

Update of the Hagedorn mass spectrum

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We present an update of the Hagedorn hypothesis of the exponential growth of the number of hadronic resonances with mass. We use the newest available experimental data for the nonstrange mesons and baryons, as well as fill in some missing states according to the observation that the high-lying states form chiral multiplets. The results show, especially for the case of the mesons, that the Hagedorn growth continues with the increasing mass, with the new states lining up along the exponential growth.

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The Hagedorn hypothesis [1–3] of the exponential growth of the number of hadronic resonances with mass is one of the most fundamental issues in particle physics. The formula for the asymptotic dependence of the density of hadronic states on mass, namely,

$$\rho(m) = f(m) \exp(m/T_H), \quad (1)$$

where $f(m)$ denotes a slowly varying function and T_H is the *Hagedorn temperature*, has gained a lot of attention due to its appealing simplicity, fundamental character, support from the experimental data and theoretical approaches, as well as because of its relevance to the phenomenology of particle production, in particular, concerning the possible phase transition from the hadron gas to the quark-gluon plasma [4–6].

The purpose of this note is to present an update of the experimental verification of Eq. (1). We supplement the data published in the Particle Data Tables [7] with the new experimental information [8,9], as well as add theoretically predicted new states belonging to chiral multiplets [10–14]. Although the appearance of some of these states has not been verified experimentally yet, their existence follows from the recent theoretical findings that the high-lying particle spectrum essentially has the features of *restored chiral symmetry* [10].

The results are shown for the nonstrange mesons and baryons, where the new data are available. The paper has no pretense of presenting new models or ideas; nevertheless, due to the fundamental nature of the problem related to basic ideas behind the formation of bound states and resonances in particle physics, the results of our simple compilation should be of interest for the community. We include the new experimental results and show that the new data are important in the verification of Eq. (1). The new results extend significantly, at least for the mesons, the range of fiducial range of the Hagedorn

hypothesis. While with the data listed in the 1998 edition of the Particle Data Tables [7] used in [15–17] the exponential growth for nonstrange mesons could be observed up to the masses of about 1.8 GeV, now it continues higher up, until about 2.3 GeV.

We start with a very brief reminder of the history of the Hagedorn hypothesis (for much more complete historical presentations we refer the reader to Hagedorn's original lecture [3] and to a tribute article by Ericson and Rafelski [18]). Equation (1) was originally proposed to explain the spectra in the p - p and π - p scattering [1,19]. Later, it was obtained from the statistical bootstrap models [2,20–22]. Subsequently, it gained convincing support from the dual string models [23–26]. It is worthwhile to recall that in the 1960s, when the original Hagedorn idea was formed, very few hadronic states were known, up to the mass of the Δ isobar. More and more states have been accumulated over the years, thus many more systematic studies were possible, such as, for instance, the analysis of Ref. [27] and of Ref. [15], where two of us (W. B. and W. F.) pointed out the different growth rate of mesons and baryons, as well as demonstrated the universality of the Hagedorn temperatures with strangeness. The faster growth of the baryon spectrum was also noted in Ref. [28]. Moreover, a complete treatment of hadronic resonances is the basic ingredient of the successful models of hadron production in heavy-ion collisions at the RHIC energies [29,30].

After many dormant years with essentially no incoming new data, a recent systematic partial wave analysis of the $\bar{p}p$ annihilation at LEAR has revealed a lot of new meson states in the mass range 1.8–2.4 GeV [8,9]. These new experimental results turned out to be in line with the proposed idea that the spontaneously broken chiral symmetry of QCD should be effectively restored in the highly excited hadrons (one terms this phenomenon as the *chiral symmetry restoration of the second kind*) [10–12]. This kind of chiral symmetry restoration implies that the excited hadron states fill out multiplets of the chiral $U(2)_L \times U(2)_R$ group. Indeed, the newly discovered me-

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son states [8,9] turned out to systematically fall into almost degenerate chiral multiplets with a few missing states yet to be discovered [13,14].

In this note we extend the analysis of Refs. [15,16] and include all mesons listed in Refs. [13,14]. We stress that in addition to the experimental states which have been reported in Refs. [8,9], we add a few still missing states (marked with the question signs in Refs. [13,14]) and reconstruct their energies according to the known energies of their chiral partners. We consider only the $J = 0$, 1, 2, and 3 states, where the experimental information is rather complete.

