Scalar fields and exact invariants in a Friedmann-Robertson-Walker spacetime

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In this work we study scalar fields in a Friedmann-Robertson-Walker space-time. We use the invariant operator formulation of Lewis and Riesenfeld in order to study the behavior of scalar fields placed in the Friedmann-Robertson-Walker space-time. In addition, we construct the coherent states in this background and establish the existence of squeezed states.

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I. INTRODUCTION

The study of quantum physical systems in curved space-time goes back to the end of the twenties and to the beginning of the thirties of the last century [1]. From that time on a lot of attention has been devoted to this subject. As examples of investigations concerning this subject we can mention those related with the determination of the vacuum expectation value of the energy-momentum tensor and the problem of creation of particles in expanding universes [2], and those connected with relativistic quantum mechanics in different background spacetimes [3]. These investigations are of considerable theoretical and experimental interest and certainly constitute an essential element to construct a consistent theory that combines quantum physics and gravity.

The gravitational effect on some physical systems has already been investigated. As examples, we can mention an experiment using a neutron interferometer in which the Newtonian gravitational effects on the phase difference of two neutron beams was measured [4]. Another gravitational effect that appears in quantum interference is the neutrino oscillations [5] which has been discussed recently. In both the examples the gravitational and quantum effects appear in the quantum interference phenomena, with a difference that in the former case the particle is nonrelativistic and in the latter case it is ultrarelativistic. Therefore, we have examples of gravitational effects on nonrelativistic and relativistic quantum systems.

Another system that has attracted great interest is the harmonic oscillator with time-dependent mass and/or frequency when placed in some specific background geometry. This system has invoked much attention mainly because it can be treated as an exactly solvable model and offers a wide range of application in the description of physical systems in different areas of physics such as plasma physics [6], gravitation [7], and quantum optics [8].

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The connection of cosmology with some process in quantum optics has received some attention. In this context, a coherent state representation for a scalar field minimally coupled to a gravitational field was constructed [9] and the language of quantum optics was used to analyze the existence of squeezed states in a cosmological model [10]. Matacz, using squeezed state formalism, derived the coherent state representation of quantum fluctuations in a expanding universe. The invariant method of Lewis and Riesenfeld also was successful used in the investigations of the scalar fields placed in an anisotropic universe [11]. The idea of adoption of the language of the squeezed states to cosmological particle creation was first introduced by Grisshchuk and Sidorov [12,13]. After him, Hu, Kang and Matacz [10] have applied squeezed states formalism to discuss the role of initial states in particle creation and have pointed out that squeeze and rotation operator were first derived by Parker [14] in his analyses of cosmological particle creation, based on the work by Kamefuchi and Umezawa [15]. Also, Matacz [16] have considered a squeezed vacuum of harmonic oscillator system with time-dependent frequency to study the coherent representation of quantum fluctuation in expanding universe.

Recently, Grisshchu and Sidorov [12] have shown that relic gravitons can be created from vacuum quantum fluctuations of the gravitational field in the course of cosmological evolution and can be interpreted as squeezed quantum states of the gravitational field, in analogous way as squeezed sates in quantum optics. In a recent article, Geralico *et al.* [17] studied classical and quantum gravitational waves using the similarities of this problem with a damped parametric oscillator, where the mass is replaced by the square of the cosmological scale factor. In this article we investigate the quantum scalar fields in a Friedmann-Robertson-Walker (FRW) background. The behavior of matter scalar fields as well as gravitational waves [12] is governed by the Einstein equations of the time-dependent type that can be mapped in an equation of the time-dependent oscillator. Thus the problem of both particle creation in a metric field fluctuation during a cosmological evolution is reduced to solve the quantum time-dependent harmonic oscillator.

We solve this problem using the dynamical invariant operator method of Lewis-Riesenfeld. We demonstrate that the problem of the field quantization in this background reduces to solve the Schrödinger equation for the harmonic oscillator with time-dependent mass and frequency. In this way, we find exactly the solution of the problem and derive its vacuum states. In addition, we construct the coherent states in this background and establish the existence of squeezed states.

