CP sensitive observables in chargino production and decay into a *W* boson

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We study *CP* sensitive observables in chargino production in electron-positron collisions with subsequent two-body decay of one chargino into a *W* boson. We identify the *CP* odd elements of the *W* boson density matrix and propose *CP* sensitive triple-product asymmetries of the chargino decay products. We calculate the density-matrix elements, the *CP* asymmetries, and the cross sections in the minimal supersymmetric standard model with complex parameters μ and M_1 for an e^+e^- linear collider with $\sqrt{s} = 800$ GeV and longitudinally polarized beams. The asymmetries can reach 7% and we discuss the feasibility of measuring these asymmetries.

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I. INTRODUCTION

In the minimal supersymmetric standard model (MSSM) [1] several supersymmetric (SUSY) parameters can be complex. In the chargino sector of the MSSM this is the Higgsino mass parameter $\mu = |\mu|e^{i\varphi_\mu}$, and in the neutralino sector of the MSSM also the U(1) gaugino mass parameter $M_1 = |M_1|e^{i\varphi_{M_1}}$ can have a physical phase [2]. Usually it is claimed that these phases, in particular φ_μ, have to be small [3,4], due to the experimental upper bounds of the electric dipole moments (EDMs) of electron and neutron. For example, in the constrained MSSM $|\varphi_\mu|$ has to be smaller than $\pi/10$ [4] for a SUSY particle spectrum of the order of a few 100 GeV. However, the EDM restrictions may be less stringent if cancellations among the different SUSY contributions occur [3]. The restrictions may disappear if also lepton flavor violating terms are included [5]. Thus, the restrictions on the phases are very model dependent and independent measurements are desirable. The study of chargino production at an e^+e^- linear collider [6] will play an important role. By measurements of the chargino masses and cross sections, a method has been developed in [7,8] to determine $\cos\varphi_\mu$, in addition to the parameters M_2, $|\mu|$, and $\tan\beta$. However, also the sign of φ_μ has to be determined unambiguously by using *CP* sensitive observables. One such observable is the chargino polarization perpendicular to the production plane [7,8]. At tree level, this polarization leads to triple-product asymmetries [9–12]. For chargino production and subsequent two-body decay of one chargino into a sneutrino, such an asymmetry can be as large as 30% [13] and it will allow us to constrain φ_μ. In the present work we will study chargino production and decay into a *W* boson. We will show that, due to the spin correlations between the chargino and the *W* boson, also an asymmetry is obtained which is sensitive to φ_{M_1}.

We study chargino production

$$e^+ + e^- \to \tilde{\chi}_i^+(p_{\chi_i^+}, \lambda_i) + \tilde{\chi}_j^-(p_{\chi_j^-}, \lambda_j), \quad (1)$$

with longitudinally polarized beams and the subsequent two-body decay

$$\tilde{\chi}_i^+ \to \tilde{\chi}_n^0(p_{\chi_n^0}, \lambda_n) + W^+(p_W, \lambda_k), \quad (2)$$

where p and λ denote momentum and helicity. We define the triple product

$$\mathcal{T}_I = \mathbf{p}_{e^-}\cdot(\mathbf{p}_{\chi_i^+} \times \mathbf{p}_W), \quad (3)$$

and the T odd asymmetry

$$\mathcal{A}_I^{\mathrm{T}} = \frac{\sigma(\mathcal{T}_I > 0) - \sigma(\mathcal{T}_I < 0)}{\sigma(\mathcal{T}_I > 0) + \sigma(\mathcal{T}_I < 0)}, \quad (4)$$

with σ the cross section of chargino production (1) and decay (2). The asymmetry $\mathcal{A}_I^{\mathrm{T}}$ is sensitive to the *CP* violating phase φ_μ. In this context it is interesting to note that asymmetries vanish if they correspond to a triple product which contains a transverse polarization vector of the e^+ and e^- beams [7,14].

In order to probe also the phase φ_{M_1}, which enters in the chargino decay process (2), we consider the subsequent hadronic decay of the *W* boson

$$W^+ \to c + \bar{s}. \quad (5)$$

The correlations between the $\tilde{\chi}_i^+$ polarization and the *W* boson polarization lead to *CP* sensitive elements of the *W* boson density matrix, which we will identify and discuss in detail. With the triple product

$$\mathcal{T}_{II} = \mathbf{p}_{e^-}\cdot(\mathbf{p}_c \times \mathbf{p}_{\bar{s}}), \quad (6)$$

which includes the momenta of the *W* decay products and thus probes the *W* polarization, we define a second T odd

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asymmetry

$$\mathcal{A}_{II}^{\mathrm{T}} = \frac{\sigma(\mathcal{T}_{II} > 0) - \sigma(\mathcal{T}_{II} < 0)}{\sigma(\mathcal{T}_{II} > 0) + \sigma(\mathcal{T}_{II} < 0)}. \quad (7)$$

Here, σ is the cross section of production (1) and decay of the chargino (2) followed by that of the W boson (5). Owing to the spin correlations, $\mathcal{A}_{II}^{\mathrm{T}}$ has CP sensitive contributions from φ_μ due to the chargino production process (1) and contributions due to φ_μ and φ_{M_1} from the chargino decay process (2). We treat the decay (5) as a standard model process.

The T odd asymmetries $\mathcal{A}_I^{\mathrm{T}}$ and $\mathcal{A}_{II}^{\mathrm{T}}$ have also absorptive contributions from s-channel resonances or final-state interactions, which do not signal CP violation. In order to eliminate these contributions, we study the two CP odd asymmetries,

$$\mathcal{A}_I = \tfrac{1}{2}(\mathcal{A}_I^{\mathrm{T}} - \bar{\mathcal{A}}_I^{\mathrm{T}}), \qquad \mathcal{A}_{II} = \tfrac{1}{2}(\mathcal{A}_{II}^{\mathrm{T}} - \bar{\mathcal{A}}_{II}^{\mathrm{T}}), \quad (8)$$

where $\bar{\mathcal{A}}_{I,II}^{\mathrm{T}}$ are the CP conjugated asymmetries for the processes $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+; \tilde{\chi}_i^- \to W^- \tilde{\chi}_n^0$ and $e^+e^- \to \tilde{\chi}_i^- \tilde{\chi}_j^+; \tilde{\chi}_i^- \to W^- \tilde{\chi}_n^0; W^- \to \bar{c}s$, respectively. As we will show, the asymmetry $\mathcal{A}_{II}$ is very sensitive to φ_{M_1}. Also triple-product asymmetries in neutralino production $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ and decay provide a handle on the determination of this phase [8,11,12,15].

In Sec. II we give our definitions and formalism used, and obtain the analytical formulas for the differential cross section and the W boson density matrix. In Sec. III we discuss general properties of the asymmetries. We present numerical results in Sec. IV, and Sec. V gives a summary and conclusions.

II. DEFINITIONS AND FORMALISM

We give the analytical formulas for the differential cross section of chargino production (1) with longitudinally polarized beams and the subsequent decay chain of one of the charginos (2) followed by the decay of the W boson

$$W^+ \to f' \bar{f}, \quad (9)$$

which may be leptonic, $f' = \nu_\ell$, $\bar{f} = \bar{\ell}$ with $\ell = e, \mu, \tau$ or hadronic, $f' = q_u$, $\bar{f} = \bar{q}_d$ with $q_u = u, d$ and $q_d = c, s$. For a schematic picture of the chargino production and decay process, see Fig. 1. In the following we will derive the W boson spin-density matrix and relate it to the asymmetries $\mathcal{A}_I^{\mathrm{T}}$ (4) and $\mathcal{A}_{II}^{\mathrm{T}}$ (7).

A. Lagrangian and couplings

The MSSM interaction Lagrangians relevant for our study are [1,16]

$$\mathcal{L}_{Z^0\ell\bar{\ell}} = -\frac{g}{\cos\theta_W} Z_\mu \bar{\ell} \gamma^\mu [L_\ell P_L + R_\ell P_R] \ell, \quad (10)$$

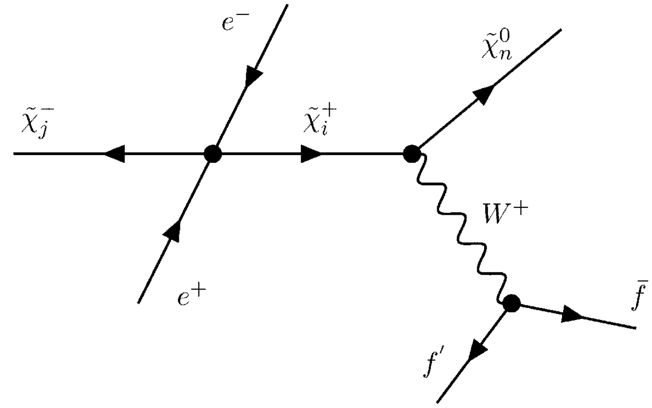

FIG. 1. Schematic picture of the chargino production and decay process.

