PHYSICAL REVIEW D **70,** 123528 (2004)

Aspects of scalar field dynamics in Gauss-Bonnet brane worlds

M. Sami*

IUCAA, Post Bag 4, Ganeshkhind, Pune 411 007, India

N. Savchenko† and A. Toporensky‡

Sternberg Astronomical Institute, Universitetski Prospect-13, Moscow 119899, Russia

(Received 21 September 2004; published 29 December 2004)

The Einstein-Gauss-Bonnet equations projected from the bulk to brane lead to a complicated Friedmann equation which simplifies to $H^2 \sim \rho^q$ in the asymptotic regimes. The Randall-Sundrum (RS) scenario corresponds to $q = 2$ whereas $q = 2/3$ and $q = 1$ give rise to high-energy Gauss-Bonnet (GB) regime and the standard general relativity (GR), respectively. Amazingly, while evolving from RS regime to high-energy GB limit, one passes through a GR-like region which has important implications for brane world inflation. For tachyon GB inflation with potentials $V(\phi) \sim \phi^p$ investigated in this paper, the scalar to tensor ratio of perturbations R is maximum around the RS region and is generally suppressed in the high-energy regime for the positive values of p. The ratio is very low for $p > 0$ at all energy scales relative to GB inflation with ordinary scalar field. The models based upon tachyon inflation with polynomial type of potentials with generic positive values of p turn out to be in the 1σ observational contour bound at all energy scales varying from GR to high-energy GB limit. The spectral index n_S improves for the lower values of p and approaches its scale invariant limit for $p = -2$ in the high-energy GB regime. The ratio R also remains small for large negative values of p, however, difference arises for models close to scale invariance limit. In this case, the tensor to scale ratio is large in the GB regime whereas it is suppressed in the intermediate region between RS and GB. Within the framework of patch cosmologies governed by $H^2 \sim \rho^q$, the behavior of ordinary scalar field near cosmological singularity and the nature of scaling solutions are distinguished for the values of $q < 1$ and $q > 1$. The tachyon dynamics, on the other hand, exhibits stable scaling solutions $\forall q$ if the adiabatic index of barotropic fluid $\gamma < 1$.

DOI: 10.1103/PhysRevD.70.123528 PACS numbers: 98.80.Cq, 04.50.+h, 98.80.Jk

I. INTRODUCTION

Being inspired by D-brane ideology in string theory, the brane world scenario *à la* Randall-Sundrum (RS) [1,2] envisages that our four-dimensional spacetime (brane) is embedded in the five-dimensional bulk. To be in line with string theory, it is assumed that all the standard model degrees of freedom reside on the brane whereas gravity can propagate into bulk. In adherence to Newtonian gravity in the low energy limit, the bulk is assumed to be anti-de Sitter allowing gravity to be localized near the brane dynamically and thereby leading to Newton's law with small corrections at large distances. The spacetime dynamics projected from the bulk to brane leads to the modified Einstein equations on the brane. The resulting Hubble equation on the Friedmann-Robertson-Walker (FRW) brane, among other things, contains high energy corrections which have important implications for early Universe physics. In particular, the prospects of inflation are enhanced in brane world cosmology. In the case of standard FRW , the steep potentials cannot support inflation and bouncing solutions. The presence of the quadratic density term (high energy corrections) in the Friedmann equation on the brane changes the expansion dynamics at early epochs [3] (see Ref. [4] for details on the dynamics of brane worlds). Consequently, the field experiences greater damping and rolls down its potential slower than it would during the conventional inflation. Thus, inflation in the brane world scenario can successfully occur for very steep potentials [5,6]. The brane assisted inflation allows to build successful models of quintessential inflation [7]. However, the recent WMAP observations and large scale galaxy clustering studies severely constrain the steep brane world inflation. For instance, the inflation driven by steep exponential potential in RS scenario is excluded by observation for the number of e-folds as large as 70 [8]. It was recently shown that Gauss-Bonnet (GB) correction in the bulk could rescue these models [9]. There is a sound theoretical reason to include the higher curvature terms in Einstein-Hilbert action [10,11]. These terms arise perturbatively as next-to-leading-order correction in effective string theory action. The Gauss-Bonnet combination is special in five dimensions as it is a unique invariant which leads to field equations of second-order linear in the highest derivative thereby ensuring a unique solution.[1]

*On leave from: Department of Physics, Jamia Millia, New Delhi 110025, India; Electronic address: sami@iucaa.ernet.in

†Electronic address: savchenko@xray.sai.msu.ru

‡Electronic address: lesha@sai.msu.ru

[1]The GB term can as well be motivated purely on classical considerations. It arises naturally as higher-order iteration of the self-interaction of gravitational field which retains the quasi-linear second-order character of the field equation. The physical realization of this iteration naturally requires a five-dimensional spacetime [12].

The Einstein-Gauss-Bonnet equations projected onto the brane lead to a complicated Hubble equation in general [13–16] (see also Ref. [17]). Interestingly, it reduces to a very simple equation $H^2 \sim \rho^q$ with $q = 1, 2, 2/3$ in limiting cases corresponding to general relativity (GR), RS, and GB regimes, respectively. In the high energy GB regime, this allows to push the spectral index n_S very close to 1 for exponential potential in case of ordinary scalar field [14]. The tachyonic inflation has been recently studied in patch cosmologies in view of observational constraints [18]. The patches corresponding to GR, RS, and GB naturally arise in the dynamical history described by the exact effective Hubble equation on the brane in presence of the GB term in the bulk. It is really interesting to carry out the detailed investigations of tachyon field inflation in the full GB dynamics which gives rise to the mentioned patches at relevant energy scales. It is also important to investigate the behavior of scalar field near singularity and look for the scaling solutions in the patch cosmologies.

In this paper we study different aspects of scalar field dynamics in brane worlds with GB term in the bulk. In Sec. II, we review the basic concepts of GB brane world cosmology. In Sec. III, we investigate the tachyon inflation in the GB background with polynomial-type potentials which corresponds to an exponential potential in a special case. This section includes the detailed description of tachyon field inflation at all the energy scales from GR to high energy GB regime.

Sec. IV is devoted to the study of noninflationary dynamics of ordinary scalar field in the background governed by the Friedmann equation $H^2 \sim \rho^q$. This section contains the description of asymptotic behavior of scalar field near singularity and the existence of scaling solutions in the background cosmology under consideration.

GAUSS-BONNET BRANE WORLDS

The Einstein-Gauss-Bonnet action for five-dimensional bulk containing a four-dimensional brane is

$$S = \frac{1}{2\kappa_5^2} \int d^5x\sqrt{-g}\{\mathcal{R} - 2\Lambda_5 + \alpha[\mathcal{R}^2 - 4\mathcal{R}_{AB}\mathcal{R}^{AB} + \mathcal{R}_{ABCD}\mathcal{R}^{ABCD}]\} + \int d^4x\sqrt{-h}(\mathcal{L}_m - \lambda). \qquad (1)$$

$\mathcal{R}$ refers to the Ricci scalar in the bulk metric g_{AB} and h_{AB} is the induced metric on the brane; α has dimensions of $(length)^2$ and is the Gauss-Bonnet coupling, while λ is the brane tension and Λ_5 (<0) is the bulk cosmological constant. The constant κ_5 contains the five-dimensional fundamental energy scale ($\kappa_5^2 = M_5^{-3}$).

A Friedmann-Robertson-Walker brane in an AdS_5 bulk is a solution to the field and junction equations [13]. The modified Friedmann equation on the (spatially flat) brane may be written as [13,15]

$$H^2 = \frac{1}{4\alpha}\left[(1 - 4\alpha\mu^2)\cosh\left(\frac{2\chi}{3}\right) - 1\right], \qquad (2)$$

$$\kappa_5^2(\rho + \lambda) = \left[\frac{2(1 - 4\alpha\mu^2)^3}{\alpha}\right]^{1/2}\sinh\chi, \qquad (3)$$

where χ is a dimensionless measure of the energy density. In order to regain general relativity at low energies, the effective four-dimensional Newton constant is defined by [15]

$$\kappa_4^2 \equiv \frac{8\pi}{M_4^2} = \frac{\kappa_5^4\lambda}{6(1 - 4\alpha\Lambda_5/9)}. \qquad (4)$$

When $\alpha = 0$, we recover the RS expression. We can fine-tune the brane tension to achieve zero cosmological constant on the brane [15]:

$$\kappa_5^4\lambda^2 = -4\Lambda_5 + \frac{1}{\alpha}\left[1 - \left(1 + \frac{4}{3}\alpha\Lambda_5\right)^{3/2}\right]. \qquad (5)$$

Eqs. (4) and (5) may be rewritten as

$$\kappa_5^4\lambda = 2\kappa_4^2(1 + 4\alpha\mu^2)(3 - 4\alpha\mu^2), \qquad (6)$$

$$\kappa_5^2\lambda = 2\mu(3 - 4\alpha\mu^2). \qquad (7)$$

These equations imply

$$\frac{\kappa_5^2}{\kappa_4^2} = \frac{1 + 4\alpha\mu^2}{\mu}. \qquad (8)$$

The modified Friedmann Eq. (2), together with Eq. (3), shows that there is a characteristic GB energy scale [16]

$$M_{\rm GB} = \left[\frac{2(1 - 4\alpha\mu^2)^3}{\alpha\kappa_5^4}\right]^{1/8}, \qquad (9)$$

such that the GB high energy regime ($\chi \gg 1$) is characterized by $\rho + \lambda \gg M_{\rm GB}^4$. If we consider the GB term in the action as a correction to RS gravity, then $M_{\rm GB}$ is greater than the RS energy scale $\lambda^{1/4}$ and this imposes a restriction on the Gauss-Bonnet coupling $\beta \equiv \alpha\mu^2$ [16]

$$\lambda < M_{\rm GB}^4 \Rightarrow \beta < 0.038. \qquad (10)$$

Expanding Eq. (2) in χ, we find three regimes for the dynamical history of the brane universe [9,14–16]:

$$\rho \gg M_{\rm GB}^4 \Rightarrow H^2 \approx \left[\frac{\kappa_5^2}{16\alpha}\rho\right]^{2/3} \qquad \text{(GB)}, \qquad (11)$$

$$M_{\rm GB}^4 \gg \rho \gg \lambda \Rightarrow H^2 \approx \frac{\kappa_4^2}{6\lambda}\rho^2 \qquad \text{(RS)}, \qquad (12)$$

$$\rho \ll \lambda \Rightarrow H^2 \approx \frac{\kappa_4^2}{3}\rho \qquad \text{(GR)}. \qquad (13)$$

In what follows we shall address the issues of tachyon inflation in the background described by (2) and (3). We shall also investigate the specific features of ordinary scalar field dynamics in the extreme regimes given by (11)–(13).