II. SCALAR FIELDS IN A FRW SPACE-TIME

The discovery of the cosmic radiation background and the observations of the expansion of the universe have established the Big Bang model as a viable model for the universe. An important principle, on which the cosmological model of the Big Bang is based, is that the universe is homogeneous and isotropic; moreover, the mass is uniformly distributed. The veracity of this hypothesis implies that the four-dimensional metric of the universe can be written as

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{1}$$

This metric is known in the literature as FRW metric. The scale factor $a(t)$ is the universal scale factor of the universe and k is the curvature of the space-time. The Gaussian curvature is given in terms of the scale factor by the relation

$$K(t) = \frac{k}{a(t)^2}, \tag{2}$$

where $k = -1, 0, 1$ means negative, zero, or positive curvature, respectively. In other words, when the curvature is negative, we say that the universe is open; when the curvature is null we say that the universe is flat and finally, when the curvature is positive, the universe is closed. If we admit that the universe is flat, i.e., $k = 0$, the metric of FRW (1) is reduced to

$$ds^2 = -dt^2 + a(t)^2 g_{ij} dx^i dx^j, \tag{3}$$

where g_{ij} is the metric of three-dimensional flat space-time. The Einstein-Hilbert action for a scalar field in a gravitational field is given by

$$S = \int d^4x\sqrt{-g}\mathcal{L}. \tag{4}$$

There are a large numbers of articles devoted to cosmological models with scalar fields. We choose a real field $\Phi(x_i, t)$, whose Lagrangian density is given by [18]

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}(\xi\mathcal{R} + m^2)\Phi^2, \tag{5}$$

where m is the mass of the field quanta, $\xi\mathcal{R}$ is the coupling between the scalar field and the gravitational field in which ξ is a numerical factor and $\mathcal{R}$ is the Ricci scalar. Following the procedure adopted for Bertoni *et al.* [19], we decompose the scalar field on a complete base $u_k(\mathbf{x}, t)$ with the spatial part written as a plane wave

$$u_k(\mathbf{x}, t) = \frac{1}{\sqrt{V}}\exp(i\mathbf{k}\cdot\mathbf{x})\frac{\phi_k^1 + i\phi_k^2}{\sqrt{2}}, \tag{6}$$

where we decompose ϕ into two parts, a real and another imaginary. The introduction of the volume guarantees the normalization. In this case, the action is rewritten as

$$S = \frac{1}{2}\sum_k\sum_{i=1,2}\int a^3 dt(\dot{\phi}_k^i - \omega_k^2\phi_k^{i2}), \tag{7}$$

where the "angular frequency" is given by

$$\omega_k^2 = \frac{k^2}{a^2} + m^2 + \xi\mathcal{R}. \tag{8}$$

For a scalar field theory, the Hamiltonian density $\mathcal{H}$ is given by

$$\mathcal{H} = \Pi\dot{\phi} - \mathcal{L}, \tag{9}$$

where Π is the canonical momentum conjugate to the field ϕ. It is obtained from the Lagrangian density by

$$\Pi = \frac{\delta\mathcal{L}}{\delta\dot{\phi}}. \tag{10}$$

On the other hand, from the action (7) we notice that the modes k and i are independent and may be considered separately, hence we need only to consider a given mode. Thus we can write the Hamiltonian density (9) as

$$\mathcal{H} = \sum_k\sum_{i=1,2}\mathcal{H}_k^i, \tag{11}$$

where for a mode i, k we have the Hamiltonian density given by

$$\mathcal{H}_k^i = \frac{1}{2a^3}(\Pi_k^{i2} + a^6\omega_k^2\phi_k^{i2}). \tag{12}$$

From the above Hamiltonian density we can find the classical equation of motion for the field ϕ as

$$\ddot{\phi}_k^i + 3\frac{\dot{a}}{a}\dot{\phi}_k^i + \omega_k^2\phi_k^i = 0. \tag{13}$$

By means of a canonical quantization, $\Pi_k^i \rightarrow -i\hbar\partial/\partial\phi_k^i$, we get the following Hamiltonian operator

$$\hat{\mathcal{H}}_k^i = \frac{1}{2a^3}\left(-\hbar^2\frac{\partial^2}{\partial\phi_k^{i2}} + a^6\omega_k^2\phi_k^{i2}\right). \tag{14}$$

