Scattering of Dirac and Majorana fermions off domain walls

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We investigate the interaction of fermions having both Dirac and left-handed and right-handed Majorana mass terms with vacuum domain walls. By solving the equations of motion in a thin-wall approximation, we calculate the reflection and transmission coefficients for the scattering of fermions off walls.

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I. INTRODUCTION

Theoretical arguments [1] and a recent analysis of Wilkinson Microwave Anisotropy Probe data [2] show that the presence of a network of low-tension domain walls in the Universe is not ruled out. Moreover, domain walls could provide a natural and nonexotic alternative to the most popular candidates of dark energy [3]. The evolution of vacuum domain walls in the early Universe is determined by their interaction with the surrounding plasma. The two most relevant effects to be considered are the particle scattering off walls and the presence of bound states near the walls, the so-called ‘‘zero modes.’’

The scattering of particles off walls (including the scattering between walls) determines the average velocity v of a wall and thus, in turn, the equation of state of a gas of domain walls, $p_w = (v^2 - 2/3)\rho_w$, where ρ_w and p_w are the energy density and pressure of the gas (see, e.g., [4]). Indeed, when particles scatter off a wall, they generate a frictional force $F = \sum_i n_i R_i \Delta p_i$, where n_i is the number density of particles of species i, R_i is their scattering probability (the reflection coefficient), and Δp_i is the momentum transfer per collision (see, e.g., [5]). Hence, defining the mean velocity of the walls as $v = \sum_i n_i R_i v_i / \sum_i n_i R_i$, the damping force can be written as $F = \mu v$, where we have defined the frictional coefficient $\mu = (\sum_i n_i R_i \Delta p_i)(\sum_i n_i v_i)/\sum_i n_i R_i$. The mean velocity of a wall is determined by balancing the tension, $f \sim \sigma/r$, where σ and r are the surface energy density and the mean curvature radius of a wall, and the friction, $f = F$. The resulting velocity is then $v \sim \sigma/r\mu$. It is clear that a full analysis of the role of domain walls in the Universe imposes the study of their interaction with particles in the primordial plasma.

The presence of zero modes localized on a domain wall can be important for the stability of the wall. In particular, fermionic zero modes may give rise to interesting phenomena as the magnetization of domain walls [6,7], and the dynamical generation of massive ferromagnetic domain walls [8]. Indeed, fermionic zero modes could drastically change both gravitational properties and cosmic evolution of a gas of domain walls [9].

The interaction of scalar particles and Dirac fermions with a domain wall has been the object of various papers in the literature (see [5] and references therein, [10–16]). Since strong evidence for neutrino masses has emerged from various neutrino oscillation experiments in recent years [17], we are motivated to investigate the interaction of Majorana fermions with domain walls (neutrinos are neutral fermions, and then can have both Majorana and Dirac masses). In a recent paper [18], Stojkovic has studied fermionic zero modes in the domain wall background, in the case in which the fermions have both Dirac and left-handed and right-handed Majorana mass terms. The aim of this paper is to study the scattering of such fermions off domain walls.

The plan of the paper is as follows. In Sec. II we introduce the Lagrangian for a single real self-interacting scalar field Φ, coupled with a fermion ψ having Dirac, left-handed and right-handed Majorana mass terms. We also derive the equations of motion. In Sec. III we calculate the reflection and transmission coefficients for the scattering of fermions off walls, in both cases in which the coupling to the scalar field Φ is or not the source of the Majorana mass terms. Finally, we summarize our results in Sec. IV.

II. LAGRANGIAN, ASYMPTOTIC STATES, AND EQUATIONS OF MOTION

We consider a simplified model in which the kink is an infinitely static domain wall in the xz plane. In this model the scalar sector giving rise to a planar wall is a real scalar field with density Lagrangian

$$\mathcal{L}_\Phi = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4}(\Phi^2 - \eta^2)^2. \qquad (1)$$

In the tree approximation, the set of vacuum states is $\langle \Phi \rangle^2 = \eta^2$, so that one may assume that there are regions with $\langle \Phi \rangle = +\eta$ and regions with $\langle \Phi \rangle = -\eta$. By continuity a region must exist in which the scalar field is out of the vacuum. This region is a domain wall [19], and the classical equation of motion admits the solution describing the transition layer between two regions with different values of $\langle \Phi \rangle$,

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$$\Phi(y) = \eta \tanh(y/\Delta), \tag{2}$$

where $\Delta = \sqrt{2\lambda}/\eta$ is the thickness of the wall [4].

