Fate of bound systems in phantom and quintessence cosmologies

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We study analytically and numerically the evolution of bound systems in universes with accelerating expansion where the acceleration either increases with time towards a Big Rip singularity (phantom cosmologies) or decreases with time (quintessence). We confirm the finding of Caldwell *et al.* [R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).] that bound structures get dissociated in phantom cosmologies but we demonstrate that this happens earlier than anticipated in Ref. [R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).]. In particular we find that the ''rip time'' when a bound system gets unbounded is not the time when the repulsive phantom energy gravitational potential due to the average $(\rho + 3p)$ balances the attractive gravitational potential of the mass M of the system. Instead, the ''rip time'' is the time when the minimum of the time-dependent effective potential (including the centrifugal term) disappears. For the Milky Way galaxy this happens approximately 180 Myrs before the Big Rip singularity instead of approximately 60 Myrs indicated in [R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).] for a phantom cosmology with $w = -1.5$. A numerical reconstruction of the dissociating bound orbits is presented.

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I. INTRODUCTION

There is mounting observational evidence that the universe has entered a phase of accelerating expansion (the scale factor obeys $\ddot{a} > 0$) and that the total amount of clustered matter in the universe is not sufficient for its small average spatial curvature. This converging observational evidence comes from a diverse set of cosmological data which includes observations of type Ia supernovae [1], large scale redshift surveys [2] and measurements of the cosmic microwave background (CMB) temperature fluctuations spectrum [3]. The observed accelerating expansion and flatness of the universe, requires either a modified theory of gravity [4] or, in the context of standard general relativity, the existence of a smooth energy component with negative pressure termed ''dark energy''[5]. This component is usually described by an equation of state parameter $w \equiv \frac{p}{\rho}$ (the ratio of the homogeneous dark energy pressure p over the energy density ρ). For cosmic acceleration, a value of $w < -\frac{1}{3}$ is required as indicated by the Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{1.1}$$

Current observational bounds [1,6] on the value of the dark energy equation of state parameter $w(t_0)$ at the present time t_0 yield

$$-1.48 < w(t_0) < -0.72 \tag{1.2}$$

at the 95% confidence level. The role of dark energy can be played by any physical field with positive energy and negative pressure which violates the strong energy condition $\rho + 3p > 0$ ($w > -\frac{1}{3}$).

Quintessence scalar fields [7] ($-1 < w < -\frac{1}{3}$) violate the strong energy condition but not the dominant energy condition $\rho + p > 0$. Their energy density scales down with the cosmic expansion and so does the cosmic acceleration rate. Phantom fields [8–10] ($w < -1$) violate the strong energy condition, the dominant energy condition and maybe physically unstable. However, they are also consistent with current cosmological data and according to recent studies [6] they are favored over their quintessence counterparts. In contrast to quintessence fields, the energy density of phantom fields increases with time and so does the predicted expansion acceleration rate $\frac{\ddot{a}}{a}$. This monotonically increasing acceleration rate of the expansion may be shown to lead to a novel kind of singularity which occurs at a finite future time and is characterized by divergences of the scale factor a, the Hubble parameter H its derivative $\dot{H}$ and the scalar curvature. This singularity has been called ''Big Smash'' [11] the first time it was discussed and ''Big Rip'' [12] (hereafter CKW) in a more recent study. Even though there are mechanisms by which the ''Big Rip'' singularity could be avoided [13] it remains an interesting possible fate of the universe.

An immediate consequence of the very rapid expansion rate as the Big Rip singularity is approached is the dissociation of bound systems due to the buildup of repulsive negative pressure in the interior of these systems. This observation was first made in [12] where a qualitative study of the dissociation times for various bound systems was also made.

The quantitative study however of the evolution of a bound system in an expanding universe remains an issue of current research. In particular the question of whether the

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expansion of the universe affects gravitationally bound systems like clusters, galaxies or planetary systems has been addressed in several early [14–17] and recent [18–20] studies. The recurrent attention paid to this issue indicates that to this point a definitive answer is still lacking. The prevalent perception however is that the physics of systems which are small compared to the radius of curvature of the cosmological background is essentially unaffected by the expansion of the universe. Even though this perception is applicable in the case of phantom cosmology, it is not applicable as the Big Rip singularity is approached and curvature increases rapidly.

A qualitative approach to the problem of bound system dissociation in phantom cosmologies was made by CKW where it was assumed that a bound system becomes unbound when the source of the repulsive phantom energy gravitational potential for a bound system of size R $[-\frac{4}{3}\pi(\rho + 3p)R^3]$ balances the attractive source of the ordinary gravitational potential (the mass M of the system). Even though this assumption is qualitatively useful as it provides a physical understanding as to why does a bound system dissociate due to phantom energy repulsion, quantitatively it leads to incorrect estimates of the dissociation times because it ignores the effects of the centrifugal barrier in the effective potential of bound systems. A quantitative analysis should make use of the particle equations of motion in the local inertial frame based on the geodesic deviation equation. Using such an equation, the time dependent effective potential of two-body bound systems may be shown to consist of three terms, the static attractive gravitational mass term, the static repulsive centrifugal term and the time-dependent repulsive dark energy term. The stability of a bound system depends on the existence of a minimum for the above effective potential. In the case of quintessence ($w > -1$) the minimum of the effective potential can not disappear at any future time because the repulsive time-dependent term decreases with time. For phantom energy however ($w < -1$) the repulsive time-dependent term increases with time and at some critical time dominates over the other two terms and destroys the minimum of the effective potential dissociating at the same time the corresponding bound system. It should be stressed that this critical time when the effective potential minimum disappears is not the time when the repulsive dark energy term balances the attractive gravitational mass term as assumed in CKW. This will be demonstrated in detail, analytically and numerically in the following sections.

