One loop predictions of the finely tuned supersymmetric standard model

Asimina Arvanitaki, Chad Davis, Peter W. Graham, and Jay G. Wacker
Institute for Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305 USA
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We study the finely tuned supersymmetric standard model, recently proposed by Arkani-Hamed and Dimopoulos, at the one loop level. The runnings of the four gaugino Yukawa couplings, the μ term, the gaugino masses, and the Higgs quartic coupling are computed. The Higgs mass is found to be 130–170 GeV for $M_s > 10^6$ GeV. Measuring the Yukawa coupling constants at the 10% level can begin to constrain the supersymmetry breaking scale. Measuring the relationships between the couplings will provide a striking signal for this model.

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I. INTRODUCTION

Recently, there has been interest in studying a version of the supersymmetric standard model (SSM) where naturalness is no longer a guiding principle [1]. This comes at a time of several growing problems associated with the standard implementation of naturalness [2]. The most pressing naturalness issue is the cosmological constant, the experimental value of which appears to be fine-tuned to one part in 10^{120} and completely dwarfs the standard hierarchy problem. While it is conceivable that these two separate fine tunings are divorced, they could also be linked with weak anthropicism [3]. There are other problems with the SSM directly related to particle physics issues, such as the nondiscovery of superpartners at LEP or Fermilab, the lack of flavor-changing neutral-current interactions, the nondiscovery of the Higgs, and the nondiscovery of proton decay. All of these increase the fine-tuning required in the SSM. Every one of these phenomenological problems is ameliorated by decoupling the scalars [1,4].

The two major successes of the SSM [5] are gauge coupling unification [6] and a viable dark matter candidate. However, removing the scalars of the SSM does not significantly alter either of these predictions. If one is willing to ignore the original motivation for the SSM and decouple all but the one scalar Higgs doublet required for electroweak symmetry breaking, then one immediately has a phenomenologically viable model without the usual concerns of the SSM. The existence of light gauginos and Higgsinos is inferred indirectly through gauge coupling unification and evidence for dark matter, which point to these states having mass in the 100 GeV to 3 TeV range.

There is a universal form of the low energy effective action for the finely tuned SSM that preserves gauge coupling unification and dark matter and has five relevant interactions—four Yukawa couplings from the gauginos and the Higgs quartic coupling. These are predicted by high energy supersymmetry from four parameters: the standard model gauge couplings g_1 and g_2, $\tan\beta$, and the scale of the scalar masses, M_s. At the LHC or a future LC (NLC) it may be possible to measure five new couplings and explain them from only two new parameters.

In this note, we calculate the one loop beta functions of these five couplings, as well as those of the μ term and the gaugino masses. We then run these couplings from their supersymmetry (SUSY) values at M_s down to the top mass m_t [7–10]. We do not compute threshold corrections because they are subdominant to the large logarithms. We define two different effective $\tan\beta$ that are related to the gaugino-Higgsino Yukawa coupling. By renormalization-group (RG) evolving these to a higher scale it is possible to determine the scale of SUSY breaking.

II. ONE LOOP BETA FUNCTIONS

The tree-level Lagrangian contains the terms

$$\begin{aligned}\mathcal{L} \supset\ & \tilde{B}(\kappa_1' h^\dagger \tilde{H}_1 + \kappa_2' h \tilde{H}_2) + \tilde{W}^a(\kappa_1 h^\dagger \tau^a \tilde{H}_1 \\ & + \kappa_2 \tilde{H}_2 \tau^a h) - \lambda |h|^4 - \mu \tilde{H}_1 \tilde{H}_2 \\ & - \tfrac{1}{2}(M_1 \tilde{B}\,\tilde{B} + M_2 \tilde{W}\,\tilde{W} + M_3 \tilde{g}\,\tilde{g}). \end{aligned} \tag{1}$$

At the SUSY breaking scale, the following relations are satisfied:

$$\begin{aligned} &\kappa_1' = \sqrt{\frac{3}{10}} g_1 \sin\beta, \qquad \kappa_2' = \sqrt{\frac{3}{10}} g_1 \cos\beta, \\ &\kappa_1 = \sqrt{2} g_2 \sin\beta, \qquad \kappa_2 = \sqrt{2} g_2 \cos\beta, \\ &\lambda = \frac{\frac{3}{5} g_1^2 + g_2^2}{8} \cos^2 2\beta. \end{aligned} \tag{2}$$

However, these couplings run in a nonsupersymmetric fashion from the SUSY breaking scale down to low energies.