Now, we consider a fermion ψ having Dirac, and left-handed and right-handed Majorana mass terms. The source of the Dirac and Majorana mass terms is the Yukawa couplings to the scalar field Φ. In terms of the chiral spinors ψ_L and ψ_R the Lagrangian density of the system is[1]

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - \frac{\lambda}{4}(\Phi^2 - \eta^2)^2 + i\bar{\psi}_L \not{\partial} \psi_L + i\bar{\psi}_R \not{\partial} \psi_R - (g_D \Phi \bar{\psi}_L \psi_R + g_L \Phi \bar{\psi}_L \psi_L^c + g_R \Phi \bar{\psi}_R \psi_R^c + \text{H.c.}), \tag{3}$$

where g_D, g_L, and g_R are the Yukawa couplings to the scalar field of the Dirac, left-handed, and right-handed Majorana fermions, respectively. In Lagrangian (3), $\psi^c = C\bar{\psi}^T$, where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix [20] and T indicates the transpose.

In the broken phase (i.e., for $y \to \pm\infty$) where Φ takes a constant value, $\langle\Phi\rangle = \pm\eta$, the scalar field gives mass to the fermion states. It is clear that the chiral fields ψ_L and ψ_R do not have a definite mass, since they are coupled by the Dirac mass term. In order to find the asymptotic states with definite masses, we have to diagonalize the mass matrix in Lagrangian (3) or, equivalently, we have to diagonalize the Dirac equation,

$$(i\not{\partial} - G\Phi)\Psi = 0, \tag{4}$$

where we have introduced the following quantities:

$$G = \begin{pmatrix} 0 & \mathcal{G} \\ \mathcal{G} & 0 \end{pmatrix}, \qquad \mathcal{G} = \begin{pmatrix} g_L & g_D \\ g_D & g_R \end{pmatrix}, \qquad \Psi = \begin{pmatrix} \psi_L \\ \psi_R^c \\ \psi_L^c \\ \psi_R \end{pmatrix}. \tag{5}$$

In the broken phase, Eq. (4) becomes

$$(i\not{\partial} \mp M)\Psi = 0, \quad \text{if } y \to \pm\infty, \tag{6}$$

where we have defined the "mass matrix" $M = \eta G$. Diagonalizing the matrix M, we get

$$(i\not{\partial} \mp \Delta)\Psi_M = 0, \quad \text{if } y \to \pm\infty, \tag{7}$$

where

$$\Delta = U^T M U = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & -m_1 & 0 \\ 0 & 0 & 0 & -m_2 \end{pmatrix}, \tag{8}$$

U is the unitary transformation which diagonalizes M, and $\Psi_M = U^T\Psi$. The eigenvalues of M, that is $\pm m_{1,2}$, are given by

$$m_{1,2} = \tfrac{1}{2}\left[m_L + m_R \pm \sqrt{4m_D^2 + (m_L - m_R)^2}\right], \tag{9}$$

where we have defined

$$m_D = g_D\eta, \qquad m_L = g_L\eta, \qquad m_R = g_R\eta. \tag{10}$$

Here, m_1 and m_2 represent the masses of the free-field propagating degrees of freedom in the theory. It can be shown (see, e.g., Ref. [17]) that the two massive fermion states are Majorana particles.

After having considered the asymptotic fermion states, it is now clear that the particle content of Lagrangian (3) consists of two Majorana fermions with masses m_1 and m_2 interacting with a vacuum domain wall (described by the scalar field Φ). The aim of this paper is to study the scattering of these two states off a wall. To this end, we use the following representation of the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{11}$$

where σ^k, $k = 1, 2, 3$ are the Pauli matrices. In this representation, a four-component fermion has left-handed and right-handed components of the form

$$\psi_L^T = (\alpha, \beta, -\alpha, -\beta), \qquad \psi_R^T = (\gamma, \delta, \gamma, \delta). \tag{12}$$

We will concentrate on the solution describing the motion of fermions perpendicular to the wall, i.e., along the y axis, and then we suppose that

$$\Phi = \Phi(y), \qquad \psi_L = \psi_L(y, t), \qquad \psi_R = \psi_R(y, t). \tag{13}$$

The Lagrangian (3), together with Eqs. (12) and (13), implies the equations of motion

$$\Phi'' - \lambda\Phi(\Phi^2 - \eta^2) = 4g_D \text{Re}(\alpha^*\gamma - \beta^*\delta), \tag{14}$$

and

$$\begin{aligned} \beta' + i\dot{\alpha} &= g_D\Phi\gamma + g_L\Phi\beta^*, \\ \alpha' - i\dot{\beta} &= g_D\Phi\delta + g_L\Phi\alpha^*, \\ \delta' + i\dot{\gamma} &= g_D\Phi\alpha + g_R\Phi\delta^*, \\ \gamma' - i\dot{\delta} &= g_D\Phi\beta + g_R\Phi\gamma^*, \end{aligned} \tag{15}$$

where $\text{Re}(x)$ is the real part of x (here, and throughout, a prime and a dot will denote differentiation with respect to y and t, respectively).

