A comparison of ultraviolet sensitivities in universal, nonuniversal, and split extra dimensional models

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(Received 24 September 2004; published 23 December 2004)

We discuss the origin of ultraviolet sensitivity in extra dimensional theories, and compare and contrast the cutoff dependences in universal, nonuniversal, and split five-dimensional models. While the gauge bosons and scalars are in the five-dimensional bulk in all scenarios, the locations of the fermions are different in different cases. In the universal model all fermions can travel in the bulk, in the nonuniversal case they are all confined at the brane, while in the split scenario some are in the bulk and some are in the brane. A possible cure from such divergences is also discussed.

I. INTRODUCTION

TeV scale higher dimensional theories [1] have been investigated from the perspectives of high energy experiments, phenomenology, string theory, cosmology, and astrophysics. From a four-dimensional (4d) point of view, a higher dimensional field appears as a tower of 4d Kaluza-Klein (KK) states labeled by (n). The multiplicity of KK states render all higher dimensional theories nonrenormalizable. These are all effective theories, parametrized by two additional quantities: the radius of compactification (R) and the ultraviolet (UV) cutoff scale $M_S = (n_S/R)$. Even within the context of such an effective framework, it is important to ask what is the UV sensitivity of such a theory, i.e., approximately up to what scale one can perform a perturbative calculation. Admittedly, TeV scale extra dimensional theories do not solve the hierarchy problem in a strict sense. But if particles with nonzero gauge quantum numbers have access to the extra dimension then experimental bounds push the inverse radius to at least a few hundred GeV or approximately a TeV. Such theories may constitute the basic building block for relatively more realistic models. Instead of looking for such realistic constructions, all we aim in this paper is to consider simple and analytically tractable, nevertheless experimentally allowed, toy scenarios and perform an illustrative analysis of how the multiplicity of equispaced KK states contribute to the nonrenormalizibility of such theories. Here we consider three scenarios, described below, and compare their UV cutoff dependences with respect to different processes. We restrict to only one extra dimension and assume a Z_2 discrete symmetry, i.e., the extra dimension is S^1/Z_2.

(1) **Universal Extra Dimension (UED):** All particles are allowed to access the extra dimension. Its implications to oblique electroweak parameters [2], flavor changing neutral current processes [3–5], $Z \to b\bar{b}$ decay [6], and other phenomenological processes [2,7] have been studied. All one-loop processes turn out to be finite because of a cancellation between wave function renormalizations and

vertex corrections. There is no dependence on M_S.

(2) **Nonuniversal Extra Dimension (NUED):** Fermions are localized at the 4d brane, while all bosons reside in the bulk. This is motivated [8] from a stringy perspective that chiral matters should be placed in the twisted sector while non-chiral states can travel in the bulk. Constraints from electroweak observables were placed on this scenario in [9]. This has also been studied in the context of $Z \to b\bar{b}$ decay and Kaon and B meson mixings [10]. In a previous publication [11], we probed the root cause of UV sensitivity in this scenario with respect to some electroweak loop processes, especially $B_d \to l^+ l^-$.

(3) **Split:** All bosons are in the bulk, but fermions are treated differently in the sense that some fermions are in the bulk but some are confined to the brane. Placing the fermions at different locations helps to induce flavor structure [12,13] by generating different Yukawa suppression factors for different fermions.[1] *We define our split scenario as the one which has the first two generations of fermions in the bulk and the third in the brane.* Roughly, this picture is motivated in $S^1/(Z_2 \times Z_2')$ GUT scenarios [14] for keeping consistencies with proton stability and b-τ unification. Neither is the way we have defined our split scenario the only consistent bottom-up approach, nor can we justify it to follow from some more fundamental consideration. All we want to emphasize is that although this scenario is less elegant and rather *ad hoc*, nevertheless it is neither more nor less realistic than either UED or NUED scenario. It is all the more important to study its UV sensitivity, which is different from that of UED or NUED, for reasons that will be clear as we go along. We consider it to be an illustrative example, and its other variants

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[1]In Ref. [13], splitting refers to the idea of placing different fermions in different places inside a thick brane.

are not expected to provide additional insights, so we do not consider them.