$$\mathcal{L}_{\gamma\tilde{\chi}_j^+\tilde{\chi}_i^-} = -eA_\mu \bar{\tilde{\chi}}_i^+ \gamma^\mu \tilde{\chi}_j^+ \delta_{ij}, \qquad e > 0, \quad (11)$$

$$\mathcal{L}_{Z^0\tilde{\chi}_j^+\tilde{\chi}_i^-} = \frac{g}{\cos\theta_W} Z_\mu \bar{\tilde{\chi}}_i^+ \gamma^\mu [O_{ij}^{\prime L} P_L + O_{ij}^{\prime R} P_R] \tilde{\chi}_j^+, \quad (12)$$

$$\mathcal{L}_{\ell\tilde{\nu}\tilde{\chi}_i^+} = -gU_{i1}^* \bar{\tilde{\chi}}_i^+ P_L \nu \tilde{\ell}_L^* - gV_{i1}^* \bar{\tilde{\chi}}_i^{+C} P_L \ell \tilde{\nu}^* + \text{H.c.}, \quad (13)$$

$$\mathcal{L}_{W^-\tilde{\chi}_i^+\tilde{\chi}_k^0} = gW_\mu^- \bar{\tilde{\chi}}_k^0 \gamma^\mu [O_{ki}^L P_L + O_{ki}^R P_R] \tilde{\chi}_i^+ + \text{H.c.}, \quad (14)$$

with the couplings

$$L_\ell = T_{3\ell} - e_\ell \sin^2\theta_W, \qquad R_\ell = -e_\ell \sin^2\theta_W, \quad (15)$$

$$O_{ij}^{\prime L} = -V_{i1}V_{j1}^* - \tfrac{1}{2}V_{i2}V_{j2}^* + \delta_{ij}\sin^2\theta_W, \quad (16)$$

$$O_{ij}^{\prime R} = -U_{i1}^*U_{j1} - \tfrac{1}{2}U_{i2}^*U_{j2} + \delta_{ij}\sin^2\theta_W, \quad (17)$$

$$O_{ki}^L = -1/\sqrt{2}(\cos\beta N_{k4} - \sin\beta N_{k3})V_{i2}^* + (\sin\theta_W N_{k1} + \cos\theta_W N_{k2})V_{i1}^*, \quad (18)$$

$$O_{ki}^R = +1/\sqrt{2}(\sin\beta N_{k4}^* + \cos\beta N_{k3}^*)U_{i2} + (\sin\theta_W N_{k1}^* + \cos\theta_W N_{k2}^*)U_{i1}, \quad (19)$$

with $i, j = 1, 2$ and $k = 1, \ldots, 4$. Here $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, g is the weak coupling constant ($g = e/\sin\theta_W$), and e_ℓ and $T_{3\ell}$ denote the charge and the third component of the weak isospin of the lepton ℓ. Furthermore, $\tan\beta = \frac{v_2}{v_1}$ where $v_{1,2}$ are the vacuum expectation values of the two neutral Higgs fields. The chargino-mass eigenstates $\tilde{\chi}_i^+ = \binom{\chi_i^+}{\bar{\chi}_i^-}$ are defined by $\chi_i^+ = V_{i1}w^+ + V_{i2}h^+$ and $\chi_j^- = U_{j1}w^- + U_{j2}h^-$ with $w^\pm$ and $h^\pm$ the two-component spinor fields of the W-ino and the charged Higgsinos, respectively. The complex unitary 2×2 matrices U_{mn}

and V_{mn} diagonalize the chargino-mass matrix $X_{\alpha\beta}$, $U^*_{m\alpha}X_{\alpha\beta}V^{-1}_{\beta n} = m_{\tilde{\chi}^+_n}\delta_{mn}$, with $m_{\tilde{\chi}^+_n} > 0$. The complex unitary 4×4 matrix N_{ij} diagonalizes the neutral gaugino-Higgsino mass matrix $Y_{\alpha\beta}$, $N^*_{i\alpha}Y_{\alpha\beta}N^\dagger_{\beta k} = m_{\tilde{\chi}^0_i}\delta_{ik}$, with $m_{\tilde{\chi}^0_i} > 0$, in the neutralino basis $\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b$.

B. Helicity amplitudes

The helicity amplitudes $T_P^{\lambda_i\lambda_j}$ for the production process are given in [16]. Those for the chargino decay (2) are

$$T^{\lambda_n\lambda_k}_{D_1,\lambda_i} = ig\bar{u}(p_{\chi^0_n},\lambda_n)\gamma^\mu[O^L_{ni}P_L + O^R_{ni}P_R]u(p_{\chi^+_i},\lambda_i)\varepsilon^{\lambda_k *}_\mu \tag{20}$$

and those for the W decay (9) are

$$T^{\lambda_{f'}\lambda_{\bar{f}}}_{D_2,\lambda_k} = i\frac{g}{\sqrt{2}}\bar{u}(p_{f'},\lambda_{f'})\gamma^\mu P_L v(p_{\bar{f}},\lambda_{\bar{f}})\varepsilon^{\lambda_k}_\mu. \tag{21}$$

The W polarization vectors $\varepsilon^{\lambda_k}_\mu$, $\lambda_k = 0, \pm 1$, are given in Appendix A. The amplitude for the whole process (1), (2), and (9) is

$$T = \Delta(\tilde{\chi}^+_i)\Delta(W^+)\sum_{\lambda_i,\lambda_k} T_P^{\lambda_i\lambda_j}T^{\lambda_n\lambda_k}_{D_1,\lambda_i}T^{\lambda_{f'}\lambda_{\bar{f}}}_{D_2,\lambda_k}, \tag{22}$$

with the chargino propagator $\Delta(\tilde{\chi}^+_i) = i/[p^2_{\chi^+_i} - m^2_{\chi^+_i} + im_{\chi^+_i}\Gamma_{\chi^+_i}]$ and the W boson propagator $\Delta(W^+) = i/[p^2_W - m^2_W + im_W\Gamma_W]$.

C. Cross section

For the calculation of the cross section for the combined process of chargino production (1) and the subsequent two-body decays (2) and (9) of $\tilde{\chi}^+_i$ we use the same spin-density matrix formalism as in [16,17]. The (unnormalized) spin-density matrix of the W boson,

$$\rho_P(W^+)^{\lambda_k\lambda'_k} = |\Delta(\tilde{\chi}^+_i)|^2\sum_{\lambda_i,\lambda'_i}\rho_P(\tilde{\chi}^+_i)^{\lambda_i\lambda'_i}\rho_{D_1}(\tilde{\chi}^+_i)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i}, \tag{23}$$

is composed of the spin-density production matrix

$$\rho_P(\tilde{\chi}^+_i)^{\lambda_i\lambda'_i} = \sum_{\lambda_j} T_P^{\lambda_i\lambda_j}T_P^{\lambda'_i\lambda_j *} \tag{24}$$

and the decay matrix of the chargino

$$\rho_{D_1}(\tilde{\chi}^+_i)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} = \sum_{\lambda_n} T^{\lambda_n\lambda_k}_{D_1,\lambda_i}T^{\lambda_n\lambda'_k *}_{D_1,\lambda'_i}. \tag{25}$$

With the decay matrix for the W decay

$$\rho_{D_2}(W^+)_{\lambda'_k\lambda_k} = \sum_{\lambda_{f'},\lambda_{\bar{f}}} T^{\lambda_{f'}\lambda_{\bar{f}}}_{D_2,\lambda_k}T^{\lambda_{f'}\lambda_{\bar{f}} *}_{D_2,\lambda'_k} \tag{26}$$

the amplitude squared for the complete process $e^+e^- \to \tilde{\chi}^+_i\tilde{\chi}^-_j$; $\tilde{\chi}^+_i \to W^+\tilde{\chi}^0_n$; $W^+ \to f'\bar{f}$ can now be written

$$|T|^2 = |\Delta(W^+)|^2\sum_{\lambda_k,\lambda'_k}\rho_P(W^+)^{\lambda_k\lambda'_k}\rho_{D_2}(W^+)_{\lambda'_k\lambda_k}. \tag{27}$$

The differential cross section is then given by

$$d\sigma = \frac{1}{2s}|T|^2 d\mathrm{Lips}(s, p_{\chi^-_j}, p_{\chi^0_n}, p_{f'}, p_{\bar{f}}), \tag{28}$$

where $d\mathrm{Lips}(s, p_{\chi^-_j}, p_{\chi^0_n}, p_{f'}, p_{\bar{f}})$ is the Lorentz invariant phase-space element, see (B1) of Appendix B. More details concerning kinematics and phase space can be found in Appendices A and B.

For the polarization of the decaying chargino $\tilde{\chi}^+_i$ with momentum $p_{\chi^+_i}$, we introduce three spacelike spin vectors $s^a_{\chi^+_i}$, $a = 1, 2, 3$, which together with $p_{\chi^+_i}/m_{\chi^+_i}$ form an orthonormal set with $s^a_{\chi^+_i}s^b_{\chi^+_i} = -\delta^{ab}$, $s^a_{\chi^+_i}p_{\chi^+_i} = 0$. Then the (unnormalized) chargino density matrix can be expanded in terms of the Pauli matrices σ^a, $a = 1, 2, 3$:

$$\rho_P(\tilde{\chi}^+_i)^{\lambda_i\lambda'_i} = 2(\delta_{\lambda_i\lambda'_i}P + \sigma^a_{\lambda_i\lambda'_i}\Sigma^a_P), \tag{29}$$

where we sum over a. With our choice of the spin vectors $s^a_{\chi^+_i}$, given in Appendix A, Σ^3_P/P is the longitudinal polarization of chargino $\tilde{\chi}^+_i$, Σ^1_P/P is the transverse polarization in the production plane, and Σ^2_P/P is the polarization perpendicular to the production plane. We give in Appendix D the analytical formulas for P and Σ^a_P in the laboratory system. To describe the polarization states of the W boson, we introduce a set of spin vectors t^c_W, $c = 1, 2, 3$, and choose polarization vectors $\varepsilon^{\lambda_k}_\mu$, $\lambda_k = 0, \pm 1$, given in Appendix A. Then we obtain for the decay matrices

$$\rho_{D_1}(\tilde{\chi}^+_i)^{\lambda_k\lambda'_k}_{\lambda'_i\lambda_i} = (\delta_{\lambda'_i\lambda_i}D^{\mu\nu}_1 + \sigma^a_{\lambda'_i\lambda_i}\Sigma^{a\mu\nu}_{D_1})\varepsilon^{\lambda_k *}_\mu\varepsilon^{\lambda'_k}_\nu \tag{30}$$

