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Gravity in the Randall-Sundrum two *D*-brane model

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We analyze the Randall-Sundrum two *D*-brane model by linear perturbation and then consider the linearized gravity on the *D* brane. The qualitative contribution from the Kaluza-Klein modes of gauge fields to the coupling to the gravity on the brane will be addressed. As a consequence, the gauge fields localized on the brane are shown not to contribute to the gravity on the brane at large distances. Although the coupling between gauge fields and gravity appears in the next order, the ordinary coupling cannot be realized.

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I. INTRODUCTION

Recently the model construction of the inflation using *D* brane has been initiated [1] in braneworld context. See Refs. [2–4] for other related issues. However, the self-gravity of *D* brane, which would be essential in considering *D*-brane cosmology, was not seriously considered there. On the other hand, a Randall-Sundrum-type (RS) [5,6] model based on *D*-brane action has been considered in Refs. [7–10]. In these papers, the bulk spacetime is described by type IIB supergravity compactified on S^5 [11] and the brane action is the Born-Infeld plus Chern-Simons action. Intuitively it is expected that the low-energy effective gravitational theory on the *D* brane is Einstein-Maxwell theory with corrections from various form fields and radion (in two *D*-brane system). However, analysis based on the long-wave approximation [12] showed that the gravity on the *D* brane does not couple to Maxwell field. This is because contribution from the Born-Infeld action is exactly canceled by those from the other form fields, which is quite nontrivial and surprising. This result would be a serious problem when we construct a cosmological model based on *D* brane.

In this paper we will reexamine the gravity on the branes by investigating the linear perturbation following Refs. [13–16]. In the previous series of papers on RS *D* braneworld [7–10], the gradient expansion (long-wave approximation) has been employed to derive the effective theory on the brane. This approximation includes the nonlinear effect but neglects the massive mode. On the other hand, the linear perturbation takes the massive mode as well as zero mode into account, although the nonlinear terms are neglected. In this sense, these two approximations are complementary.

Furthermore, in the previous works, the form fields B_2 and C_2 are assumed to be closed, that is, $dB_2 = dC_2 = 0$, for simplicity. However, such assumption kills the transverse tensor part of the form fields, which might be a reason for the noncoupling of the gravity and the Maxwell field. In this paper, on the other hand, we will not impose such an assumption. As seen below, however, the contribution from the zero mode of gauge fields does not still have the usual form and is negligible at large distances. This is consistent with the result from long-wave approximation discussed in the previous paper [10] and the appendix of this paper.

The rest of this paper is organized as follows. In Sec. II, we describe the model which we consider here. In Sec. III, we formulate the Arnowitt-Deser-Misner (ADM) formalism for the current model. In Sec. IV, the linearized gravitational equation on the brane is derived. The contributions from massive modes of the form fields are also considered. Finally we will give summary and discussion in Sec. V. In the appendix, for a comparison with the result obtained in linear perturbation analysis, we rederive the gravitational equation on the brane using the gradient expansion without assumption of $dB_2 = dC_2 = 0$.

II. MODEL

We consider the Randall-Sundrum model in type IIB supergravity compactified on S^5. The brane is described by Born-Infeld and Chern-Simons actions. So we begin with the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-\mathcal{G}}\Big[{}^{(5)}R - 2\Lambda - \frac{1}{2}|H|^2 - \frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}|\tilde{F}|^2 - \frac{1}{2}|\tilde{G}|^2\Big] + S^{(+)}_{\rm brane} + S^{(+)}_{\rm CS} + S^{(-)}_{\rm brane} + S^{(-)}_{\rm CS}, \quad (1)$$

where $H_{MNK} = \frac{1}{2}\partial_{[M}B_{NK]}$, $F_{MNK} = \frac{1}{2}\partial_{[M}C_{NK]}$, $G_{K_1K_2K_3K_4K_5} = \frac{1}{4!}\partial_{[K_1}D_{K_2K_3K_4K_5]}$, $\tilde{F} = F + \chi H$ and $\tilde{G} = G + C\wedge H$. $M, N, K = 0, 1, 2, 3, 4$. B_{MN} and C_{MN} are two-form fields, and $D_{K_1K_2K_3K_4}$ is a four-form field. χ is a scalar field. $\mathcal{G}_{MN}$ is the metric of five-dimensional spacetime.

$S^{(\pm)}_{\rm brane}$ is given by Born-Infeld action

$$S^{(+)}_{\rm brane} = \gamma_{(+)} \int d^4x \sqrt{-\det(h + \mathcal{F}^{(+)})}, \tag{2}$$

$$S^{(-)}_{\rm brane} = \gamma_{(-)} \int d^4x \sqrt{-\det(q + \mathcal{F}^{(-)})}, \tag{3}$$

where $h_{\mu\nu}$ and $q_{\mu\nu}$ are the induced metric on the $D_\pm$ brane and

$$\mathcal{F}^{(\pm)}_{\mu\nu} = B^{(\pm)}_{\mu\nu} + (|\gamma_{(\pm)}|)^{-1/2} F^{(\pm)}_{\mu\nu}. \tag{4}$$

$F_{\mu\nu}$ is the U(1) gauge field (Maxwell field) on the brane. Here $\mu, \nu = 0, 1, 2, 3$ and $\gamma_{(\pm)}$ are $D_\pm$-brane tension.

$S^{(\pm)}_{\rm CS}$ is Chern-Simons action

$$S^{(+)}_{\rm CS} = \gamma_{(+)} \int d^4x \sqrt{-h} \epsilon^{\mu\nu\rho\sigma} \bigg[\frac{1}{4} \mathcal{F}^{(+)}_{\mu\nu} C^{(+)}_{\rho\sigma} + \frac{\chi}{8} \mathcal{F}^{(+)}_{\mu\nu} \mathcal{F}^{(+)}_{\rho\sigma} + \frac{1}{24} D^{(+)}_{\mu\nu\rho\sigma} \bigg], \tag{5}$$

$$S^{(-)}_{\rm CS} = \gamma_{(-)} \int d^4x \sqrt{-q} \epsilon^{\mu\nu\rho\sigma} \bigg[\frac{1}{4} \mathcal{F}^{(-)}_{\mu\nu} C^{(-)}_{\rho\sigma} + \frac{\chi}{8} \mathcal{F}^{(-)}_{\mu\nu} \mathcal{F}^{(-)}_{\rho\sigma} + \frac{1}{24} D^{(-)}_{\mu\nu\rho\sigma} \bigg]. \tag{6}$$

Here the brane charges, the factors in front of the integration, are set equal to the brane tensions. Therefore, our model contains Bogomol'nyi-Prasad-Sommerfield (BPS) state of D branes.