Note that Eq. (13) is a classical equation for the harmonic oscillator with time-dependent mass and frequency. It may be regarded as obtained from the Hamiltonian

$$H(t) = \frac{p^2}{2m(t)} + \frac{1}{2}m(t)\omega^2(t)q^2(t), \tag{15}$$

for the mode $k_i (i = 1, 2, 3)$, with the mass $m(t) = a^3$ and $\omega^2(t)$ as defined previously. In order to quantize this system, let us impose to each mode of the field the canonical commutation relation $[q, p] = i\hbar$, where the $p's$ and $q's$ corresponding to different modes commute. In fact the modes of oscillation of scalar waves in the time-dependent background can be described in analogy with a system of parametric oscillator. Although the physical idea is not new, the mapping of Eq. (14) in Eq. (15) gives an alternative description which can draw upon developments in quantum optic. Using this analogy of Eq. (14) with Eq. (15) we can quantize Eq. (15) with well-known methods. This same physics idea is contained in Parker's [14] and Zeldovich [20] papers. Several authors have used squeezed and coherent states as probes to analyze some cosmological problem such as cosmological particle creation [12], entropy generation [21] detection of gravitational waves [22], and inflationary scenario [23]. We use this analogy and employ the Lewis-Riesenfeld methods to solve the Schödinger equation for this Hamiltonian; in this way we have quantized the Hamiltonian of the system under consideration.

III. SCALAR FIELDS AND EXACT INVARIANTS

In this section we will use the Lewis and Riesenfeld invariant method to obtain the exact solution of the fields that satisfies the Hamiltonian (12). The quantization of Hamiltonian is obtained solving the following Schrödinger equation

$$i\hbar\frac{\partial\psi(\phi_k^i, t)}{\partial t} = \mathcal{H}_k^i\psi(\phi_k^i, t), \tag{16}$$

for the Hamiltonian (12). Now, according to the invariant operator method of Lewis and Riesenfeld [24], a solution of the Schrödinger equation with time-dependent Hamiltonian is found if a nontrivial Hermitian operator $I_k^i(t)$ exists and satisfies the equation

$$\frac{dI_k^i(t)}{dt} = \frac{1}{i\hbar}[\mathcal{H}_k^i, I_k^i] + \frac{\partial I_k^i}{\partial t} = 0. \tag{17}$$

The above condition (17) allows us to write the solutions of the time-dependent Schrödinger equation as

$$\psi_\lambda(\phi_k^i, t) = e^{i\alpha_\lambda(t)}\chi_\lambda(\phi_k^i, t), \tag{18}$$

where $\chi_\lambda(\phi_k^i, t)$ is an eigenfunction of $\hat{I}_k^i(t)$ with a time-dependent eigenvalue λ and α_λ is a phase function which satisfies the equation

$$\hbar\frac{d\alpha_\lambda}{dt} = \left\langle\chi_\lambda \,\middle|\, i\hbar\frac{\partial}{\partial t} - \mathcal{H}_k^i \,\middle|\, \chi_\lambda\right\rangle. \tag{19}$$

On the other hand, it is known that an invariant for the system specified by the Hamiltonian (12) that satisfies the condition (17) is given by [25–27]

$$I_k^i(t) = \frac{1}{2}\left[\left(\frac{\phi_k^i}{\rho_k}\right)^{1/2} + (\rho_k\Pi_k^i - a^3\dot{\rho}_k\phi_k^i)^2\right], \tag{20}$$

where $\rho_k(t)$ is a real function satisfying the following auxiliary nonlinear equation

$$\ddot{\rho}_k + 3\frac{\dot{a}}{a}\dot{\rho}_k = \frac{1}{a^6\rho_k^3}. \tag{21}$$

The eigenstates of $I_k^i(t)$ corresponding to the time-independent eigenvalue λ_n are solutions of the equation

$$I_k^i(t)\chi_n(\phi_k^i, t) = \lambda_n\chi_n(\phi_k^i, t), \tag{22}$$

where the eigenfunctions $\chi_n(\phi_k^i, t)$ satisfy the relation $<\chi_{n'}|\chi_n> = \delta_{n'n}$. In Eq. (22) we have suppressed the index k for χ_n. In order to solve the eigenvalue problem (22) we introduce the unitary transformation

$$\chi_n'(\phi_k^i, t) = \mathcal{U}\chi_n(\phi_k^i, t), \tag{23}$$

with the unitary operator $\mathcal{U}$ given by [27]

$$\mathcal{U} = \exp\left(-i\frac{a^3\dot{\rho}_k}{2\hbar\rho_k}\phi_k^{i2}\right). \tag{24}$$

