n\cdot\partial b(x,y) = -\delta^4(x-y).\end{aligned} \qquad (73)$$

The first equation is easy to solve as in the case of the axial-type gauges and leads to

$$a(x,y) = -\int\frac{d^4k}{(2\pi)^4}\frac{e^{ik\cdot(x-y)}}{k^2}. \qquad (74)$$

The equation for the longitudinal component, on the other hand, is quite different from that in (57) and this brings in new features for the light-cone gauge. For example, we note that to be able to solve [see (73)]

$$\left[1 - \frac{1}{\xi}\delta(\tilde{n}\cdot x - \tau)\right]n\cdot\partial b(x,y) = -\frac{1}{n\cdot\partial}\delta^4(x-y), \qquad (75)$$

we need a prescription for $\frac{1}{n\cdot\partial}$. Let us represent

$$\frac{1}{n\cdot\partial}\delta(\tilde{n}\cdot x - \tilde{n}\cdot y) = \Theta(\tilde{n}\cdot x - \tilde{n}\cdot y), \qquad (76)$$

where Θ represents a generalized step function satisfying

$$n\cdot\partial\Theta(\tilde{n}\cdot x - \tilde{n}\cdot y) = \delta(\tilde{n}\cdot x - \tilde{n}\cdot y). \qquad (77)$$

For example, we can have the naive representation of (76) as the ordinary step function or an alternating step function if we choose the principal value prescription or the Mandelstam-Leibbrandt prescription [10]. This prescription dependence, even after fixing the residual gauge symmetry, is a new feature of the light-cone gauge and reflects the fact that there is still some underlying global invariance of the theory, such as the one discussed in the last section, which leads to this arbitrariness. It is easy to check that the global transformation of (10) becomes only an on-shell symmetry of the quadratic action because of the $\delta(\tilde{n}\cdot x - \tau)$ term in the gauge fixing action. However, if we generalize the transformation of (10) as

$$\delta A_{\mu} = \zeta n_{\mu}\left[1 - \frac{1}{\xi}\delta(\tilde{n}\cdot x - \tau)\right]\partial\cdot A, \qquad (78)$$

this defines a global symmetry of the quadratic part of the action (68) and this is the origin of the arbitrariness (prescription dependence) in the definition of the propagator. One needs to treat $\delta(x)$ and in particular $\delta(0)$ in this derivation in a regularized manner from a definition such as

$$\delta(x) = \lim_{\eta\to 0}\frac{1}{\sqrt{\pi}\eta}\exp\left(-\frac{x^2}{\eta^2}\right), \qquad (79)$$

with the understanding that the limit $\eta \to 0$ has to be taken only at the end.

It is well known that, in the light-cone gauge, prescriptions such as the principal value for the unphysical poles lead to incorrect results and the only consistent prescription that works correctly is the Mandelstam-Leibbrandt prescription [10]. Therefore, choosing this prescription, we can write

$$\begin{aligned}\Theta(\tilde{n}\cdot x - \tilde{n}\cdot y) &= \lim_{\eta\to 0}\int\frac{d(n\cdot k)}{2\pi i(n\cdot\tilde{n})}\frac{e^{i(n\cdot k)(\tilde{n}\cdot x - \tilde{n}\cdot y)}}{(n\cdot k) + i(\tilde{n}\cdot k)\eta} \\ &= -\frac{1}{(n\cdot\tilde{n})}\epsilon(\tilde{n}\cdot k)\theta(-\tilde{n}\cdot k(\tilde{n}\cdot x - \tilde{n}\cdot y)).\end{aligned} \qquad (80)$$

With such a prescription, it is straightforward to check that the solution of (75) or (73) has the form

$$b(x,y) = -\epsilon(n\cdot\tilde{n})\bigg[\tilde{n}\cdot x\Theta(\tilde{n}\cdot x-\tilde{n}\cdot y) + \tilde{n}\cdot y\Theta(\tilde{n}\cdot y-\tilde{n}\cdot x) - \frac{n\cdot\tilde{n}\Theta(\tau-\tilde{n}\cdot x)\Theta(\tau-\tilde{n}\cdot y)}{[\xi-\delta(0)]}\bigg] \times \delta^2(x_\perp - y_\perp)\delta(n\cdot x - n\cdot y), \tag{81}$$

where the $\delta(0)$ term is necessary to impose the correct boundary condition satisfied by the propagator and should be understood in a regularized manner from a representation such as in (79). The longitudinal part of the propagator is now uniquely determined from (81) and the Mandelstam-Leibbrandt prescription (80) then defines the unphysical poles of the transverse part of the propagator as well.