III. TACHYON INFLATION ON THE GAUSS-BONNET BRANE

It was recently suggested that rolling tachyon condensate, in a class of string theories, might have interesting cosmological consequences. It was shown by Sen [19] that the decay of D branes produces a pressureless gas with finite energy density that resembles classical dust (see also Ref. [20] on the related theme). Attempts have been made to construct viable cosmological model using rolling tachyon field as a suitable candidate for inflaton, dark matter, or dark energy [21]. As for the inflation, the rolling tachyon models are faced with difficulties related to the requirement of enough inflation and the right level of density perturbations. It seems to be impossible to meet these requirements if we stick to string theory tachyons as the string inspired effective potentials do not contain any free parameter to ensure enough slow roll and the COBE normalized level of density perturbations. In what follows we shall consider the tachyonic potentials in purely phenomenological context to obtain viable models of inflation. Unfortunately, even after this relaxation, the tachyonic models face difficulties associated with reheating [22] and the formation of caustics/kinks [23], and we do not address these problems in this paper. We should, however, note that the model based upon the rolling massive scalar field on $\bar{D}_3$ brane is free from these difficulties [24], perhaps except the formation of caustics, which requires further investigation.[2]

The tachyonic field is described by the following action

$$S = \int d^4x\left\{\sqrt{-g}\left(\frac{R}{2\kappa^2}\right) - V(\phi)\sqrt{-\det(g_{ab} + \partial_a\phi\partial_b\phi)}\right\}. \tag{14}$$

In a spatially flat FRW background, the energy momentum tensor which follows from (14) for the Born-Infeld scalar ϕ acquires the diagonal form $T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p)$. The energy density ρ and the pressure p, in this case, are given by [we use the signature $(-, +, +, +)$],

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{15}$$

$$p = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{16}$$

The equation of motion of the rolling scalar field follows from Eq. (14)

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_\phi}{V(\phi)} = 0, \tag{17}$$

which is equivalent to the conservation equation

$$\frac{\dot{\rho}}{\rho} + 3H(1+w) = 0. \tag{18}$$

We now describe inflation on the brane assuming slow roll approximation, $\dot{\phi}^2 \ll V$ and $|\ddot{\phi}| \ll H|\dot{\phi}|$. The energy density becomes $\rho \sim V(\phi)$ and using Eqs. (3) and (5) we obtain [for the weak GB coupling defined by (10)]

$$V \simeq \sqrt{\left(\frac{\lambda}{3\alpha\kappa_4^2}\right)}\sinh\chi. \tag{19}$$

The slow roll parameters in this case become

$$\epsilon = \left(\frac{2\lambda}{\kappa_4^2}\frac{V_\phi^2}{V^4}\right)\epsilon_{\rm GB}, \qquad \eta = \left[\frac{2\lambda}{\kappa_4^2 V^2}(\ln V)_{\phi\phi}\right]\eta_{\rm GB}, \tag{20}$$

where the GB corrections to the RS values are given by

$$\begin{aligned} \epsilon_{\rm GB} &= \left[\frac{2}{27}\frac{\sinh(2\chi/3)\tanh\chi\sinh^2\chi}{[\cosh(2\chi/3)-1]^2}\right], \\ \eta_{\rm GB} &= \left[\frac{2}{9}\frac{\sinh^2\chi}{\cosh(2\chi/3)-1}\right]. \end{aligned} \tag{21}$$

The number of e-folds of inflationary expansion, $\mathcal{N} = \int H dt$, is obtained using (2) and (17), which is given by

$$\mathcal{N} = 3\int_{\chi_e}^{\chi_N} H^2\frac{V}{V_\chi}\left(\frac{d\phi}{d\chi}\right)^2 d\chi, \tag{22}$$

which using Eqs. (17) and (19) takes the form

$$\mathcal{N}(\chi) = -\frac{3}{4\alpha}\int_{\chi_N}^{\chi_{\rm end}} d\chi\left(\frac{d\phi}{d\chi}\right)^2[\cosh(2\chi/3)-1]\tanh\chi. \tag{23}$$

We should note that we have used the weak coupling nature of GB correction while writing Eqs. (19), (20), and (23).

A. Inflation with Polynomial-type Potential

We shall now assume that the potential for Born-Infeld scalar field is

$$V(\phi) = V_0\phi^p, \tag{24}$$

where V_0 and p are constants. We are mainly interested in the cases of $p = 2$ (massive inflaton), $p = 4$ (massless inflaton) and $p \to \infty$ (exponential potential). For the potential (24) two slow roll parameters can be written as

$$\epsilon = \frac{4\lambda p^2 V_0^{2/p}}{27\kappa_4^2 A^{2(p+1)/p}} f(\chi), \qquad \eta = -\frac{4\lambda p V_0^{2/p}}{9\kappa_4^2 A^{2(p+1)/p}} g(\chi), \tag{25}$$

where $A = \sqrt{3\lambda/\alpha\kappa_4^2}$ and $f(\chi)$, $g(\chi)$ are given by

[2] We thank A Starobinsky for his comment on the problem of caustics formation.

$$f(\chi) = \frac{\sinh(2\chi/3)\tanh\chi(\sinh\chi)^{-2/p}}{[\cosh(2\chi/3) - 1]^2},$$
$$g(\chi) = \frac{(\sinh\chi)^{-2/p}}{[\cosh(2\chi/3) - 1]}. \tag{26}$$

A comment on the behavior of slow roll parameters is in order. As pointed out in Ref. [9], both ϵ and η exhibit a peculiarity for $p = 6$ (in case of ordinary scalar field inflation) in the region $\chi \ll 1$: they are increasing functions of χ for $p < 6$ whereas the situation is reversed for $p > 6$. It turns out that the $p = 6$ case also gets distinguished for large values of χ, i.e., in the GB regime where the dynamics is described by a simple equation $H^2 \sim \rho^{2/3}$. Indeed, in region $\chi \gg 1$, the slow roll parameters behave as

$$\epsilon, \eta \propto \chi^{(p-6)/3p} \quad \text{(ordinary scalar field)}, \tag{27}$$

$$\epsilon, \eta \propto \chi^{-(3+p)/3p} \quad \text{(tachyon field)}. \tag{28}$$

It is clear from Eqs. (27) and (28) that in the GB regime, the slow roll parameters exhibit a specific behavior in case $p = 6$ for ordinary scalar field whereas the similar behavior is realized for tachyon field if $p = -3$ [25] (see Ref. [26] which deals with similar problem in case of RS and standard GR). After a brief remark on the scalar field dynamic in patch cosmology, we return to the full dynamics described by (2) and (3). We now compute the number of inflationary e-foldings for polynomial potential (24)

$$\mathcal{N} = -\frac{3V_0^{-2/p}}{8\alpha p^2 A^{-2/p}} \int_{\chi_N}^{\chi_{\rm end}} d\chi \frac{[\cosh(2\chi/3) - 1]}{[\sinh(\chi)]^{2(p-1)/p}} \times \sinh(2\chi). \tag{29}$$

For a general p, it is not possible to get a close analytical expression for $\mathcal{N}$. However, for particular values $p \pm 2$, ∞, the integral in (29) can be computed analytically. For one of the values of interest $p = 4$, we shall opt for the numerical computation of the integral. It will be instructive to present the expression for the number of e-foldings , in general, as follows

$$\mathcal{N} = \frac{3}{8\alpha p^2 V_0^{2/p} A^{-2/p}} [F(\chi)]_{\chi_e}^{\chi_N}. \tag{30}$$

In order to estimate the maximum number of e-foldings, we can assume that inflation ends in the RS regime ($\chi \ll 1$) which allows us to write (30) as

$$\mathcal{N} = \frac{3}{8\alpha p^2 V_0^{2/p} A^{-2/p}} \left[F(\chi_N) - \frac{2p}{9(1+p)} \chi_e^{2(p+1)/p} \right]. \tag{31}$$

We observe that the slow roll parameter ϵ scales as $\chi^{2(p+1)/p}$ for $\chi \ll 1$ which helps to estimate the value χ_e at the end of inflation

$$\chi_e^{2(p+1)/p} = \frac{2\lambda p^2 V_0^{2/p}}{\kappa_4^2 A^{2(p+1)/p}}. \tag{32}$$

Using Eqs. (30) and (32), we can express χ_e through $F(\chi_N)$ as

$$\chi_e^{2(p+1)/p} = \frac{9(p+1)}{2[2\mathcal{N}(p+1)+1]} F(\chi_N), \tag{33}$$

which for $p \to \infty$ reduces the expression for χ_e obtained in Ref. [27] for exponential potential. We now give the analytical expressions for the function F for $p = \pm 2, \infty$

$$F(\chi) = \frac{4}{5}(6\cosh(2\chi/3) - 1)\sinh^3(\chi/3) \qquad (p = 2),$$
$$F(\chi) = 3\cosh(2\chi/3) - \ln(1 + 2\cosh(2\chi/3)) + 2\ln(\sinh(\chi/3)) - 2\ln(\sinh(\chi)) + 3(\ln(3) - 1) \qquad (p = \infty),$$
$$F(\chi) = 4\frac{\arctan[2\sinh(\chi/3)/\sqrt{3}]}{\sqrt{3}} - \frac{4\sinh(\chi/3)}{1 + 2\cosh(2\chi/3)} \qquad (p = -2), \tag{34}$$

whereas for other values of p, the function $F(\chi)$ should be evaluated numerically.

The slow roll parameters can now be cast entirely as a known function of χ_N

$$\epsilon = \frac{(p+1)F(\chi_N)}{3[2\mathcal{N}(p+1)+p]} \times \left(\frac{\sinh(2\chi_N/3)\tanh\chi_N(\sinh\chi_N)^{-2/p}}{[\cosh(2\chi_N/3) - 1]^2}\right),$$
$$\eta = -\frac{(p+1)F(\chi_N)}{p[2\mathcal{N}(p+1)+p]}\left(\frac{(\sinh\chi_N)^{-2/p}}{[\cosh(2\chi_N/3) - 1]}\right) \tag{35}$$

which for $p \to \infty$ corresponds to the case of exponential potential; the slow roll parameter η vanishes in this limit and (35) reduces to the expression obtained in Ref. [27].