All of the following results are given with SU(5) normalization of the hypercharge. The beta function for the Higgs quartic coupling is

$$16\pi^2\beta_\lambda = +24\lambda^2 - 6y_{\rm top}^4 + 12\lambda y_{\rm top}^2 + \frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 - \frac{9}{5}\lambda g_1^2 - 9\lambda g_2^2 - \frac{5}{8}(\kappa_1^4 + \kappa_2^4) - \frac{1}{4}\kappa_1^2\kappa_2^2 - 2(\kappa_1'^2 + \kappa_2'^2)^2 - (\kappa_1\kappa_1' + \kappa_2\kappa_2')^2 + 3\lambda(\kappa_1^2 + \kappa_2^2) + 4\lambda(\kappa_1'^2 + \kappa_2'^2). \tag{3}$$

The beta function for the top Yukawa coupling is

$$16\pi^2\beta_{y_{\rm top}} = \frac{9}{2}y_{\rm top}^3 - y_{\rm top}\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right) + \frac{3}{4}y_{\rm top}(\kappa_1^2 + \kappa_2^2) + y_{\rm top}(\kappa_1'^2 + \kappa_2'^2). \tag{4}$$

As a check, when all κ's are set to zero, these β functions reproduce those of the standard model [11]. The beta function for the bino Yukawa coupling is

$$16\pi^2\beta_{\kappa_1'} = 3\kappa_1' y_{\rm top}^2 - \kappa_1'\left(\frac{9}{20}g_1^2 + \frac{9}{4}g_2^2\right) + \frac{5}{2}\kappa_1'^3 + 4\kappa_1'\kappa_2'^2 + \frac{9}{8}\kappa_1'\kappa_1^2 + \frac{3}{4}\kappa_1'\kappa_2^2 + \frac{3}{2}\kappa_1\kappa_2\kappa_2' \tag{5}$$

and similarly for κ_2' after changing $\kappa_1 \leftrightarrow \kappa_2$ and $\kappa_1' \leftrightarrow \kappa_2'$.

The beta function for the wino Yukawa coupling is

$$16\pi^2\beta_{\kappa_1} = 3y_{\rm top}^2\kappa_1 - \kappa_1\left(\frac{9}{20}g_1^2 + \frac{33}{4}g_2^2\right) + \frac{11}{8}\kappa_1^3 + \frac{3}{2}\kappa_1\kappa_1'^2 + \frac{1}{2}\kappa_1\kappa_2^2 + \kappa_1\kappa_2'^2 + 2\kappa_1'\kappa_2\kappa_2' \tag{6}$$

and similarly for κ_2 after changing $\kappa_1 \leftrightarrow \kappa_2$ and $\kappa_1' \leftrightarrow \kappa_2'$.

The beta function for the μ term is

$$16\pi^2\beta_\mu = -\mu\left(\frac{9}{10}g_1^2 + \frac{9}{2}g_2^2\right) + \frac{3}{2}\kappa_1\kappa_2M_2 + 2\kappa_1'\kappa_2'M_1 + \frac{3}{8}\mu(\kappa_1^2 + \kappa_2^2) + \frac{1}{2}\mu(\kappa_1'^2 + \kappa_2'^2). \tag{7}$$

The beta functions for the gaugino masses are

$$16\pi^2\beta_{M_1} = 8\mu\kappa_1'\kappa_2' + 2M_1(\kappa_1'^2 + \kappa_2'^2), \tag{8}$$

$$16\pi^2\beta_{M_2} = -12g_2^2M_2 + 2\mu\kappa_1\kappa_2 + \tfrac{1}{2}M_2(\kappa_1^2 + \kappa_2^2), \tag{9}$$

$$16\pi^2\beta_{M_3} = -18g_3^2M_3. \tag{10}$$

In the following sections, we run the Yukawa couplings and the mass terms from the SUSY breaking scale down to the low scale. We examine the behavior of the various parameters at the low scale as a function of M_s.

III. HIGGS MASS

The Higgs quartic coupling at M_s depends only on $\cos 2\beta$ and M_s and can easily be run down with the beta functions of the previous section. We find that the Higgs is heavier than in the usual SSM with low-scale SUSY breaking [10,12]. The dimensionful A terms and μ term are around the weak/dark matter scale and are small in comparison to the SUSY breaking scale. They give finite threshold effects to the Higgs quartic coupling that are $\mathcal{O}(A^2/M_s^2)$ and can be neglected in this model. We have used a top mass of 178.0 ± 4.3 GeV [13]. The $\overline{MS}$ top Yukawa coupling was set to $y_t = 0.99 \pm 0.02$ by the relation [10,11]

$$m_t = y_t v\left(1 + \frac{16}{3}\frac{g_3^2}{16\pi^2} - 2\frac{y_t^2}{16\pi^2}\right). \tag{11}$$

For a SUSY breaking scale of 10^9 GeV, we find that the Higgs mass varies from 140 to 165 GeV as $\cos 2\beta$ goes from 0 to 1 at the high scale. The Higgs mass as a function of M_s is shown in Fig. 1 for $\tan\beta = 1$ and $\tan\beta = 50$. For values of $\tan\beta$ between 1 and 50, the Higgs mass is between the bounds shown. The Higgs quartic coupling is insensitive to $\tan\beta$ for large $\tan\beta$.