III. BASIC EQUATIONS

In this section we write down the basic equations and boundary conditions based on the ADM formalism along the direction transverse to the brane. Then let us perform (1 + 4) decomposition

$$ds^2 = \mathcal{G}_{MN} dx^M dx^N = e^{2\phi(y,x)} dy^2 + g_{\mu\nu}(y,x) dx^\mu dx^\nu, \tag{7}$$

where y is the coordinate orthogonal to the brane. D_+ brane and D_- brane are supposed to locate at $y = y^{(+)} = 0$ and $y = y^{(-)} = y_0$.

The spacelike "evolutional" equations to the y direction for the extrinsic curvature of the brane,

$$K_{\mu\nu} = \frac{1}{2} e^{-\phi} \partial_y g_{\mu\nu}, \tag{8}$$

and other fields are

$$e^{-\phi} \partial_y K = {}^{(4)}R - \kappa^2 \bigg({}^{(5)}T^\mu_\mu - \frac{4}{3} {}^{(5)}T^M_M \bigg) - K^2 - e^{-\phi} D^2 e^\phi, \tag{9}$$

$$e^{-\phi} \partial_y \tilde{K}^\mu_\nu = {}^{(4)}\tilde{R}^\mu_\nu - \kappa^2 \bigg({}^{(5)}T^\mu_\nu - \frac{1}{4} \delta^\mu_\nu {}^{(5)}T^\alpha_\alpha \bigg) - K \tilde{K}^\mu_\nu - e^{-\phi} [D^\mu D_\nu e^\phi]_{\rm traceless}, \tag{10}$$

$$\partial^2_y \chi + D^2 \chi + e^\phi K \partial_y \chi - \frac{1}{2} H_{y\alpha\beta} \tilde{F}^{y\alpha\beta} = 0, \tag{11}$$

$$\partial_y X^{y\mu\nu} + e^\phi K X^{y\mu\nu} + D_\alpha \phi H^{\alpha\mu\nu} + D_\alpha H^{\alpha\mu\nu} + \frac{1}{2} F_{y\alpha\beta} \tilde{G}^{y\alpha\beta\mu\nu} = 0, \tag{12}$$

$$\partial_y \tilde{F}^{y\mu\nu} + e^\phi K \tilde{F}^{y\mu\nu} + D_\alpha \phi \tilde{F}^{\alpha\mu\nu} + D_\alpha \tilde{F}^{\alpha\mu\nu} - \frac{1}{2} H_{y\alpha\beta} \tilde{G}^{y\alpha\beta\mu\nu} = 0, \tag{13}$$

$$\partial_y \tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = e^\phi K \tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{14}$$

where $X^{y\mu\nu} := H^{y\mu\nu} + \chi \tilde{F}^{y\mu\nu}$ and the energy-momentum tensor is

$$\kappa^{2(5)} T_{MN} = \frac{1}{2} \bigg[\nabla_M \chi \nabla_N \chi - \frac{1}{2} g_{MN} (\nabla\chi)^2 \bigg] + \frac{1}{4} [H_{MKL} H_N{}^{KL} - g_{MN} |H|^2] + \frac{1}{4} [\tilde{F}_{MKL} \tilde{F}_N{}^{KL} - g_{MN} |\tilde{F}|^2] + \frac{1}{96} \tilde{G}_{MK_1K_2K_3K_4} \tilde{G}_N{}^{K_1K_2K_3K_4} - \Lambda g_{MN}. \tag{15}$$

$\tilde{K}^\mu_\nu$ and ${}^{(4)}\tilde{R}^\mu_\nu$ are the traceless parts of K^μ_ν and ${}^{(4)}R^\mu_\nu$, respectively. Here D_μ is the covariant derivative with respect to $g_{\mu\nu}$.

The Hamiltonian and momentum constraints and constraints for the form fields on $y =$ const. hypersurfaces are

$$-\frac{1}{2} \bigg[{}^{(4)}R - \frac{3}{4} K^2 + \tilde{K}^\mu_\nu \tilde{K}^\nu_\mu \bigg] = \kappa^{2(5)} T_{yy} e^{-2\phi}, \tag{16}$$

$$D_\nu K^\nu_\mu - D_\mu K = \kappa^{2(5)} T_{\mu y} e^{-\phi}, \tag{17}$$

$$D_\alpha (e^\phi X^{y\alpha\mu}) + \frac{1}{6} e^\phi F_{\alpha_1\alpha_2\alpha_3} \tilde{G}^{y\alpha_1\alpha_2\alpha_3\mu} = 0, \tag{18}$$

$$D_\alpha (e^\phi \tilde{F}^{y\alpha\mu}) - \frac{1}{6} e^\phi H_{\alpha_1\alpha_2\alpha_3} \tilde{G}^{y\alpha_1\alpha_2\alpha_3\mu} = 0, \tag{19}$$

$$D^\alpha (e^{-\phi} \tilde{G}_{y\alpha\mu_1\mu_2\mu_3}) = 0. \tag{20}$$

Under Z_2 symmetry, the junction conditions at the brane located $y = y^{(\pm)}$ are

$$[K_{\mu\nu} - g_{\mu\nu} K]_{y=y^{(\pm)}} = \mp \frac{\kappa^2}{2} \gamma_{(\pm)} (g_{\mu\nu} - T^{(\pm)}_{\mu\nu}) + O(T^2_{\mu\nu}) \tag{21}$$

$$H_{y\mu\nu}(y^{(\pm)}, x) = \mp \kappa^2 \gamma_{(\pm)} e^\phi \mathcal{F}^{(\pm)}_{\mu\nu}, \tag{22}$$

$$\tilde{F}_{y\mu\nu}(y^{(\pm)}, x) = \mp \frac{\kappa^2}{2} \gamma_{(\pm)} e^\phi \epsilon_{\mu\nu\alpha\beta} \mathcal{F}^{(\pm)\alpha\beta}, \tag{23}$$

$$\tilde{G}_{y\mu\nu\alpha\beta}(y^{(\pm)}, x) = \mp\kappa^2\gamma_{(\pm)}e^{\phi}\epsilon_{\mu\nu\alpha\beta}, \tag{24}$$

$$\partial_y\chi(y^{(\pm)}, x) = \mp\frac{\kappa^2}{8}\gamma_{(\pm)}e^{\phi}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{(\pm)}_{\mu\nu}\mathcal{F}^{(\pm)}_{\alpha\beta}. \tag{25}$$