As mentioned above, the cases corresponding to $p \pm 2$ and $p = \infty$ (exponential potential) can be treated analytically. In the case of $p = 4$, we get complicated combinations of hypergeometric functions; it is not very illuminating to produce them in the text and we have studied this case numerically. We have ensured that the numerics in the case of $p = \pm 2$ and $p = \infty$ produces our analytical results. In Figs. 1 and 2, we have plotted the slow roll parameters ϵ and η for three cases. We observe that for large values of p, the slow roll parameter ϵ has minimum in the intermediate region which increases and approaches a constant value as we move towards the GB regime (large values of χ_N). The minimum becomes less and less pronounced for smaller values of p. The slow roll parameter $\eta \equiv 0$ in the case of the exponential potential, whereas in other two cases, it represents a monotonically increasing

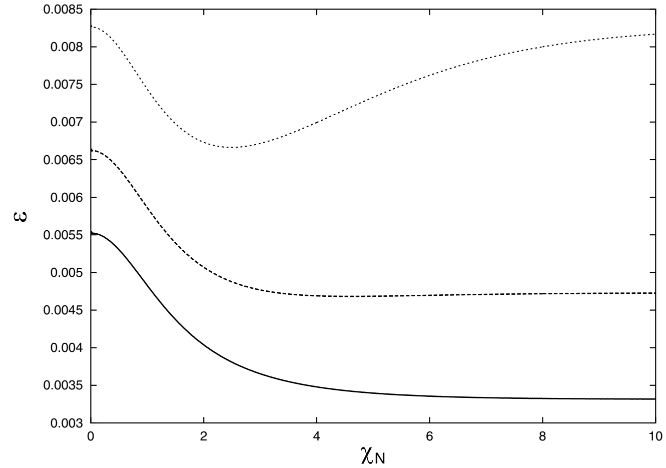

FIG. 1. The slow roll parameter ϵ as a function of χ_N for the number of e-folds $\mathcal{N} = 60$ in the case of the potential (24). The solid line corresponds to $p = 2$, the dashed and the dotted lines correspond to $p = 4$ and the exponential potential, respectively.

function of χ_N approaching a constant value in the GB regime (see Fig. 2). It is interesting to compare these features with GB inflation in the case of ordinary scalar field. In the latter case, the slow roll parameters are monotonously increasing function of χ_N for large values of p in contrast to the GB tachyonic inflation where they assume a minimum value in the intermediate region and then gradually approach a constant value. Second, numerical values of these parameters, at all energy scales and for $\forall p > 0$, remain much smaller than their counterparts associated with ordinary scalar field GB inflation.

B. Perturbation from Gauss-Bonnet Inflation

Hwang and Noh [28] provided the formalism to evaluate the perturbation spectra for the general action

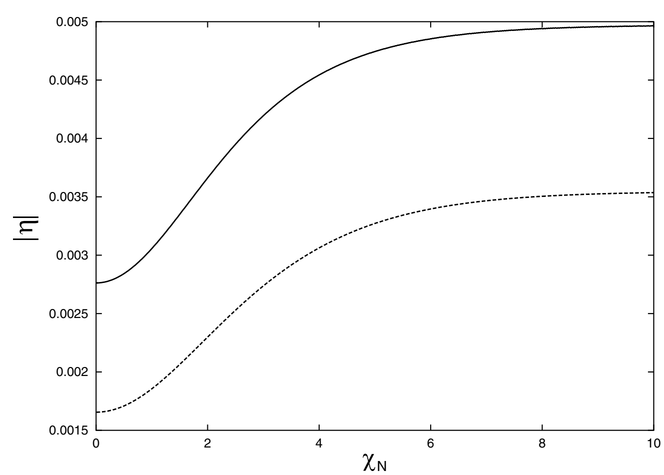

FIG. 2. Plot of η as a function of χ_N for the number of e-folds $\mathcal{N} = 60$. The dashed line corresponds to $p = 4$ whereas the solid line corresponds to $p = 2$; $\eta \equiv 0$ in the case of the exponential potential.

$$S = \int d^4x\sqrt{-g}\frac{1}{2}f(R, \phi, X), \tag{36}$$

which includes our action (14). Here the function f depends upon the Ricci scalar R, a scalar field ϕ, and its derivative $X = (\nabla\phi)^2/2$. The Born-Infeld scalar field corresponds to the case with

$$f = \frac{R}{\kappa^2} + 2\Lambda - 2V(\phi)\sqrt{1 + 2X}. \tag{37}$$

The amplitude of density perturbations in this case is given by[28,29]

$$A_S^2 = \left(\frac{H^2}{2\pi^2\dot{\phi}}\right)^2\frac{1}{Z_S}, \tag{38}$$

where $Z_S \equiv -(f_{,X}/2 + f_{,XX}X) = V(1 - \dot{\phi}^2)^{-3/2}$. Under the slow roll approximation, the power spectrum of curvature perturbations is estimated to be [28]

$$A_S^2 = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2\frac{1}{V} \tag{39}$$

The extra piece of V occurring in (39) leads to the modified expression for spectral index n_S in case of tachyon field

$$n_S - 1 = \frac{d\ln A_S^2}{d\ln k}\bigg|_{k=aH} = -[4 + \theta(\chi)]\epsilon + 2\eta, \tag{40}$$

where $\theta(\chi)$ is given by

$$\theta(\chi) = 2\left(1 - \frac{3\mathcal{G}(\chi)}{2}\right), \qquad \mathcal{G}(\chi) = \frac{[\cosh(2\chi/3) - 1]}{\sinh(2\chi/3)}\coth(\chi). \tag{41}$$

We have used Eqs. (2) and (19) in deriving (41). The function $\theta(\chi)$ encodes the GB effects for tachyon inflation. It interpolates between 1 and -1 [$\mathcal{G}(\chi)$ varies from 0 to 1) as χ varies from RS to GB limit (Fig. 3) which is in confirmation with the findings of Refs. [18,25] in extreme limits.

The tensor perturbations in brane world with Gauss-Bonnet term in the bulk were recently studied in Ref. [16]. The amplitude of tensor perturbations was shown to be given by

$$A_T^2 = \left[\kappa^4\frac{H^2}{4\pi^2}\right]\mathcal{F}_\beta^2(H/\mu), \tag{42}$$

where the function $\mathcal{F}_\beta$ contains the information about the GB term

$$\mathcal{F}_\beta^{-2} = \sqrt{1 + x^2} - \left(\frac{1 - \beta}{1 + \beta}\right)\sinh^{-1}x^{-1} \qquad (x \equiv H/\mu). \tag{43}$$

The dimensionless variables x and χ associated with energy scales are related to each other via the Eqs. (2) and (3).

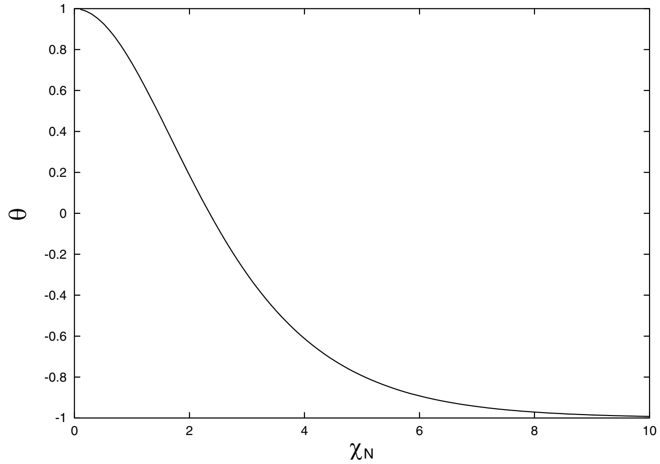

FIG. 3. Plot of function θ versus the energy scale χ_N. The function interpolates between 1 and -1 as χ_N runs from RS region to GB regime.

The tensor spectral index in this case is

$$n_T = \frac{d\ln A_T^2}{d\ln k}\bigg|_{k=aH} = -\epsilon G_\beta(x), \tag{44}$$

where $G_\beta(x)$ is given by

$$G_\beta(x) = 1 - \frac{x\mathcal{F}_\beta^2[1-(1-\beta)\sqrt{1+x^2}\sinh^{-1}x^{-1}]}{(1+\beta^2)\sqrt{1+x^2}}. \tag{45}$$

The tensor to scalar ratio is defined as

$$R = 16\frac{A_T^2}{A_S^2}. \tag{46}$$

Following Ref. [16], we have the expression for the tensor to scalar ratio R

$$R = -8Q(x)n_T, \qquad Q(x) = \left(\frac{1+\beta+2\beta x^2}{1+\beta+\beta x^2}\right), \tag{47}$$

where Q carries the information of GB correction. It determines the size of breaking of degeneracy of the consistency relation in Gauss-Bonnet brane world inflation. We finally express the ratio of perturbations through the spectral index using Eqs. (40), (44), and (47) as

$$R = \mathcal{D}(\chi_N)\left[\frac{8}{4+\theta}(1-n_S) + \frac{16}{(4+\theta)}\eta\right], \tag{48}$$

where $\mathcal{D}(\chi_N) = Q(\chi_N)G_\beta(\chi_N)$. The evolution of functions $\mathcal{D}$, Q and G is shown in Fig. 4. Knowing the slow roll parameters and the functions $\mathcal{D}(\chi_N)$ and $\theta(\chi)$, we can evaluate the spectral index n_S and the ratio R. In Figs. 5 and 8, we have displayed their dependence on the dimensionless energy scale χ_N. The spectral index rises to maximum in the intermediate region and then gradually decreases approaching a constant value in GB regime ($\chi_N \gg 1$). It

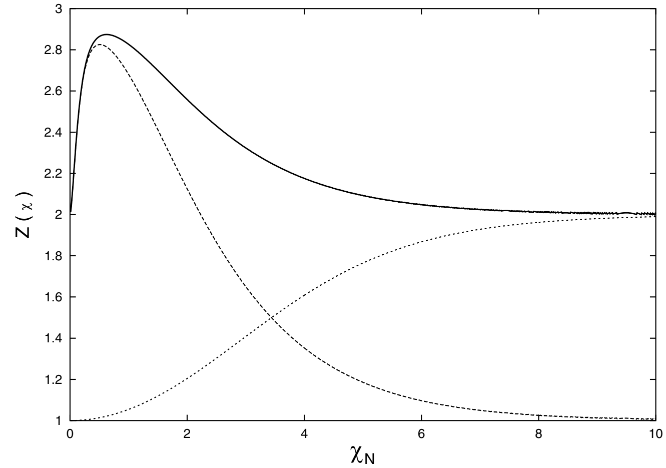

FIG. 4. Evolution of function $Z = Q$ (dotted line), G (dashed line), $\mathcal{D}$ (solid line) with the energy scale χ_N. The degeneracy factor Q evolves from 1 to 2 as χ_N varies from RS ($\chi_N \ll 1$) to GB regime, $\chi_N \gg 1$ ($Q = 1$ in the standard GR case corresponding to $\beta = 0$). G interpolates between 2 and 1; it expresses the variation of the ratio n_T/ϵ as the energy scale changes from lower to higher values. The function $\mathcal{D}$ peaks around the RS regime and tends to a constant value for large χ_N.

improves in general for lower values of p ($p > 0$). In case of the exponential potential, the maximum value of the spectral index is nearly equal to 0.97 for $\mathcal{N} = 60$ which is consistent with the result obtained earlier in [27].