Experimental uncertainties in y_t and g_3 lead to an uncertainty in the prediction of the Higgs mass as shown by the wide bands in Fig. 1. The error in the top mass dominates while the uncertainty due to g_3 is approximately one-tenth as large. As a test of the theoretical uncertainty, each $\mathbf{5} \oplus \bar{\mathbf{5}}$ fermion added in at the TeV scale increases the Higgs mass by 0.2% for $M_s = 10^9$ GeV.

A future linear collider may be able to measure the Higgs mass to a precision of 100 MeV, the top mass to 200 MeV, and α_s to 1% [14]. The narrow bands in Fig. 1 show the uncertainty in the Higgs mass prediction using these more precise measurements and assuming the current central value. The small error on the Higgs mass measurement could allow the most precise determination of the SUSY breaking scale within the context of this model. If $\tan\beta$ is measured to 50%, M_s will be known to within an

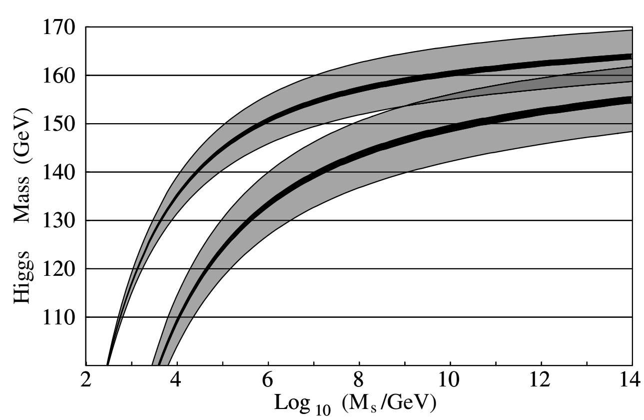

FIG. 1. The Higgs mass as a function of the SUSY breaking scale $\log_{10}(M_s/\text{GeV})$. The upper bands are for $\tan\beta(M_s) = 50$ and the lower ones are $\tan\beta(M_s) = 1$. The width of each gray band is the experimental uncertainty, mainly due to m_t. The width of each black band is the uncertainty when expected improvements from a future linear collider are taken into account.

order of magnitude. Although the bands in Fig. 1 asymptote at high scales, making M_s difficult to determine from the Higgs mass, we do not expect M_s to be greater than 10^{13} GeV [1].

IV. YUKAWA COUPLINGS AND MASS TERMS

The gaugino couplings are set at M_s by Eq. (2) and RG evolved to m_t. There are two separate low energy definitions of $\tan\beta$,

$$\tan\beta_{\rm low}(m) = \frac{\kappa_1(m)}{\kappa_2(m)}, \qquad \tan\beta'_{\rm low}(m) = \frac{\kappa'_1(m)}{\kappa'_2(m)}, \quad (12)$$

that run from equal values at the SUSY breaking scale. Running up from the weak scale to the point where they unify provides a clear determination of the SUSY breaking scale (Fig. 2). If the couplings could be measured to 10% at a future LC [15], this would determine M_s to within a few orders of magnitude. Note that there are fixed points in the evolution of some of the $\tan\beta$'s at $\tan\beta = 0, 1, \infty$. However, the gaugino couplings do change as M_s is changed and therefore can provide a useful measure of M_s even when $\tan\beta_{\rm low}$ does not change significantly with M_s.

The Yukawa couplings run significantly from their supersymmetric values (Fig. 3). We find that, for $\tan\beta \gtrsim 5$, the ratios $\kappa(m_t)/\kappa(M_s)$ are relatively unaffected by changes in $\tan\beta$. The four Yukawa couplings and the Higgs quartic are five independently measurable parameters that are determined by the scale of SUSY breaking and $\tan\beta$. Thus, this model predicts that these five couplings will satisfy three relations at the low scale.