The structure of this paper is the following: In Sec. II we use a metric that interpolates between the static Schwarzschild at small scales and a general time-dependent Friedmann universe metric at large scales to derive the geodesics of a test particle in the Newtonian limit. We then focus on the particular Friedmann universe that contains a two-component cosmic fluid (matter and dark energy) and derive the form of the scale factor for any value of constant w (the equation of state parameter). Using this form of the scale factor in the previously derived geodesic equation we find the equation of motion for a two-body bound system in an expanding quintessence or phantom cosmological setup. In Sec. III we study this equation of motion and derive the evolution of the radius of two-body bound systems with distance and mass scales corresponding to the Solar System, the Milky Way Galaxy and the Coma Cluster. We also derive analytically the dissociation time as a function of w for a bound system of given mass and radius and test our result using numerical simulations of the above three types of bound systems (Solar System, Milky Way and Coma Cluster). Our analytical result for the time difference "Big Rip time - Dissociation time" differs from the corresponding result of CKW by a factor of 3.08 (we find the dissociation to occur earlier). The time evolution of the bound system effective potential is also shown. Our results are extended to the case of planar mass sources and to the case of quintessence ($w > -1$). In the later case it is shown that the size change of the bound system due to the expansion is negligible. Finally in Sec. IV we conclude, summarizing our main results and propose possible extensions of this work.

II. GEODESICS IN EXPANDING SCHWARZSCHILD METRIC

In order to investigate the effects of expansion on local bound systems we must find the geodesics of test particles in the appropriate metric that describes the spacetime in the vicinity of a point mass M placed in an expanding background. Such a metric should interpolate between a static Schwarzschild metric at small distances from M and a time-dependent Friedmann spacetime at large distances. The detailed form of this interpolation is not unique and there are different approaches to this problem in the literature [14,15,18,21]. In the Newtonian limit (weak field, low velocities) such an interpolating metric takes the form:

$$ds^2 = \left(1 - \frac{2GM}{a(t)\rho}\right) \cdot dt^2 - a(t)^2 \cdot [d\rho^2 + \rho^2 \cdot (d\theta^2 + sin^2\theta d\varphi^2)], \tag{2.1}$$

where ρ is the comoving radial coordinate. Using

$$r = a(t) \cdot \rho \tag{2.2}$$

the geodesics corresponding to the line element (2.1) take the form

$$-\left(\ddot{r} - \frac{\ddot{a}}{a} r\right) - \frac{GM}{r^2} + r\dot{\varphi}^2 = 0 \tag{2.3}$$

and

$$r^2\dot{\varphi} = L, \tag{2.4}$$

where L is the constant angular momentum per unit mass. Therefore the radial equation of motion for a test particle in the Newtonian limit considered is

$$\ddot{r} = \frac{\ddot{a}}{a} r + \frac{L^2}{r^3} - \frac{GM}{r^2}. \tag{2.5}$$

The same equation of motion is obtained in the Newtonian limit by other interpolations even though the details in other limits may vary [18].

There is another simple and intuitive (but not rigorous) way to derive the same equation of motion by using Gauss's law for gravity with gravitational sources the mass M and the integral of the homogeneous source $\rho + 3p$. This approach leads to

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} - \frac{4\pi G}{3}(\rho + 3p)\vec{r}. \tag{2.6}$$

Using now the Friedmann Eq. (1.1) this reduces to Eq. (2.5).

Therefore the dynamics of a subluminal test particle bound in the gravitational field of a mass M (or equivalently a two-body bound system) in an expanding universe can be described by the geodesic equation of motion (2.5). In what follows we will study the implications of this equation for two-body bound systems in various cosmologies.