C. Asymptotic Scale Invariance in GB Tachyon Inflation

As seen in Fig. 5 , the spectral index n_S improves for lower values of the exponent p. It would really be interesting to compare this situation with the standard inflationary scenario in presence of the GB correction in the bulk. In

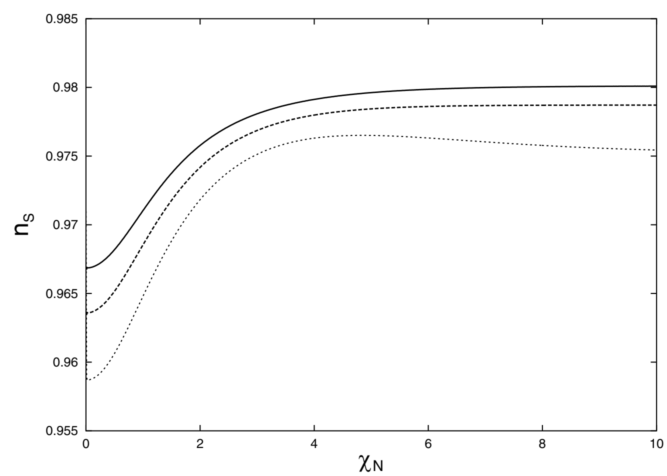

FIG. 5. Plot of the spectral index n_S versus the dimensionless energy scale χ_N for the number of e-folds $\mathcal{N} = 60$. Solid line corresponds to $p = 2$, dashed to $p = 4$, and dotted line to $p = \infty$ (exponential potential).

this case, the spectral index shows a very different behavior relative to the tachyonic GB inflation for exponential potential. It monotonously increases and approaches 1 for large χ_N[14]. Actually, the exponential potential is special to ordinary GB inflation which is related to the fact that scale invariance is exact in this case if the background dynamics is governed by the Hubble equation $H^2 \sim \rho^{2/3}$[14]. And this is certainly not true for tachyon field as it is governed by different dynamics. Interestingly, exact scaling for tachyon GB inflation is realized by a field potential very different from the exponential function. Indeed, let us consider the slow roll parameters in the background described by $H^2 \sim \rho^q$

$$\epsilon = \frac{q}{6H^2}\left(\frac{V_{,\phi}}{V}\right)^2, \qquad \eta = \frac{1}{3H^2}(\ln V)_{,\phi\phi}, \tag{49}$$

which for the power law type of potential $V \sim \phi^p$ leads to the following expression for the spectral index n_S in the asymptotic limit $\chi_N \gg 1$

$$n_S - 1 = -\frac{1}{3H^2}\left(\frac{(4+\theta)pq}{2} + 2\right)\frac{p}{\phi^2}. \tag{50}$$

In deriving Eq. (50), we have used Eq. (40). It should be noted that the general expressions of slow roll parameters (20) reduce to (49) in the limits of small χ with $q = 2$ and large χ with $q = 2/3$ and that Eq. (50) is valid in the asymptotic regimes $\chi_N \ll 1$ (RS regime) and $\chi_N \gg 1$ (GB regime). For scale invariance of spectrum, the right-hand side (RHS) of (50) should vanish leading to the simple relation

$$p = -\frac{4}{q(4+\theta)}, \tag{51}$$

which gives rise to $p = -2$ for GB patch ($q = 2/3$) and $p = -2/5$ in case of RS patch ($q = 2$), in agreement with the result obtained in[18]. Here we have taken into account that $\theta(\chi) \to \pm 1$ in the limits of $\chi_N \ll 1$ and $\chi_N \gg 1$, respectively. Our treatment of the full dynamics confirms this feature in the high energy GB regime (see Fig. 6). We have also considered models corresponding to larger inverse powers than the inverse square potential. We find that the numerical values of n_S for $p \leq -3$ are lower as compared to the case of an exponential potential (Fig. 7) and approach the latter in the limit of large negative p. The crossing takes place for $p > -3$ allowing the scale invariant limit to be reached for $p = -2$.

D. Tensor to Scalar Ratio of Perturbations R

The behavior of the tensor to scalar ratio of perturbations is dictated by the features possessed by the functions $\mathcal{D}(\chi_N)$ and n_S. The ratio R is plotted in Fig. 8. The function R peaks around the RS regime which subsequently decreases to minimum and increases thereafter approaching a constant value in the GB regime. This is a very important feature of GB inflation common to both tachyonic as well as nontachyonic models. The RS value of the ratio R is generally larger relative to the case of GR[18]. The minimum of the function R is attributed to the fact that while passing from RS regime characterized by $H^2 \sim \rho^2$ to the high energy GB limit with $H^2 \sim \rho^{2/3}$, there is an intermediate region which mimics the GR like behavior. In the case of lower values of p, the minimum of R is not distinguished. The numerical values of R as a function of χ_N are generally smaller for less steep potentials. We find that the tensor to scalar ratio of perturbations is very low for all the values of the exponent $p > 0$ at all the energy scales thereby providing support to the recent analysis of Ref. [18] in the limiting cases. The tachyonic model of

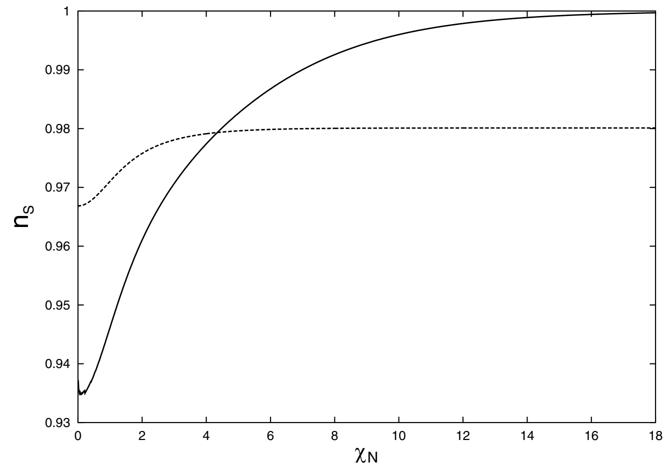

FIG. 6. Spectral index n_S versus the dimensionless energy scale χ_N for the number of e-folds $\mathcal{N} = 60$ in the case of potential $V \sim \phi^p$. Solid line corresponds to $p = -2$, dashed to $p = 2$. The spectral index for inverse square potential is seen approaching the scale invariance limit ($n_S = 1$) in the GB regime.

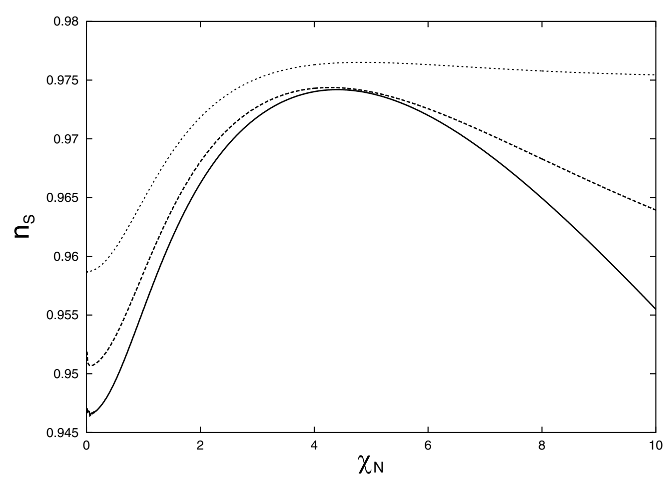

FIG. 7. Plot of spectral index n_S versus energy scale χ_N for the number of e-folds $\mathcal{N} = 60$ in the case of potential $V \sim \phi^p$ with $p = -3$ (solid line) and $p = -4$ (dashed line). The dotted line corresponds to the exponential potential.

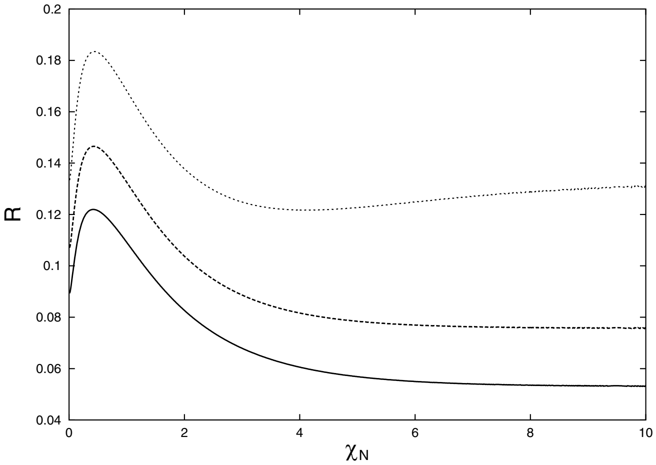

FIG. 8. The tensor to scalar ratio of perturbations R is shown as a function of the dimensionless scale χ_N for $\mathcal{N} = 60$. The solid line corresponds to the case of $p = 2$. The dashed and dotted lines correspond to $p = 4$ and the exponential potential, respectively.