The crucial issue that controls the UV sensitivities is the question of KK momentum conservation. For compact direction, momentum becomes discrete (n/R) but still remains conserved.[2] In UED scenario it is always conserved. In split and NUED scenarios, the *localization* of some or all fermions at the brane causes KK number nonconservation for a brane-localized interaction. The issue of KK number conservation or nonconservation is intimately linked to the occurrence of a single or multiple KK sum in a loop integral involving KK modes in the internal lines. This aspect constitutes the prime criterion to judge whether the theory would be well-behaved or UV sensitive. An illustrative example is given below.

Consider a conventional bosonic 4d propagator and its KK-towering. The modification is as follows:

$$(k_E^2 + M^2)^{-1} \longrightarrow \sum_{n=-\infty}^{\infty} (k_E^2 + M^2 + n^2/R^2)^{-1}$$
$$= (\pi R/k_E') \coth(\pi R k_E'), \qquad (1)$$

where k_E is an Euclidean four momentum and $k_E' = \sqrt{k_E^2 + M^2}$. For large argument coth function goes like unity. This means that a sum over KK modes reduces the power of k_E in the denominator. Therefore, if a 4d loop diagram is log divergent in the UV limit, then KK-towering for only one propagator turns it linearly divergent. If two propagators are separately KK-summed (i.e., n_1 and n_2 are independent), then a logarithmically diver-

gent diagram becomes quadratically divergent, and so on. On the other hand, if n_1 and n_2 are not independent by virtue of KK number conservation, then the divergence will be less than quadratic. A word of caution is in order in this context. Although an infinite KK summation before actually performing the 4d momentum (k_E) integration is analytically doable providing intuitive insight into the increasing hardness of the divergence caused by the towering, the actual numerical coefficient of the divergence turns out to be physicswise incorrect if infinite summation is performed first and the 4d integration second (this point is emphasized in a specific case in footnote 3). Towards the end, we discuss a possible mechanism which can cure such divergences. Incidentally, we do assume that gravitational interaction becomes important at a scale which is considerably higher than M_S so that we can safely neglect their contribution.

II. THE SPLIT SCENARIO

In this section, we explicitly write down the KK expansions of the higher dimensional gauge, scalar, and fermion fields in the split scenario, when viewed from 4d perspective.

(1) The extra dimension (y) is compactified on a circle of radius $R = M_c^{-1}$ and y is identified with $-y$, i.e., it corresponds to an orbifold S^1/Z_2.

(2) The gauge bosons $A^M(x, y)$, a generic scalar doublet $\phi(x, y)$, and the first two generations of quarks and leptons are 5d fields. The 5d fields can be Fourier expanded in terms of 4d KK fields as:

$$A^\mu(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}} A_{(0)}^\mu(x) + \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} A_{(n)}^\mu(x) \cos\frac{ny}{R}, \qquad A^5(x, y) = \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} A_{(n)}^5(x) \sin\frac{ny}{R},$$

$$\phi^+(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}} \phi_{(0)}^+(x) + \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \phi_{(n)}^+(x) \cos\frac{ny}{R}, \qquad \phi^-(x, y) = \frac{2}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \phi_{(n)}^-(x) \sin\frac{ny}{R},$$

$$\mathcal{Q}_i(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}} \left\{ (u_i, d_i)_L(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_L \mathcal{Q}_{iL}^{(n)}(x) \cos\frac{ny}{R} + P_R \mathcal{Q}_{iR}^{(n)}(x) \sin\frac{ny}{R} \right] \right\},$$

$$\mathcal{U}_i(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}} \left\{ u_{iR}(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_R \mathcal{U}_{iR}^{(n)}(x) \cos\frac{ny}{R} + P_L \mathcal{U}_{iL}^{(n)}(x) \sin\frac{ny}{R} \right] \right\},$$