As a warm up exercise let us consider the evolution of a bound system in an expanding universe with scale factor

$$a(t) \sim t^{\alpha} \tag{2.7}$$

where $\alpha = \text{const}$. Let us assume that at some initial time t_0 the test particle is at circular orbit with radius r_0 and $\dot{\varphi}(t_0) = \omega_0 = \frac{GM}{r_0^3}$. Then the equation of motion (2.5) may be written in dimensionless form as

$$\ddot{\bar{r}} - \frac{\bar{\omega}_0^2}{\bar{r}^3} + \frac{\bar{\omega}_0{}^2}{\bar{r}^2} - \frac{\alpha(\alpha - 1)}{\bar{t}^2}\bar{r} = 0, \tag{2.8}$$

where $\bar{r} \equiv \frac{r}{r_0}$, $\bar{\omega}_0 \equiv \omega_0 t_0$ and $\bar{t} \equiv \frac{t}{t_0}$. In what follows we will omit the bar ($\bar{\ }$) for convenience but we shall work in dimensionless form. Typically for gravitationally bound systems in the universe and cosmological timescales we have

$$\omega_0^2 = \frac{GM}{r_0^3} t_0^2 \gg 1 \tag{2.9}$$

(e.g., for galaxies $\omega_0 \simeq O(100)$) and we may therefore consider the last term in Eq. (2.8) as a perturbation perturbing the circular orbit $r_0 = 1$ to $r = 1 + \delta r(t)$. It is then easy to show that (see also [19])

$$\delta r(t) = \frac{\alpha(\alpha - 1)}{\omega_0^2 t^2}. \tag{2.10}$$

The radius of the orbit tends to increase for $\alpha \in (0, 1)$ (decelerating expansion) while the perturbation δr is negative because the homogeneous *attractive* gravitational source ($\rho + 3p > 0$ in this decelerating case) within a sphere of radius r decreases with time. For $\alpha > 1$ (accelerating expansion) the perturbation δr is positive but it is decreasing with time because the homogeneous *repulsive* gravitational source ($\rho + 3p < 0$ in this accelerating case) within a sphere of radius r decreases with time.

To better understand physically the behavior of the perturbed orbit we use the Friedmann Eq. (1.1). This equation shows that the time-dependent perturbing term of Eq. (2.8) comes from the homogeneous gravitational source $S = \frac{4\pi}{3}(\rho + 3p)r_0^3$ within the unperturbed radius r_0. For an equation of state $p = w\rho$ we have

$$a \sim t^{2/(3(w+1))} = t^{\alpha} \tag{2.11}$$

$$\rho \sim a^{-3(1+w)} \sim t^{-2} \tag{2.12}$$

The case $w > -\frac{1}{3}$ corresponds to $\alpha \in (0, 1)$ and the radius perturbation (2.10) is negative and decreases with time. This is to be expected because the gravitational source $S \sim \rho + 3p$ is positive (attractive) and decreases with time (Eq. (2.12)). Thus δr is negative (attractive perturbation) but decreases with time (the energy density decreases with time and so does the homogeneous gravitational source). For $-1 < w < -\frac{1}{3}$ we have $\alpha > 1$ and the radius perturbation (2.10) is positive decreasing with time. This is understood because the gravitational source S is negative (repulsive) and decreases with time. Thus δr is positive (repulsive perturbation) but decreases with time. Finally for $w < -1$ we have $\alpha < 0$. In this case the universe has an expanding phase for $t < 0$, a singularity (Big Rip) at $t = 0$ and a contracting phase at $t > 0$. Our present expanding phase is identified with the first phase ($t < 0$) and the radius perturbation is positive and *increasing* with time. This is to be expected because the gravitational source S is negative (repulsive) but also increases with time as t approaches the singularity at $t = 0$ (see Eq. (2.12)). Thus δr is positive (repulsive perturbation) and increases with time.

This perturbative approach shows that the bound system radial perturbation increases only in a phantom ($w < -1$, $\alpha < 0$) expanding background but it can give no hint on whether a bound system will eventually become unbound or not. A nonperturbative approach is required to address this question in detail. Such an approach should lead to the derivation of the full time evolution of the radius of a two-body bound system. As discussed in the next section this can be achieved by either explicitly solving the equation of motion (2.5) or by finding the time evolution of the minimum of the effective potential derived from Eq. (2.8).

Before proceeding with this more appropriate approach we will improve on the estimate of the scale factor evolution by considering a two-component (instead of one-component) fluid in a Friedmann universe. Assuming the fluid components to be matter with energy density ρ_m and dark energy with density ρ_x and equation of state $p_x = w\rho_x$ we may write the Friedmann equations as [9]

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}[\rho_m + \rho_x] = H_0^2[\Omega_m^0(a_0/a)^3 + \Omega_x^0(a_0/a)^{3(1+w)}] \tag{2.13}$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}[\rho_m + \rho_x(1+3w)] = -\frac{4\pi G}{3}\rho_x[\Omega_x^{-1} + 3w] = -\frac{4\pi G}{3}\rho_x\left[\frac{\Omega_m^0}{\Omega_x^0}\left(\frac{a_0}{a}\right)^{-3w} + 1 + 3w\right].$$

The transition from matter (decelerating) to dark energy (accelerating) phase occurs at the transition redshift

$$1 + z_m = \frac{a_0}{a_m} = \left[\frac{-(3w+1)\Omega_x^0}{\Omega_m^0}\right]^{-1/3w}. \tag{2.14}$$