In the above

$$T^{(\pm)\mu}{}_{\nu} = \mathcal{F}^{(\pm)\mu\alpha}\mathcal{F}^{(\pm)}_{\nu\alpha} - \frac{1}{4}\delta^{\mu}_{\nu}\mathcal{F}^{(\pm)}_{\alpha\beta}\mathcal{F}^{(\pm)\alpha\beta}. \tag{26}$$

As seen in the junction condition for χ, χ is quadratic in $\mathcal{F}$ so that it can be omitted in the linear approximation. For the same reason, we can omit the quadratic term in Eq. (21).

IV. LINEARIZED GRAVITY

A. Background

We assume that the background bulk spacetime is five-dimensional anti-de Sitter spacetime produced by the negative cosmological constant Λ and the four-form field $D_{K_1K_2K_3K_4}$. Other form fields and radion ϕ are assumed to vanish at the zeroth order.

The metric for the background space time is given by

$$\overset{(0)}{g}_{\mu\nu} = a^2(y)\eta_{\mu\nu} = e^{-2y/\ell}\eta_{\mu\nu}, \tag{27}$$

where ℓ is the curvature radius of anti-de Sitter spacetime. For this background, the equations for the four-form field (14), (20), and (24) are solved exactly as,

$$\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = -a^4\kappa^2\gamma\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{28}$$

where $\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}$ is the Levi-Civita tensor with respect to $\eta_{\mu\nu}$ while one in the previous section is with respect to $g_{\mu\nu}$. Hereafter we adopt the former convention. The tensions on the two branes must have the same magnitude and the opposite signature in order for the junction conditions (21) and (24) to be satisfied, and

$$\gamma \equiv \gamma_{(+)} = -\gamma_{(-)}. \tag{29}$$

Then the bulk energy-momentum tensor at the zeroth order is,

$$\kappa^{2(5)}T_{MN} = -\left(\frac{1}{4}\kappa^4\gamma^2 + \Lambda\right)g_{MN}, \tag{30}$$

Here we must impose the Randall-Sundrum tuning,

$$\left(\frac{1}{4}\kappa^4\gamma^2 + \Lambda\right) + \frac{1}{6}\kappa^4\gamma^2 = \Lambda + \frac{5}{12}\kappa^4\gamma^2 = 0, \tag{31}$$

so that the brane is Minkowski at the zeroth order. Finally, from Eqs. (30) and (31), the curvature radius ℓ is expressed in terms of the brane tension:

$$\frac{1}{\ell} = -\frac{1}{6}\kappa^2\gamma. \tag{32}$$

B. Linear perturbation

First we consider the linear perturbation for the evolutional equation of $B_{\mu\nu}$ and $C_{\mu\nu}$, and later their backreaction to the gravity will be studied. In linear order Eqs. (12) and (13) become

$$\partial_y H_{y\mu\nu} + a^{-2}\partial_\alpha H^{\alpha}{}_{\mu\nu} + \frac{3}{\ell}F_{y\alpha\beta}\epsilon_{\mu\nu}{}^{\alpha\beta} = 0, \tag{33}$$

and

$$\partial_y F_{y\mu\nu} + a^{-2}\partial_\alpha F^{\alpha}{}_{\mu\nu} - \frac{3}{\ell}H_{y\alpha\beta}\epsilon_{\mu\nu}{}^{\alpha\beta} = 0, \tag{34}$$

where $F^{\alpha}{}_{\mu\nu} = \eta^{\alpha\beta}F_{\beta\mu\nu}$. Hereafter $F^{\alpha}{}_{\mu\nu} = \eta^{\alpha\beta}F_{\beta\mu\nu}$, $H^{\alpha}{}_{\mu\nu} = \eta^{\alpha\beta}H_{\beta\mu\nu}$, and so on.

The constraint Eqs. (18) and (19) are rewritten as

$$F_{\mu\nu\alpha} = \frac{\ell}{6}\epsilon_{\mu\nu\alpha}{}^{\beta}\partial^{\rho}H_{y\rho\beta}, \tag{35}$$

and

$$H_{\mu\nu\alpha} = -\frac{\ell}{6}\epsilon_{\mu\nu\alpha}{}^{\beta}\partial^{\rho}F_{y\rho\beta}. \tag{36}$$

Here note that we can impose the following gauge conditions

$$B_{y\mu} = C_{y\mu} = 0, \tag{37}$$

using the gauge transformations

$$B_{MN} \to B'_{MN} = B_{MN} + \partial_M\int_0^y dy' B_{yN}(y', x) - \partial_N\int_0^y dy' B_{yM}(y', x), \tag{38}$$

$$C_{MN} \to C'_{MN} = C_{MN} + \partial_M\int_0^y dy' C_{yN}(y', x) - \partial_N\int_0^y dy' C_{yM}(y', x). \tag{39}$$

Then

$$H_{y\mu\nu} = \partial_y B_{\mu\nu} \quad \text{and} \quad F_{y\mu\nu} = \partial_y C_{\mu\nu}. \tag{40}$$

$B_{\mu\nu}$ and ${}^{*}C_{\mu\nu}$ are decomposed to

$$B_{\mu\nu} = B^{TT}_{\mu\nu} + \partial_\mu B^{T}_{\nu} - \partial_\nu B^{T}_{\mu}, \tag{41}$$

and

$${}^{*}C_{\mu\nu} := \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}C_{\alpha\beta} = {}^{*}C^{TT}_{\mu\nu} + \partial_\mu{}^{*}C^{T}_{\nu} - \partial_\nu{}^{*}C^{(T)}_{\mu}, \tag{42}$$

where $\partial^{\mu}B^{TT}_{\mu\nu} = \partial^{\mu*}C^{TT}_{\mu\nu} = \partial^{\mu}B^{T}_{\mu} = \partial^{\mu}C^{T}_{\mu} = 0$.