and

$$\rho_{D_2}(W^+)_{\lambda'_k\lambda_k} = D^{\mu\nu}_2\varepsilon^{\lambda_k}_\mu\varepsilon^{\lambda'_k *}_\nu, \tag{31}$$

with

$$D_1^{\mu\nu} = g^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)[2p_{\chi_i^+}^\mu p_{\chi_i^+}^\nu - (p_{\chi_i^+}^\mu p_W^\nu + p_{\chi_i^+}^\nu p_W^\mu) - \tfrac{1}{2}(m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - m_W^2)g^{\mu\nu}] + 2g^2\mathrm{Re}(O_{ni}^{R*}O_{ni}^L)m_{\chi_i^+}m_{\chi_n^0}g^{\mu\nu} \pm ig^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)\epsilon^{\mu\alpha\nu\beta}p_{\chi_i^+\alpha}p_{W\beta}, \quad (32)$$

$$\Sigma_{D_1}^{a\mu\nu} = \pm g^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)m_{\chi_i^+}[s_{\chi_i^+}^{a,\mu}(p_{\chi_i^+}^\nu - p_W^\nu) + s_{\chi_i^+}^{a,\nu}(p_{\chi_i^+}^\mu - p_W^\mu) + (s_{\chi_i^+}^a p_W)g^{\mu\nu}] - ig^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)m_{\chi_i^+}\epsilon^{\mu\alpha\nu\beta}s_{\chi_i^+\alpha}^a(p_{\chi_i^+\beta} - p_{W\beta}) + 2ig^2\mathrm{Re}(O_{ni}^{R*}O_{ni}^L)m_{\chi_n^0}\epsilon^{\mu\alpha\nu\beta}s_{\chi_i^+\alpha}^a p_{\chi_i^+\beta} - 2ig^2\mathrm{Im}(O_{ni}^{R*}O_{ni}^L)m_{\chi_n^0}(s_{\chi_i^+}^{a,\mu}p_{\chi_i^+}^\nu - s_{\chi_i^+}^{a,\nu}p_{\chi_i^+}^\mu); \qquad (\epsilon_{0123} = 1), \quad (33)$$

and

$$D_2^{\mu\nu} = g^2(-2p_{\bar f}^\mu p_{\bar f}^\nu + p_W^\mu p_{\bar f}^\nu + p_{\bar f}^\mu p_W^\nu - \tfrac{1}{2}m_W^2 g^{\mu\nu}) \overset{-}{\underset{(+)}{}} ig^2\epsilon^{\mu\alpha\nu\beta}p_{W\alpha}p_{\bar f\beta}, \quad (34)$$

where here, and in the following, the lower signs hold for the conjugated processes, here $\tilde\chi_i^- \to W^-\tilde\chi_n^0$ and $W^- \to \bar f' f$, respectively. In (30) and (31) we use the expansion [18]

$$\varepsilon_\mu^{\lambda_k}\varepsilon_\nu^{\lambda_k'*} = \frac{1}{3}\delta^{\lambda_k'\lambda_k}I_{\mu\nu} - \frac{i}{2m_W}\epsilon_{\mu\nu\rho\sigma}p_W^\rho t_W^{c\sigma}(J^c)^{\lambda_k'\lambda_k} - \frac{1}{2}t_{W\mu}^c t_{W\nu}^d(J^{cd})^{\lambda_k'\lambda_k}, \quad (35)$$

summed over c, d, and $\epsilon_{0123} = 1$. J^c, $c = 1, 2, 3$, are the 3×3 spin-1 matrices with $[J^c, J^d] = i\epsilon_{cde}J^e$. The matrices

$$J^{cd} = J^cJ^d + J^dJ^c - \tfrac{4}{3}\delta^{cd}, \quad (36)$$

with $J^{11} + J^{22} + J^{33} = 0$, are the components of a symmetric, traceless tensor. An explicit form of J^c and J^{cd} is given in Appendix C. The completeness relation of the polarization vectors

$$\sum_{\lambda_k}\varepsilon_\mu^{\lambda_k*}\varepsilon_\nu^{\lambda_k} = -g_{\mu\nu} + \frac{p_{W\mu}p_{W\nu}}{m_W^2} \quad (37)$$

is guaranteed by

$$I_{\mu\nu} = -g_{\mu\nu} + \frac{p_{W\mu}p_{W\nu}}{m_W^2}. \quad (38)$$

The second term of (35) describes the vector polarization and the third term describes the tensor polarization of the W boson. The decay matrices can be expanded in terms of the spin matrices J^c and J^{cd}. The first term of the decay matrix ρ_{D_1} (30), which is independent of the chargino polarization, then is

$$D_1^{\mu\nu}\varepsilon_\mu^{\lambda_k*}\varepsilon_\nu^{\lambda_k'} = D_1\delta^{\lambda_k\lambda_k'} + {}^cD_1(J^c)^{\lambda_k\lambda_k'} + {}^{cd}D_1(J^{cd})^{\lambda_k\lambda_k'}, \quad (39)$$

summed over c, d, with

$$D_1 = \frac{1}{6}g^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)\left[m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - 2m_W^2 + \frac{(m_{\chi_i^+}^2 - m_{\chi_n^0}^2)^2}{m_W^2}\right] - 2g^2\mathrm{Re}(O_{ni}^{R*}O_{ni}^L)m_{\chi_i^+}m_{\chi_n^0}, \quad (40)$$

$${}^cD_1 = \pm g^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)m_W(t_W^c p_{\chi_i^+}), \quad (41)$$

$${}^{cd}D_1 = -g^2(|O_{ni}^R|^2 + |O_{ni}^L|^2)[(t_W^c p_{\chi_i^+})(t_W^d p_{\chi_i^+}) + \tfrac{1}{4}(m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - m_W^2)\delta^{cd}] + g^2\mathrm{Re}(O_{ni}^{R*}O_{ni}^L)m_{\chi_i^+}m_{\chi_n^0}\delta^{cd}. \quad (42)$$

As a consequence of the completeness relation (37), the diagonal coefficients are linearly dependent

$${}^{11}D_1 + {}^{22}D_1 + {}^{33}D_1 = -\tfrac{3}{2}D_1. \quad (43)$$

For large chargino momentum $\mathbf{p}_{\chi_i^+}$, the W boson will mainly be emitted in the direction of $\mathbf{p}_{\chi_i^+}$, i.e., $\hat{\mathbf{p}}_{\chi_i^+} \approx \hat{\mathbf{p}}_W$, with $\hat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. Therefore, for high energies we have $(t_W^{1,2}p_{\chi_i^+}) \approx 0$ in (42), resulting in ${}^{11}D_1 \approx {}^{22}D_1$.

For the second term of ρ_{D_1} (30), which depends on the polarization of the decaying chargino, we obtain

$$\Sigma_{D_1}^{a\mu\nu}\varepsilon_\mu^{\lambda_k*}\varepsilon_\nu^{\lambda_k'} = \Sigma_{D_1}^a\delta^{\lambda_k\lambda_k'} + {}^c\Sigma_{D_1}^a(J^c)^{\lambda_k\lambda_k'} + {}^{cd}\Sigma_{D_1}^a(J^{cd})^{\lambda_k\lambda_k'}, \quad (44)$$

summed over c, d, with

$$\Sigma_{D_1}^a = \pm\frac{2}{3}g^2(|O_{ni}^R|^2 - |O_{ni}^L|^2)m_{\chi_i^+}(s_{\chi_i^+}^a p_W) \times\left[\frac{m_{\chi_i^+}^2 - m_{\chi_n^0}^2}{2m_W^2} - 1\right], \quad (45)$$

$$
\begin{aligned}
{}^{c}\Sigma_{D_1}^{a} = & \frac{g^2}{m_W}(|O_{ni}^{R}|^2 + |O_{ni}^{L}|^2) m_{\chi_i^+}\Big[(t_W^c p_{\chi_i^+})(s_{\chi_i^+}^a p_W) \\
& + \frac{1}{2}(t_W^c s_{\chi_i^+}^a)(m_{\chi_n^0}^2 - m_{\chi_i^+}^2 + m_W^2)\Big] \\
& - \frac{2g^2}{m_W}\mathrm{Re}(O_{ni}^{R*} O_{ni}^{L}) m_{\chi_n^0}\Big[(t_W^c p_{\chi_i^+})(s_{\chi_i^+}^a p_W) \\
& + \frac{1}{2}(t_W^c s_{\chi_i^+}^a)(m_{\chi_n^0}^2 - m_{\chi_i^+}^2 - m_W^2)\Big] \\
& + \frac{2g^2}{m_W}\mathrm{Im}(O_{ni}^{R*} O_{ni}^{L}) m_{\chi_n^0}\epsilon_{\mu\nu\rho\sigma} s_{\chi_i^+}^{a\mu} p_{\chi_i^+}^{\nu} p_W^{\rho} t_W^{c\sigma}, \quad (46)
\end{aligned}
$$

$$
\begin{aligned}
{}^{cd}\Sigma_{D_1}^{a} = & \pm\frac{1}{2}g^2(|O_{ni}^{R}|^2 - |O_{ni}^{L}|^2) m_{\chi_i^+}[(s_{\chi_i^+}^a p_W)\delta^{cd} \\
& - (t_W^c p_{\chi_i^+})(t_W^d s_{\chi_i^+}^a) - (t_W^d p_{\chi_i^+})(t_W^c s_{\chi_i^+}^a)]. \quad (47)
\end{aligned}
$$