Under this unitary transformation the eigenvalue equation is mapped to

$$I_k'^i(t)\chi_n'(\phi_k^i, t) = \lambda_n\chi_n'(\phi_k^i, t), \tag{25}$$

where

$$I_k'^i(t) = \mathcal{U}I_k^i(t)\mathcal{U}^\dagger. \tag{26}$$

By making use of Eq. (20) and (24) and after a direct calculation, we obtain that the operator $I_k'^i(t)$ in Eq. (26) can be expressed as

$$I_k'^i(t) = -\frac{\hbar^2}{2}\rho_k^2\frac{\partial^2}{\partial\phi_k^{i2}} + \frac{1}{2}\left(\frac{\phi_k^i}{\rho_k}\right)^2. \tag{27}$$

Setting $\sigma_k = \phi_k^i/\rho_k$, we can rewrite Eq. (25) in the form

$$\left[-\frac{\hbar^2}{2}\frac{\partial^2}{\partial\sigma_k^2} + \frac{\sigma_k^2}{2}\right]\varphi_n(\sigma_k) = \lambda_n\varphi_n(\sigma_k), \tag{28}$$

where φ_n is related to χ_n' by

$$\chi_n'(\phi_k^i, t) = \frac{1}{\sqrt{\rho_k}}\varphi_n(\sigma_k). \tag{29}$$

The solution of Eq. (28) is well-known and it is given by

$$\varphi_n(\sigma_k) = \left[\frac{1}{(\pi\hbar)^2 n! 2^n}\right]^{1/2}\exp(-\sigma_k^2/2\hbar)H_n(\sigma_k/\sqrt{\hbar}), \tag{30}$$

where

$$\lambda_n = \hbar(n + 1/2), \tag{31}$$

and H_n is the Polynomial of Hermite of order n. From the above results we can write the eigenfunctions χ_n as

$$\chi_n(\phi_k^i, t) = \left[\frac{1}{(\pi\hbar)^2 n! 2^n}\right]^{1/2} \exp\left[\frac{ia^3}{2\hbar}\left(\frac{\dot{\rho}_k}{\rho_k} + \frac{i}{a^3\rho_k^2}\right)\phi_k^{i2}\right] \times H_n\left(\frac{1}{\sqrt{\hbar}}\frac{\phi_k^i}{\rho_k}\right). \quad (32)$$

Now, after a straightforward evaluation of the matrix element of the right of Eq. (33) we get the phase functions as [27]

$$\alpha_n(t) = -(n + 1/2)\int_0^t \frac{1}{a(t')^3\rho(t')_k^2}dt'. \quad (33)$$

Therefore, the exact solutions of the Schrödinger Eq. (16) are

$$\psi_n(\phi_k^i, t) = \exp[i\alpha_n(t)]\left[\frac{1}{(\pi\hbar)^{1/2} n! 2^n \rho_k}\right]^{1/2} \times \exp\left[\frac{ia^3}{2\hbar}\left(\frac{\dot{\rho}_k}{\rho_k} + \frac{i}{a^3\rho_k^2}\right)\phi_k^{i2}\right] H_n\left(\frac{1}{\sqrt{\hbar}}\frac{\phi_k^i}{\rho_k}\right). \quad (34)$$

where the phase functions are given by Eq. (19). Now by analogy with the harmonic oscillator, we obtain the vacuum state corresponding to $n = 0$ which is given by

$$\psi_0(\phi_k^i, t) = \exp[i\alpha_0(t)]\left[\frac{1}{\rho_k^2\pi\hbar}\right]^{1/4} \times \exp\left[-\left(\frac{1}{\rho_k^2} - \frac{ia^3\dot{\rho}_k}{\rho_k}\right)\frac{\phi_k^{i2}}{2\hbar}\right], \quad (35)$$

where

$$\alpha_0(t)) = -\frac{1}{2}\int_0^t \frac{1}{a^3(t')\rho_k^2(t')}dt'. \quad (36)$$

The result (35) differs from that obtained in [19] for a phase factor $\alpha_0(t)$. Note that the above vacuum state is quite distinct from the time-independent harmonic oscillator one. Finally, we obtain that the general solution can be expressed as

$$\psi(\phi_k^i, t) = \sum_n c_n \psi_n(\phi_k^i, t), \quad (37)$$

with c_n constant.