In momentum space, the field equations are

$$\partial_y^2 B^{(m)TT}_{\mu\nu} - a^{-2}k^2 B^{(m)TT}_{\mu\nu} + \frac{6}{\ell}\partial_y{}^{*}C^{(m)TT}_{\mu\nu} = 0, \tag{43}$$

$$\partial_y^{2*}C^{(m)TT}_{\mu\nu} + \frac{6}{\ell}\partial_y B^{(m)TT}_{\mu\nu} = 0, \qquad (44)$$

$$\partial_y^2 B^{(m)T}_{\mu} + \frac{6}{\ell}\partial_y{}^* C^{(m)T}_{\mu} = 0, \qquad (45)$$

and

$$\partial_y^{2*}C^{(m)T}_{\mu} - a^{-2}k^{2*}C^{(m)T}_{\mu} + \frac{6}{\ell}\partial_y B^{(m)T}_{\mu} = 0. \qquad (46)$$

The constraint equations become

$$B^{(m)TT}_{\mu\nu} + \frac{\ell}{6}\partial_y{}^* C^{(m)TT}_{\mu\nu} = 0, \qquad (47)$$

and

$${}^*C^{(m)T}_{\mu} + \frac{\ell}{6}\partial_y B^{(m)T}_{\mu} = 0, \qquad (48)$$

which are consistent with Eqs. (44) and (45).

Substituting Eq. (47) into Eq. (43), we obtain

$$\partial_y^2 B^{(m)TT}_{\mu\nu} - a^{-2}k^2 B^{(m)TT}_{\mu\nu} - \frac{36}{\ell^2}B^{(m)TT}_{\mu\nu} = 0. \qquad (49)$$

In the same way, Eq. (44) with Eq. (48) leads to

$$\partial_y^{2*}C^{(m)T}_{\mu} - a^{-2}k^{2*}C^{(m)T}_{\mu} - \frac{36}{\ell^2}{}^*C^{(m)T}_{\mu} = 0. \qquad (50)$$

The junction conditions are

$$\partial_y B^{TT}_{\mu\nu}(y^{(\pm)},x) = -\partial_y{}^* C^{TT}_{\mu\nu}(y^{(\pm)},x) = -\kappa^2\gamma B^{(\pm)TT}_{\mu\nu}, \qquad (51)$$

and

$$\partial_y B^{T}_{\mu}(y^{(\pm)},x) = -\partial_y{}^* C^{T}_{\mu}(y^{(\pm)},x) = -\kappa^2\gamma A^{(\pm)T}_{\mu}. \qquad (52)$$

In the above $\mathcal{F}^{(\pm)}_{\mu\nu}$ is decomposed to

$$\begin{aligned}\mathcal{F}^{(\pm)}_{\mu\nu}(x) &= B^{(\pm)TT}_{\mu\nu} + \partial_\mu B^{(\pm)T}_{\nu} - \partial_\nu B^{(\pm)T}_{\mu} + F^{(\pm)}_{\mu\nu}\\ &= B^{(\pm)TT}_{\mu\nu} + \partial_\mu A^{(\pm)T}_{\nu} - \partial_\nu A^{(\pm)T}_{\mu},\end{aligned} \qquad (53)$$

where $A^{(\pm)T}_{\mu} := A^{(\pm)}_{\mu} + B^{(\pm)T}_{\mu}$ and $F^{(\pm)}_{\mu\nu} = \partial_\mu A^{(\pm)}_{\nu} - \partial_\nu A^{(\pm)}_{\mu}$. It should be noted that the superscript $(\pm)$ denotes the value evaluated on the $D_\pm$ brane.

Next we focus on the perturbed metric

$$\begin{aligned}ds^2 &= (1+2\phi)dy^2 + (\gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu\\ &= (1+2\phi)dy^2 + (a^2\eta_{\mu\nu} + h^{TT}_{\mu\nu} - \gamma_{\mu\nu}\psi)dx^\mu dx^\nu,\end{aligned} \qquad (54)$$

where $h^{TT}_{\mu\nu}$ is the transverse-traceless part of $h_{\mu\nu}$. Since the bulk background spacetime is anti-de Sitter spacetime, the Green function is exactly the same as that of the Randall-Sundrum model. The difference is just presence of the bulk form fields. Therefore we follow the argument of Refs. [13,14] and then the perturbation $\bar{h}_{\mu\nu}$ in the Gaussian normal coordinate can be computed as

$$\bar{h}_{\mu\nu} = h^{TT}_{\mu\nu} - \gamma_{\mu\nu}\Big[\psi - \frac{2}{\ell}\hat{\xi}^5(x)\Big], \qquad (55)$$

where $\hat{\xi}^5(x)$ is a brane-bending mode (radion field) and

$$\begin{aligned}h^{TT}_{\mu\nu}(y,x) = &-2\kappa^2\int d^4x' G_R(y,x;0,x')|\gamma|\Sigma^{(+)}_{\mu\nu}(x')\\ &+ 2\kappa^2\int d^4x' G_R(y,x;y_0,x')|\gamma|\Sigma^{(-)}_{\mu\nu}(x')\\ &- 2\kappa^2\int dy' d^4x' G_R(y,x;y',x')\delta^{(5)}T_{\mu\nu}(y',x'),\end{aligned} \qquad (56)$$

where

$$\Sigma^{(\pm)}_{\mu\nu} = T^{(\pm)}_{\mu\nu} + \frac{1}{\kappa^2}\Big(\partial_\mu\partial_\nu\hat{\xi}^{5(\pm)} - \frac{1}{4}\gamma_{\mu\nu}\partial^2\hat{\xi}^{5(\pm)}\Big). \qquad (57)$$