At $z > z_m$ the solution of the Friedmann equation is

$$a^{3/2}(t) = \frac{3}{2}\xi t, \tag{2.15}$$

where ξ is a constant. In the accelerating phase $z < z_m$ the corresponding solution is approximated by

$$a^{3(1+w)/2}(t) = \frac{3}{2}\chi t + c, \tag{2.16}$$

where c is an integration constant and $\chi = H_0\sqrt{\Omega_x^0}$. Extending the expressions (2.15) and (2.16) up to t_m and matching for the evaluation of the integration constants ξ and c leads to the expression for the scale factor valid for $t > t_m$ $(z < z_m)$

$$a(t) = \frac{a(t_m)}{[-w + (1+w)t/t_m]^{-2/[3(1+w)]}} \quad \text{for} \quad t > t_m. \tag{2.17}$$

For phantom energy $(w < -1)$ the scale factor diverges at a finite time

$$t_* = \frac{w}{1+w}t_m > 0 \tag{2.18}$$

leading to the Big Rip singularity. Since $\rho_x \sim a^{-3(1+w)}$ it is easy to see that the phantom energy also diverges at t_* as

$$\rho_x(t) = \frac{\rho(t_m)}{[-w + (1+w)t/t_m]^2}. \tag{2.19}$$

Using the expression (2.17) for the scale factor in the two-component fluid universe we are in position to study in detail the evolution of bound orbits in phantom and quintessence cosmologies. This task will be undertaken in the following section.

III. BOUND SYSTEM EVOLUTION

Substituting the derived form of the scale factor in Eq. (2.17) to the equation of motion (2.8) of a two-body gravitating system in an expanding universe we obtain

$$\ddot{r} - \frac{\omega_0^2}{r^2}\left(1 - \frac{1}{r}\right) + \frac{2}{9}\frac{(1+3w)r}{[-w + (1+w)t]^2} = 0. \tag{3.1}$$

As in Eq. (2.8) this equation has been made dimensionless by setting $\frac{r}{r_0} \rightarrow r$ and $\frac{t}{t_m} \rightarrow t$. The time-dependent last term proportional to

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = \frac{2}{9}\frac{(1+3w)}{[-w + (1+w)t]^2} \tag{3.2}$$

expresses the gravitating effects of the dark energy included within a sphere of radius r while the dimensionless parameter ω_0^2 is defined as

$$\omega_0^2 = \frac{GM}{r_0^3}t_m^2. \tag{3.3}$$

We shall use the dimensionless equation of motion (3.1) with initial conditions $r(t=1) = r_{\rm min}$ and $\dot{r}(t=1) \simeq 0$ ($r_{\rm min}$ is the minimum of the effective potential at $t = 1$) to study the evolution of the radius of a two-body bound system which is initially at circular orbit. It will be shown that in the phantom case $(w < -1)$ the increasing repulsive effects of the time-dependent term of Eq. (3.1) lead to a dissociation of the bound system at a critical time $t_{\rm rip}$. A simplified qualitative approach to this question was made by CKW where no reference was made to the equation of motion as it was assumed that the dissociation occurs when the time-dependent dark energy gravitational source within the initial radius balances the attractive gravitational source $\frac{\omega_0^2}{r_{\rm min}^2}$. This approach is only qualitative and in many cases can lead to incorrect results for two reasons:

(i) It does not take into account the centrifugal force $\frac{\omega_0^2}{r_{\rm min}^3}$

(ii) It implicitly assumes that the radius of the system just before dissociation is the same as the initial radius $r_{\rm min}$.

Nevertheless for comparison with our later quantitative exact result we will rederive the result of CKW. The balance condition of CKW may be written as

$$\omega_0^2 \equiv \left(\frac{2\pi}{T}\right)^2 = -\frac{2}{9}\frac{(1+3w)}{[-w + (1+w)t]^2}, \tag{3.4}$$

which leads to

$$t_* - t_{\rm rip} = \frac{T\sqrt{2|1+3w|}}{6\pi|1+w|}, \tag{3.5}$$

where t_* is the Big Rip singularity time given by (2.19). This is the result of CKW to be compared with our quantitative result derived in what follows.

The time-dependent effective potential that determines the dynamics of the bound system is easily derived from

Eq. (3.1) to be

$$V_{\rm eff} = -\frac{\omega_0^2}{r} + \frac{\omega_0^2}{2r^2} - \frac{1}{2}\lambda(t)^2 r^2, \tag{3.6}$$

where

$$\lambda(t) = \frac{\sqrt{2|1+3w|}}{3[-w+(1+w)t]} \tag{3.7}$$

with $w < -1$. At $t = 1$ the system is assumed to be in circular orbit with radius given by the minimum $r_{\rm min}(t)$ of the effective potential of Eq. (3.6). The location of $r_{\rm min}(t)$ is time-dependent and approximates the radius of the system at any given time. It is the solution of the equation

$$q(t)^2 r_{\rm min}^4 = r_{\rm min} - 1, \tag{3.8}$$

where

$$q(t) \equiv \frac{\lambda(t)}{\omega_0}. \tag{3.9}$$

It may be shown (using, e.g., Mathematica [22]) that this equation has a solution only for

$$q(t)^2 \leq \frac{27}{256} \equiv q_c. \tag{3.10}$$

Therefore the time $t_{\rm rip}$ when the minimum of the potential (3.6) disappears and the system becomes unbound is given by the solution of the equation

$$q(t_{\rm rip})^2 = \frac{27}{256}. \tag{3.11}$$

It is straightforward to solve Eq. (3.11) for $t_{\rm rip}$ and find

$$t_* - t_{rip} = \frac{16\sqrt{3}}{9}\frac{T\sqrt{2|1+3w|}}{6\pi|1+w|}. \tag{3.12}$$

This result differs from the corresponding result of CKW by the factor $\frac{16\sqrt{3}}{9} \simeq 3$.