In the following we shall analyze the simple case in which the backreaction of the fermion field ψ on the domain wall configuration is null. Indeed, we make the ansatz: $\beta = \alpha^*$ and $\gamma = \delta^*$, which is compatible with Eq. (15), and makes null the right-hand side of Eq. (14). [This, in turn, means that the wall profile is given by Eq. (2).] Moreover, writing α and δ as a sum of positive and negative energy states,

$$\begin{aligned} \alpha(y, t) &= \alpha_+(y)e^{-iEt} + \alpha_-(y)e^{iEt}, \\ \delta(y, t) &= \delta_+(y)e^{-iEt} + \delta_-(y)e^{iEt}, \end{aligned} \tag{16}$$

[1]For a full discussion of validity of Lagrangian (3) in phenomenologically relevant models in neutrino physics, see Ref. [18] and references therein.

and inserting into Eq. (15) we get

$$\begin{aligned}
\alpha_-^{*\prime} + E\alpha_+ &= g_D\Phi\delta_-^* + g_L\Phi\alpha_+,\\
\delta_-^{*\prime} - E\delta_+ &= g_D\Phi\alpha_-^* + g_R\Phi\delta_+,\\
\alpha_+^{*\prime} - E\alpha_- &= g_D\Phi\delta_+^* + g_L\Phi\alpha_-,\\
\delta_+^{*\prime} + E\delta_- &= g_D\Phi\alpha_+^* + g_R\Phi\delta_-.
\end{aligned} \tag{17}$$

Starting from Eq. (17), we will calculate, in the next section, the reflection and transmission coefficients for the scattering of fermions off walls.

III. SCATTERING

We will work in "thin-wall approximation" [5]; that is to say we suppose that the thickness of the wall is vanishingly small, $\Delta \to 0$. In this case, the wall profile takes the simple form $\Phi = \eta\,\mathrm{sgn}(y)$, where $\mathrm{sgn}(x)$ is the sign function. The thin-wall approximation is valid whenever the wavelength of scattered particles is much greater than the thickness of the wall. This approximation allows us to find analytical solutions to the equations of motion and does not affect the main results of our analysis. In the thin-wall approximation, the solution of the system (17) for $y > 0$ is easily found:

$$\begin{aligned}
\alpha_+ &= c_1e^{ip_1y} + c_2e^{ip_2y} + c_3e^{-ip_1y} + c_4e^{-ip_2y}, \qquad \alpha_-^* = ix_1c_1e^{ip_1y} + ix_2c_2e^{ip_2y} - ix_1c_3e^{-ip_1y} - ix_2c_4e^{-ip_2y},\\
\delta_+ &= ix_3c_1e^{ip_1y} + ix_4c_2e^{ip_2y} - ix_3c_3e^{-ip_1y} - ix_4c_4e^{-ip_2y}, \qquad \delta_-^* = x_5c_1e^{ip_1y} + x_6c_2e^{ip_2y} + x_5c_3e^{-ip_1y} + x_6c_4e^{-ip_2y}.
\end{aligned} \tag{18}$$

Here c_i are integration constants,

$$p_{1,2} = \sqrt{E^2 - m_{1,2}^2}, \tag{19}$$

with $m_{1,2}$ given by Eq. (9), and

$$\begin{aligned}
x_{1,2} &= \frac{p_{1,2}(E + m_{2,1})}{(E + m_L)(E + m_R) - m_D^2},\\
x_{3,4} &= \frac{p_{1,2}[(E + m_L)(m_R - m_L \pm m_1 \mp m_2) - 2m_D^2]}{2m_D[(E + m_L)(E + m_R) - m_D^2]},\\
x_{5,6} &= \frac{m_R - m_L \pm m_1 \mp m_2}{2m_D}.
\end{aligned} \tag{20}$$

The solution in the case $y < 0$ is obtained from Eq. (18), by the substitutions $c_i \to d_i$ and $x_i \to y_i$, where d_i are new integration constants and

$$\begin{aligned}
y_{1,2} &= \frac{p_{1,2}(E + m_{2,1})}{(E - m_L)(E - m_R) - m_D^2},\\
y_{3,4} &= \frac{p_{1,2}[(E - m_L)(m_R - m_L \mp m_1 \pm m_2) + 2m_D^2]}{2m_D[(E - m_L)(E - m_R) - m_D^2]},\\
y_{5,6} &= x_{5,6}.
\end{aligned} \tag{21}$$