In addition to these states, we also consider the states with hidden strangeness, i.e., composed of the $\bar{s}s$ pairs. These states could not be seen in $\bar{p}p$. Hence, here our procedure is somewhat more speculative. We assume that any isosinglet $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$, which is experimentally seen in $\bar{p}p$, should be accompanied by an $\bar{s}s$ state with the mass approximately 200 MeV higher. Hence, given the complete amount of the $\bar{n}n$ states listed in Refs. [13,14], we add the corresponding $\bar{s}s$ states.

Rather than comparing the density of states $\rho(m)$ itself to the data, it is customary to form the accumulated number of states of mass lower than m ,

$$N_{\text{exp}}(m) = \sum_i g_i \Theta(m - m_i), \quad (2)$$

where $g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy of the i th state, and m_i is its mass. The theoretical counterpart of Eq. (2) is

$$N_{\text{theor}}(m) = \int_0^m \rho(m') dm'. \quad (3)$$

Working with $N(m)$ rather than $\rho(m)$ conveniently avoids the need of building histograms, but clearly it is a purely technical issue and the conclusions drawn below remain unchanged if one decides to work with $\rho(m)$ itself.

The results of our compilation for nonstrange mesons are shown in Fig. 1. The lines with steps correspond to Eq. (2). Above $m = 1.8$ GeV the curves split into three, with the lower one representing the compilation of Ref. [15] based on the 1998 review of the Particle Data Group (PDG) [7]. The middle curve contains in addition the new states listed in Refs. [13,14], while the top curve includes the new states as well as the hidden-strangeness states described above. It is clear from Fig. 1 that the included new states nicely line up along the exponential growth, thus extending the range of the Hagedorn hypothesis seen in the data. We also note that adding up the hidden-strangeness states has a much smaller effect than adding the states of Refs. [13,14], which is simply due to a lower isospin degeneracy factor.

The thin solid lines in Fig. 1 show the results of the exponential fits with $f(m) = \text{const}$ in Eqs. (1) and (3), which is the simplest choice. While for the old data the

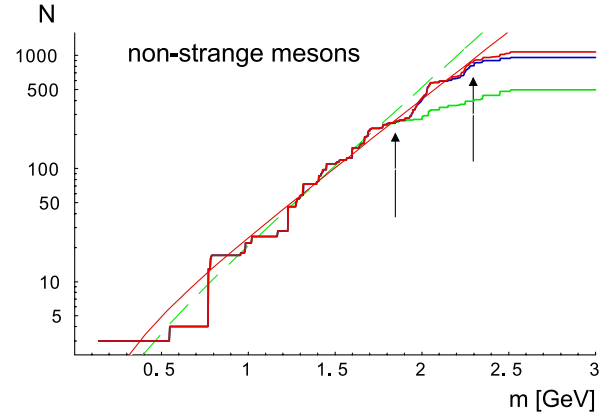


FIG. 1 (color online). Accumulated spectrum of nonstrange mesons plotted as a function of mass (steplike lines). The lower curve at high m corresponds to particles listed in the Particle Data Tables of Ref. [7], while the higher two curves include the new experimental and theoretical states as described in the text. The middle curve includes the states listed in Refs. [13,14], while the top curve adds in addition the states with hidden strangeness. The thin dashed (solid) line corresponds to the exponential fit to the spectra of the old (new) data. The arrows indicate the approximate upper values in m of the validity of the Hagedorn hypothesis for the old and new data, respectively.

least-squares method yields $\rho(m) = (2.84/\text{GeV}) \times \exp[m/(314 \text{ MeV})]$, with the states of Refs. [13,14] included, we obtain $\rho(m) = (4.73/\text{GeV}) \exp[m/(367 \text{ MeV})]$, and with the additional $\bar{s}s$ states we get $\rho(m) = (4.52/\text{GeV}) \exp[m/(362 \text{ MeV})]$. The fit was made up to $m = 1.8$ GeV with the old data and up to $m = 2.3$ GeV with the new data. The higher value for T_H obtained with the new data corresponds to the lower slope in Fig. 1. Certainly, the values of the fitted parameters should be taken with care, since they also reflect the assumed fitting range in m . It should also be noted that adding more states in the range around 2 GeV, when experimentally found, would increase the slope, thus decreasing T_H .