IV. COHERENT AND SQUEEZED STATES

In this section, we construct coherent states and establish the existence of squeezed states for the system we are considering. To do so, let us consider the time-dependent creation and annihilation operators as

$$a_k^i = \left(\frac{1}{2\hbar}\right)^{1/2}\left[\left(\frac{\phi_k^i}{\rho_k} + i\rho_k\Pi_k^i\right)\right], \quad (38a)$$

$$a_k^{i\dagger} = \left(\frac{1}{2\hbar}\right)^{1/2}\left[\left(\frac{\phi_k^i}{\rho_k} - i\rho_k\Pi_k^i\right)\right], \quad (38b)$$

with $[a_k^i, a_k^{i\dagger}] = \delta_{k'k}$. In terms of these operators the expression for $I_k^{i'}$, Eq. (27), can be rewritten as

$$I_k^{i'} = \hbar\left(a_k^i a_k^{i\dagger} + \frac{1}{2}\right), \quad (39)$$

which is invariant, i.e., it satisfies Eq. (17). The coherent states for the invariant (39) have the form [27,28]

$$\varphi_\alpha(\sigma_k, t) = \exp(-|\alpha|^2/2)\sum_n \frac{\alpha^n}{\sqrt{n!}}\exp[i\alpha_n(t)]\varphi_n(\sigma_k), \quad (40)$$

where $\alpha_n(t)$ is given by Eq. (33) and α is a arbitrary complex number. Hence, we find that the coherent states for the system described by the Hamiltonian (12) are given by [27,28]

$$\chi_\alpha(\phi_k^i, t) = \frac{1}{\sqrt{\rho_k}}\exp\left(\frac{ia^3\dot{\rho}_k}{2\hbar\rho_k}\right)\varphi_\alpha(\sigma_k, t). \quad (41)$$

The eigenstates (41) satisfy the eigenvalue equation

$$b_k^i\chi_\alpha(\phi_k^i, t) = \alpha(t)\chi_\alpha(\phi_k^i, t), \quad (42)$$

where

$$b_k^i(t) = \mathcal{U}^\dagger a_k^i \mathcal{U} = \left(\frac{1}{2\hbar}\right)^{1/2}\left[\left(\frac{\phi_k^i}{\rho_k}\right) + i(\rho_k\Pi_k^i - a^3\dot{\rho}_k\phi_k^i)^2\right], \quad (43)$$

and

$$\alpha(t) = \alpha\exp(2i\alpha_0). \quad (44)$$

Note that the operators $b_k^i(t)$ and $b_k^{i\dagger}(t)$ factorize the invariant (20) as

$$I_k^i = \hbar\left(b_k^{i\dagger}b_k^i + \frac{1}{2}\right). \quad (45)$$

The expectation value of I_k^i and ϕ_k^i in the state $\chi_\alpha(\phi_k^i, t)$ are, respectively, given by

$$\langle I_k^i\rangle = \hbar(|\alpha|^2 + 1/2), \quad (46)$$

and

$$\langle\phi_k^i\rangle = \sqrt{2\hbar|\alpha|^2\rho_k}\sin[\Omega(t) + \delta], \quad (47)$$

where $\Omega(t) = -2\alpha_0(t)$ and δ is the argument of the complex number α. We further note that $<\phi_k^i>$ is solution of Eq. (13). After a straightforward calculation we find that the uncertainties in ϕ_k^i and π_k^i are

$$(\Delta\phi_k^i)^2 = \frac{\hbar}{2}\rho_k^2, \tag{48a}$$

$$(\Delta\Pi_k^i)^2 = \frac{\hbar}{2}\left(\frac{1}{\rho_k^2} + a^6\dot{\rho}_k^2\right). \tag{48b}$$