Returning to the expression for the chiral spinor fields ψ_L and ψ_R, we have

$$\psi_L = \psi_L^{(+)} + \psi_L^{(-)}, \qquad \psi_R = \psi_R^{(+)} + \psi_R^{(-)}, \tag{22}$$

where $\psi_L^{(\pm)}$ and $\psi_R^{(\pm)}$ are explicitly given by

$$\psi_L^{(\pm)} = \begin{pmatrix} \alpha_\pm \\ \alpha_\mp^* \\ -\alpha_\pm \\ -\alpha_\mp^* \end{pmatrix} e^{\mp iEt}, \qquad \psi_R^{(\pm)} = \begin{pmatrix} \delta_\mp^* \\ \delta_\pm \\ \delta_\mp^* \\ \delta_\pm \end{pmatrix} e^{\mp iEt}. \tag{23}$$

For definiteness we consider solutions for which we have incident fermion states from the left ($y < 0$) which are scattered into reflected waves going to the left and transmitted waves going to the right ($y > 0$). Therefore, the fermions are represented by incoming and reflected waves to the left of the wall and by transmitted waves to the right. Hence, taking into account Eqs. (18) and (23) we obtain the transmitted, incident, and reflected left-handed wave functions:

$$\begin{aligned}
(\psi_L^{(\pm)})^{\mathrm{tran}} &= (c_1u_{L,1}^{(\pm)}e^{\pm ip_1y} + c_2u_{L,2}^{(\pm)}e^{\pm ip_2y})e^{\mp iEt},\\
(\psi_L^{(\pm)})^{\mathrm{inc}} &= (d_1v_{L,1}^{(\pm)}e^{\pm ip_1y} + d_2v_{L,2}^{(\pm)}e^{\pm ip_2y})e^{\mp iEt},\\
(\psi_L^{(\pm)})^{\mathrm{refl}} &= (d_3v_{L,3}^{(\pm)}e^{\mp ip_1y} + d_4v_{L,4}^{(\pm)}e^{\mp ip_2y})e^{\mp iEt},
\end{aligned} \tag{24}$$

with the condition $c_3 = c_4 = 0$. Here, we have introduced the spinors

$$\begin{aligned}
(u_{L,1}^{(+)})^T &= (1, ix_1, -1, -ix_1),\\
(u_{L,2}^{(+)})^T &= (1, ix_2, -1, -ix_2),\\
(u_{L,3}^{(+)})^T &= (1, -ix_1, -1, ix_1),\\
(u_{L,4}^{(+)})^T &= (1, -ix_2, -1, ix_2).
\end{aligned} \tag{25}$$

The spinors $v_{L,i}^{(+)}$ are obtained from $u_{L,i}^{(+)}$ by the replacements $x_i \to y_i$, while $u_{L,i}^{(-)} = Cu_{L,i}^{(+)}$ and $v_{L,i}^{(-)} = Cv_{L,i}^{(+)}$, where $i = 1, 2, 3, 4$, and C is the charge conjugation matrix. The transmitted, incident, and reflected right-handed wave functions are obtained from Eq. (24) by the substitutions $u_{L,i}^{(\pm)} \to u_{R,i}^{(\pm)}$, $v_{L,i}^{(\pm)} \to v_{R,i}^{(\pm)}$, where

$$\begin{aligned}
(u_{R,1}^{(+)})^T &= (x_5, ix_3, x_5, ix_3),\\
(u_{R,2}^{(+)})^T &= (x_6, ix_4, x_6, ix_4),\\
(u_{R,3}^{(+)})^T &= (x_5, -ix_3, x_5, -ix_3),\\
(u_{R,4}^{(+)})^T &= (x_6, -ix_4, x_6, -ix_4).
\end{aligned} \tag{26}$$

The spinors $v_{R,i}^{(+)}$ are obtained from $u_{R,i}^{(+)}$ by the replacements $x_i \to y_i$, while $u_{R,i}^{(-)} = Cu_{R,i}^{(+)}$ and $v_{R,i}^{(-)} = Cv_{R,i}^{(+)}$. By imposing continuity of $\alpha_\pm(y)$ and $\delta_\pm(y)$ in $y = 0$, we get

$$
\begin{aligned}
c_1 + c_2 &= d_1 + d_2 + d_3 + d_4,\\
x_1c_1 + x_2c_2 &= y_1d_1 + y_2d_2 - y_1d_3 - y_2d_4,\\
x_3c_1 + x_4c_2 &= y_3d_1 + y_4d_2 - y_3d_3 - y_4d_4,\\
x_5c_1 + x_6c_2 &= y_5d_1 + y_6d_2 + y_5d_3 + y_6d_4.
\end{aligned}
\tag{27}
$$

Solving the above system with respect to c_1, c_2, d_3, and d_4, we obtain

$$
\begin{aligned}
c_1 &= \left(1 + \frac{m_1}{E}\right)d_1, \qquad d_3 = \frac{m_1}{E}d_1,\\
c_2 &= \left(1 + \frac{m_2}{E}\right)d_2, \qquad d_4 = \frac{m_2}{E}d_2.
\end{aligned}
\tag{28}
$$