The first and second terms in the right-hand side of Eq. (56) come from the D_+ and D_- brane, respectively. The third one is the contribution from the bulk fields. In the above, $\delta^{(5)}T_{\mu\nu}$ is the projected bulk stress tensor in the linear order. G_R is the five-dimensional retarded Green function

$$G_R(y,x;y',x') = G^{(0)}_R(y,x;y',x') + G^{(\mathrm{KK})}_R(y,x;y',x'), \qquad (58)$$

where

$$G^{(0)}_R(y,x;y',x') = -\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(x-x')}\frac{1}{\ell(1-a_0^2)} \times\frac{a(y)^2a(y')^2}{\mathbf{k}^2-(\omega+i\epsilon)^2}, \qquad (59)$$

and

$$G^{(\mathrm{KK})}_R(y,x;y',x') = -\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(x-x')} \times\int dm\frac{u_m(y)u_m(y')}{m^2+\mathbf{k}^2-(\omega+i\epsilon)^2}. \qquad (60)$$

$G^{(0)}_R$ is the truncated retarded Green function for zero mode. $u_m(y)$ are the mode functions which are expressed by Bessel functions, $u_m(y)\propto J_1(m\ell)N_2(mz) - N_1(m\ell)J_2(mz)$.

In the present model, the equation for $\hat{\xi}^5$ becomes

$$\partial^2\hat{\xi}^5(x) = \frac{\kappa^2}{6}T^{(+)} = 0, \qquad (61)$$

that is, the radion is a massless scalar field.

The equation for ψ comes from the Hamiltonian constraint equation:

$$-\frac{3}{2}\frac{1}{a^2}\partial^2\psi = \kappa^2\delta^{(5)}T^y_y, \qquad (62)$$

where

$$\kappa^2\delta^{(5)}T^y_y = \frac{1}{8}(H_{y\alpha\beta}H_y{}^{\alpha\beta} + \tilde{F}_{y\alpha\beta}\tilde{F}_y{}^{\alpha\beta}) - \frac{1}{24}(H_{\mu\alpha\beta}H^{\mu\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}^{\mu\alpha\beta}). \qquad (63)$$

The relation between ϕ and ψ comes from the traceless part of the (μ, ν) component of five-dimensional Einstein equation

$$\psi - \phi \sim \kappa^2 (1/\partial^2)^2 \partial^\mu \partial^\nu [\delta^{(5)} T_{\mu\nu}]_{\text{traceless}}. \tag{64}$$

C. Zero-mode truncation

In order to see the low-energy effective (gravitational) theory on the D brane, we will truncate zero mode carefully. Let us focus on the zero mode for $B_{\mu\nu}$ and $C_{\mu\nu}$. Introducing new variables

$$\Psi^{(\pm)}_{\mu\nu} := B^{(0)}_{\mu\nu} \pm {}^*C^{(0)}_{\mu\nu}, \tag{65}$$

we obtain two linearly independent equations

$$\partial_y^2 \Psi^{(\pm)}_{\mu\nu} \pm \frac{6}{\ell} \partial_y \Psi^{(\pm)}_{\mu\nu} = 0. \tag{66}$$

The junction conditions are written by

$$\partial_y \Psi^{(+)}_{\mu\nu}(y^{(\pm)}, x) = 0, \tag{67}$$

$$\partial_y \Psi^{(-)}_{\mu\nu}(y^{(\pm)}, x) = -2\kappa^2 \gamma \mathcal{F}^{(\pm)}_{\mu\nu}. \tag{68}$$

First it is easy to see that the solutions to the equations for gauge fields are

$$\Psi^{(+)}_{\mu\nu}(y, x) = \alpha_{\mu\nu}(x), \tag{69}$$

and

$$\Psi^{(-)}_{\mu\nu}(y, x) = \frac{2}{\ell} a^{-6}(y) \mathcal{F}^{(+)}_{\mu\nu}(x) + \beta_{\mu\nu}(x), \tag{70}$$

using the junction condition at $y = 0$. $\alpha_{\mu\nu}(x)$ and $\beta_{\mu\nu}(x)$ are constants of integration:

$$H^{(0)}_{y\mu\nu}(y, x) = \partial_y B^{(0)}_{\mu\nu}(y, x) = -\kappa^2 \gamma a^{-6}(y) \mathcal{F}^{(+)}_{\mu\nu}, \tag{71}$$

$$\tilde{F}^{(0)}_{y\mu\nu}(y, x) = \partial_y C^{(0)}_{\mu\nu}(y, x) = -\frac{\kappa^2\gamma}{2} a^{-6}(y) \epsilon_{\mu\nu}{}^{\alpha\beta} \mathcal{F}^{(+)}_{\alpha\beta}. \tag{72}$$

The remaining junction condition then implies the relation between gauge fields on the two branes

$$a_0^6 \mathcal{F}^{(-)}_{\mu\nu} = \mathcal{F}^{(+)}_{\mu\nu}, \tag{73}$$

and

$$T^{(-)}_{\mu\nu} = a_0^{-14} T^{(+)}_{\mu\nu}. \tag{74}$$

We also can compute the bulk stress tensor as

$$\begin{aligned}
\kappa^2 \delta^{(5)} T_{\mu\nu} &= \frac{1}{2} a^{-2}\left(H_{\mu y\alpha} H_\nu{}^{y\alpha} - \frac{1}{4}\eta_{\mu\nu} H_{y\alpha\beta} H^{y\alpha\beta}\right) + \frac{1}{2} a^{-2}\left(\tilde{F}_{\mu y\alpha} \tilde{F}_\nu{}^{y\alpha} - \frac{1}{4}\eta_{\mu\nu} \tilde{F}_{y\alpha\beta} \tilde{F}^{y\alpha\beta}\right) \\
&\quad + \frac{1}{4} a^{-4}\left(H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu} H_{\alpha\beta\rho} H^{\alpha\beta\rho}\right) + \frac{1}{4} a^{-4}\left(\tilde{F}_{\mu\alpha\beta} \tilde{F}_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu} \tilde{F}_{\alpha\beta\rho} \tilde{F}^{\alpha\beta\rho}\right) \\
&= \left(\frac{6}{\ell}\right)^2 a^{-14} T^{(+)}_{\mu\nu} + \frac{1}{4} a^{-4}\left(H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu} H_{\alpha\beta\rho} H^{\alpha\beta\rho}\right) + \frac{1}{4} a^{-4}\left(\tilde{F}_{\mu\alpha\beta} \tilde{F}_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu} \tilde{F}_{\alpha\beta\rho} \tilde{F}^{\alpha\beta\rho}\right),
\end{aligned} \tag{75}$$