Thus, the uncertainty product is expressed by

$$(\Delta\phi_k^i)(\Delta\Pi_k^i) = \frac{\hbar}{2}(1 + a^6\rho_k^2\dot{\rho}_k^2)^{1/2}, \tag{49}$$

which coincides with that obtained in [19]. Here, we observe that the uncertainty relation (49), in general, does not attain its minimum value. This occurs because the states $\chi_\alpha(\phi_k^i, t)$ are equivalent to well-known squeezed states [29–33]. To see this more clearly, let us consider the operators A_k^i and $A_k^{i\dagger}$ defined by

$$A_k^i = \left(\frac{a^3\omega_k}{2\hbar}\right)^{1/2}\left(\phi_k^i + i\frac{\Pi_k^i}{a^3\omega_k}\right), \tag{50a}$$

$$A_k^{i\dagger} = \left(\frac{a^3\omega_k}{2\hbar}\right)^{1/2}\left(\phi_k^i - i\frac{\Pi_k^i}{a^3\omega_k}\right). \tag{50b}$$

These operators factor the Hamiltonian (12) as

$$\mathcal{H}_k^i = \hbar\omega_k\left(A_k^{i\dagger}A_k^i + \frac{1}{2}\right), \tag{51}$$

and are related to the operators b_k^i and $b_k^{i\dagger}$ by the Bogolubov transformations [17,32,33]

$$b_k^i = \mu(t)A_k^i + \nu(t)A_k^{i\dagger}, \tag{52a}$$

$$b_k^{i\dagger} = \mu^*(t)A_k^{i\dagger} + \nu^*(t)A_k^i, \tag{52b}$$

whose coefficients are expressed by

$$\mu(t) = \left(\frac{1}{4\omega_k a^3}\right)^{1/2}\left[\frac{1}{\rho_k} - ia^3\dot{\rho}_k + a^3\rho_k\omega_k\right], \tag{53a}$$

$$\nu(t) = \left(\frac{1}{4\omega_k a^3}\right)^{1/2}\left[\frac{1}{\rho_k} - ia^3\dot{\rho}_k - a^3\rho_k\omega_k\right]. \tag{53b}$$

A straightforward calculation shows that

$$|\mu(t)|^2 - |\nu(t)|^2 = 1. \tag{54}$$

Therefore, from Eqs. (42), (52), and (53), we see that the coherent states $\chi_\alpha(\phi_k^i, t)$ are, by definition, equal to the well-known squeezed states [30–33]. On the other hand, in terms of coefficients $\mu(t)$ and $\nu(t)$, we can rewrite the uncertainties in ϕ_k^i and π_k^i for the state $\chi_\alpha(\pi_k^i, t)$ as [29]

$$(\Delta\phi_k^i)^2 = \frac{\hbar}{2\omega_k a^3}|\mu - \nu|^2, \tag{55a}$$

$$(\Delta\Pi_k^i)^2 = \frac{a^3\hbar\omega_k}{2}|\mu + \nu|^2, \tag{55b}$$

whence

$$(\Delta\phi_k^i)(\Delta\Pi_k^i) = \frac{\hbar}{2}|\mu - \nu||\nu + \mu|. \tag{56}$$

Note that the relation (56) is equivalent to Eq. (49).

V. CONCLUDING REMARKS

In this article we analyze the scalar quantum field in a FRW background using the time-dependent Lewis-Riesenfeld invariants. We also construct coherent states and establish the existence of squeezed states in this background. It is interesting note that even in this context we obtain states of quantum fields which have unequal uncertainties in two quadrature phases. This point is due to the fact that the squeezed sates are capable of improving the signal in a process of detection. We claim the attention that study of quantum fields in a curved FRW space-time can be used in the analysis of the detection of relic gravitons created from zero point fluctuations of the gravitational field in the course of cosmological expansion [12]. In this way, they may provide extremely valuable information on the physical condition in early universe.

In conclusion, we would like to remember that the Bogolubov transformation is the basic concept in the theory of particles creation in external fields. On the other hand, the created particles do exist in squeezed quantum states [12] and we have shown that with the help of the method of Lewis and Riesenfeld the obtention of squeezed states is simplified. Further, the mapping of Eq. (14) in Eq. (15) is responsible for this facility and it seems that this one can be made for any system described by quadratic Hamiltonians [16]. Thus, this approach may be used to describe scalar field, gravitons, or gauge invariants cosmological perturbations[16,34]. Finally, we observe that the invariant method used in this paper can be employed in the study of a variety of physical system where the language of squeezed states can be employed, as, for example, in the analyzes of production of particle in inflationary models [23] in which the inflaton can be represented in coherent and squeezed state formalism [35] and to investigate the Berry's [36,37] quantum phase in gravitational waves [12]. We hope to report on these possibilities in a future article.

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