$$\mathcal{D}_i(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}} \left\{ d_{iR}(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[P_R \mathcal{D}_{iR}^{(n)}(x) \cos\frac{ny}{R} + P_L \mathcal{D}_{iL}^{(n)}(x) \sin\frac{ny}{R} \right] \right\},$$

$$(2)$$

where $i = 1, 2$ correspond to first two generations. Above, $x \equiv x^\mu$ ($\mu = 0, 1, 2, 3$ denote the four noncompact space-time coordinates), y denotes the fifth (extra) compactified coordinate, and $M = 0, 1, 2, 3, 5$. The fields $\phi_{(n)}^\pm(x)$ are the 4d KK scalar fields, $A_{(n)}^\mu(x)$ are the 4d KK gauge fields, and $A_{(n)}^5(x)$ are the 4d KK scalar fields in the adjoint

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[2]In the UED scenario, what is actually conserved is the KK parity [15]. As a consequence, there can be mixing only among even states or only among odd states, and that too only at the orbifold fixed points. We do not consider such mixings as their effects for our processes would be tiny.

representation of the gauge group. The field $A^5(x, y)$ depends on sine of y to ensure its absence on the brane ($y = 0$). The fields $\mathcal{Q}$, $\mathcal{U}$, and $\mathcal{D}$ describe the 5d states whose zero modes are the 4d Standard Model (SM) quarks. The KK expansions of the weak-doublet and weak-singlet leptons $\mathcal{L}$ and $\mathcal{E}$ are not shown for brevity.

(3) The third generation of the fermions are 4d fields localized at the orbifold fixed point ($y = 0$):

$$\mathcal{Q}_3(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}}[(t, b)_L(x)],$$

$$\mathcal{U}_3(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}}[t_R(x)], \tag{3}$$

$$\mathcal{D}_3(x, y) = \frac{\sqrt{2}}{\sqrt{2\pi R}}[b_R(x)],$$

and similarly for the leptons.

(4) The Higgs components $\chi^{\pm}_{(n)}$ and $\chi^3_{(n)}$ mix with the adjoint scalar fields ($W^{\pm 5}_{(n)}$ and $Z^5_{(n)}$) to produce Goldstone bosons ($G^0_{(n)}$, $G^{\pm}_{(n)}$) and three additional physical scalar modes ($a^0_{(n)}$, $a^{\pm}_{(n)}$), where

$$G^0_{(n)} = \frac{1}{M_{Z_{(n)}}}\left[M_Z \chi^3_{(n)} - \frac{n}{R}Z^5_{(n)}\right],$$

$$a^0_{(n)} = \frac{1}{M_{Z_{(n)}}}\left[\frac{n}{R}\chi^3_{(n)} + M_Z Z^5_{(n)}\right],$$

$$G^{\pm}_{(n)} = \frac{1}{M_{W_{(n)}}}\left[M_W \chi^{\pm}_{(n)} - \frac{n}{R}W^{\pm 5}_{(n)}\right], \tag{4}$$

$$a^{\pm}_{(n)} = \frac{1}{M_{W_{(n)}}}\left[\frac{n}{R}\chi^{\pm}_{(n)} + M_W W^{\pm 5}_{(n)}\right],$$

where $M_n^2 = M^2 + n^2/R^2$, M being the zero mode W or Z mass ($M_{W(Z)}$), and M_n corresponds to a generic nth mode mass. Clearly, at the brane, only the $\chi_{(n)}$ states are nonvanishing.

We identify two cases which require separate treatments. Case (a): We restrict our discussion to the first two generations of fermions. Then all interactions and Feynman rules are exactly like in UED discussed in detail in [2,3]. The KK number is conserved at all vertices. We, however, note that the mixing of KK fermions within the same generation is controlled by an angle α_n, given by $\tan 2\alpha_n = m_0/M_n$. Since the zero mode masses (m_0) of the first two generations of fermions are negligible, we take this mixing to be vanishing. Case (b): At least one third generation fermion is involved. The Lagrangian contains $[\delta(y) + \delta(y - \pi R)]$ to ensure the localization of this interaction *only* on the brane. There is no KK number conservation as a result of breakdown of

translational invariance. Unlike in the UED case or case (a), the operator structure of gauge boson-fermion interaction in case (b) is SM like.