where $H^{\alpha\beta\rho} = \eta^{\alpha\mu}\eta^{\beta\nu}\eta^{\rho\sigma} H_{\mu\nu\sigma}$. From Eq. (56) with Eqs. (61), (74), and (75) we finally obtain the following linearized equation on branes

$$\begin{aligned}
\partial^2 \bar{h}_{\mu\nu}(y, x) &= \partial^2 h^{TT}_{\mu\nu} - \gamma_{\mu\nu}\partial^2\psi \\
&= -2\kappa^2 \frac{a^2(y)}{\ell(1 - a_0^2)} |\gamma| \Sigma^{(+)}_{\mu\nu} + 2\kappa^2 \frac{a^2(y) a_0^2}{\ell(1 - a_0^2)} |\gamma| \Sigma^{(-)}_{\mu\nu} - 2\frac{6}{\ell^2} \frac{a^2(y)(a_0^{-12} - 1)}{1 - a_0^2} T^{(+)}_{\mu\nu} \\
&\quad - 2\frac{a_0^2}{\ell(1 - a_0^2)} \int_0^{y_0} dy \frac{1}{4} a^{-2} [H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} + \tilde{F}_{\mu\alpha\beta} \tilde{F}_\nu{}^{\alpha\beta}]_{\text{traceless}} + \frac{2}{3}\kappa^2 \eta_{\mu\nu} (\delta^{(5)} T^y_y)^{(0)} \\
&= -\frac{2a^2(y)}{\ell(1 - a_0^2)} |\gamma| \partial_\mu\partial_\nu (\hat{\xi}^{5(+)} - a_0^2 \hat{\xi}^{5(-)}) - 2\frac{a_0^2}{\ell(1 - a_0^2)} \int_0^{y_0} dy \frac{1}{4} a^{-2} [H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} + \tilde{F}_{\mu\alpha\beta} \tilde{F}_\nu{}^{\alpha\beta}]_{\text{traceless}} \\
&\quad + \frac{2}{3}\kappa^2 \eta_{\mu\nu} (\delta^{(5)} T^y_y)^{(0)} \\
&= -\frac{2a^2(y)}{\ell(1 - a_0^2)} |\gamma| \partial_\mu\partial_\nu (\hat{\xi}^{5(+)} - a_0^2 \hat{\xi}^{5(-)}) + O(H^2_{\mu\nu\alpha}).
\end{aligned} \tag{76}$$

The gauge fields localized on the branes do not appear as usual. The appropriate contribution from the boundary stress tensor is exactly canceled out by that from the bulk stress tensor. The first term in the last line represents the physical degree of freedom of the distance between two branes.[1] In the above $(\cdots)^{(0)}$ represents the zero-mode part. For example, $(\delta^{(5)}T_y^y)^{(0)}$ is

$$(\delta^{(5)}T_y^y)^{(0)} = -\frac{1}{24}(H_{\mu\alpha\beta}H^{\mu\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}^{\mu\alpha\beta}). \quad (77)$$

As mentioned before the difference between the usual RS model and the present one is the contribution from the bulk form field. In the present model, the brane carries the charge so that the form fields have nontrivial configuration with the source of the brane charge in the bulk. The whole configuration of spacetimes and gauge fields is fixed to realize the minimum energy in some sense. As a consequence the bulk-boundary cancellation occurs in the current setup with BPS.

One might be interested in the case with stabilized radion. However it would be unlikely that the stabilization improves the result because the radion affects only the trace part in the coupling between the gravity and matters. Anyway, the cancellation would be by virtue of the BPS condition.

D. Massive modes of form fields

The mode function for $B_{\mu\nu}^{(m)TT}$ etc., satisfying the junction condition at $y = y^{(+)} = 0$, is

$$\psi_m = \sqrt{m\ell e^{y/\ell}}\,\frac{\alpha_m J_6(m\ell e^{y/\ell}) - \beta_m N_6(m\ell e^{y/\ell})}{\sqrt{\alpha_m^2 + \beta_m^2}}, \quad (78)$$

where

$$\alpha_m = m\ell N_5(m\ell) - 6N_6(m\ell), \quad (79)$$

and

$$\beta_m = m\ell J_5(m\ell) - 6J_6(m\ell). \quad (80)$$

m should be quantized by the junction condition at $y = y^{(-)} = y_0$. For $m\ell \gg 1$ and $m\ell e^{y^{(-)}/\ell} \gg 1$, we obtain the mass spectrum of

$$m_n^{(\pm)} \simeq \frac{n\pi}{\ell(1 - e^{y_0/\ell})}. \quad (81)$$

After determination of correct normalization, for $m\ell \ll 1$, we can evaluate the contribution from massive modes to the right-hand side of Eq. (56) as

$$|\kappa^2\delta^{(5)}T_{\mu\nu}| \simeq \frac{1}{\ell^2}\left(\frac{r_0}{r}\right)^3\left(\frac{\ell}{r}\right)^{12}|T_{\mu\nu}^{(+)}| \ll \frac{1}{\ell^2}|T_{\mu\nu}^{(+)}|, \quad (82)$$

where r_0 is spatial scale of support of form field. Thus even if we consider the contribution from the massive modes, they will be negligible at low-energy scale.

[1] The redshift factor in the front of $\hat{\xi}^{5(-)}$ is expected in Ref. [14].