IV. CONCLUSION

In this paper we have analyzed the question of residual gauge fixing in the path integral approach in a systematic manner and have determined the completely gauge fixed propagator in the axial-type gauges as well as in the light-cone gauge. In both the cases, the propagator can be defined without fixing the residual gauge symmetry, but then one has to specify a prescription for handling the unphysical poles in the propagator. In the case of the axial-type gauges, the residual gauge fixing determines the propagator completely without any problem of unphysical poles. In the light-cone gauge, however, there is still a prescription dependence in the propagator even after fixing the residual gauge symmetry. This reflects the existence of a global invariance (78) of the quadratic part of the theory which is the source of the arbitrariness in the definition of the propagator. However, if we take the Mandelstam-Leibbrandt prescription (80) which is the conventional prescription in the light-cone gauge, it determines both the transverse as well the longitudinal parts of the propagator completely. We note here that the completely gauge fixed propagator in the path integral approach in axial-type gauges as well as the light-cone gauge in general continue to be different from that in the Hamiltonian formalism (for similar gauge fixing terms). Namely, in the Hamiltonian formalism the propagator is doubly transverse (this is true in axial-type gauges as well) while the completely gauge fixed propagator in the path integral approach is, in general, transverse with respect to n^μ, but has a longitudinal component with respect to p^μ. Such a difference is, in fact, natural and can be easily understood on physical grounds as follows. In the Hamiltonian formalism, the constraints can be set strongly equal to zero (after calculating the Dirac brackets) thereby eliminating certain components of the fields. This leads to analogues of nonlocal "instantaneous Coulomb" type interaction terms in the Hamiltonian. On the other hand, in the path integral formalism, one does not explicitly eliminate components of the fields and correspondingly, such interactions arise only through the exchange of longitudinal gluons (longitudinal components of the gauge propagator) and the ghosts. The longitudinal components of the propagator are, therefore, absolutely essential in the path integral approach together with the ghost terms (ghosts are necessary to cancel out any dependence of physical quantities on τ) as has been stressed within the context of the calculation of the Wilson line in the temporal gauge [9].

In summary, we note that in this paper, our goal has been to compare the form of the path integral propagator with that obtained from a Hamiltonian analysis. To that end, we have chosen the residual gauge fixing to be the covariant gauge in a manner completely parallel with the Hamiltonian analysis. As we have shown, with this choice in the light-cone gauge, there is a residual global symmetry of the free action (both in the Hamiltonian as well as the path integral formalisms). In fact, there is as well an Abelian local gauge invariance

$$A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu\hat{\theta}(n\cdot x), \tag{82}$$

of the free action where $\hat{\theta}(n\cdot x)$ depends only on $n\cdot x$. The presence of these residual symmetries leads to the prescription dependence in the propagator, both in the Hamiltonian as well as in the path integral formalisms. The prescription dependence can be eliminated in the path integral formalism much like in the axial-type gauges if one chooses a residual gauge fixing term which leaves no further global/local invariance in the free action. The choice of such a residual gauge in the context of the light-cone gauge is presently under study and the results will be reported in future.