III. UV DIVERGENCES FOR DIFFERENT PROCESSES IN DIFFERENT MODELS

A. $Z\bar{b}d$ vertex

The effective $Z\bar{b}d$ vertex is constructed from triangle and self energy diagrams in which the up-type quarks and W boson circulate inside the loops. For clarity, we use W to denote the transverse part of the $W^{\pm}$ boson, while ϕ refers to its longitudinal component and the additional scalars ($\phi \equiv G^{\pm}$, $a^{\pm}$). By W (or ϕ) mediated graphs we mean loops with internal $W_{(n)}$ (or $\phi_{(n)}$) KK bosons. We assign an index i to indicate the three generations of up-type quarks inside the loop, and use α to label the different diagrams for each i. We employ a notation $d_\alpha^{W(\phi)}$ which captures the relevant tree-level couplings of the Z boson for the diagram labeled "α" (i.e., Zii or $ZW(\phi)W(\phi)$ for triangles, and self energies on the d and b legs) and the associated loop factors. The relevant Cabibbo-Kobayashi-Maskawa (CKM) elements appearing in the loop vertices are separately denoted by $\xi_i \equiv V_{id}V_{ib}^*$.

When we calculate the amplitudes, we integrate over the loop momentum (k) and sum over the KK modes ($k_5 = n/R$). For individual loop graphs we encounter two kinds of cutoffs: the 4d momentum cutoff denoted by Λ (scaled to be a dimensionless number) and the KK momentum cutoff by n_s, and we expect $\Lambda \sim n_s$. Operationally, we perform the 4d integration first and then do the KK summation.[3]

The W and ϕ mediated loop amplitudes have the following structures:

$$A_W = \sum_n A_{W_{(n)}} = \sum_n \sum_i \sum_\alpha \xi_i d_\alpha^W [\ln\Lambda + f_{i\alpha}(x_i, n)],$$

$$A_\phi = \sum_n A_{\phi_{(n)}} = \sum_n \sum_i \sum_\alpha \xi_i x_i d_\alpha^\phi [\ln\Lambda + g_{i\alpha}(x_i, n)], \tag{5}$$

where $x_i \equiv m_{0i}^2/M_W^2$, $f_{i\alpha}$, and $g_{i\alpha}$ are finite pieces obtained from the individual KK modes for the W and ϕ mediated graphs, respectively. It is important to observe at this point that the relation $\sum_i \xi_i = 0$ ensures through

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[3]We stress that the operational ordering of performing the 4d loop momentum integration first and then doing the KK summation is technically more correct than the other way around. As we stressed the 4d momentum cutoff Λ should be of the order of the KK momentum cutoff n_s. If we do the infinite KK summation first as in Eq. (1) and then perform the 4d momentum integration we include contributions from scales above $\sim \Lambda$ which we have observed lead to physicswise meaningless results. A similar conclusion has also been drawn in [16] in a different context.

Glashow-Iliopoulos-Maiani (GIM) mechanism the cancellation of the $\ln\Lambda$ dependence in the W mediated graphs, while the arrangement between wave function renormalizations and vertex corrections leading to $\sum_\alpha d_\alpha^\phi = 0$ guarantees the absence of the net $\ln\Lambda$ dependence in ϕ mediated loops. We emphasize that Eq. (5) is a kind of master equation which can cover all the three extra dimensional scenarios under consideration.

(1) **SM:** The Eq. (5) for $n = 0$ describes the SM situation. As stated above, the GIM mechanism ($\sum_i \xi_i = 0$) and the relationship between vertex corrections and wave function renormalizations ($\sum_\alpha d_\alpha^\phi = 0$) together ensure the finiteness of the effective $Z\bar{b}d$ vertex.