In order to test this result numerically we must solve the equation of motion (3.1) for a specific bound system, obtain numerically the orbits $r(t)$ and compare the analytical prediction for the dissociation time with the corresponding time visualized numerically. For concreteness we consider a phantom cosmology with $w = -1.2$, $\Omega_m^0 = 0.3$ and $\Omega_x^0 = 0.7$. Our goal is to evaluate the dimensionless angular velocity ω_0 corresponding to specific systems in the context of this cosmology and use it to calculate numerically the corresponding future radial evolution. We will consider the scales corresponding to three bound gravitational systems: the solar system ($M = 2 \times 10^{33} gr$, $r_0 = 7 \times 10^{15} cm$, $\omega_0 = 3.5 \times 10^6$), the Milky Way galaxy ($M = 2 \times 10^{45} gr$, $r_0 = 5 \times 10^{22} cm$, $\omega_0 = 182$) and the Coma Cluster ($M = 6 \times 10^{48} gr$, $r_0 = 9 \times 10^{24} cm$, $\omega_0 = 4.15$). In evaluating ω_0 for the above systems we have used the value of t_m in the particular phantom cosmology considered

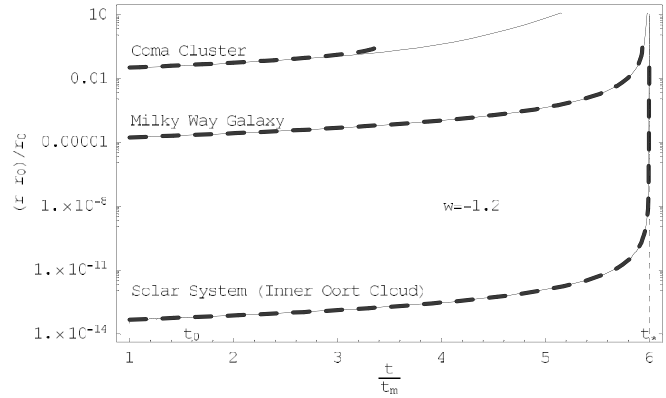

FIG. 1. The numerical evolution of the radius (continuous line) and the analytical evolution of the effective potential minimum (dashed line) for the three bound systems.

$$t_m = \left(H_0\sqrt{\Omega_m^0}\right)^{-1}\frac{2}{3}(1+z_m)^{-3/2} \simeq 1.8 \times 10^{17} h^{-1}\text{sec}. \tag{3.13}$$

We have evaluated the evolution of the radius of the above systems using two methods: calculation of the effective potential minimum (thick dashed lines of Fig. 1) using Eq. (3.8) evolved until the minimum disappears and explicit numerical evolution of the equation of motion (3.1) (continuous line) evolved up to the dissociation time as obtained by CKW (Eq. (3.5)). The numerical evolution started at $t = t_m$ ($t = 1$ in the dimensionless form) with initial orbit radius at the minimum of the effective potential and negligible radial velocity chosen such as to minimize radial oscillations. As seen in Fig. 1 there is very good agreement between the numerical evolution of the radius (continuous line) and the analytical evolution of the effective potential minimum (dashed line) for the three bound systems considered. The value of the present time t_0 is also indicated on the time axis of Fig. 1. In Fig. 2 we show the evolution of the dimensionless form of the effective potential around the time $t_{\rm rip}$ when the minimum disappears for a

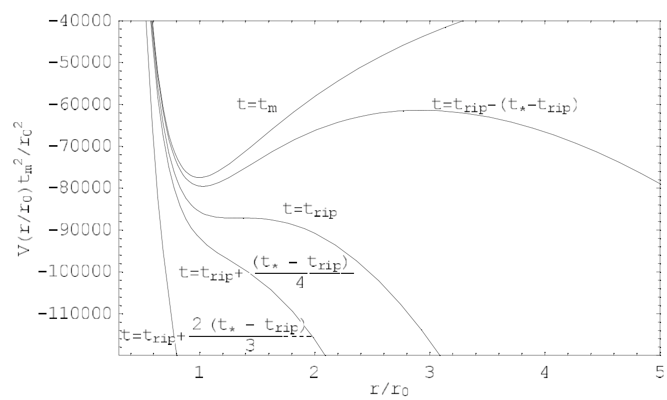

FIG. 2. The evolution of the dimensionless form of the effective potential around the time trip for the Milky Way galaxy.