The total current is defined in terms of the asymptotic fermion states discussed in Sec. II:

$$
\begin{aligned}
J^\mu_{(\pm)} &= \Psi_M^{(\pm)\dagger}\gamma^0\gamma^\mu\Psi_M^{(\pm)} = \Psi^{(\pm)\dagger}\gamma^0\gamma^\mu\Psi^{(\pm)}\\
&= [\psi_L^{(\pm)\dagger}, (\psi_R^c)^{(\pm)\dagger}, (\psi_L^c)^{(\pm)\dagger}, \psi_R^{(\pm)\dagger}]\\
&\times \begin{pmatrix} \gamma^0\gamma^\mu & 0 & 0 & 0\\ 0 & \gamma^0\gamma^\mu & 0 & 0\\ 0 & 0 & \gamma^0\gamma^\mu & 0\\ 0 & 0 & 0 & \gamma^0\gamma^\mu \end{pmatrix}\begin{pmatrix} \psi_L^{(\pm)}\\ (\psi_R^c)^{(\pm)}\\ (\psi_L^c)^{(\pm)}\\ \psi_R^{(\pm)} \end{pmatrix},
\end{aligned}
\tag{29}
$$

where the second equality holds because U is a unitary matrix. Here "$\pm$" refers to positive and negative energy states, and $\Psi_M^{(\pm)} = U^T\Psi^{(\pm)}$. It should be noted that the conjugate wave functions are $(\psi_L^c)^{(\pm)} = C(\bar{\psi}_L^{(\mp)})^T$ and $(\psi_R^c)^{(\pm)} = C(\bar{\psi}_R^{(\mp)})^T$.

Because we are considering the motion of fermions perpendicular to the wall, the relevant currents are those perpendicular to the kink, i.e., $J^2_{(\pm)}$. Taking into account the expressions for the chiral wave functions and Eq. (29), we get the transmitted, incident, and reflected currents:

$$
\begin{aligned}
(J^2_{(\pm)})^{\rm tran} &= 8[(x_1 + x_3x_5)c_1^2 + (x_2 + x_4x_6)c_2^2] = \chi^T\mathcal{T}\chi,\\
(J^2_{(\pm)})^{\rm inc} &= 8[(y_1 + y_3y_5)d_1^2 + (y_2 + y_4y_6)d_2^2] = \chi^T\chi,\\
(J^2_{(\pm)})^{\rm refl} &= -8[(y_1 + y_3y_5)d_3^2 + (y_2 + y_4y_6)d_4^2]\\
&= -\chi^T\mathcal{R}\chi,
\end{aligned}
\tag{30}
$$

where we have introduced the vector[2]

$$
\chi^T = 2\sqrt{2}\left(\sqrt{y_1 + y_3y_5}\,d_1, \sqrt{y_2 + y_4y_6}\,d_2\right), \tag{31}
$$

and the "reflection and transmission matrices"

$$
\mathcal{R} = \begin{pmatrix} m_1^2/E^2 & 0\\ 0 & m_2^2/E^2 \end{pmatrix}, \qquad \mathcal{T} = \begin{pmatrix} p_1^2/E^2 & 0\\ 0 & p_2^2/E^2 \end{pmatrix}. \tag{32}
$$

[2]It is straightforward to check that the quantities $y_1 + y_3y_5$ and $y_2 + y_4y_6$ are positive definite.

Note that $\mathcal{R} + \mathcal{T} = 1$. For an incident particle, the reflection and transmission coefficients are given as the ratios of the corresponding reflected and transmitted currents. From Eq. (30) we get

$$
R = -\frac{(J^2)^{\rm refl}}{(J^2)^{\rm inc}} = \frac{\chi^T\mathcal{R}\chi}{\chi^T\chi}, \qquad T = \frac{(J^2)^{\rm tran}}{(J^2)^{\rm inc}} = \frac{\chi^T\mathcal{T}\chi}{\chi^T\chi}. \tag{33}
$$

Taking into account Eq. (32), the unitary relation, $R + T = 1$, follows immediately. For $m_L = m_R = 0$, it is straightforward to check that $R = m_D^2/E^2$, as it should be [5].

It should be noted that, since d_1^2 and d_2^2 are directly proportional to the amplitudes of the free-field incident wave functions (i.e., the incident asymptotic fermion states), by a suitable normalization of wave functions we can take d_1 and d_2 such that $d_1^2 + d_2^2 = 1$. Let us observe that the two incident fermion states of momenta p_1 and p_2 are scattered in different ways. Indeed taking $d_1 = 0$ we get $R = m_2^2/E^2$, while for $d_2 = 0$ we have $R = m_1^2/E^2$. We see that the interaction with vacuum domain walls is able to produce a local asymmetry in the distribution of the two Majorana fermion states of masses m_1 and m_2.