A similar expansion for the W decay matrix (31) results in

$$\rho_{D_2}(W^+)_{\lambda_k'\lambda_k} = D_2\delta^{\lambda_k'\lambda_k} + {}^{c}D_2(J^c)^{\lambda_k'\lambda_k} + {}^{cd}D_2(J^{cd})^{\lambda_k'\lambda_k}, \quad (48)$$

where we sum over c, d, with

$$D_2 = \tfrac{1}{3}g^2 m_W^2, \quad (49)$$

$${}^{c}D_2 = \underset{(+)}{-} g^2 m_W (t_W^c p_{\bar f}), \quad (50)$$

$${}^{cd}D_2 = g^2[(t_W^c \cdot p_{\bar f})(t_W^d p_{\bar f}) - \tfrac{1}{4}m_W^2\delta^{cd}]. \quad (51)$$

The diagonal coefficients are linearly dependent

$${}^{11}D_2 + {}^{22}D_2 + {}^{33}D_2 = -\tfrac{3}{2}D_2. \quad (52)$$

Inserting the density matrices (29) and (30) into (23) leads to

$$
\begin{aligned}
\rho_P(W^+)^{\lambda_k\lambda_k'} = & 4|\Delta(\tilde\chi_i^+)|^2[(PD_1 + \Sigma_P^a\Sigma_{D_1}^a)\delta^{\lambda_k\lambda_k'} + (P\,{}^{c}D_1 \\
& + \Sigma_P^a\,{}^{c}\Sigma_{D_1}^a)(J^c)^{\lambda_k\lambda_k'} + (P\,{}^{cd}D_1 \\
& + \Sigma_P^a\,{}^{cd}\Sigma_{D_1}^a)(J^{cd})^{\lambda_k\lambda_k'}], \quad (53)
\end{aligned}
$$

summed over a, c, d. Inserting then (31) and (53) into (27) leads to the decomposition of the amplitude squared in its scalar (first term), vector (second term) and tensor part (third term):

$$
\begin{aligned}
|T|^2 = & 4|\Delta(\tilde\chi_i^+)|^2|\Delta(W^+)|^2\{3(PD_1 + \Sigma_P^a\Sigma_{D_1}^a)D_2 \\
& + 2(P\,{}^{c}D_1 + \Sigma_P^a\,{}^{c}\Sigma_{D_1}^a)^{c}D_2 + 4[(P\,{}^{cd}D_1 \\
& + \Sigma_P^a\,{}^{cd}\Sigma_{D_1}^a)^{cd}D_2 - \tfrac{1}{3}(P\,{}^{cc}D_1 + \Sigma_P^a\,{}^{cc}\Sigma_{D_1}^a)^{dd}D_2]\}, \quad (54)
\end{aligned}
$$

summed over a, c, d.

D. Density matrix of the W boson

The mean polarization of the W bosons in the laboratory system is given by the 3×3 density matrix $\langle\rho(W^+)\rangle$, obtained by integrating (53) over the Lorentz invariant phase-space element $d\mathrm{Lips}(s, p_{\chi_j^-}, p_{\chi_n^0}, p_W)$ [see (B1)] and normalizing by the trace:

$$
\begin{aligned}
\langle\rho(W^+)^{\lambda_k\lambda_k'}\rangle &= \frac{\int\rho_P(W^+)^{\lambda_k\lambda_k'}d\mathrm{Lips}}{\int\mathrm{Tr}\{\rho_P(W^+)^{\lambda_k\lambda_k'}\}d\mathrm{Lips}} \\
&= \frac{1}{3}\delta^{\lambda_k\lambda_k'} + V_c(J^c)^{\lambda_k\lambda_k'} + T_{cd}(J^{cd})^{\lambda_k\lambda_k'}, \quad (55)
\end{aligned}
$$

summed over c, d. The vector and tensor coefficients V_c and T_{cd} are given by

$$V_c = \frac{\int|\Delta(\tilde\chi_i^+)|^2(P\,{}^{c}D_1 + \Sigma_P^a\,{}^{c}\Sigma_{D_1}^a)d\mathrm{Lips}}{3\int|\Delta(\tilde\chi_i^+)|^2 PD_1 d\mathrm{Lips}}, \quad (56)$$

$$T_{cd} = T_{dc} = \frac{\int|\Delta(\tilde\chi_i^+)|^2(P\,{}^{cd}D_1 + \Sigma_P^a\,{}^{cd}\Sigma_{D_1}^a)d\mathrm{Lips}}{3\int|\Delta(\tilde\chi_i^+)|^2 PD_1 d\mathrm{Lips}}, \quad (57)$$

with sum over a. The density matrix in the circular polarization basis (A11) is given by

$$\langle\rho(W^+)^{--}\rangle = \tfrac{1}{2} - V_3 + T_{33}, \quad (58)$$

$$\langle\rho(W^+)^{00}\rangle = -2T_{33}, \quad (59)$$

$$\langle\rho(W^+)^{-0}\rangle = \frac{1}{\sqrt2}(V_1 + iV_2) - \sqrt2(T_{13} + iT_{23}), \quad (60)$$

$$\langle\rho(W^+)^{-+}\rangle = T_{11} - T_{23} + 2iT_{12}, \quad (61)$$

$$\langle\rho(W^+)^{0+}\rangle = \frac{1}{\sqrt2}(V_1 + iV_2) + \sqrt2(T_{13} + iT_{23}), \quad (62)$$

where we have used $T_{11} + T_{22} + T_{33} = -\frac{1}{2}$.

III. T ODD ASYMMETRIES

From (53) we obtain for asymmetry $\mathcal{A}_I^{\mathrm{T}}$ (4):

$$
\begin{aligned}
\mathcal{A}_I^{\mathrm{T}} &= \frac{\int\mathrm{Sign}[\mathcal{T}_I]\mathrm{Tr}\{\rho_P(W^+)^{\lambda_k\lambda_k'}\}d\mathrm{Lips}}{\int\mathrm{Tr}\{\rho_P(W^+)^{\lambda_k\lambda_k'}\}d\mathrm{Lips}} \\
&= \frac{\int|\Delta(\tilde\chi_i^+)|^2\mathrm{Sign}[\mathcal{T}_I]\Sigma_P^2\Sigma_{D_1}^2 d\mathrm{Lips}}{\int|\Delta(\tilde\chi_i^+)|^2 PD_1 d\mathrm{Lips}}, \quad (63)
\end{aligned}
$$

with $d\mathrm{Lips}(s, p_{\chi_j^-}, p_{\chi_n^0}, p_W)ds_{\chi_i^+}\sum_{\pm}d\mathrm{Lips}(s_{\chi_i^+}, p_{\chi_n^0}, p_{\bar W}^{\pm})$, given in (B1). In the numerator of (63), only the spin correlations $\Sigma_P^2\Sigma_{D_1}^2$ perpendicular to the production plane remain, since only this term contains the triple product $\mathcal{T}_I = \mathbf{p}_{e^-}\cdot(\mathbf{p}_{\chi_i^+}\times\mathbf{p}_W)$. In the denominator only the term PD_1 remains, and all spin correlations vanish due to the integration over the complete phase space. Note that $\mathcal{A}_I^{\rm T} \propto \Sigma_{D_1}^2 \propto (|O_{ni}^R|^2 - |O_{ni}^L|^2)$ [see (45)] and thus $\mathcal{A}_I^{\rm T}$ may be reduced for $|O_{ni}^R| \approx |O_{ni}^L|$. Moreover, $\mathcal{A}_I^{\rm T}$ will be small for $m_{\chi_i^+}^2 - m_{\chi_n^0}^2 \approx 2m_W^2$.

For the asymmetry $\mathcal{A}_{II}^{\rm T}$ (7), we obtain from (54)

$$\mathcal{A}_{II}^{\rm T} = \frac{\int \mathrm{Sign}[\mathcal{T}_{II}]|T|^2 d\mathrm{Lips}}{\int |T|^2 d\mathrm{Lips}} = \frac{\int |\Delta(\tilde{\chi}_i^+)|^2 |\Delta(W^+)|^2 \mathrm{Sign}[\mathcal{T}_{II}] 2\Sigma_P^{a\,c}\Sigma_{D_1}^{a}{}^{c}D_2 d\mathrm{Lips}}{\int |\Delta(\tilde{\chi}_i^+)|^2 |\Delta(W^+)|^2 3PD_1D_2 d\mathrm{Lips}}, \tag{64}$$

summed over a and c, with $d\mathrm{Lips} = d\mathrm{Lips}(s, p_{\chi_j^-}, p_{\chi_n^0}, p_{f'}, p_{\bar{f}})$, defined in (B1). In the numerator only the vector part of $|T|^2$ remains because only the vector part contains the triple product $\mathcal{T}_{II} = \mathbf{p}_{e^-} \cdot (\mathbf{p}_c \times \mathbf{p}_{\bar{s}})$. In the denominator the vector and tensor parts of $|T|^2$ vanish due to phase-space integration. Owing to the correlations between the $\tilde{\chi}_i^+$ and the W boson polarization, $\Sigma_P^{a\,c}\Sigma_{D_1}^a$, there are contributions to the asymmetry $\mathcal{A}_{II}^{\rm T}$ from the chargino production process (1), and/or from the chargino decay process (2). The contribution from the production is given by the term with $a = 2$ in (64) and it is proportional to the transverse polarization of the chargino perpendicular to the production plane, Σ_P^2 (29). For $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_i^+$, we have $\Sigma_P^2 = 0$. The contributions from the decay, which are the terms with $a = 1, 3$ in (64), are proportional to

$$^c\Sigma_{D_1}^{a}{}^{c}D_2 \supset -2g^4 m_{\chi_n^0} \mathrm{Im}(O_{ni}^{R*}O_{ni}^L) \times (t_W^c p_{\bar{f}})\epsilon_{\mu\nu\rho\sigma} s_{\chi_i^+}^{a,\mu} p_{\chi_i^+}^{\nu} p_W^{\rho} t_W^{c\sigma}, \tag{65}$$

which contains the ϵ tensor [see the last term of (46)]. Thus, $\mathcal{A}_{II}^{\rm T}$ can be enhanced (reduced) if the contributions from production and decay have the same (opposite) sign. Note that the contributions from the decay would vanish for a two-body decay of the chargino into a scalar particle instead of a W boson.