In this place the reader may be a bit surprised with the quoted high values of T_H , much higher than the typically cited values in the range of 200 MeV. The issue, as discussed in detail in Ref. [17], has to do with the choice of the “slowly varying” function $f(m)$. The point is that typical model predictions for this function are not so slowly varying in the range of data. For instance, with the original Hagedorn choice $f(m) = \text{const}/[m^2 + (500 \text{ MeV})^2]^{5/4}$ we get much lower values for T_H . With this form we obtain for the bottom to top curves of Fig. 1 the following values: $T_H = 196, 230,$ and 228 MeV, respectively. The choice of the fitting range in m is as stated above.

Now we pass to the case of the nonstrange baryons. With the help of identification of states in chiral multiplets [11], we add the missing states (marked with the question signs in [11]) on top of the states from PDG

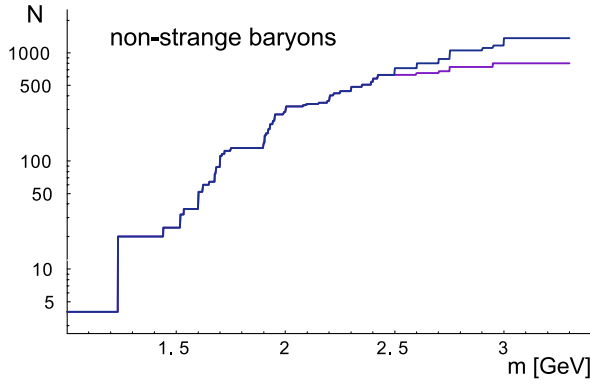


FIG. 2 (color online). Accumulated spectrum of nonstrange baryons plotted as a function of m . The lower curve at high m corresponds to older data of Ref. [7], while the higher curve includes the new states as described in the text.

[7] used in Ref. [15]. In this way we fill the chiral multiplets. The results of this procedure are shown in Fig. 2. We note that the effect of including these baryon states is less important than in the analogous procedure for the mesons. In the present case, we do not show the fit to the exponential formula, since it is difficult to line up the results along one straight line in a sufficiently broad range of m . Actually, with the present data one may see a straight line up to about $m = 2$ GeV, and possibly another straight line, with a lower slope, above. However, this may well be an artifact of missing data in the high-mass range.

Indeed, the parity doublets in N and Δ can be associated with the $(0, 1/2) \oplus (1/2, 0)$ representations for the nucleon spectrum, the $(0, 3/2) \oplus (3/2, 0)$ multiplets in the Δ spectrum, and with the $(1, 1/2) \oplus (1/2, 1)$ representations which combine the doublets in the nucleon and delta spectra. If all these multiplets are realized in nature, then the number of the states in the region above 2 GeV should be much larger than given in PDG. Unfortunately, this region has never been systematically explored in experiments.

The optimization for nonstrange baryons with the old data yields for the exponential form $\rho(m) = (1.33/\text{GeV}) \exp[m/(322 \text{ MeV})]$, and for the Hagedorn form $\rho(m) = (0.43/\text{GeV})/[m^2 + (500 \text{ MeV})^2]^{5/4} \exp[m/(219 \text{ MeV})]$. The fits were made in the mass range up to $m = 2.4$ GeV. Making use of the new data to produce the fits requires extending the range above 2.5 GeV, which, as has been pointed out, would carry ambiguities.

We now come back to the meson case of Fig. 1 wishing to present the data in a somewhat different manner. The problem of the presentation in the log scale, as in Fig. 1, is that the low-mass states are sparse, while the high-mass states are jammed up. For that reason in Fig. 3 we present the ratio of the experimental function (2) to the model function (3), with the choice $f(m) = \text{const}$ and the parameters at the fitted values quoted in the text. The ratio is