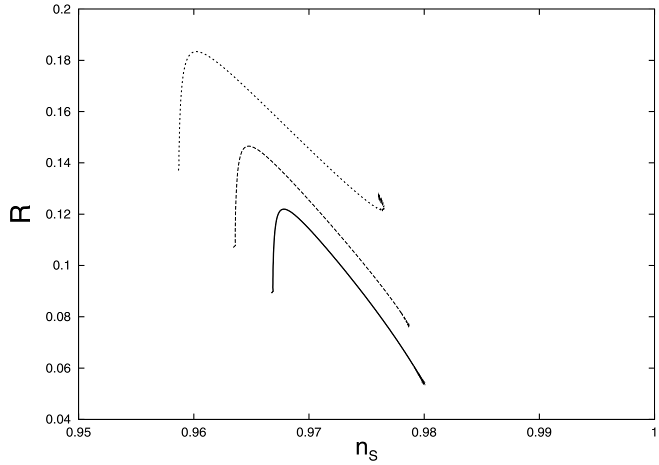

FIG. 9. The tensor to scalar ratio of perturbations R is shown on the (R, n_S) plane for $\mathcal{N} = 60$. The solid line corresponds to the case of $p = 2$. The dashed and dotted lines correspond to $p = 4$ and the exponential potential, respectively.

inflation with polynomial-type potentials is within the 1σ contour bonds at all energy scales for $p > 0$ (see Fig. 9 and the observational contours given in Ref. [9]; also see Ref. [30] on the related theme). In the case of the runaway potentials for small negative values of p, the tensor to scalar ratio becomes large for large values of χ_N and it is suppressed in the intermediate region (Fig. 10). Thus, there is a possibility for these models to be consistent with observation in the intermediate region between RS and GB which is analogous to ordinary scalar field GB inflation with steep potentials[9]. Finally, we should remark that the dimensionless density scale cannot increase indefinitely, it is restricted by the quantum gravity limit which corresponds to $\rho < \kappa_5^{-8/3}$

$$\frac{\alpha^3 \lambda}{\kappa_4^2} > 48\sinh^6(\chi_N). \tag{52}$$

Using Eqs. (39), (32), and (33) along with COBE normalized value of density perturbations, we can express $\alpha^3 \lambda/\kappa_4^2$, entirely, as a function of energy scale χ_N and the number of e-folds $\mathcal{N}$. The constraint (52), then leads to an upper bound on the variable χ_N. In the case of ordinary scalar field GB inflation, it was very important to find these bounds as the tensor to scalar ratio R, in general, is a monotonously increasing function of χ_N which becomes large in high energy GB regime. In our case, as mentioned above, the ratio remains very low for all values of χ_N in case of any generic positive value of p. However, it is true that it makes sense to consider only those values of the energy scale which are consistent with (52). The upper bounds on χ_N in our model lies between 6–7 in these cases.

IV. ISSUES OF SCALAR FIELD DYNAMICS IN $H^2 \sim \rho^q$ COSMOLOGY

So far, three particular models of the form $H^2 \sim \rho^q$ have been considered in the literature. These include: standard cosmology ($q = 1$), the Randall-Sundrum brane ($q = 2$), and the Gauss-Bonnet brane ($q = 2/3$). The scalar field dynamics in these three cases exhibits several important differences. In order to understand the connections between the power index q in the generalized Friedmann equation and particular properties of corresponding scalar field dynamics it is necessary to examine the problem in the general cosmological background. The general description of the dynamics seems to be possible in a number of

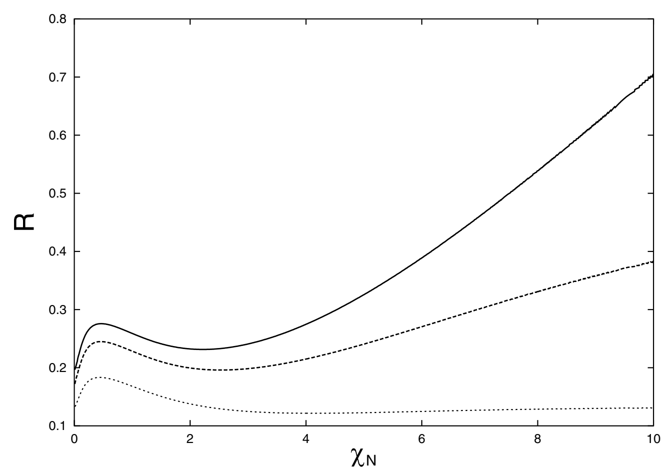

FIG. 10. The energy scale dependence of R for the model described in Fig. 7. The solid line corresponds to the case of $p = -3$, dashed and dotted lines correspond to $p = -4$ and the exponential potential, respectively. R takes minimum value in the intermediate region between RS and GB regimes.

interesting physical situations. In what follows we shall describe asymptotic behavior of ordinary scalar field near a cosmological singularity and investigate the possibilities for the existence of scaling solutions for the standard as well as the tachyon field.

A. Asymptotic behavior near singularity

An interesting example of different dynamics in the standard and brane cosmologies is related to the behavior of the scalar field near a cosmological singularity. It is known that in the standard case the scalar field diverges near a singularity [31] while it remains finite in the brane world [32]. We shall consider the behavior of scalar field near singularity and study the asymptotic solutions in a cosmological background governed by $H^2 \sim \rho^q$. Considering this problem in the general case we start with a massless field. The equation of motion

$$\ddot{\phi} + 3H\dot{\phi} = 0, \tag{53}$$

in the background described by $H^2 \sim \rho^q$ gives $H \sim \dot{\phi}^q$ which leads to

$$\ddot{\phi} + \dot{\phi}^{1+q} = 0. \tag{54}$$

Equation (54) easily integrates and gives

$$\phi = A(t - t_0)^{1-1/q}, \tag{55}$$

where A and t_0 are constants of integration. We observe that the standard cosmology, $q = 1$ [in this case we cannot use (55) for which the asymptotic has the known form $\phi \sim \ln(t/t_0)$], is an exceptional case which divides all possible asymptotics into two classes. For $q < 1$ both ϕ and $\dot{\phi}$ diverge near a cosmological singularity. The GB brane belongs to this class with the asymptotic $\phi \to 1/\sqrt{t - t_0}$. On the contrary, $q > 1$ leads to nonsingular ϕ and singular $\dot{\phi}$ [$\dot{\phi}$ cannot be nonsingular because the power index in (55) is always less than unity]. The well-known example of this dynamics is provided by the Randall-Sundrum brane with $\phi \to \sqrt{t - t_0}$.

It is known that in the standard case the scalar field potential $V(\phi)$ is not important during the cosmological collapse unless it is steeper than exponent (see [33] for detail). In the general case the role of potential depends on the sign of $q - 1$. For $q > 1$ the asymptotic $\phi \to$ const, $\dot{\phi} \to \infty$ prevents the potential from playing an important role in the cosmological collapse. If $q < 1$, however, steep enough potential would destroy the regime (55). For the scalar field growing as

$$\phi \sim (t - t_0)^{-a}, \tag{56}$$

the kinetic energy behaves as

$$\dot{\phi}^2 \sim (t - t_0)^{-2-2a}. \tag{57}$$

Assuming the power law form of potential $V(\phi) \sim \phi^b$ and using Eqs. (56) and (57), it is easy to see that the potential becomes important provided that

$$b > \frac{2(1 + a)}{a}. \tag{58}$$

Using Eq. (55) we then find the critical value of the power index in the potential

$$b = \frac{2}{1 - q}. \tag{59}$$

For steeper potentials it is impossible to neglect $V(\phi)$ which makes the asymptotic (55) invalid and the scalar field in a contracting Universe enters into a regime of oscillations, similar to that described in [33].

We should emphasize that (59) expresses an important condition for inflation [25] and can be understood from a slightly different perspective. Indeed, the constancy of the slow roll parameters for $V(\phi) \sim \phi^b$

$$\epsilon, \eta \sim \phi^{b(1-q)-2} \tag{60}$$

immediately leads to (59) thereby ensuring the power law inflation. The similar situation arises for $V(\phi) \sim \phi^{-2/q}$ in case of a tachyon field.

B. Scaling solutions

In this subsection we shall investigate the cosmological dynamics of a scalar field in presence of ordinary matter. We are mainly interested in scaling solutions, which can exist in this model. By scaling solution we mean the situation in which the scalar field energy density scales exactly as the power of the scale factor, $\rho_\phi \sim a^{-n}$, while the energy density of the perfect fluid [with equation of state $p_m = (\gamma - 1)\rho_m$], being the dominant component, scales as a (possible) different power, $\rho_m \sim a^{-m}$, $m = 3\gamma$.

We will follow the method of Refs. [34] and[35] (see also Ref. [36] on the related theme), where scaling solutions have been found in the standard and brane cosmology.

1. Standard scalar field

Supposing that the scalar field energy density behaves as $\rho_\phi \sim a^{-n}$, then using the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{61}$$

we see that the ratio of scalar field kinetic energy density and total scalar field energy density remains constant

$$\frac{\dot{\phi}^2/2}{\rho_\phi} = \frac{n}{6}. \tag{62}$$

In the case of matter dominance we have

$$a(t) \sim t^{2/qm}. \tag{63}$$

Then using Eqs. (63) and (61) we get

$$\ddot{\phi} = -\frac{6}{qm}\frac{1}{t}\dot{\phi} - \frac{dV}{d\phi} \tag{64}$$

and Eq. (62) gives

$$\dot{\phi} \sim t^{-n/qm}. \tag{65}$$

Equation (65) readily integrates to yield

$$\phi = At^{1-n/qm}. \tag{66}$$

Substituting (66) into (64) and solving equation for $V(\phi)$ we find the potentials, which allows the scaling behavior

$$V(\phi) = \frac{2[6(\beta - 2) - qm\beta]}{(\beta - 2)^2 qm\beta} A^{2-\beta}\phi^{\beta}, \tag{67}$$

where

$$\beta = \frac{2n}{n - qm}. \tag{68}$$

For a given potential $V(\phi) \sim \phi^{\beta}$ the scalar field energy density scales as a^n, where

$$n = \frac{q\beta}{\beta - 2} m. \tag{69}$$

For stability analysis of this solution we use new variables

$$\tau = \ln t, \qquad u(\tau) = \frac{\phi(\tau)}{\phi_0(\tau)}, \qquad p(\tau) = u'(\tau), \tag{70}$$

where $\phi_0(\tau)$ is the exact solution given by (66) and the prime denotes the derivative with respect to τ.