The relative statistical error of $\mathcal{A}_I^{\rm T}$ is $\delta\mathcal{A}_I^{\rm T} = \Delta\mathcal{A}_I^{\rm T}/|\mathcal{A}_I^{\rm T}| = 1/(|\mathcal{A}_I^{\rm T}|\sqrt{N})$, where $N = \mathcal{L}\cdot\sigma$ is the number of events for the integrated luminosity $\mathcal{L}$ and the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-) \times \mathrm{BR}(\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0)$. For the CP asymmetry $\mathcal{A}_I$ (8), we have $\Delta\mathcal{A}_I = \Delta\mathcal{A}_I^{\rm T}/\sqrt{2}$. The statistical significance, with which the asymmetry can be measured, is then given by $S_I = |\mathcal{A}_I|\sqrt{2\mathcal{L}\cdot\sigma}$. A similar result is obtained for the asymmetry $\mathcal{A}_{II}$ with $S_{II} = |\mathcal{A}_{II}|\sqrt{2\mathcal{L}\cdot\sigma}$ and the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-) \times \mathrm{BR}(\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0) \times \mathrm{BR}(W^+ \to c\bar{s})$.

Note that in order to measure $\mathcal{A}_I$ the momentum of $\tilde{\chi}_i^+$, i.e., the production plane, has to be kinematically reconstructed. This could be accomplished by measuring the decay of the other chargino $\tilde{\chi}_j^-$, if the masses of the charginos and the masses of their decay products are known. If the other chargino is a $\tilde{\chi}_1^-$ it can decay into $W^-\tilde{\chi}_1^0$ or into $\ell^-\tilde{\nu}_\ell$. If the W^- decays hadronically, the event can be reconstructed after applying a suitable acoplanarity cut. Possible combinatoric problems can in principle be solved by charm tagging and applying energy-momentum constraints. If the decay $\tilde{\chi}_1^- \to \ell^-\tilde{\nu}_\ell$ occurs and the mass of $\tilde{\nu}_\ell$ is known, the reconstruction of the whole event is possible. If the other chargino is a $\tilde{\chi}_2^-$ even the decays into the Z boson $\tilde{\chi}_2^- \to \tilde{\chi}_1^- Z^0$ and the lightest neutral Higgs boson $\tilde{\chi}_2^- \to \tilde{\chi}_1^- H_1^0$ can be used for an event selection.

For the measurement of $\mathcal{A}_{II}$, the flavors of the quarks c and $\bar{s}$ have to be distinguished, which will be possible by flavor tagging of the c quark [19,20]. The main techniques for charm tagging are the reconstruction of the secondary vertex from decaying charm hadrons and lepton tagging. In [20] the decay $W \to c\bar{s}$ at LEP2 energies has been studied. A charm tagging efficiency of about 18% has been obtained. In principle, for the decay $W \to u\bar{d}$ also an asymmetry similar to $\mathcal{A}_{II}$ can be considered, if it is possible to distinguish between the u and $\bar{d}$ jet, for instance, by measuring the average charge. Also, it is clear that detailed Monte Carlo studies taking into account background and detector simulations are necessary to predict the expected accuracies. However, this is beyond the scope of the present work.

IV. NUMERICAL RESULTS

We study the dependence of $\mathcal{A}_I$, $\mathcal{A}_{II}$ (8), and the density matrix $\langle\rho(W^+)\rangle$ (55), on the MSSM parameters $\mu = |\mu|e^{i\varphi_\mu}$, $M_1 = |M_1|e^{i\varphi_{M_1}}$, $\tan\beta$, and the universal scalar mass parameter m_0. We will allow $\varphi_{M_1} \in [\pi, -\pi]$; however, we take into account $|\varphi_\mu| \lesssim 0.1\pi$ in some of the plots, as suggested from the EDM analyses [3,4]. In order to show the full phase dependence of the asymmetries, however, we will relax the EDM restrictions in some of the examples studied.

The feasibility of measuring the asymmetries depends also on the cross sections $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-) \times \mathrm{BR}(\tilde{\chi}_i^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$, which we will discuss in our scenarios. We choose a center of mass energy of $\sqrt{s} = 800$ GeV and longitudinally polarized beams with $(P_{e^-}, P_{e^+}) = (-0.8, +0.6)$. This choice enhances sneutrino exchange in the chargino production process, which results in larger cross sections and asymmetries. For the calculation of the branching ratios $\mathrm{BR}(\tilde{\chi}_i^+ \to W^+\tilde{\chi}_1^0)$ and widths $\Gamma_{\chi_i^+}$, we include the two-body decays,

$$\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0, \tilde{e}_L^+\nu_e, \tilde{\mu}_L^+\nu_\mu, \tilde{\tau}_{1,2}^+\nu_\tau, e^+\tilde{\nu}_e, \mu^+\tilde{\nu}_\mu, \tau^+\tilde{\nu}_\tau, \tag{66}$$

and neglect three-body decays. For the W boson decay, we take the experimental value $\mathrm{BR}(W^+ \to c\bar{s}) = 0.31$ [21]. In order to reduce the number of parameters, we assume the relation $|M_1| = 5/3M_2\tan^2\theta_W$ and use the

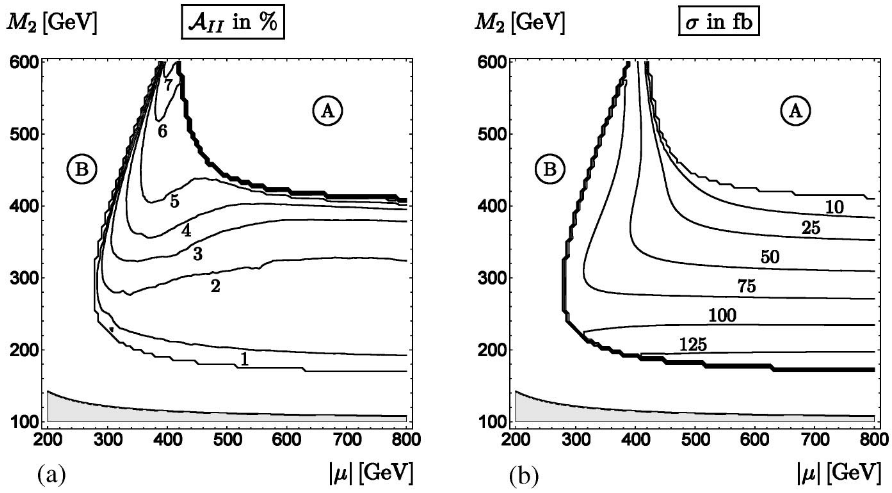

FIG. 2. Contour lines of the asymmetry $\mathcal{A}_{II}$ (a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ (b), in the $|\mu|$-M_2 plane for $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A is kinematically forbidden by $m_{\chi_1^+} + m_{\chi_1^-} > \sqrt{s}$, the area B by $m_W + m_{\chi_1^0} > m_{\chi_1^+}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

approximate solutions of the renormalization group equations for the slepton and sneutrino masses, $m_{\tilde{\ell}_L}^2 = m_0^2 + 0.79M_2^2 + m_Z^2\cos 2\beta(-1/2 + \sin^2\theta_W)$ and $m_{\tilde{\nu}_\ell}^2 = m_0^2 + 0.79M_2^2 + m_Z^2/2\cos 2\beta$ [22]. In the stau sector [23], we fix the trilinear scalar coupling parameter to $A_\tau = 250$ GeV.

A. Production of $\tilde{\chi}_1^+\tilde{\chi}_1^-$

For the production $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_i^-$ of a pair of charginos, the polarization perpendicular to the production plane vanishes, and thus $\mathcal{A}_I = 0$. However, $\mathcal{A}_{II}$ need not be zero and is sensitive to φ_μ and φ_{M_1}, because this asymmetry has contributions from the chargino decay process. For $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$, we show in Fig. 2(a) the $|\mu|$-M_2 dependence of $\mathcal{A}_{II}$, which can reach values of 5%–7% for $M_2 \gtrsim 400$ GeV. We also studied the φ_μ dependence of $\mathcal{A}_{II}$ in the $|\mu|$-M_2 plane. For $\varphi_{M_1} = 0$, $\varphi_\mu = 0.1\pi(0.5\pi)$ and the other parameters as given in the caption of Fig. 2, we find $|\mathcal{A}_{II}| < 2\%(7\%)$.

In Fig. 2(b), we show the contour lines of the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ in the $|\mu|$-M_2 plane for $(\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0)$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ reaches more than 400 fb. For our choice of $m_0 = 300$ GeV, $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$ is the only allowed two-body decay channel.

In Fig. 3(a), we plot the contour lines of $\mathcal{A}_{II}$ for $|\mu| = 350$ GeV and $M_2 = 400$ GeV in the φ_μ-φ_{M_1} plane.