In the upper left panel of Fig. 1 we plot the reflection coefficient versus the energy at fixed d_1, d_2, m_D, and m_L, for different values of m_R. In the upper right (lower) panel we fix m_L (m_D) and m_R, and vary m_D (m_L). These figures show that the reflection coefficient rapidly decreases as the energy of the incident particles increases, as expected. Indeed, from Eq. (33) we get that $R \simeq A/E^2$ for $E \gg m_{1,2}$, where A is a constant depending on d_1, d_2, m_1, and m_2. If $m_L \ll m_D \ll m_R$ and $d_1 = d_2$, then $A = m_2^2$. Moreover, at fixed energy, fixing two of the three masses m_D, m_L, m_R, the reflection coefficient is an increasing function of the remaining mass parameter.[3] The essential properties of R above discussed does not change if we take $d_1 \neq d_2$. Indeed, the only effect to take $d_1 > d_2$ ($d_1 < d_2$) is, at fixed energy and mass parameters, to increase (decrease) the reflection coefficient.

As pointed out in Ref. [18], the Majorana masses could arise from the coupling to a scalar field which undergoes a phase transition above the phase transition of the field Φ. In this case, the source of the Majorana masses is not the coupling with Φ, and the Majorana mass terms are spatially homogeneous. In this case we set

$$
g_L\Phi \to m_L, \qquad g_R\Phi \to m_R, \tag{34}
$$

in Lagrangian (3). In thin-wall approximation, the solution of the equations of motion is, for $y > 0$, equal to Eq. (18), while for $y < 0$ is

[3]Because the most relevant phenomenological model for neutrino masses is the so-called "seesaw mechanism" [17] in which $m_L = 0$ and $m_D \ll m_R$, in our figures we have taken $m_L \leq m_D \leq m_R$.