Then we have the system of two first-order differential equations

$$\begin{aligned} u' &= p, \\ p' &= \frac{2}{\beta - 2}\left(\frac{6}{qm} - \frac{\beta}{\beta - 2}\right)(u - u^{\beta - 1}) \\ &\quad - \left(\frac{2 + \beta}{2 - \beta} + \frac{6}{qm}\right)p, \end{aligned} \tag{71}$$

and scaling solution corresponds to a critical point $(u, p) = (1, 0)$. Linearizing (71) about this point we find the eigenvalues of these coupled equations

$$\begin{aligned} \lambda_{1,2} &= \frac{\beta + 2}{2(\beta - 2)} - \frac{3}{qm} \\ &\quad \pm \sqrt{\left(\frac{\beta + 2}{2(\beta - 2)} \frac{3}{qm}\right)^2 + \frac{2\beta}{\beta - 2} - \frac{12}{qm}}. \end{aligned} \tag{72}$$

The condition for stability is given by the negativity of the real parts of both eigenvalues.

Now we consider some properties of these scaling solutions in more detail. First of all, positivity of the potential (67) requires

$$\frac{1}{\beta} < \frac{6 - qm}{12}, \tag{73}$$

and the stability condition requires additionally

$$\frac{\beta + 2}{\beta - 2} > \frac{6}{qm}. \tag{74}$$

We now discuss some consequences of (73) and (74). First of all, we should point out that since $m = 3\gamma$ with γ being the equation of state of ordinary matter, the value for m is bounded in the interval $m \in [0, 6]$. Because of an additional degree of freedom, we have a more complicated situation than the one discussed in [34,35]. To describe it in detail, it would be convenient to discuss two cases—for positive and negative β—separately.

A. The case of $\beta < 0$

The stability condition (74) gives no further restrictions to the condition for existence of scaling solutions

$$\beta < 2\frac{6 + qm}{6 - qm}. \tag{75}$$

They exist for

$$\beta < \frac{12}{6 - qm}. \tag{76}$$

From this equation one can see that for $q \leq 1$, regardless the value of m, we always have scaling solution. The standard cosmology and Gauss-Bonnet brane belong to this class. For $q > 1$ the denominator in (76) can be negative, restricting the range of β suitably for the scaling solution. A known example is the Randall-Sundrum brane ($q = 2$), where the scaling solutions exist for $\beta < 6/(3 - m)$ [35].

B. The case of $\beta > 0$

In this case one can rewrite (73) as follows

$$\beta > \frac{12}{6 - qm}. \tag{77}$$

As the region $0 < \beta < 2$ is already excluded by (77) (since $qm > 0$) we can rewrite (74) as

$$\beta > 2\frac{6 + qm}{6 - qm}. \tag{78}$$

The Eq. (78) is more restrictive.

Thus we find that for $q \leq 1$ scaling solutions exists for $\forall m$ if β is large enough. On the other hand, if $q > 1$, then there exists no scaling solutions for the matter with $m > 6/q$, or, equivalently, $\gamma > 2/q$. On the Randall-Sundrum brane, scaling solutions with $\beta > 0$ are absent if $m > 3$ [35].

We summarize our results in Table I.

TABLE I. Existence of stable scaling solutions for different indexes q and β.

	$q < 1$	$q > 1$
$\beta < 0$	Exists for all β	Exists only for $\beta < \frac{12}{6-qm}$
$\beta > 0$	Exists $\forall$ m, but $\beta > 2\frac{6+qm}{6-qm}$	Does not exist for $m > \frac{6}{q}$

For a zone with negative β where there are no scaling solutions the stable kinetic-term-dominated solution exists. Its explicit form for our model $H^2 \sim \rho^q$ is

$$\phi \sim t^{1-6/qm}. \tag{79}$$

The Eq. (69) gives also the following: Consider first the case of $\beta < 0$. For $q \leq 1$ the numerator is always less than the denominator. It means that the scalar field energy density always drops less rapidly than the matter density. For $q > 1$ it happens only if $\beta > 2/(\alpha - 1)$. On the other hand, if $\beta > 0$ and $q \geq 1$, field energy density scales faster than matter. For $q < 1$ it happens if $\beta > 2/(1 - q)$, i.e., for power law potentials which cannot support inflation.

2. Tachyon field

We would now investigate the existence of tachyon field scaling solution for matter dominance in the general cosmological background described by $H^2 \sim \rho^q$. Assuming $\rho_\phi \sim a^{-n}$ for tachyon field energy density, one can obtain from using Eq. (18)

$$\dot{\phi}^2 = \frac{n}{3}, \tag{80}$$

which integrates to yield

$$\phi = \sqrt{\frac{n}{3}}t. \tag{81}$$

Since the matter energy density scales as the power of the scale factor $\rho_m \sim a^{-m}$, in case of matter dominance we have

$$a(t) \sim t^{2/qm}. \tag{82}$$

Substituting (81) and (82) into (17) and solving this equation for $V(\phi)$ we find the potentials, which allow the scaling behavior for tachyon field

$$V(\phi) \sim \phi^{-\beta}, \tag{83}$$

where

$$\beta = \frac{2n}{qm}. \tag{84}$$

As for the stability of the solution, we use the same new variables as in case of usual scalar field

$$\tau = \ln t, \qquad u(\tau) = \frac{\phi(\tau)}{\phi_0(\tau)}, \qquad p(\tau) = u'(\tau), \tag{85}$$

where $\phi_0(\tau)$ is the exact solution given by (81). Then we can get the system of two first-order differential equations as analogue of system (71). And linearizing these equations about critical point $(u, p) = (1, 0)$ corresponding to a scaling solution, we find the eigenvalues of this system

$$\lambda_{1,2} = \frac{1}{2}\Bigg[\beta - \frac{6}{qm} - 1 \pm \sqrt{1 + 6\Big(\beta - \frac{6}{qm}\Big) + \Big(\beta - \frac{6}{qm}\Big)^2}\Bigg]. \tag{86}$$

The condition for stability is as usual given by the negativity of the real parts of both eigenvalues.

The condition of stability for tachyon scaling solution gives

$$0 < \beta < \frac{6}{qm}, \tag{87}$$

which readily leads to $n < 3$ or equivalently $\gamma < 1$ if the tachyon field mimics the background (solutions with this property are often called *trackers*). It should be noted that the condition for existence of stable scaling solution for tachyon is independent of q. Our result is in agreement with Ref. [37] which investigates the scaling solutions in case of standard GR ($q = 1$). In general, our findings for tachyon field are consistent with the results of Ref. [38] which provides a unified framework to investigate the scaling solutions for a variety of dynamical systems.

V. CONCLUSIONS

In this paper we have examined different aspects of scalar field dynamics in Gauss-Bonnet brane worlds. We have presented detailed investigation of tachyon inflation in GB background. Our analysis is quite general and deals with tachyon field dynamics at all energy scales from GR to GB regimes for polynomial-type potentials. The information of GB correction is encoded in the functions $\mathcal{D}$, θ and the slow roll parameters. We find that the spectral index reaches a maximum value in the intermediate region between RS and GB regimes which improves for lower values of the exponent in case of positive p. Our analytical results and numerical treatment of the full GB dynamics show that tachyonic inflation with inverse square potential leads to scale invariant spectrum in the high energy GB limit which is in agreement with the asymptotic analysis of Ref. [18]. The combined effect of GB term on the tensor to scalar ratio, encoded in $\mathcal{D}(\chi)$, $\theta(\chi)$ and the slow roll parameters, is such that R peaks around the RS regime and exhibits a minimum in the intermediate region. While evolving the energy scales from RS to high energy GB patch, the background dynamics gradually changes from $H^2 \sim \rho^2$ to $H^2 \sim \rho^{2/3}$ mimicking the GR-like features ($H^2 \sim \rho$) in the intermediate region. This is an important property of GB correction which manifests in both tachyonic and standard scalar field dynamics. We have shown that the tensor to scalar ratio is generally very low in the case of GB tachyonic inflation at all energy scales for polynomial-type potentials with generic positive values of p. We find similar features in the case inflation is driven by inverse power law potentials with large negative powers. When p is close to the scale invariant limit , the tensor to scalar ratio becomes large in the high energy GB regime whereas it is suppressed in the intermediate region making these models consistent with observation.

In Sec. IV, we have examined the generalized dynamics with Friedmann equation $H^2 \sim \rho^q$ for an arbitrary q. This allowed us to explain some known differences between the standard cosmology ($q = 1$) and a Randall-Sundrum brane ($q = 2$) in the framework of a unified picture as well as to obtain new results in the case of Gauss-Bonnet brane ($q = 2/3$). In the generalized background cosmology, we have investigated the asymptotic behavior of scalar field near cosmological singularity and studied scaling solutions in the regime when a perfect fluid energy density dominates. In the case of ordinary scalar field, we have demonstrated that the underlying field dynamics exhibits distinct features depending whether $q < 1$ or $q > 1$ which, in particular, distinguishes the Gauss-Bonnet and Randall-Sundrum brane worlds. For the tachyon system, we have shown that the existence of stable scaling solutions $\forall\ q$ is guaranteed if the adiabatic index of barotropic fluid $\gamma < 1$.

ACKNOWLEDGMENTS

We thank Gianluca Calcagni and Shinji Tsujikawa for their critical comments and for pointing out the reason for the mismatch we had with their recent work in the asymptotic regime. M. S. acknowledges the useful discussion with N. Dadhich, T. Padmanabhan, and V. Sahni. The work of A. T. was supported by RFBR Grant No. 02-02-16817 and Scientific School Grant No. 2338.2003.2 of the Russian Ministry of Science and Technology.