bound system corresponding to the Milky Way galaxy. As the repulsive term destroys the minimum there is a small increase of the location of the minimum and then a sudden disappearance and dissociation of the system. The difference between our quantitative prediction for $t_{\rm rip}$ (indicated by the end of the dashed lines in Fig. 1) and the corresponding qualitative estimate of CKW (indicated by the end of the continuous lines) is more prominent for the Coma Cluster and less so for the Milky Way. The actual values in years for $t_* - t_{\rm rip}$ are shown in Table I along with the qualitative prediction of CKW. The corresponding dissociation times $t_{\rm rip}$ in units of t_m are shown in Table II. For the case $w = -1.5$ considered by CKW, the corresponding predictions for the Milky Way dissociation are $t_* - t_{\rm rip} \simeq 166$ Myrs and $(t_* - t_{\rm rip})_{CKW} \simeq 54$ Myrs.

Using the radial equation of motion (3.1) along with the conservation of angular momentum

$$r^2\dot{\varphi} = r_0^2\omega_0, \tag{3.14}$$

it is straightforward to obtain numerically the full trajectory corresponding to the evolution of the three bound

systems and visualize the dissociation process. This dissociation is demonstrated in Figs. 3 and 4 for a two-body bound system corresponding to the Milky Way galaxy. In particular, in Fig. 3 the system is evolved until the effective potential minimum disappears while the evolution in Fig. 4 corresponds to the same system but lasts until the phantom energy repulsive gravitational force balances the attractive gravity of bound matter. Clearly the dissociation time is the evolution time of Fig. 3 while the evolution in Fig. 4 continuous well after the dissociation as expected based on our analysis. Notice the radial (instead of tangential) motion followed after dissociation which is due to the dominant repulsive gravity of phantom energy.

The numerical evolution shown in Figs. 1 and 2 has assumed a phantom cosmology with $w = -1.2$. The dissociation time $t_{\rm rip}$ however is sensitive on the value of w as shown in Eq. (3.12). To demonstrate this dependence Fig. 5 shows the dependence of the relative rip time difference defined as

$$\frac{t_* - t_{\rm rip}}{T} \tag{3.15}$$

(where T is the rotation period of the bound system) on w

TABLE I. The dissociation times differences $t_* - t_{\rm rip}$ for three bound systems in years as predicted by Eqs. (3.12) and (3.5). The value $w = -1.2$ was assumed.

System	$t_* - t_{\rm rip}$ (yrs)	$(t_* - t_{\rm rip})_{CKW}$ (yrs)
Solar System	1.88×10^4	6.11×10^3
Milky Way	3.59×10^8	1.17×10^8
Coma Cluster	1.58×10^{10}	5.14×10^9

TABLE II. The dissociation times $t_{\rm rip}$ for the three bound systems in units of t_m. For $w = -1.2$, $t_m \simeq 5.65h^{-1}$ Gyrs.

System	$t_{\rm rip}/t_m$
Solar System	6.00
Milky Way	5.94
Coma Cluster	3.19

for $w < -1$. The continuous curve defines $t_{\rm rip}$ as the time when the effective potential minimum disappears while the corresponding definition for the dashed curve is the time when the total gravity force vanishes as in CKW. Clearly the two curves differ significantly and the difference becomes more pronounced as w approaches the value $w = -1$.

Our discussion so far was based on the assumption of phantom cosmologies ($w < -1$). This has been due to the fact that for $w > -1$ (quintessence) the dark energy density decreases with time and can therefore not destroy the effective potential minimum. Quintessence can only cause negligible evolution (decrease) of the radius of bound systems. This evolution can be easily obtained by the perturbative treatment of Sec. II by setting $\alpha = \frac{2}{3(w+1)}$. To demonstrate the negligible effect of expansion on bound systems for nonphantom cosmologies we have plotted the effective potential for a Milky Way scale system with $w = -0.9$ at times $t = t_m$ and $t = 3t_0$ (Fig. 6). The corresponding plot for a Coma Cluster scale system is shown in Fig. 7. The decrease of the radius is in both cases minor but in the Coma Cluster case where ω_0 is of $O(1)$ it is somewhat

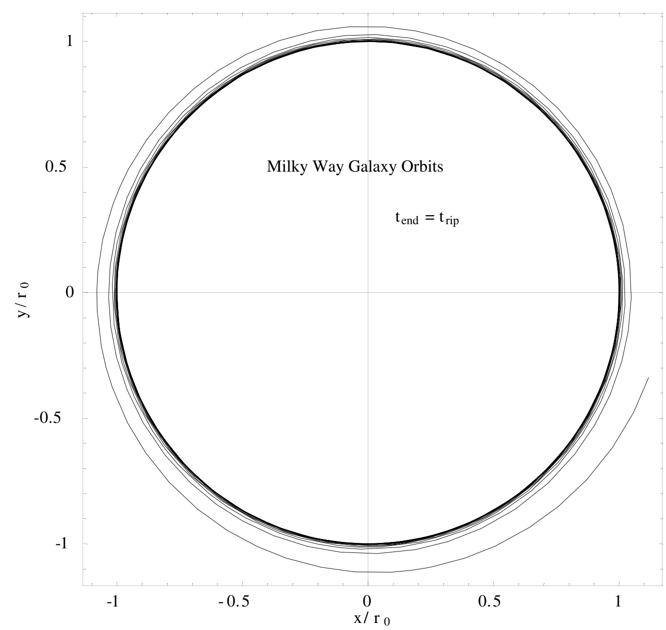

FIG. 3. The evolution of the the system of the Milky Way galaxy until the effective potential minimum disappears.