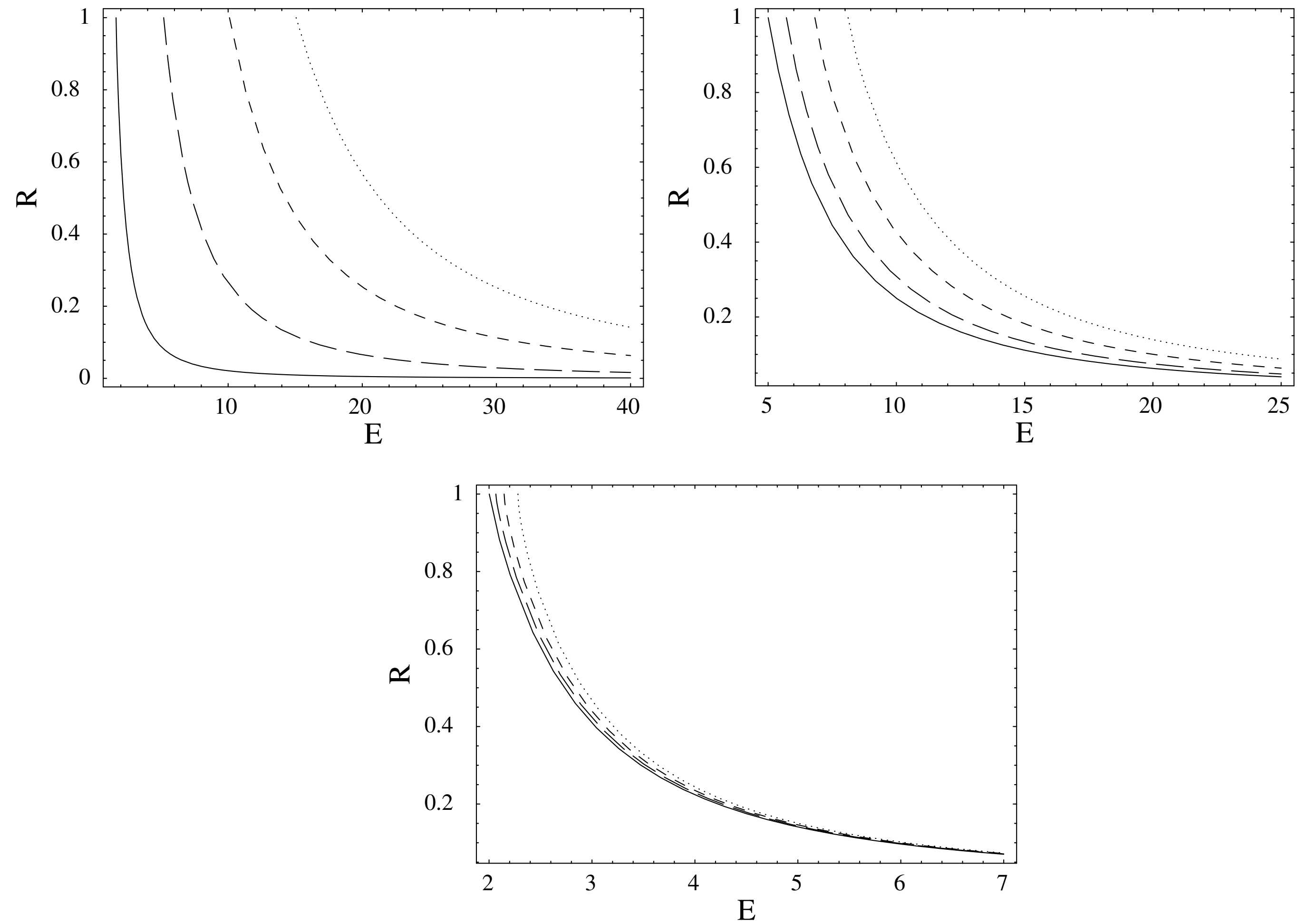

FIG. 1. Reflection coefficient versus the energy for the case of nonconstant Majorana mass terms, with $d_1 = d_2 = 1/\sqrt{2}$. Upper left panel: R in the case $m_D = 1$ and $m_L = 0.1$, for four different values of m_R: $m_R = 1$ (solid line), $m_R = 5$ (long-dashed line), $m_R = 10$ (short-dashed line), $m_R = 15$ (dotted line). Upper right panel: R in the case $m_L = 0.1$ and $m_R = 5$, for four different values of m_D: $m_D = 0.1$ (solid line), $m_D = 2$ (long-dashed line), $m_D = 3.5$ (short-dashed line), $m_D = 5$ (dotted line). Lower panel: R in the case $m_D = 1$ and $m_R = 2$, for four different values of m_L: $m_L = 0.01$ (solid line), $m_L = 0.3$ (long-dashed line), $m_L = 0.6$ (short-dashed line), $m_L = 1$ (dotted line).

$$\begin{aligned} \alpha_+ &= d_1 e^{ip_1 y} + d_2 e^{ip_2 y} + d_3 e^{-ip_1 y} + d_4 e^{-ip_2 y}, \qquad \alpha_-^* = ix_1 d_1 e^{ip_1 y} + ix_2 d_2 e^{ip_2 y} - ix_1 d_3 e^{-ip_1 y} - ix_2 d_4 e^{-ip_2 y}, \\ \delta_+ &= -ix_3 d_1 e^{ip_1 y} - ix_4 d_2 e^{ip_2 y} + ix_3 d_3 e^{-ip_1 y} + ix_4 d_4 e^{-ip_2 y}, \\ \delta_-^* &= -x_5 d_1 e^{ip_1 y} - x_6 d_2 e^{ip_2 y} - x_5 d_3 e^{-ip_1 y} - x_6 d_4 e^{-ip_2 y}, \end{aligned} \tag{35}$$

where d_i are constants of integration, $p_{1,2}$ and $m_{1,2}$ are the same as in Eq. (19), and x_i are given by Eq. (20). Taking into account the expressions for $\psi_L^{(\pm)}$ and $\psi_R^{(\pm)}$ [see Eq. (23)], and Eq. (35), we obtain the transmitted, incident, and reflected left-handed wave functions:

$$\begin{aligned} (\psi_L^{(\pm)})^{\rm tran} &= (c_1 u_{L,1}^{(\pm)} e^{\pm ip_1 y} + c_2 u_{L,2}^{(\pm)} e^{\pm ip_2 y}) e^{\mp iEt}, \\ (\psi_L^{(\pm)})^{\rm inc} &= (d_1 u_{L,1}^{(\pm)} e^{\pm ip_1 y} + d_2 u_{L,2}^{(\pm)} e^{\pm ip_2 y}) e^{\mp iEt}, \\ (\psi_L^{(\pm)})^{\rm refl} &= (d_3 u_{L,3}^{(\pm)} e^{\mp ip_1 y} + d_4 u_{L,4}^{(\pm)} e^{\mp ip_2 y}) e^{\mp iEt}, \end{aligned} \tag{36}$$

where the spinors $u_{L,i}^{(\pm)}$ are given by Eq. (25). The transmitted, incident, and reflected right-handed wave functions are obtained from Eq. (36) by the substitutions $u_{L,i}^{(\pm)} \to u_{R,i}^{(\pm)}$, and $d_i \to -d_i$, where the spinors $u_{R,i}^{(\pm)}$ are given by Eq. (26).