V. SUMMARY AND DISCUSSION

In this paper we derived the linearized gravitational equation on the D brane and then it turned out that the gauge fields do not couple to the gravity on the brane at zero modes in a usual way. Instead, an unusual coupling appears and it is negligible at large distances. We also discussed the contribution from the Kaluza-Klein modes which was shown to be also negligible at large distances.

The model which we considered is minimum extension of the Randall-Sundrum-type model to the supergravity-like one. Therein Z_2 symmetry and RS tuning are assumed. In this model, RS tuning corresponds to the condition of equality of brane tension and charge. It is likely that D brane in BPS state does not provide us the realistic model for the braneworld. As analyzed in Ref. [9], on the other hand, it was shown that the coupling of the gravity to the gauge fields will appear and the coupling constant is proportional to the cosmological constant. Therefore non-BPS state will be important for braneworld cosmology. We can also consider the cases without Z_2 symmetry. For example, a model considered in Ref. [1] does not have Z_2 symmetry. So the careful study of the self-gravitational effect for such a mode will be important.

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APPENDIX: LONG-WAVE APPROXIMATION

In this appendix, we approximately solve the bulk field equations by long-wave approximation (gradient expansion [12]) and derive the effective theory on the brane. The equation obtained here will include the nonlinear effect. Thus we can obtain the same result with one obtained in Sec. IV if we linearize the equation. This appendix can be regarded as the extension of previous work [10] into the general cases where we do not impose $dC_2 = dB_2 = 0$.

In the case with bulk fields we must carefully use the geometrical projection method [17] because the projected Weyl tensor $E_{\mu\nu}$ contains the leading effect from the bulk fields.

The bulk metric is written again as,

$$ds^2 = e^{2\phi(x)}dy^2 + g_{\mu\nu}(y, x)dx^\mu dx^\nu. \quad (A1)$$

The induced metric on the brane will be denoted by $h_{\mu\nu} := g_{\mu\nu}(0, x)$ and then

$$g_{\mu\nu}(y, x) = a^2(y, x)[h_{\mu\nu}(x) + \overset{(1)}{g}_{\mu\nu}(y, x) + \cdots]. \quad (A2)$$

In the above $\overset{(1)}{g}_{\mu\nu}(0,x) = 0$ and $a(0,x) = 1$. In a similar way, the extrinsic curvature is expanded as

$$K^\mu_\nu = \overset{(0)}{K}{}^\mu_\nu + \overset{(1)}{K}{}^\mu_\nu + \overset{(2)}{K}{}^\mu_\nu + \cdots. \tag{A3}$$

The small parameter is $\epsilon = (\ell/L)^2 \ll 1$, where L and ℓ are the curvature scale on the brane and the bulk anti-de Sitter curvature scale, respectively.

It is easy to obtain the zeroth order solutions. Without derivation they are given by the Randall-Sundrum setup; Eqs. (32) and (31). Then

$$\overset{(0)}{K}{}^\mu_\nu = -\frac{1}{\ell}\delta^\mu_\nu, \tag{A4}$$

$$\overset{(0)}{g}_{\mu\nu} = a^2(y,x)h_{\mu\nu}(x) = e^{-2d(y,x)/\ell}h_{\mu\nu}(x), \tag{A5}$$

where

$$d(y,x) = \int_0^y dy e^{\phi(x)}. \tag{A6}$$

$\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}$ is also given by Eq. (28).

Next we consider the first order equations. The first order equations for $\tilde{F}_{y\mu\nu}$ and $H_{y\mu\nu}$ are

$$\partial_y \overset{(1)}{\tilde{F}}_{y\mu\nu} - \frac{1}{2a^4}\overset{(1)}{H}_{y\alpha\beta}\tilde{G}_{y\rho\sigma\mu\nu}h^{\alpha\rho}h^{\beta\sigma} = 0, \tag{A7}$$

and

$$\partial_y \overset{(1)}{H}_{y\mu\nu} + \frac{1}{2a^4}\overset{(1)}{\tilde{F}}_{y\alpha\beta}\tilde{G}_{y\rho\sigma\mu\nu}h^{\alpha\rho}h^{\beta\sigma} = 0. \tag{A8}$$

Together with the junction conditions on D_+ brane the solutions are given by

$$\overset{(1)}{H}_{y\mu\nu}(y,x) = -\kappa^2\gamma a^{-6}e^{\phi}\mathcal{F}^{(+)}_{\mu\nu}, \tag{A9}$$

and

$$\overset{(1)}{\tilde{F}}_{y\mu\nu}(y,x) = -\frac{\kappa^2}{2}\gamma a^{-6}e^{\phi}\epsilon_{\mu\nu\rho\sigma}\mathcal{F}^{(+)}_{\alpha\beta}h^{\rho\alpha}h^{\sigma\beta}. \tag{A10}$$

The remaining junction conditions on D_- brane imply the relation between $\mathcal{F}^{(+)}_{\mu\nu}$ and $\mathcal{F}^{(-)}_{\mu\nu}$ as

$$\mathcal{F}^{(-)}_{\mu\nu} = a_0^{-6}\mathcal{F}^{(+)}_{\mu\nu}, \tag{A11}$$

and then

$$T^{(-)}_{\mu\nu} = a_0^{-14}T^{(+)}_{\mu\nu}, \tag{A12}$$

where $a_0 = a(y_0,x) = e^{-d_0(x)/\ell}$ and $d_0(x) := d(y_0,x)$.

Let us first substitute the junction conditions for $H_{y\mu\nu}$ and $\tilde{F}_{y\mu\nu}$ on the D_+ brane into the constraint equations of Eqs. (18) and (19). Then we see

$$\mathcal{D}^\mu\left(\mathcal{F}^{(+)}_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}C^{\alpha\beta}\right) = 0, \tag{A13}$$

$$\epsilon^{\mu\nu\alpha\beta}\mathcal{D}_\nu(\mathcal{F}^{(+)}_{\alpha\beta} - B_{\alpha\beta}) = 0, \tag{A14}$$

where $\mathcal{D}_\mu$ is the covariant derivative with respect to $h_{\mu\nu}$.