(2) **UED:** As mentioned above, the $\ln\Lambda$ dependence cancels out mode by mode, so what remains to be seen is whether the KK sum over the finite pieces from individual modes yields a finite or a divergent result. In this scenario

$$A_{W_{(n)}} \sim 1/n^2, \qquad A_{\phi_{(n)}} \sim 1/n^2, \qquad (6)$$

in the large n limit, and hence the KK summation over these individually finite pieces also yields a finite effective $Z\bar{b}d$ vertex.

(3) **NUED:** Since the external legs are all zero mode states, there is still a single KK index running inside the loop. In this case the n dependence arises only from the bosonic excitations. Again, the amplitudes in the large n limit go like $1/n^2$. Here also the effective $Z\bar{b}d$ vertex is finite.

(4) **Split:** We recall that the first two generations of fermions have KK excitations, while the third generation is brane-localized. Still, an inspection on the diagrams reveals that there is only a single KK index running in the loop. The net $\ln\Lambda$ dependence cancels exactly for the same reason cited in the context of the master Eq. (5). Now we ask the question whether KK summing over the finite parts of the individual modes yields a divergent or a finite result. For that we again give a look at the master equation. The large n behavior of the W mediated graphs is different now from the UED case, while in the same limit the ϕ mediated loops behave as in UED. More specifically,

$$A_{W_{(n)}} \sim n^2/n^2, \qquad A_{\phi_{(n)}} \sim 1/n^2, \qquad (7)$$

where in the first case the appearance of n^2 in the numerator is a consequence of the split nature, i.e., a relative localization of different generations of fermions. The net divergence therefore appears only from this W mediated part after the KK summation, and it is not difficult to see that it is

a linear divergence. Taking $R^{-1} \sim 1$ TeV $\gg m_t$, the effective $Z\bar{b}d$ vertex looks like ($n_s \sim \Lambda$)

$$\Gamma_\mu^{Zbd} = \frac{ig}{\cos\theta_W}\left(\frac{g^2}{16\pi^2} F_S \xi_t\right)\gamma_\mu P_L,$$

with $\quad F_S \simeq \left(\frac{3}{4} - \frac{5}{6}\sin^2\theta_W\right)\Lambda.$ $\qquad (8)$

B. $B - \bar{B}$ mixing

(1) **SM:** The relevant box diagrams are all finite.

(2) **UED:** Since KK number is conserved, there is only a single KK number in the loop, hence a single KK summation. After one performs the summation, it is not difficult to see from power counting that each such box is finite. The bottom line is that the finiteness owes to the single summation.

(3) **NUED:** There are two independent KK indices attached to the two internal bosons. Their propagators can be summed independently, and thus each such box (be it W mediated or ϕ mediated) is log divergent. The divergence from the ones involving two internal KK W's sums up to zero on account of GIM mechanism. But the divergences from the boxes having two KK ϕ's just add up (the ones having one W and one ϕ are finite anyway). The dominant contribution goes as $\xi_t^2 x_t^2 \ln\Lambda$. The double KK summation involved is the deciding factor behind this divergence.

(4) **Split:** A point to observe is that two of the four vertices in the box which connect to the external b quarks necessarily violate the KK number since the third generation quark is brane-localized. The remaining two vertices may or may not violate KK number (depending on whether the internal fermion is t quark or u, c quarks). The result is that any such box involves two independent KK summations, and hence each of them turns out to be log divergent. The root of the net divergence not only lies in the ϕ mediated boxes, the W-boxes also sum up to a net divergence due to incomplete GIM cancellation owing to the placement of different fermion generations in different locations.