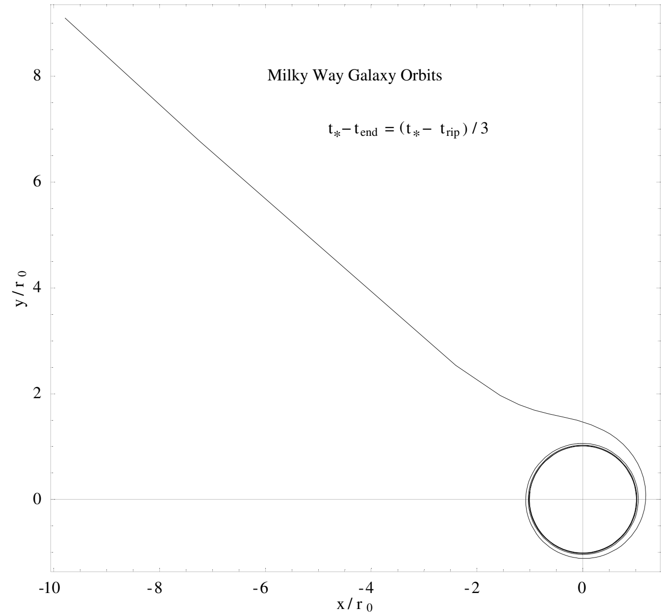

FIG. 4. The same system as in the previous figure but the evolution lasts until the phantom energy balances the attractive gravity of bound matter.

more prominent as expected from the perturbative result (2.10).

Finally it is of some interest to investigate the evolution of bound systems with planar geometry. Consider a test particle at a distance h from a surface with surface density σ in an expanding universe background. Using Gauss's law for simplicity it is straightforward to show that the equation of motion for the particle is

$$\ddot{h} = -2\pi G\sigma + \frac{3}{2}\frac{\ddot{a}}{a}h. \tag{3.16}$$

Using Eq. (2.17) for the scale factor in a phantom cosmology this may be written as

$$t_m^2\ddot{h} = -h_0 + \beta h, \tag{3.17}$$

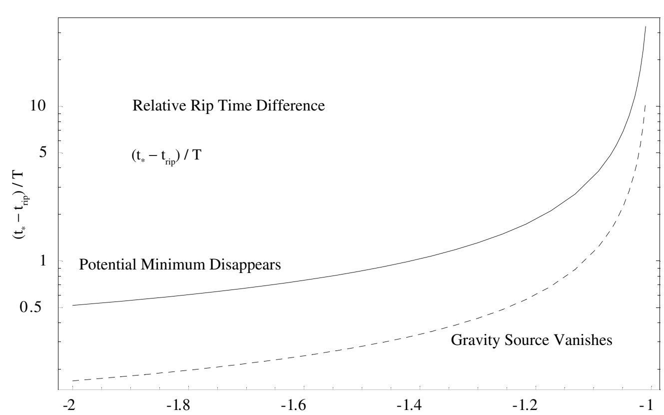

FIG. 5. The dependence on w of the relative rip time difference.

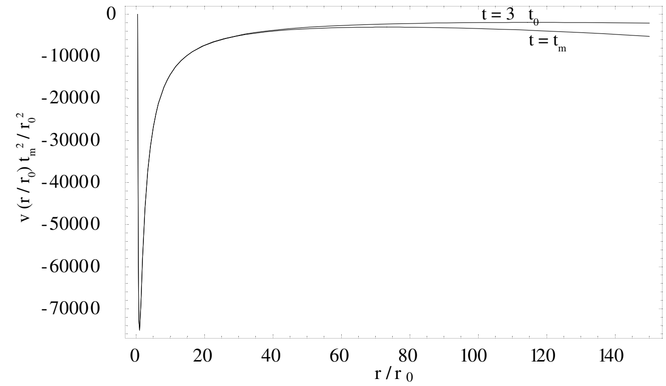

FIG. 6. The effective potential for the Milky Way galaxy with $w = -0.9$ at times $t = t_m$ and $t = 3t_0$.