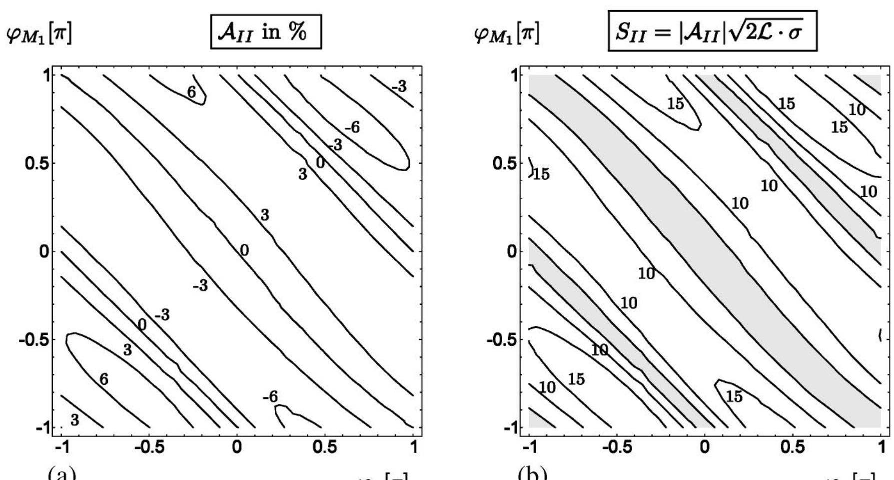

FIG. 3. Contour lines of the asymmetry $\mathcal{A}_{II}$ (a) and the statistical significance S_{II} (b) for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$; $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$; $W^+ \to c\bar{s}$, in the φ_μ-φ_{M_1} plane for $|\mu| = 350$ GeV, $M_2 = 400$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$, and $\mathcal{L} = 500$ fb^{-1}. In the gray shaded area of (b) we have $S_{II} < 5$.

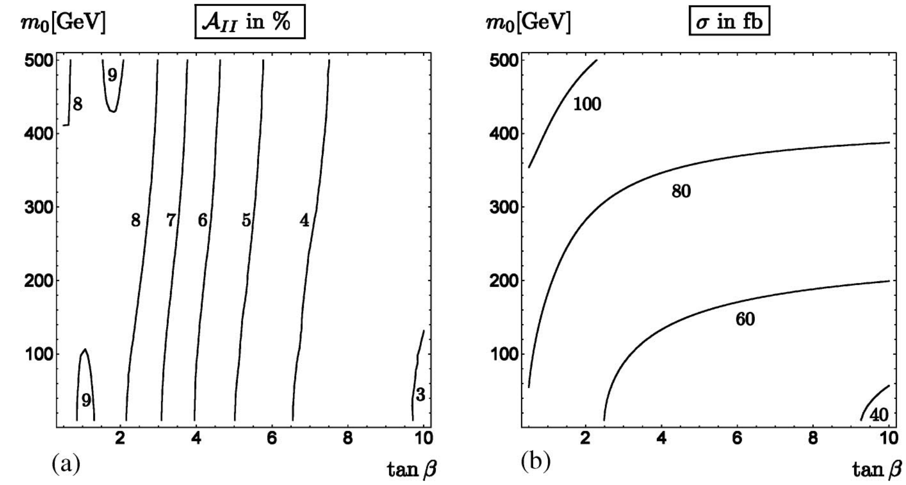

FIG. 4. Contour lines of the asymmetry $\mathcal{A}_{II}$ (a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$ (b), in the $\tan\beta$-m_0 plane for $(\varphi_{M_1}, \varphi_\mu) = (0.7\pi, 0)$, $M_2 = 400$ GeV, $|\mu| = 350$ GeV, $\sqrt{s} = 800$ GeV, and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

Figure 3(a) shows that $\mathcal{A}_{II}$ is essentially depending on the sum $\varphi_\mu + \varphi_{M_1}$. However, maximal phases of $\varphi_{M_1} = \pm 0.5\pi$ and $\varphi_\mu = \pm 0.5\pi$ do not lead to the highest values of $|\mathcal{A}_{II}| \gtrsim 6\%$, which are reached for $(\varphi_{M_1}, \varphi_\mu) \approx (\pm 0.8\pi, \pm 0.6\pi)$. The reason for this is that the spin-correlation terms $\Sigma_P^{a\,c}\Sigma_{D_1}^{a\ \,c}D_2$ in the numerator of $\mathcal{A}_{II}$ (64) are products of CP odd and CP even factors. The CP odd (CP even) factors have a sinelike (cosinelike) phase dependence. Therefore, the maximum of the CP asymmetry $\mathcal{A}_{II}$ may be shifted to smaller or larger values of the phases. In the φ_μ-φ_{M_1} region shown in Fig. 3(a), the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) \times \mathrm{BR}(W^+ \to c\bar{s})$, with $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 W^+) = 1$, does not depend on φ_{M_1} and ranges between 74 fb for $\varphi_\mu = 0$ and 66 fb for $\varphi_\mu = \pi$.

In Fig. 3(b), we show the contour lines of the significance $S_{II} = |\mathcal{A}_{II}|\sqrt{2\mathcal{L}\cdot\sigma}$, defined in Sec. III. For $\mathcal{L} = 500\ \mathrm{fb}^{-1}$ and for, e.g., $(\varphi_{M_1}, \varphi_\mu) \approx (\pi, 0.1\pi)$, we have $S_{II} \approx 8$ and thus $\mathcal{A}_{II}$ could be measured even for small φ_μ.

In Figs. 4(a) and 4(b), we show the $\tan\beta$-m_0 dependence of $\mathcal{A}_{II}$ and σ for $(\varphi_{M_1}, \varphi_\mu) = (0.7\pi, 0)$. The asymmetry is rather insensitive to m_0 and shows strong dependence on $\tan\beta$ and decreases with increasing $\tan\beta \gtrsim 2$. The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ increases with increasing m_0 and decreasing $\tan\beta$. For $m_0 \lesssim 200$ GeV, the branching ratio $\mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) < 1$, since the decay channels of $\tilde{\chi}_1^+$ into sleptons and/or sneutrinos open.

In Fig. 5(a), we show the φ_μ dependence of the vector (V_i) and tensor (T_{ij}) elements of the density matrix $\langle\rho(W^+)\rangle$ for $\varphi_{M_1} = \pi$ [see (56) and (57)]. In Fig. 5(b), we show their dependence on φ_{M_1} for $\varphi_\mu = 0$. In both figures, the element V_2 is CP odd, while T_{13}, T_{11}, T_{22} and

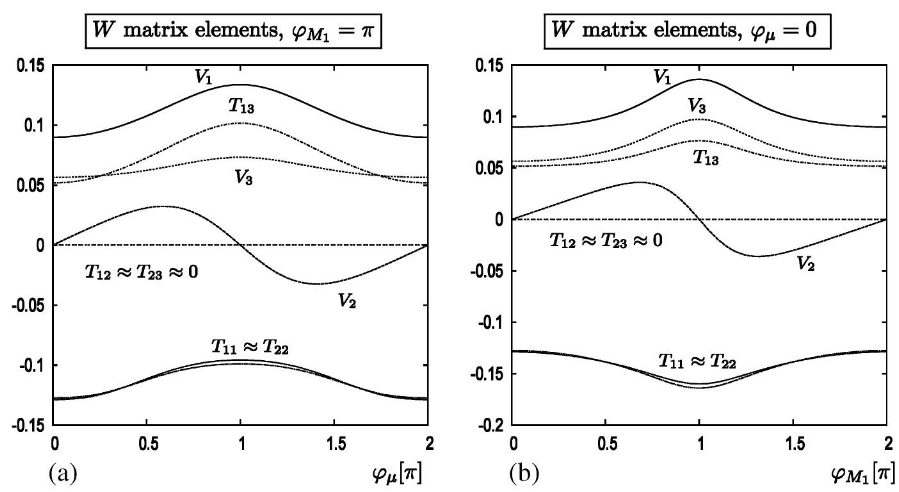

FIG. 5. Dependence of vector (V_i) and tensor (T_{ij}) elements of the W^+ density-matrix $\langle\rho(W^+)\rangle$ on φ_μ (a) and on φ_{M_1} (b), for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-; \tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$, for $|\mu| = 350$ GeV, $M_2 = 400$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

V_1, V_3 show a *CP* even behavior. As discussed in Sec. II C, the tensor elements T_{11} and T_{22} are almost equal and have the same order of magnitude as V_1 and V_3, whereas the other elements T_{12}, $|T_{23}| < 10^{-5}$ are small. In the *CP* conserving case $(\varphi_{M_1}, \varphi_\mu) = (0, 0)$ and $M_2 = 400$ GeV, $|\mu| = 350$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ the density matrix reads

$$\langle\rho(W^+)\rangle = \begin{pmatrix} \langle\rho^{--}\rangle & \langle\rho^{-0}\rangle & \langle\rho^{-+}\rangle \\ \langle\rho^{0-}\rangle & \langle\rho^{00}\rangle & \langle\rho^{0+}\rangle \\ \langle\rho^{+-}\rangle & \langle\rho^{+0}\rangle & \langle\rho^{++}\rangle \end{pmatrix} = \begin{pmatrix} 0.200 & -0.010 & -0.001 \\ -0.010 & 0.487 & 0.137 \\ -0.001 & 0.137 & 0.313 \end{pmatrix}. \tag{67}$$

In the *CP* violating case, e.g., for $(\varphi_{M_1}, \varphi_\mu) = (0.7\pi, 0)$ and the other parameters as above, the density matrix has imaginary parts due to a nonvanishing V_2:

$$\langle\rho(W^+)\rangle = \begin{pmatrix} 0.219 & -0.010 + 0.025i & 0.002 \\ -0.010 - 0.025i & 0.405 & 0.171 + 0.025i \\ 0.002 & 0.171 - 0.025i & 0.376 \end{pmatrix} \tag{68}$$

Imaginary parts of the density matrix are thus an indication of *CP* violation.