As an illustrative example, we present the functional dependence of the KK modes in the amplitude (after the 4d loop momentum integration) when there are two top quarks in the internal lines of a box ($y_n \equiv 1 + n^2/(R^2 M_W^2)$):

$$I(n, m) = \frac{f(x_t, y_n) - f(x_t, y_m)}{y_n - y_m}, \qquad \text{where} \qquad (9)$$

$$f(x_t, y_n) = \frac{x_t(0.5x_t - y_n)\ln x_t + 0.5y_n^2 \ln y_n - 0.75y_n^2 - x_t(0.25x_t - y_n)}{2(x_t - y_n)^2}. \tag{10}$$

Summing over n and m yields $I \sim \ln n_s \sim \ln\Lambda$. Other combinations of the internal quark lines also lead to log divergence. The net divergence structure expectedly reads $\sim \xi_t^2 \ln\Lambda$ for the W mediated boxes and $\sim \xi_t^2 x_t^2 \ln\Lambda$ for the ϕ mediated boxes. We do not display here the exact coefficients, they are the results of the above and other more complicated summations and integrations. On the other hand, the boxes containing one $W_{(n)}$ and one $\phi_{(m)}$ are finite after KK summation.

C. The ρ parameter

(1) **SM:** Each of the W and Z self-energy diagrams with fermion loops is quadratically divergent, and each loop with internal bosons is log divergent. But the net contribution is UV finite having the well-known expression which is approximately

$$(\Delta\rho)_{\mathrm{SM}} \sim \frac{\alpha}{\pi}\left[\frac{m_t^2}{M_Z^2} - \ln\frac{M_h}{M_Z}\right]. \tag{11}$$

(2) **UED:** The contributions coming from higher KK modes decouple as their inverse square masses. The net contribution from each KK mode is finite. It is interesting to note that for a given KK mode, the contribution of M_h to $\Delta\rho$ appears quadratically as opposed to its logarithmic dependence in the SM contribution. The contribution from the nth KK mode approximately reads [2]

$$(\Delta\rho)_n \sim \frac{\alpha}{\pi}\left[\frac{m_t^4}{M_Z^2 M_n^2} - \frac{M_h^2}{7M_n^2} - \frac{M_W^2}{M_n^2}\right]. \tag{12}$$

Clearly, the KK summation leads to a finite result for $\Delta\rho$.

(3) **NUED:** Since the fermions are all in the brane, new contributions would come only from the KK bosons. The net contribution is exactly the same as Eq. (12) but without the first term, i.e., without the fermionic part.

(4) **Split:** The bosonic (gauge boson and Higgs) contribution would expectedly be the same as in UED (the second and third terms in Eq. (12)), and hence KK summation over the bosonic excitations leads to a finite result. But the fermionic contribution is tricky, just because different fermions are located at different places. First, it is important to recall that the Z boson self-energy diagrams receive nonzero contribution purely from the *axial* part of the fermionic coupling. The $Z_\mu^{(0)}(\bar{\mathcal{U}})_i^{(n)}\mathcal{U}_i^{(n)}$ and $Z_\mu^{(0)}\bar{\mathcal{Q}}_i^{(n)}\mathcal{Q}_i^{(n)}$ couplings being purely vectorial [2] do not contribute to the Z boson self-energy diagrams. The $Z_\mu^{(0)}(\bar{\mathcal{U}})_i^{(n)}\mathcal{Q}_i^{(n)}$ ($i = 1, 2$) couplings are purely axial but still do not contribute to the extent the mixing angle (α_n) can be ignored for the first two generation of fermions. In the same limit, the W self-energy graphs also receive vanishing contribution from the first two generations of fermions. Only those W boson self-energy diagrams in which one quark is a localized state, i.e., $t(b)$, and the other is a bulk state, i.e., d or s (u or c), remain *unmatched* in the sense that there are no corresponding Z self-energy diagrams which could have possibly cancelled their divergences. Since each diagram is divergent, these *unmatched* divergences from the W self-energy graphs survive in the absence of their Z counterpart. The net divergence to $\Delta\rho$ arising from the surviving diagrams after the KK summation is observed to go like Λ^3 *modulo* the suppression factor $(1 - |V_{tb}|^2)$. Since in the SM each *individual* fermionic graph has a quadratic divergence, a single KK summation enhances the degree of divergence by one order. The ρ parameter thus offers the most serious constraint to the split scenario.