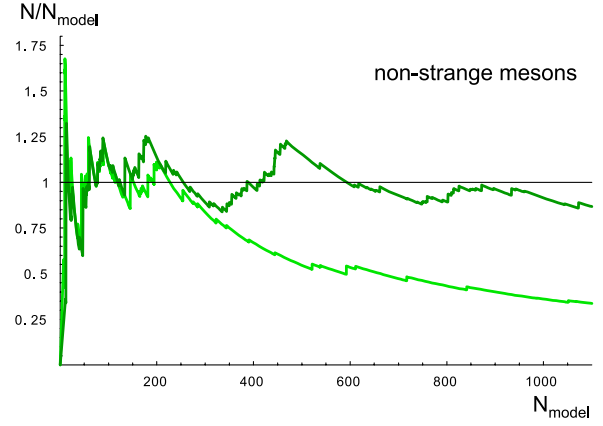


FIG. 3 (color online). The ratio of the accumulated spectrum of nonstrange baryons to the exponential fit, plotted as a function of the model number of states, N_{model} . The lower curve at high N_{model} corresponds to the older data of Ref. [7], while the higher curve includes the new states as described in the text. We note a sizable increase of the validity range of the Hagedorn hypothesis, which now holds for about 900 states, compared to 250 with the old data alone.

plotted as a function of the accumulated number of model states, N_{model} . If the Hagedorn hypothesis complies to the data, this ratio should be equal to unity. Indeed, this is so with the new data up to about 900 states, while with the old data it was true only up to about 250 states. Again, we see vividly that the inclusion of the new states significantly increases the range of validity, or verification, of Eq. (1). Figure 4 shows an analogous analysis for baryons, where we observe a much larger departure of the model from experiment than for mesons.

Finally, for the reader's convenience we overlay our results for the mesons and baryons in one plot of Fig. 5. As pointed out in Ref. [15], up to $m = 2$ GeV we note a faster growth rate for baryons than for mesons, which means two distinct Hagedorn temperatures for mesons and baryons. This is a prediction of dual string models

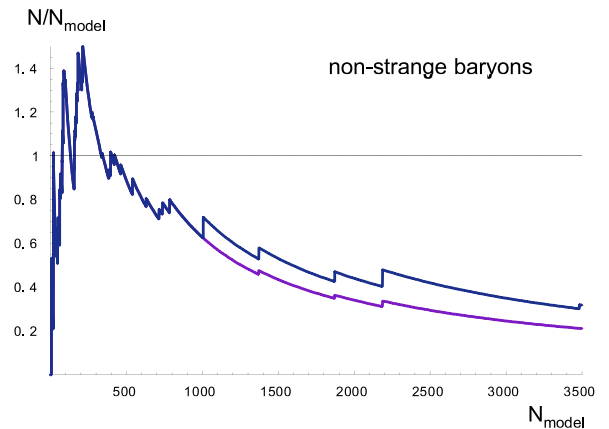


FIG. 4 (color online). Same as Fig. 3 for nonstrange baryons.

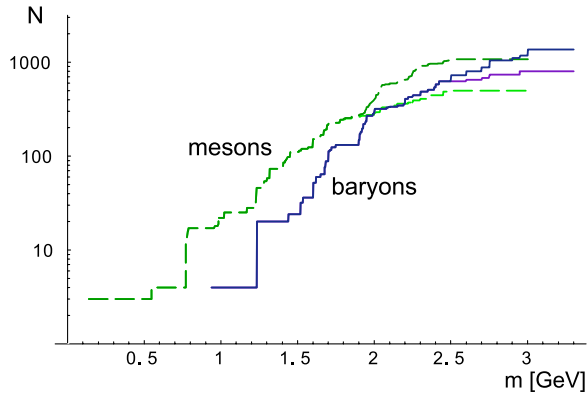


FIG. 5 (color online). Comparison of mesons (dashed lines) and baryons (solid lines) of Figs. 1 and 2.

(see Ref. [17] for a discussion). For higher masses this feature is no longer obvious, with more experimental information needed to clarify the issue.

In conclusion, we list our main observations:

- (i) The newly observed meson states lead to a continued exponential growth on the number of states with mass, in accordance to the Hagedorn

hypothesis, which now reaches up to masses of about 2.

- (ii) For the baryons the situation is less clear, with the exponential growth seen up to about 2 GeV.
- (iii) The inclusion of the missing states based on the identification of chiral multiplets helps to comply to the Hagedorn hypothesis at higher masses. We have shown that with the new states included the hypothesis hold for about first 900 states (with the spin degeneracy counted), while with the old data it hold only for about first 250 states.
- (iv) Certainly, more experimental data in the high-mass range are highly desired to investigate further and with greater detail the hadron spectroscopy.

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