Using these results the evolutional equation for the traceless part of the extrinsic curvature is

$$e^{-\phi}\partial_y \overset{(1)}{\tilde{K}}{}^\mu_\nu = -\overset{(0)}{K}\overset{(1)}{\tilde{K}}{}^\mu_\nu + \tilde{R}^\mu_\nu(g) - \kappa^4\gamma^2 a^{-16}T^{(+)\mu}_\nu - e^{-\phi}[D^\mu D_\nu e^\phi]_{\text{traceless}}, \tag{A15}$$

where

$$\tilde{R}^\mu{}_\nu(g) = \frac{1}{a^2}\left[R^\mu_\nu(h) + \frac{2}{\ell}\mathcal{D}^\mu\mathcal{D}_\nu d + \frac{2}{\ell^2}\mathcal{D}^\mu d\mathcal{D}_\nu d\right]_{\text{traceless}}, \tag{A16}$$

and

$$D^\mu D_\nu e^\phi = \frac{1}{a^2}\mathcal{D}^\mu\mathcal{D}_\nu e^\phi + \frac{1}{a^2\ell}(\mathcal{D}^\mu e^\phi\mathcal{D}_\nu d + \mathcal{D}^\mu d\mathcal{D}_\nu e^\phi - \delta^\mu_\nu\mathcal{D}^\alpha d\mathcal{D}_\alpha e^\phi). \tag{A17}$$

$R^\mu_\nu(h) = h^{\mu\alpha}R_{\alpha\nu}(h)$ is the Ricci tensor with respect to $h_{\mu\nu}$ and $T^\mu_\nu = h^{\mu\alpha}T_{\alpha\nu}$.

The solution is summarized as

$$\overset{(1)}{\tilde{K}}{}^\mu_\nu(y,x) = -\frac{\ell}{2a^2}{}^{(4)}\tilde{R}^\mu_\nu(h) + \frac{1}{2}\kappa^2\gamma a^{-16}T^{(+)\mu}{}_\nu - a^{-2}\left[\mathcal{D}^\mu\mathcal{D}_\nu d - \frac{1}{\ell}\mathcal{D}^\mu d\mathcal{D}_\nu d\right]_{\text{traceless}} + \frac{\chi^\mu_\nu(x)}{a^4}, \tag{A18}$$

where χ^μ_ν is the "constant of integration."

The solution to the trace part of the extrinsic curvature is

$$\overset{(1)}{K}(y,x) = -\frac{\ell}{6a^2}{}^{(4)}R(h) - \frac{1}{a^2}\mathcal{D}^2 d + \frac{1}{a^2\ell}(\mathcal{D}d)^2. \tag{A19}$$

On the D_+ brane Eqs. (A18) and (A19) become

$${}^{(4)}\tilde{R}^\mu_\nu(h) = \frac{2}{\ell}\chi^\mu_\nu(x) \tag{A20}$$

and

$$0 = \overset{(1)}{K}(0, x) = -\frac{\ell}{6}{}^{(4)}R(h). \tag{A21}$$

They correspond to the Einstein equation on the brane obtained in Ref. [17] and χ^μ_ν is projected Weyl tensor $E_{\mu\nu}$. For the moment, $\chi^\mu_\nu(x)$ is an unknown term.

On D_- brane, Eq. (A18) becomes

$$\frac{\kappa^2}{2}\gamma T^{(-)\mu}{}_\nu = -\frac{\ell}{2a_0^2}{}^{(4)}\tilde{R}^\mu_\nu(h) + \frac{\kappa^2}{2}a_0^{-16}\gamma T^{(+)\mu}{}_\nu - \frac{1}{a_0^2}\left[\mathcal{D}^\mu\mathcal{D}_\nu d_0 - \frac{1}{\ell}\mathcal{D}^\mu d_0\mathcal{D}_\nu d_0\right]_{\text{traceless}} + \frac{\chi^\mu_\nu(x)}{a_0^4} \tag{A22}$$

and

$$0 = \overset{(1)}{K}(y_0, x) = -\frac{1}{a_0^2}\mathcal{D}^2 d_0 + \frac{1}{a_0^2\ell}(\mathcal{D}d_0)^2. \tag{A23}$$

All together we obtain the Einstein equation on D_+ brane

$$(a_0^{-2} - 1)G_{\mu\nu}(h) = \frac{2}{\ell}\left[\mathcal{D}_\mu\mathcal{D}_\nu d_0 - \frac{1}{\ell}\mathcal{D}_\mu d_0\mathcal{D}_\nu d_0\right]_{\text{traceless}}. \tag{A24}$$

The equation for radion becomes

$$\mathcal{D}^2 d_0 - \frac{1}{\ell}(\mathcal{D}d_0)^2 = 0. \tag{A25}$$

Thus the contribution from the gauge fields to gravity on the brane does not exist at low-energy scale. Let us mention the relation between this analysis and linear perturbation. The radion perturbation corresponds to the brane bending introduced in Eq. (55), that is, $\partial_\mu\partial_\nu d_0 \sim \partial_\mu\partial_\nu(\hat{\xi}^{5(+)} - a_0^2\hat{\xi}^{5(-)})$. Then Eqs. (A24) and (A25) without nonlinear terms coincide with Eqs. (76) and (61) without $O(H^2_{\mu\nu\alpha})$ terms, respectively. Thus the results from linear analysis and long-wave approximation are consistent, as they should be.

Finally, let us define

$$\Psi = 1 - e^{-2d_0/\ell} \quad \text{and} \quad \omega(\Psi) = \frac{3}{2}\frac{\Psi}{\Psi - 1}. \tag{A26}$$

Then the effective Einstein equations are rewritten as

$$G_{\mu\nu}(h) = \left[\frac{1}{\Psi}\mathcal{D}_\mu\mathcal{D}_\nu\Psi + \frac{\omega}{\Psi^2}\mathcal{D}_\mu\Psi\mathcal{D}_\nu\Psi\right]_{\text{traceless}} \tag{A27}$$

and equation of motion for Ψ is

$$\mathcal{D}^2\Psi + \frac{1}{2\omega + 3}\frac{d\omega}{d\Psi}(\mathcal{D}\Psi)^2 = 0, \tag{A28}$$

which are equivalent to the Brans-Dicke theory.

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