B. Production of $\tilde{\chi}_1^+\tilde{\chi}_2^-$

For the production of an unequal pair of charginos, $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$, their polarization perpendicular to the production plane is sensitive to the phase φ_μ, which leads to a nonvanishing asymmetry $\mathcal{A}_I$ (63). We will study the decay of the lighter chargino $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$. For $|M_2| = 250$ GeV and $\varphi_{M_1} = 0$, we show in Fig. 6(a) the $|\mu|$-φ_μ dependence of $\mathcal{A}_I$, which attains values up to 4%. Note that $\mathcal{A}_I$ is not maximal for $\varphi_\mu = 0.5\pi$, but is rather sensitive for phases in the regions $\varphi_\mu \in [0.7\pi, \pi]$ and $\varphi_\mu \in [-0.7\pi, -\pi]$. As mentioned before, values of φ_μ close to the *CP* conserving points $\varphi_\mu = 0, \pm\pi$ are suggested by EDM analyses. For $\varphi_\mu = 0.9\pi$ and $|\mu| = 350$ GeV, the statistical significance is $S_I = |\mathcal{A}_I|\sqrt{2\mathcal{L}\cdot\sigma} \approx 1.5$ with $\mathcal{L} = 500$ fb^{-1}. Thus $\mathcal{A}_I$ could be measured at a confidence level larger than 68% $(S_I = 1)$.

In Fig. 6(b), we show contour lines of the corresponding cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0)$ in the $|\mu|$-φ_μ plane for the parameters as above. The cross section shows a *CP* even behavior, which has been used in [7,8,14] to constrain $\cos\varphi_\mu$. In our scenario we have considered the decay of the lighter chargino $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$ since for our choice $m_0 = 300$ GeV we have $\mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) = 1$. For the decay of $\tilde{\chi}_2^+$, one would have to take into account also the decays into the Z boson and the lightest neutral Higgs boson, which would reduce $\mathrm{BR}(\tilde{\chi}_2^+ \to W^+\tilde{\chi}_1^0) \approx 0.2$.

The asymmetry $\mathcal{A}_{II}$ is also sensitive to the phase φ_{M_1}. We show the φ_μ-φ_{M_1} dependence of $\mathcal{A}_{II}$, choosing the parameters as above, in Fig. 7(a). In Fig. 7(b), we show the contour lines of the significance $S_{II} = |\mathcal{A}_{II}|\sqrt{2\mathcal{L}\cdot\sigma}$ for $\mathcal{L} = 500$ fb^{-1}. For $(\varphi_{M_1}, \varphi_\mu) \approx (\pi, 0.1\pi)$ we have

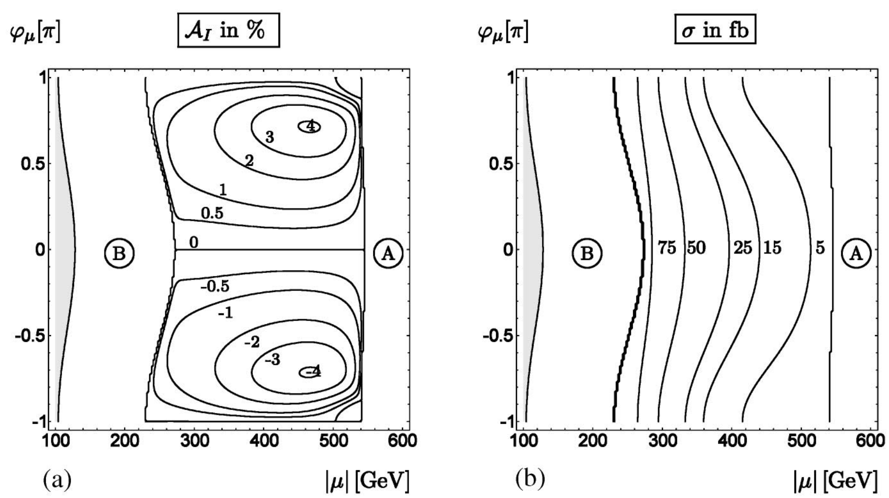

FIG. 6. Contour lines of the asymmetry $\mathcal{A}_I$ (a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0)$ (b), in the $|\mu|$-φ_μ plane for $\varphi_{M_1} = 0$, $M_2 = 250$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A is kinematically forbidden by $m_{\chi_2^+} + m_{\chi_1^-} > \sqrt{s}$, the area B by $m_W + m_{\chi_1^0} > m_{\chi_1^+}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

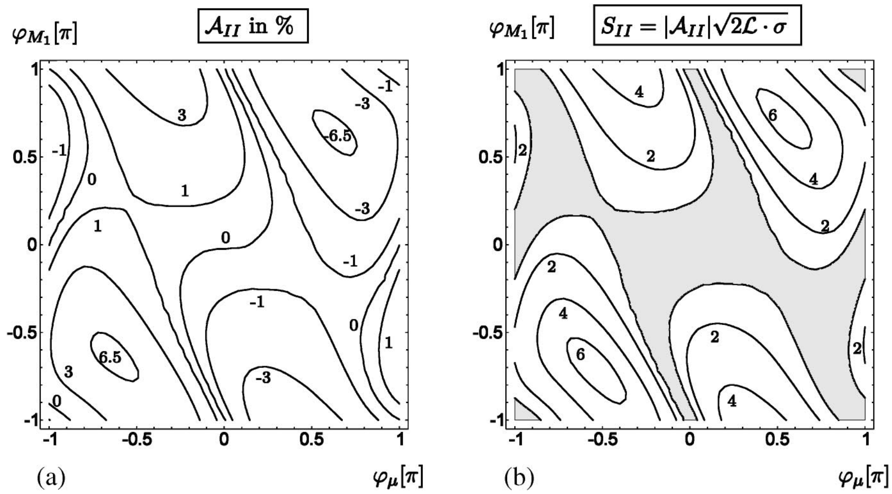

FIG. 7. Contour lines of the asymmetry $\mathcal{A}_{II}$ (a) and the statistical significance S_{II} (b) for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$; $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0$; $W^+ \to c\bar{s}$, in the φ_μ-φ_{M_1} plane for $|\mu| = 350$ GeV, $M_2 = 250$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$, and $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. In the gray shaded area of (b) we have $S_{II} < 1$.

$S_{II} \approx 2.4$ and thus $\mathcal{A}_{II}$ could be accessible even for small phases by using polarized beams.

V. SUMMARY AND CONCLUSIONS

We have proposed and analyzed *CP* sensitive observables in chargino production, $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$, with subsequent two-body decay, $\tilde{\chi}_i^+ \to W^+\chi_n^0$. We have defined the *CP* asymmetry $\mathcal{A}_I$ of the triple product $\mathbf{p}_{e^-}(\mathbf{p}_{\tilde{\chi}_i^+} \times \mathbf{p}_W)$. In the MSSM with complex parameters μ and M_1, we have shown that $\mathcal{A}_I$ can reach 4% and that even for $\varphi_\mu \approx 0.9\pi$ the asymmetry could be accessible in the process $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_2^-$. Further we have analyzed the *CP* sensitive density-matrix elements of the *W* boson. The phase φ_{M_1} enters in the decay $\tilde{\chi}_i^+ \to W^+\chi_n^0$ due to correlations of the chargino and the *W* boson spins, which can be probed via the hadronic decay $W^+ \to c\bar{s}$. Moreover, the triple product $\mathbf{p}_{e^-}(\mathbf{p}_c \times \mathbf{p}_{\bar{s}})$ defines the *CP* asymmetry $\mathcal{A}_{II}$, which can be as large as 7% for $\tilde{\chi}_1^+\tilde{\chi}_1^-$ or $\tilde{\chi}_1^+\tilde{\chi}_2^-$ production. By analyzing the statistical errors of $\mathcal{A}_I$ and $\mathcal{A}_{II}$, we found that the phases φ_μ and φ_{M_1} could be strongly constrained in future e^+e^- collider experiments in the 800 GeV range with high luminosity and longitudinally polarized beams.

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APPENDIX A: COORDINATE FRAME AND SPIN VECTORS

We choose a coordinate frame in the laboratory system such that the momentum of the chargino $\tilde{\chi}_j^-$ points in the z direction (in our definitions we follow closely [16]). The scattering angle is $\theta\angle(\mathbf{p}_{e^-}, \mathbf{p}_{\chi_j^-})$ and the azimuth ϕ can be chosen zero. The momenta are

$$p_{e^-}^\mu = E_b(1, -\sin\theta, 0, \cos\theta), \quad p_{e^+}^\mu = E_b(1, \sin\theta, 0, -\cos\theta), \tag{A1}$$

$$p_{\chi_i^+}^\mu = (E_{\chi_i^+}, 0, 0, -q), \qquad p_{\chi_j^-}^\mu = (E_{\chi_j^-}, 0, 0, q), \tag{A2}$$

with the beam energy $E_b = \sqrt{s}/2$ and

$$E_{\chi_i^+} = \frac{s + m_{\chi_i^+}^2 - m_{\chi_j^-}^2}{2\sqrt{s}}, \qquad E_{\chi_j^-} = \frac{s + m_{\chi_j^-}^2 - m_{\chi_i^+}^2}{2\sqrt{s}}, \quad q = \frac{\lambda^{1/2}(s, m_{\chi_i^+}^2, m_{\chi_j^-}^2)}{2\sqrt{s}}, \tag{A3}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$. For the description of the polarization of chargino $\tilde{\chi}_i^+$, we choose three spin vectors:

$$s_{\tilde{\chi}_i^+}^{1,\mu} = (0, -1, 0, 0), \qquad s_{\tilde{\chi}_i^+}^{2,\mu} = (0, 0, 1, 0), \quad s_{\tilde{\chi}_i^+}^{3,\mu} = \frac{1}{m_{\tilde{\chi}_i^+}}(q, 0, 0, -E_{\tilde{\chi}_i^+}). \tag{A4}$$

Together with $p_{\chi_i^+}^\mu/m_{\chi_i^+}$ they form an orthonormal set. For the two-body decay $\tilde{\chi}_i^+ \to W^+\tilde{\chi}_n^0$ the decay angle $\theta_1\angle(\mathbf{p}_{\chi_i^+}, \mathbf{p}_W)$ is constrained by $\sin\theta_1^{\max} = q^0/q$ for $q > q^0$, where $q^0 = \lambda^{1/2}(m_{\chi_i^+}^2, m_W^2, m_{\chi_n^0}^2)/2m_W$ is the chargino momentum if the *W* boson is produced at rest. In this case there are two solutions

$$|\mathbf{p}_W^{\pm}| = \frac{(m_{\chi_i^+}^2 + m_W^2 - m_{\chi_n^0}^2)q\cos\theta_1 \pm E_{\chi_i^+}\sqrt{\lambda(m_{\chi_i^+}^2, m_W^2, m_{\chi_n^0}^2) - 4q^2m_W^2(1-\cos^2\theta_1)}}{2q^2(1-\cos^2\theta_1) + 2m_{\chi_i^+}^2}. \tag{A5}$$

If $q^0 > q$, θ_1 is not constrained and there is only the physical solution $|\mathbf{p}_W^+|$ left.