APPENDIX: VARIOUS ENERGY SCALES, THE RS BRANE TENSION, THE GB ENERGY SCALE AND ALL THAT

The GB brane world contains different energy scales discussed in [16] which we summarize here for the sake of completeness. The GB term may be thought of as the lowest-order stringy correction to the five-dimensional Einstein-Hilbert action, with coupling constant $\alpha > 0$. In this case, $\alpha|\mathcal{R}^2| \ll |\mathcal{R}|$, so that

$$\alpha \ll \ell^2, \tag{A1}$$

where ℓ is the bulk curvature scale, $|\mathcal{R}| \sim \ell^{-2}$. The RS-type models are recovered for $\alpha = 0$. The five-dimensional field equations following from the bulk action are

$$\mathcal{G}_{ab} = -\Lambda_5{}^{(5)}g_{ab} + \frac{\alpha}{2}\mathcal{H}_{ab}, \tag{A2}$$

$$\begin{aligned}\mathcal{H}_{ab} = {}&[\mathcal{R}^2 - 4\mathcal{R}_{cd}\mathcal{R}^{cd} + \mathcal{R}_{cdef}\mathcal{R}^{cdef}]^{(5)}g_{ab}\\ &- 4[\mathcal{R}\mathcal{R}_{ab} - 2\mathcal{R}_{ac}\mathcal{R}^{c}{}_{b} - 2\mathcal{R}_{acbd}\mathcal{R}^{cd}\\ &+ \mathcal{R}_{acde}\mathcal{R}_{b}{}^{cde}].\end{aligned} \tag{A3}$$

An AdS_5 bulk satisfies the five-dimensional field equations, with

$$\bar{\mathcal{R}}_{abcd} = -\frac{1}{\ell^2}[{}^{(5)}\bar{g}^{(5)}_{ac}\bar{g}_{bd} -{}^{(5)}\bar{g}^{(5)}_{ad}\bar{g}_{bc}], \tag{A4}$$

$$\bar{\mathcal{G}}_{ab} = \frac{6}{\ell^2}{}^{(5)}\bar{g}_{ab} = -\Lambda_5^{(5)}\bar{g}_{ab} + \frac{\alpha}{2}\bar{\mathcal{H}}_{ab}, \tag{A5}$$

$$\bar{\mathcal{H}}_{ab} = \frac{24^{(5)}}{\ell^4}\bar{g}_{ab}. \tag{A6}$$

It follows that

$$\Lambda_5 = -\frac{6}{\ell^2} + \frac{12\alpha}{\ell^4}, \tag{A7}$$

$$\frac{1}{\ell^2} \equiv \mu^2 = \frac{1}{4\alpha}\left[1 - \sqrt{1 + \frac{4}{3}\alpha\Lambda_5}\right], \tag{A8}$$

where we choose in Eq. (A8) the branch with an RS limit, and μ is the energy scale associated with ℓ. This reduces to the RS relation $1/\ell^2 = -\Lambda_5/6$ when $\alpha = 0$. Note that there is an upper limit to the GB coupling from Eq. (A8):

$$\alpha < \frac{\ell^2}{4}, \tag{A9}$$

which, in particular, ensures that $\Lambda_5 < 0$.

A Friedmann-Robertson-Walker brane in an AdS_5 bulk is a solution to the field and junction equations. The modified Friedmann equation on the (spatially flat) brane is

$$\kappa_5^2(\rho + \lambda) = 2\sqrt{H^2 + \mu^2}[3 - 4\alpha\mu^2 + 8\alpha H^2]. \tag{A10}$$

This may be rewritten as

$$H^2 = \frac{1}{4\alpha}\left[(1 - 4\alpha\mu^2)\cosh\left(\frac{2\chi}{3}\right) - 1\right], \tag{A11}$$

$$\kappa_5^2(\rho + \lambda) = \left[\frac{2(1 - 4\alpha\mu^2)^3}{\alpha}\right]^{1/2}\sinh\chi, \tag{A12}$$

where χ is a dimensionless measure of the energy density. Note that the limit in Eq. (A9) is necessary for H^2 to be non-negative.

When $\rho = 0 = H$ in Eq. (A10) we recover the expression for the critical brane tension which achieves zero cosmological constant on the brane,

$$\kappa_5^2\lambda = 2\mu(3 - 4\alpha\mu^2). \tag{A13}$$

The effective four-dimensional Newton constant is given by

$$\kappa_4^2 = \frac{\mu}{(1 + 4\alpha\mu^2)}\kappa_5^2. \tag{A14}$$

When Eq. (A1) holds, this implies $M_5^3 \approx M_4^2/\ell$. The modified Friedmann Eq. (A11), together with Eq. (A12), shows that there is a characteristic GB energy scale,

$$M_{\rm GB} = \left[\frac{2(1-4\alpha\mu^2)^3}{\alpha\kappa_5^4}\right]^{1/8}, \tag{A15}$$

such that the GB high energy regime ($\chi \gg 1$) is $\rho + \lambda \gg M_{\rm GB}^4$. If we consider the GB term in the action as a correction to RS gravity, then $M_{\rm GB}$ is greater than the RS energy scale $M_\lambda = \lambda^{1/4}$, which marks the transition to RS high energy corrections to four-dimensional general relativity. By Eq. (A13), this requires $3\beta^3 - 12\beta^2 + 15\beta - 2 < 0$ where $\beta \equiv 4\alpha\mu^2$. Thus (to two significant figures),

$$M_\lambda < M_{\rm GB} \Rightarrow \alpha\mu^2 < 0.038, \tag{A16}$$

which is consistent with Eq. (A1).

Expanding Eq. (A11) in χ, we find three regimes for the dynamical history of the brane universe:

(1) the GB regime,

$$\rho \gg M_{\rm GB}^4 \Rightarrow H^2 \approx \left[\frac{\kappa_5^2}{16\alpha}\rho\right]^{2/3}, \tag{A17}$$

(2) the RS regime,

$$M_{\rm GB}^4 \gg \rho \gg \lambda \equiv M_\lambda^4 \Rightarrow H^2 \approx \frac{\kappa_4^2}{6\lambda}\rho^2, \tag{A18}$$

(3) and the four-dimensional regime,

$$\rho \ll \lambda \Rightarrow H^2 \approx \frac{\kappa_4^2}{3}\rho. \tag{A19}$$

The GB regime, when the GB term dominates gravity at the highest energies, above the brane tension, can usefully be characterized as

$$H^2 \gg \alpha^{-1} \gg \mu^2, \qquad H^2 \propto \rho^{2/3}. \tag{A20}$$

The brane energy density should be limited by the quantum gravity limit, $\rho < M_5^4$, in the high energy regime. By Eq. (A17),

$$\rho < M_5^4 \Rightarrow H < \left(\frac{\pi M_5}{2\alpha}\right)^{1/3}. \tag{A21}$$

In addition, since $\rho \gg M_{\rm GB}^4$, we have

$$M_5 \gg M_{\rm GB} \Rightarrow \alpha \gg \frac{2}{(8\pi M_5)^2}. \tag{A22}$$

Combining these two equations leads to

$$M_{\rm GB}^4 \ll \rho < M_5^4 \Rightarrow H \ll 4\pi^{3/2}M_5. \tag{A23}$$

Comparing Eqs. (A22) and (A16), we also find that

$$\ell \gg \frac{1}{8\pi M_5}. \tag{A24}$$

The mass scales M_5 and M_4 are related

$$M_5^3 \simeq \sqrt{\frac{4\pi}{3}}\lambda^{1/2}M_4. \tag{A25}$$

Since the brane energy density is limited by quantum gravity limit, the dimensionless energy scale χ cannot exceed certain maximum value $\chi_{\rm max}$. Using COBE normalized value of density perturbations we found $\chi_{\rm max} \simeq 6$. As for the brane tension λ, it is typically of the order of $10^{-5}M_4^4$.

Eqs. (A6) and (A12) allow to relate the scales M_5 and $M_{\rm GB}$

$$M_5^4 \simeq M_{\rm GB}^4 \sinh(\chi_{\rm max}). \tag{A26}$$

The typical estimates for various scales are

$$M_\lambda \simeq 10^{-5}M_4, \qquad M_5 \simeq 10^{-3}M_4, \qquad M_{\rm GB} \simeq 10^{-4}M_4. \tag{A27}$$

These estimates are consistent with the bounds on various scales in the problem quoted above. We once again emphasize that we treat here the GB term perturbatively such that the smooth limit to RS brane world exists.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999); **83**, 3370 (1999).

[2] S. W. Hawking, T. Hertog, and H. Reall, Phys. Rev. D **62**, 043501 (2000); S. Nojiri, S. D. Odintsov, and S. Zerbini, Phys. Rev. D **62**, 064006 (2000); S. Nojiri and S. Odintsov, Phys. Lett. B **484**, 119 (2000); L. Anchordoqui, C. Nunez, and K. Olsen, J. High Energy Phys. 10 (2000) 050; S. Nojiri and S. Odintsov, Phys. Lett. B **494**, 135 (2000); S. Gubser, Phys. Rev. D **63**, 084017 (2001); T. Shiromizu and D. Ida, Phys. Rev. D **64**, 044015 (2001).

[3] J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999); N. Kaloper, Phys. Rev. D **60**, 123506 (1999); T. Shiromizu, K. Maeda, and M. Sasaki, Phys. Rev. D **62**, 024012 (2000); P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Phys. Lett. B **477**, 285 (2000); E. E. Flanagan, S.-H. Tye, and I. Wasserman, Phys. Rev. D **62**, 044039 (2000).

[4] Roy Maartens, Living Rev. Relativity **7**, 1 (2004).

[5] R. Maartens, D. Wands, B. A. Bassett, and I. P. C. Heard, Phys. Rev. D **62**, 041301 (2000).

[6] E. J. Copeland, A. R. Liddle, and J. E. Lidsey, Phys. Rev. D **64**, 023509 (2001).

[7] P. J. Peebles and A. Vilenkin, Phys. Rev. D **59**, 063505 (1999); E. J. Copeland, A. R. Liddle, and J. E. Lidsey, Phys. Rev. D **64**, 023509 (2001); G. Huey and Lidsey, Phys. Lett. B **514**, 217 (2001); V. Sahni, M. Sami, and T. Souradeep, Phys. Rev. D **65**, 023518 (2002); A. S. Majumdar, Phys. Rev. D **64**, 083503 (2001); J. W. F.

Valle, Astropart. Phys. **18**, 287 (2002); K. Dimopoulos, Nucl. Phys. B, Proc. Suppl. **95**, 70 (2001); M. Sami, N. Dadhich, and Tetsuya Shiromizu, Phys. Lett. **68B**, 118 (2003); M. Sami and V. Sahni, Phys. Rev. D **70**, 083513 (2004); M. Sami and N. Dadhich, hep-th/0405016.

[8] S. Tsujikawa and A. R. Liddle, J. Cosmol. Astropart. Phys. 03 (2004) 001.

[9] Shinji Tsujikawa, M. Sami, and Roy Maartens, Phys. Rev. D **70**, 063525 (2004).

[10] D. Lovelock, J. Math. Phys. (N.Y.) **12**, 498 (1971); N. Deruelle and J. Madore, Mod. Phys. Lett. A **1**, 237 (1986); N. Deruelle and L. Farina-Busto, Phys. Rev. D **41**, 3696 (1990).