Finally, using the calculated β functions, the running values of μ and the gaugino masses can be found. As a simple example, we set all four masses equal to 100 GeV at M_s and then run them down to the low scale. As shown in Fig. 4, the gluino mass increases greatly at the low scale (to $\sim$400 GeV for $M_s = 10^9$ GeV). M_2 and μ increase modestly while M_1 decreases slightly. The running of the gluino mass depends only on g_3 and M_3, so the ratio $M_3(m_t)/M_3(M_s)$ is independent of the specific values chosen for M_3 and $\tan\beta$. Although the runnings of μ, M_1, and M_2 are more complicated, they are relatively insensitive to changes of $\tan\beta$. We expect μ and the gaugino masses to be of the same order as the weak scale [1].

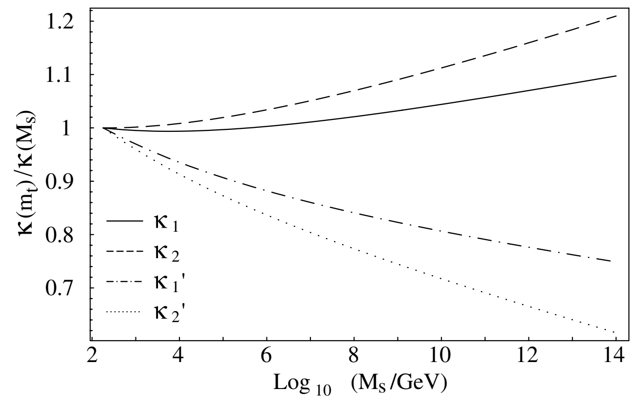

FIG. 3. The ratio $\kappa(m_t)/\kappa(M_s)$ as a function of M_s for fixed $\tan\beta(M_s) = 5$.

V. CONCLUSION

We have computed the one loop leading log running for the finely tuned SSM where the scalars are much heavier than the weak scale. We find that the Higgs mass is in the 140–165 GeV range at $M_s = 10^9$ GeV, depending on $\tan\beta$. The Higgs mass should be calculated at the two loop level including one loop threshold effects for a more exact prediction. The gaugino Yukawa couplings were found to run significantly, and, if measured to 10% accuracy at an NLC, could determine the scale of SUSY breaking to within a few orders of magnitude. A measurement of

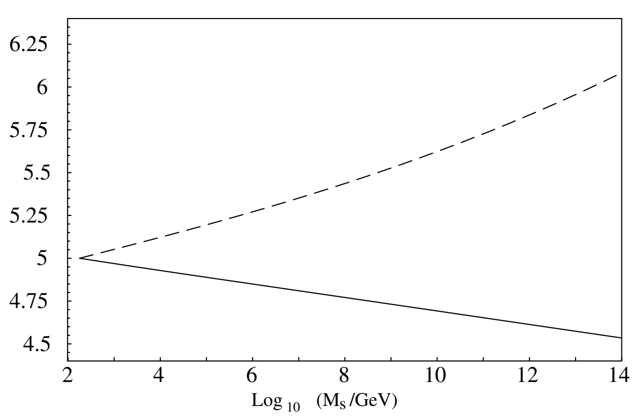

FIG. 2. The solid line shows $\tan\beta_{\rm low}(m_t)$ as a function of M_s. The dashed line is for $\tan\beta'_{\rm low}(m_t)$. Here $\tan\beta(M_s) = 5$.

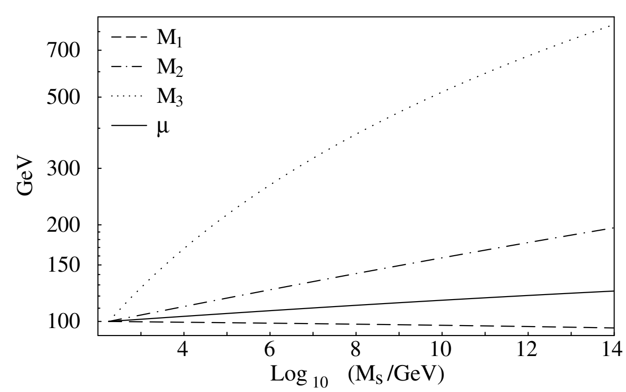

FIG. 4. The gaugino masses and μ evaluated at m_t as a function of M_s for fixed $\tan\beta(M_s) = 5$.

the Higgs mass and $\tan\beta$ could provide an even better estimate of M_s but does not verify the model. More work is needed to determine how effectively the LHC and NLC will be able to extract the gaugino Yukawa couplings, but a measurement of the relationships between these couplings would provide a phenomenal signal of high scale supersymmetry.

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