where the derivative is with respect to $\frac{t}{t_m}$,

$$h_0 = 2\pi G\sigma t_m^2 \tag{3.18}$$

and

$$\beta(t) = \frac{|1 + 3w|}{3[-w + \frac{t}{t_m}(1 + w)]^2}. \tag{3.19}$$

Dividing by h_0 and setting $\frac{t}{t_m} \to t$, $\frac{h}{h_0} \to h$, Eq. (3.17) may be written in dimensionless form as

$$\ddot{h} = -1 + \beta h. \tag{3.20}$$

The effective potential corresponding to this equation has obviously no centrifugal term and differs significantly from the corresponding effective potential of the spherically symmetric case. It is of the form

$$V_{\rm eff} = -\frac{1}{2}\beta\left(h - \frac{1}{\beta}\right)^2 \tag{3.21}$$

and is shown schematically in Fig. 8. It is a reversed harmonic oscillator with a time-dependent unstable equilibrium point at $h_{eq} = \frac{1}{\beta(t)}$. For quintessence $\beta(t)$ de-

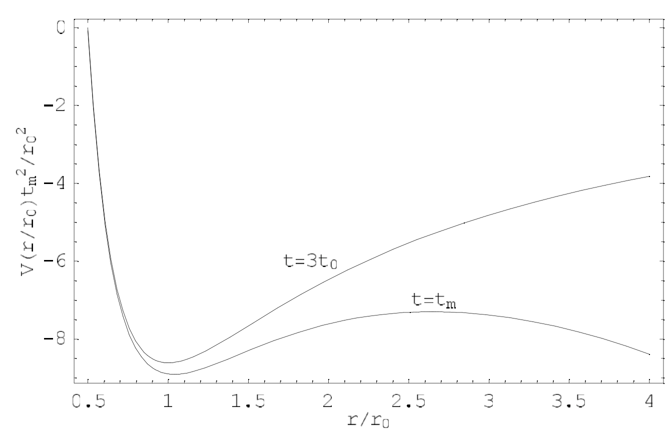

FIG. 7. The effective potential for the Coma Cluster with $w = -0.9$ at times $t = t_m$ and $t = 3t_0$.

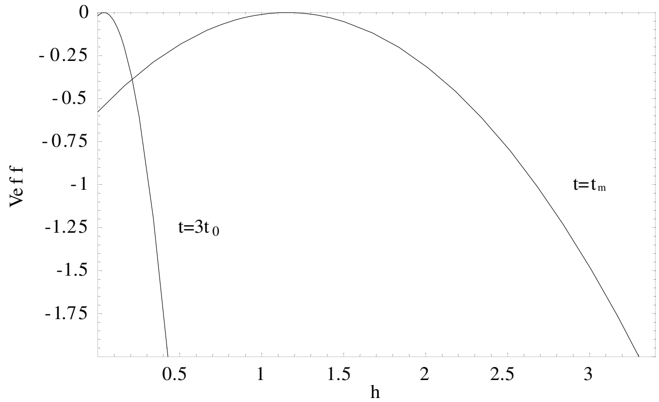

FIG. 8. The effective potential corresponding to bound systems with planar geometry for $t = t_m, 3t_0$ and $w = -1.2$. There is reflection symmetry of the potential with respect to $h = 0$.

creases with time and therefore the location of the equilibrium point increases with time. Points initially on the left of the equilibrium point will remain bound on the attractive side of the potential. Points initially on the right of the equilibrium (unbound) may also eventually end up bound on the left side of the equilibrium point. For phantom cosmologies $\beta(t)$ increases with time. This implies that the unstable equilibrium scale $h_{\rm eq}(t)$ decreases with time and therefore all scales will eventually become larger than $h_{\rm eq}(t)$ and dissociate. The dimensionless scale $h_d(t_{\rm rip})$ that dissociates at the time $t_{\rm rip}$ is found by solving the equation

$$h_d = \frac{1}{\beta(t_{\rm rip})} = -\frac{3[-w + t_{\rm rip}(1+w)]^2}{1+3w} \tag{3.22}$$

with solution

$$t_* - t_{\rm rip} = \frac{T}{8\sqrt{3}} \frac{\sqrt{2|1+3w|}}{|1+w|}, \tag{3.23}$$

where $T = 4\sqrt{2h_d}$ is the period of the oscillating test mass. Notice the similarity of this result with the corresponding result found for spherically symmetric systems (3.12).

IV. CONCLUSION-OUTLOOK

We have studied the evolution of bound systems in expanding backgrounds and focused on the case of accelerated expansion powered by phantom energy ($w < -1$). We have found the radial time dependence of bound systems in phantom cosmologies and determined the time when these systems dissociate due to the repulsive effects of phantom energy, as a function of the equation of state parameter w. A universal behavior was found for the dissociation time for different geometries of bound systems. We have also plotted the bound system trajectory around the time of dissociation and demonstrated that the bound systems explode radially outward after dissociation. Our results were compared with previous corresponding results in the literature and were found to be in qualitative but not in quantitative agreement.

In the present study we have assumed a constant equation of state parameter w. The extension of our results to the case of a redshift dependent w ($w(z)$) is straightforward and consists a potentially interesting extension of this work. A potential improvement to the accuracy of our results may come by using a more accurate metric for the interpolation between the Schwarzschild and the Friedmann metric [18]. Such an improvement would be more important for strongly bound high velocity systems.

The Mathematica [22] file used for the production of the figures of the paper can be downloaded from [23] or sent by e-mail upon request.

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