[11] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985); B. Zwiebach, Phys. Lett. **156B**, 315 (1985); A. Sen, Phys. Rev. Lett. **55**, 1846 (1985); R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. **B293**, 385 (1987); N. E. Mavromatos and J. Rizos, Phys. Rev. D **62**, 124004 (2000); I. P. Neupane, J. High Energy Phys. 09 (2000) 040; I. P. Neupane, Phys. Lett. B **512**, 137 (2001); K. A. Meissner and M. Olechowski, Phys. Rev. Lett. **86**, 3708 (2001); Y. M. Cho, I. Neupane, and P. S. Wesson, Nucl. Phys. **B621**, 388 (2002); N. Deruelle and M. Sasaki, Prog. Theor. Phys. **110**, 441 (2003).

[12] N. Dadhich, gr-qc/0407003.

[13] C. Charmousis and J-F. Dufaux, Classical Quantum Gravity **19**, 4671 (2002).

[14] J. E. Lidsey and N. J. Nunes, Phys. Rev. D, **67**, 103510 (2003).

[15] Kei-ichi Maeda and Takashi Torii, Phys. Rev. D **69**, 024002 (2004).

[16] J.-F. Dufaux, J. Lidsey, R. Maartens, and M. Sami, Phys. Rev. D **70**, 083525 (2004).

[17] Shin'ichi Nojiri and Sergei D. Odintsov, J. High Energy Phys. 07 (2000) 049; B. Abdesselam and N. Mohammedi, Phys. Rev. D **65**, 084018 (2002); J. E. Lidsey, Shin'ichi Nojiri, and S. D. Odintsov, J. High Energy Phys. 06 (2002) 026; S. C. Devis, Phys. Rev. D **67**, 024030 (2003); E. Gravanis and S. Willison, Phys. Lett. B **562**, 118 (2003); S. Nojiri, S.Odintsov, and S. Ogushi, Phys. Rev. D **65**, 023521 (2002); J. P. Gregory and A. Padilla, Classical Quantum Gravity **20**, 4221 (2003); H. M. Lee, hep-th/0010193; K. H. Kim and Y. S. Myung, J. Cosmol. Astropart. Phys. 04 (2004) 009; N. Deruelle and C. Germani, Nuovo Cimento Soc. Ital. Fis., B **118**, 977 (2003); Nathalie Deruelle and John Madore, gr-qc/0305004; Kyong Hee Kim and Yun Soo Myung, astro-ph/0408278; Kyong Hee Kim and Yun Soo Myung, astro-ph/0406387; Yun Soo Myung, Phys. Lett. B **601**, 1 (2004).

[18] G. Calcagni and Shinji Tsujikawa, Phys. Rev. D **70**, 103514 (2004); Maeda, Phys. Rev. D **64**, 123525 (2001); M. Sami, Mod. Phys. Lett. A **18**, 691 (2003); T. D. Saini, S. Raychaudhury, V. Sahni, and Alexei A. Starobinsky, Phys. Rev. Lett. **85**, 1162 (2000).

[19] A. Sen, J. High Energy Phys. 04 (2002) 048; J. High Energy Phys. 07 (2002) 065; Mod. Phys. Lett. A **17**, 1797 (2002).

[20] M. R. Garousi, Nucl. Phys. **B584**, 284 (2000); M. R. Garousi, J. High Energy Phys. 04 (2003) 027; E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras, and S. Panda, J. High Energy Phys. 05 (2000) 009.

[21] G. W. Gibbons, Phys. Lett. B **537**, 1 (2002); M. Fairbairn and M. H. G. Tytgat, Phys. Lett. B **546**, 1 (2002); S. Mukohyama, Phys. Rev. D **66**, 024009 (2002); A. Feinstein, Phys. Rev. D **66**, 063511 (2002); D. Choudhury, D. Ghoshal, D. P. Jatkar, and S. Panda, Phys. Lett. B **544**, 231 (2002); G. Shiu and I. Wasserman, Phys. Lett. B **541**, 6 (2002); T. Padmanabhan and T. R. Choudhury, Phys. Rev. D **66**, 081301 (2002); A. Ishida and S. Uehara, J. High Energy Phys. 02 (2003) 050; 10 (2002) 034; A. Mazumdar, S. Panda, and A. Perez-Lorenzana, Nucl. Phys. **B614**, 101 (2001); Y. S. Piao, R. G. Cai, X. m. Zhang, and Y. Z. Zhang, Phys. Rev. D **66**, 121301 (2002); Z. K. Guo, Y. S. Piao, R. G. Cai, and Y. Z. Zhang, Phys. Rev. D **68**, 043508 (2003); B. Wang, E. Abdalla, and R. K. Su, Mod. Phys. Lett. A **18**, 31 (2003); S. Mukohyama, Phys. Rev. D **66**, 123512 (2002); J. g. Hao and X. z. Li, Phys. Rev. D **66**, 087301 (2002); M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **67**, 063511 (2003); A. Sen, Int. J. Mod. Phys. A **18**, 4869 (2003); C. j. Kim, H. B. Kim, and Y. b. Kim, Phys. Lett. B **552**, 111 (2003); J. S. Bagla, H. K. Jassal, and T. Padmanabhan, Phys. Rev. D **67**, 063504 (2003); F. Leblond and A. W. Peet, J. High Energy Phys. 04 (2003) 048; T. Matsuda, Phys. Rev. D **67**, 083519 (2003); A. Das and A. DeBenedictis, Gen. Relativ. Gravit. **36**, 1741 (2004); A. DeBenedictis and A. Das, Nucl. Phys. **B653**, 279 (2003); M. Majumdar and A. C. Davis, Phys. Rev. D **69**, 103504 (2004); S. Nojiri and S. D. Odintsov, Phys. Lett. B **571**, 1 (2003); D. Bazeia, F. A. Brito, and J. R. S. Nascimento, Phys. Rev. D **68**, 085007 (2003); L. R. W. Abramo and F. Finelli, Phys. Lett. B **575**, 165 (2003); L. Raul Abramo, Fabio Finelli, and Thiago S. Pereira, astro-ph/0405041; V. Gorini, A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Rev. D **69**, 123512 (2004); M. B. Causse, astro-ph/0312206; P. K. Suresh, gr-qc/0309043; F. Piazza and S. Tsujikawa, J. Cosmol. Astropart. Phys. 07 (2004) 004; Jian-gang Hao and Xin-zhou Li, Phys. Rev. D **69**, 107303 (2004); Xin-zhou Li, Dao-jun Liu, Jian-gang Hao, Dao-jun Liu, and Xin-zhou Li, hep-th/0207146; Zong-Kuan Guo and Yuan-Zhong Zhang, J. Cosmol. Astropart. Phys. 08 (2004) 010; Xin-zhou Li, Jian-gang Hao, and Dao-jun Liu, Chin. Phys. Lett. **19**, 1584 (2002); Dao-jun Liu and Xin-zhou Li, astro-ph/0402063 [Phys. Rev. D (to be published)]; J. M. Aguirregabiria and Ruth Lazkoz, Phys. Rev. D **69**, 123502 (2004); Mod. Phys. Lett. A **19**, 927 (2004); Kenji Hotta, Prog. Theor. Phys. **112**, 653 (2004); Xin-He Meng and Peng Wang, Classical Quantum Gravity **21**, 2527 (2004); Peng Wang and Xin-He Meng, hep-th/0406170; Luis R. Abramo, Fabio Finelli, and Thiago S. Pereira, astro-ph/0405041; Joris Raeymaekers, J. High Energy Phys. 10 (2004) 057; Kamal L. Panigrahi, Phys. Lett. B **601**, 64 (2004); Chanju Kim, Hang Bae Kim, Yoonbai Kim, O-Kab Kwon, and Chong Oh Lee, hep-th/0404242; Ahmad Ghodsi and Amir E. Mosaffa, hep-th/0408015; S. K. Srivastava, gr-qc/0409074.

[22] L. Kofman and A. Linde, J. High Energy Phys. 07 (2002) 004; M. Sami, Pravabati Chingangbam, and Tabish Qureshi, Phys. Rev. D **66**, 043530 (2002); M. Sami, Pravabati Chingangbam, and Tabish Qureshi, Pramana **62**, 765 (2004).

[23] A. Frolov, Lev Kofman, and Alexei Starobinsky, Phys. Lett. B **545**, 8 (2002); Gary Felder, Lev Kofman, and Alexei Starobinsky, J. High Energy Phys. 09 (2002) 026; Neil Barnaby, J. High Energy Phys. 07 (2004) 025.

[24] M. R.Garousi, M. Sami, and Shinji Tsujikawa, Phys. Rev. D **70**, 043536 (2004); M. R. Garousi, M. Sami, and Shinji Tsujikawa, hep-th/0405012.

[25] Gianluca Calcagni, Phys. Rev. D **69**, 103508 (2004).

[26] T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002); K.-I Maeda, Phys. Rev. D **64**, 123525 (2001); Shuntaro Mizuno, Kei-ichi Maeda, and Kohta Yamamoto, Phys. Rev. D **67**, 023516 (2003); M. Sami, Mod. Phys. Lett. A **18**, 691 (2003).

[27] B. C. Paul and M. Sami, Phys. Rev. D **70**, 027301 (2004).

[28] J. c. Hwang and H. Noh, Phys. Rev. D **66**, 084009 (2002).

[29] D. A. Steer and F. Vernizzi, Phys. Rev. D **70**, 043527 (2004).

[30] S. Tsujikawa and B. Gumjudpai, Phys. Rev. D **69**, 123523 (2004).

[31] S. Foster, Classical Quantum Gravity **15**, 3485 (1998).

[32] A. V. Toporensky, P. V. Tretyakov, and V. O. Ustiansky, Astronomy Letters **29**, 1 (2003).

[33] S. Foster, gr-qc/9806113.

[34] A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999).

[35] N. Yu. Savchenko and A. V. Toporensky, Classical Quantum Gravity **20**, 2553 (2003).

[36] E. J. Copeland, A. R. Liddle, and D. Wands, Phys. Rev. D **57**, 4686 (1998); A. de la Macorra and G. Piccinelli, Phys. Rev. D **61**, 123503 (2000); Shuntaro Mizuno, Seung-Joo Lee, and Edmund J. Copeland, Phys. Rev. D **70**, 043525 (2004); S. C. C. Ng, N. J. Nunes, and F. Rosati, Phys. Rev. D **64**, 083510 (2001).

[37] Aguirregabiria and R. Lazkoz, Phys. Rev. D **69**, 123502 (2004).

[38] Shinji Tsujikawa and M. Sami, Phys. Lett. B **603**, 113 (2004).