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APPENDIX A: PROPAGATOR AND THE INVERSE TWO POINT FUNCTION IN THE LIGHT-CONE GAUGE

In this appendix, we will show very briefly that the propagator in the naive light-cone gauge in the path integral approach corresponds to the inverse of the two point function given in (4). Let us consider the generating function (7) in the presence of sources.

$$Z[J^{\mu}, J] = N \int \mathcal{D}A_{\mu} \mathcal{D}F e^{i(S+S_{\text{source}})},$$
$$S = S_{\text{inv}} - \int d^4x \text{Tr} F n \cdot A, \tag{A1}$$
$$S_{\text{source}} = \int d^4x \text{Tr}(J^{\mu}A_{\mu} + JF).$$

Here we have absorbed the nondynamical Faddeev-Popov determinant into the normalization constant and have exponentiated the delta function constraint with the help of an auxiliary field.

It is straightforward to determine the classical fields from the form of the action S in (A1) and they take the forms

$$\begin{aligned} F^c &= \frac{1}{n \cdot \partial} \partial_{\mu} J^{\mu}, \\ A^c_{\mu} &= \frac{1}{\Box}\left(g_{\mu\nu} - \frac{n_{\mu}\partial_{\nu} + n_{\nu}\partial_{\mu}}{n \cdot \partial}\right) J^{\nu} + \frac{1}{n \cdot \partial} \partial_{\mu} J \\ &= (\Gamma^{(\text{PI})})^{-1}_{\mu\nu}(n, \partial) J^{\nu} + \frac{1}{n \cdot \partial} \partial_{\mu} J, \end{aligned} \tag{A2}$$

where $(\Gamma^{(\text{PI})})^{-1}_{\mu\nu}(n, \partial)$ is the inverse of the two point function given in (4) in the coordinate space. Shifting the fields in the generating function (A1) by

$$A_{\mu} \to A_{\mu} + A^c_{\mu}, \qquad F \to F + F^c,$$

we obtain

$$Z[J^{\mu}, J] = e^{i \int d^4x \text{Tr}[(1/2)J^{\mu}(\Gamma^{(\text{PI})})^{-1}_{\mu\nu}(n,\partial)J^{\nu} + J(1/n\cdot\partial)\partial_{\mu}J^{\mu}]} \times N \int \mathcal{D}A_{\mu} \mathcal{D}F e^{iS}. \tag{A3}$$

This determines that the propagator for the gauge field in the naive light-cone gauge in the path integral approach has the form

$$D^{(\text{PI})}_{\mu\nu} = -\frac{1}{Z} \frac{\delta^2 Z}{\delta J^{\mu} \delta J^{\nu}} \bigg|_{J^{\mu}, J=0} = (\Gamma^{(\text{PI})})^{-1}_{\mu\nu}(n, \partial), \tag{A4}$$

as claimed in the text.

APPENDIX B: SOME USEFUL FORMULAS

In this appendix, we compile some formulas that are quite useful in dealing with light-cone variables. First, let us assume that the lightlike vectors n^{μ}, $\tilde{n}^{\mu}$ have vanishing components along $i = 1, 2$ (transverse) directions. In that case, we can introduce a new set of coordinates

$$\bar{x}^{\alpha} = (n \cdot x, x^i, \tilde{n} \cdot x) = L^{\alpha}_{\mu} x^{\mu}, \tag{B1}$$

where x^{μ} represents the usual Minkowski coordinates and

$$L^{\alpha}_{\mu} = \begin{pmatrix} n^0 & 0 & 0 & -n^3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \tilde{n}^0 & 0 & 0 & -\tilde{n}^3 \end{pmatrix}. \tag{B2}$$

The metric tensors for the new coordinates take the forms

$$\bar{g}^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & n \cdot \tilde{n} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ n \cdot \tilde{n} & 0 & 0 & 0 \end{pmatrix},$$
$$\bar{g}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{n\cdot\tilde{n}} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{n\cdot\tilde{n}} & 0 & 0 & 0 \end{pmatrix}. \tag{B3}$$

The integration measure correspondingly can be written as

$$\int d^4x = \frac{1}{|n \cdot \tilde{n}|} \int d^4\bar{x}. \tag{B4}$$

Furthermore, the delta functions would transform as

$$\delta^4(x) = |n \cdot \tilde{n}| \delta^4(\bar{x}). \tag{B5}$$

These are some of the formulas that have been used in the derivations in the paper.

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