Taking into account Eqs. (29) and (36), we get the transmitted, incident, and reflected currents:

$$\begin{aligned} (J^2_{(\pm)})^{\rm tran} &= 8[(x_1 + x_3 x_5) c_1^2 + (x_2 + x_4 x_6) c_2^2], \\ (J^2_{(\pm)})^{\rm inc} &= 8[(x_1 + x_3 x_5) d_1^2 + (x_2 + x_4 x_6) d_2^2], \\ (J^2_{(\pm)})^{\rm refl} &= -8[(x_1 + x_3 x_5) d_3^2 + (x_2 + x_4 x_6) d_4^2]. \end{aligned} \tag{37}$$

Now, imposing continuity of $\alpha_\pm(y)$ and $\delta_\pm(y)$ in $y = 0$, we get

$$\begin{aligned} c_1 + c_2 &= d_1 + d_2 + d_3 + d_4, \\ x_1 c_1 + x_2 c_2 &= x_1 d_1 + x_2 d_2 - x_1 d_3 - x_2 d_4, \\ x_3 c_1 + x_4 c_2 &= -x_3 d_1 - x_4 d_2 + x_3 d_3 + x_4 d_4, \\ x_5 c_1 + x_6 c_2 &= -x_5 d_1 - x_6 d_2 - x_5 d_3 - x_6 d_4. \end{aligned} \tag{38}$$

Solving the above system with respect to c_1, c_2, d_3, d_4, and inserting the solution into Eq. (37), we obtain, after some manipulations, the reflection and transmission coefficients:

$$\begin{aligned} R &= \frac{2m_D^2 E^2}{2m_D^2 E^2 + p_1 p_2 (E^2 + p_1 p_2 - m_1 m_2)}, \\ T &= \frac{p_1 p_2 (E^2 + p_1 p_2 - m_1 m_2)}{2m_D^2 E^2 + p_1 p_2 (E^2 + p_1 p_2 - m_1 m_2)}. \end{aligned} \tag{39}$$

The unitary condition follows immediately from Eq. (37) and for $m_L = m_R = 0$ we get $R = m_D^2/E^2$, as it should be [5].

It is interesting to note that in the case of spatially homogeneous Majorana mass terms the reflection and transmission coefficients do not depend on the amplitudes of the two incident asymptotic fermion states of momenta p_1 and p_2. In fact, the dependence due to the amplitudes of this states factorizes in the expression of the currents in such a way that the reflection and transmission coefficients do not show any explicit dependence on d_1 and d_2. Since the two Majorana fermion states of masses m_1 and m_2 are scattered in the same way, there is no production of local asymmetry of any kind.

The behavior of R as a function of one of the tree mass parameters (keeping constant the other two) is the same as in the case of nonconstant Majorana mass terms, while for large values of energy, $E \gg m_{1,2}$, the reflection coefficient decreases as $R \simeq m_D^2/E^2$.

IV. CONCLUSIONS

We studied the interaction of fermions having both Dirac and left-handed and right-handed Majorana mass terms with kink domain walls. The source of the Dirac mass term was taken to be the coupling to the scalar field Φ that gives rise to a wall. In regards to the source of the Majorana mass terms, we analyzed two possible cases. In the first case we assumed that the Majorana masses are generated by the coupling to the scalar field Φ, in the second one, the Majorana mass terms were taken to be spatially homogeneous. We found the asymptotic fermion states with definite masses, m_1 and m_2, which represent the free-field propagating degrees of freedom in the theory.

By solving the Dirac equation in thin-wall approximation, we calculated the reflection and transmission coefficients for the scattering of such fermions off walls. The peculiar properties of the reflection coefficient R were analyzed in both cases of nonconstant and constant Majorana mass terms. In the case of nonconstant Majorana mass terms, the fermion states with definite masses scatter with different probabilities. Indeed, if the incident state consists of a state of definite mass m_1 or m_2, then his scattering probability is $R = m_1^2/E^2$ or $R = m_2^2/E^2$, respectively. In the case in which the incident state is a superposition of the two definite mass states, then the reflection coefficient has a quite complicated expression. However, for high energy of the incident particles, it is given by $R \simeq m_2^2/E^2$ (in the limit $m_L \ll m_D \ll m_R$).

In the case of constant Majorana mass terms, the fermion states with definite masses scatter in the same way. We found that for high energy of the incident particles, the reflection coefficient is $R \simeq m_D^2/E^2$.

We conclude by stressing that the reflection coefficients we found in this paper are important for determining the equation of state of a gas of domain walls which could be present in our Universe. The scattering of Majorana particles off vacuum domain walls, together with the presence of localized zero modes, could strongly influence the cosmic evolution of a gas of domain walls. However, the discussion of this last issue is beyond the aim of this paper and will be the object of future investigations.

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