IV. A SUMMARY OF UV DIVERGENCES AND A POSSIBLE REMEDY

Our primary aim in this paper has been to study the UV cutoff dependences in the split scenario and probe the root cause behind them. In the process, we compare and contrast three scenarios: split, UED, and NUED, to make a judgement of their relative effectiveness. The following points are worth noting.

(1) If one takes more than one extra dimension, all such models give divergent results at any order (even the tree amplitude diverges if one can perform a summation over the KK propagator).

(2) Suppose we restrict to one extra dimension only, and remain within one loop. Then in the UED scenario all results are finite, while in the NUED picture, quite a few processes are UV cutoff sensitive. The core issue is whether KK number is conserved or not, which relates the origin of divergence to the occurrence of more than one KK summation within a one-loop integral. In the split scenario, the UV divergences are more severe in some cases (the contribution to $\Delta\rho$ being a glaring example). The appearance of divergence

in the split scenario is caused not only due to the occurrence of more than one KK summation in a one-loop integral, but also as a result of incomplete GIM cancellation since some of the fermions have KK excitations while some are brane-localized.

(3) If one goes beyond one loop, then even insisting on one extra dimension only, *all* such models will become UV sensitive with varying degree of cutoff dependence.

A comparison of UV sensitivities of different models for different processes has been summarized in Table I.

A possible remedy:

(1) A prescription for a possible cure from the occurrence of such divergence may be advanced by supplementing these models with some ideas of brane fluctuations advocated in [17]. A bold and rather far-fetched assumption will be to attribute some dynamics at the orbifold fixed points which may render the couplings at the loop vertices to be KK label (n) dependent. If in such a situation a typical coupling g at a loop vertex is replaced by $g(n)$, given by $g(n) \sim ga(n)\exp(-cn^2/M_S^2 R^2)$, then the exponential suppression will lead to a well-behaved integral at the UV end. Whether or not it can actually be realized in a concrete scenario is beyond the scope of this paper.

(2) Very recently, an idea which goes by the name of "minimal length scenario" has emerged which does not admit a length scale smaller than the string length ($l_p \sim \hbar/M_S$) [18]. Be that as it may, even though the momentum (p) can go to infinity, the wave vector (k) is restricted from above by the requirement that the Compton wavelength ($2\pi/k$) cannot be smaller than l_p. This means that the usual relation $p = \hbar k$ has to be replaced, and a simple but quite *ad hoc* ansatz would be to invoke $l_p k(p) = \tanh(p/M_S)$ and $l_p \omega(E) = \tanh(E/M_S)$. Admitting this relation also for the compactified direction means an insertion of a factor $(\partial k/\partial p) =$

TABLE I. Ultraviolet divergences in different cases, with one extra dimension.

Processes	UED	NUED	Split
$Z\bar{b}d$ vertex	Finite	Finite	$\sim \Lambda$
$B - \bar{B}$ mixing	Finite	$\sim \ln\Lambda$	$\sim \ln\Lambda$
$(\Delta\rho)_{\text{KK fermion}}$	Finite	No Contribution	$\sim \Lambda^3$

$\text{sech}^2(p/M_S)$ inside a KK summation where p is equivalent to the KK mass M_n. Since $\text{sech}(x) \sim \exp(-x)$ for large x, one obtains an exponential suppression of the effective coupling (or form factor) for higher KK states.[4] We must confess that from a pure field theoretic perspective all we did is a simple trading of the cutoff parameter Λ in favor of an *ad hoc* form factor which contains a minimal length l_p in the way shown. One should note however that if we agree to go by the above hypothesis then the appearance of an exponentially suppressed form factor ensures a smooth loop integration up to infinity at all orders.

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[4]There is a slight difference between the string and minimal length form factors. While the string form factors are rather of $\exp(-p^2)$ type, the minimal length principle, as advocated in [18], generates a form factor which has a linear dependence on $(-|p|)$ in the exponential.

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