The momenta in the laboratory system are

$$p_W^{\pm,\mu} = (E_W^{\pm}, -|\mathbf{p}_W^{\pm}|\sin\theta_1\cos\phi_1, |\mathbf{p}_W^{\pm}|\sin\theta_1\sin\phi_1, -|\mathbf{p}_W^{\pm}|\cos\theta_1), \tag{A6}$$

$$p_{\bar f}^{\mu} = (E_{\bar f}, -|\mathbf{p}_{\bar f}|\sin\theta_2\cos\phi_2, |\mathbf{p}_{\bar f}|\sin\theta_2\sin\phi_2, -|\mathbf{p}_{\bar f}|\cos\theta_2), \tag{A7}$$

$$E_{\bar f}^{\mu} = |\mathbf{p}_{\bar f}| = \frac{m_W^2}{2(E_W^{\pm} - |\mathbf{p}_W^{\pm}|\cos\theta_{D_2})}, \tag{A8}$$

with $\theta_2 \angle(\mathbf{p}_{\chi_i^+}, \mathbf{p}_{\bar f})$ and the decay angle $\theta_{D_2}\angle(\mathbf{p}_W, \mathbf{p}_{\bar f})$ given by

$$\cos\theta_{D_2} = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\phi_2 - \phi_1). \tag{A9}$$

The spin vectors t_W^c, $c = 1, 2, 3$, of the W boson in the laboratory system are chosen as

$$t_W^{1,\mu} = \left(0, \frac{\mathbf{p}_W^2 \times \mathbf{p}_W^3}{|\mathbf{p}_W^2 \times \mathbf{p}_W^3|}\right), \qquad t_W^{2,\mu} = \left(0, \frac{\mathbf{p}_{e^-} \times \mathbf{p}_W}{|\mathbf{p}_{e^-} \times \mathbf{p}_W|}\right),$$
$$t_W^{3,\mu} = \frac{1}{m_W}\left(|\mathbf{p}_W|, E_W\frac{\mathbf{p}_W}{|\mathbf{p}_W|}\right). \tag{A10}$$

The spin vectors and p_W^{μ}/m_W form an orthonormal set.

The polarization vectors ε^{λ_k} for helicities $\lambda_k = -1, 0, +1$ of the W boson are defined by

$$\varepsilon^- = \frac{1}{\sqrt{2}}(t_W^1 - it_W^2); \qquad \varepsilon^0 = t_W^3;$$
$$\varepsilon^+ = -\frac{1}{\sqrt{2}}(t_W^1 + it_W^2). \tag{A11}$$

APPENDIX B: PHASE-SPACE

The Lorentz invariant phase-space element for the chargino production (1) and the decay chain (2)–(9) can be decomposed into the two-body phase-space elements:

$$d\text{Lips}(s, p_{\chi_j^-}, p_{\chi_n^0}, p_{f'}, p_{\bar f}) = \frac{1}{(2\pi)^2}d\text{Lips}(s, p_{\chi_i^+}, p_{\chi_j^-})ds_{\chi_i^+}\sum_{\pm}d\text{Lips}(s_{\chi_i^+}, p_{\chi_n^0}, p_W^{\pm})ds_W d\text{Lips}(s_W, p_{f'}, p_{\bar f}), \tag{B1}$$

$$d\text{Lips}(s, p_{\chi_i^+}, p_{\chi_j^-}) = \frac{q}{8\pi\sqrt{s}}\sin\theta d\theta, \tag{B2}$$

$$d\text{Lips}(s_{\chi_i^+}, p_{\chi_n^0}, p_W^{\pm}) = \frac{1}{2(2\pi)^2}\frac{|\mathbf{p}_W^{\pm}|^2}{2|E_W^{\pm}q\cos\theta_1 - E_{\chi_i^+}|\mathbf{p}_W^{\pm}||}d\Omega_1, \tag{B3}$$

$$d\text{Lips}(s_W, p_{f'}, p_{\bar f}) = \frac{1}{2(2\pi)^2}\frac{|\mathbf{p}_{\bar f}|^2}{m_W^2}d\Omega_2, \tag{B4}$$

with $s_{\chi_i^+} = p_{\chi_i^+}^2$, $s_W = p_W^2$, and $d\Omega_i = \sin\theta_i d\theta_i d\phi_i$. We use the narrow width approximation for the propagators: $\int|\Delta(\tilde\chi_i^+)|^2 ds_{\chi_i^+} = \pi/(m_{\chi_i^+}\Gamma_{\chi_i^+})$, $\int|\Delta(W)|^2 ds_W = \frac{\pi}{m_W\Gamma_W}$. The approximation is justified for $(\Gamma_{\chi_i^+}/m_{\chi_i^+})^2 \ll 1$, which holds in our case with $\Gamma_{\chi_i^+} \lesssim \mathcal{O}(1\text{ GeV})$.

APPENDIX C: SPIN MATRICES

In the basis (A11) the spin matrices J^c and the tensor components J^{cd} are [12]:

$$J^1 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \qquad J^2 = \begin{pmatrix} 0 & \frac{i}{\sqrt{2}} & 0 \\ -\frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \qquad J^3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{C1}$$

$$J^{11} = \begin{pmatrix} -\frac{1}{3} & 0 & 1 \\ 0 & \frac{2}{3} & 0 \\ 1 & 0 & -\frac{1}{3} \end{pmatrix}, \qquad J^{22} = \begin{pmatrix} -\frac{1}{3} & 0 & -1 \\ 0 & \frac{2}{3} & 0 \\ -1 & 0 & -\frac{1}{3} \end{pmatrix}, \qquad J^{33} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix}, \tag{C2}$$

$$J^{12} = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \qquad J^{23} = \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ 0 & -\frac{i}{\sqrt{2}} & 0 \end{pmatrix}, \qquad J^{13} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}. \tag{C3}$$

APPENDIX D: CHARGINO PRODUCTION MATRICES

We give the analytical formulas for $P, \Sigma_P^1, \Sigma_P^2, \Sigma_P^3$ of the chargino production matrix $\rho_P(\tilde{\chi}_i^+)^{\lambda_i\lambda_i'} = 2(\delta_{\lambda_i\lambda_i'}P + \sigma^a_{\lambda_i\lambda_i'}\Sigma_P^a)$ (29), in the laboratory system. Covariant expressions for these functions can be found in [16].

A. Chargino production

The coefficient P is independent of the chargino polarization. It can be composed into the different contributions from the production channels:

$$P = P(\gamma\gamma) + P(\gamma Z) + P(\gamma\tilde{\nu}) + P(ZZ) + P(Z\tilde{\nu}) + P(\tilde{\nu}\,\tilde{\nu}), \tag{D1}$$

which read

$$P(\gamma\gamma) = \delta_{ij}2e^4|\Delta(\gamma)|^2(c_L + c_R)E_b^2(E_{\chi_i^+}E_{\chi_j^-} + m_{\chi_i^+}m_{\chi_j^-} + q^2\cos^2\theta), \tag{D2}$$

$$P(\gamma Z) = \delta_{ij}2\frac{e^2g^2}{\cos^2\theta_W}E_b^2\mathrm{Re}\{\Delta(\gamma)\Delta(Z)^*[(L_ec_L - R_ec_R) \times (O_{ij}^{\prime R*} - O_{ij}^{\prime L*})2E_bq\cos\theta + (L_ec_L + R_ec_R) \times (O_{ij}^{\prime L*} + O_{ij}^{\prime R*})(E_{\chi_i^+}E_{\chi_j^-} + m_{\chi_i^+}m_{\chi_j^-} + q^2\cos^2\theta)]\}, \tag{D3}$$

$$P(\gamma\tilde{\nu}) = \delta_{ij}e^2g^2E_b^2c_L\mathrm{Re}\{V_{i1}^*V_{j1}\Delta(\gamma)\Delta(\tilde{\nu})^*\}(E_{\chi_i^+}E_{\chi_j^-} + m_{\chi_i^+}m_{\chi_j^-} - 2E_bq\cos\theta + q^2\cos^2\theta), \tag{D4}$$

$$P(ZZ) = \frac{g^4}{\cos^4\theta_W}|\Delta(Z)|^2E_b^2[(L_e^2c_L - R_e^2c_R)(|O_{ij}^{\prime R}|^2 - |O_{ij}^{\prime L}|^2)2E_bq\cos\theta + (L_e^2c_L + R_e^2c_R)(|O_{ij}^{\prime L}|^2 + |O_{ij}^{\prime R}|^2)(E_{\chi_i^+}E_{\chi_j^-} + q^2\cos^2\theta) + (L_e^2c_L + R_e^2c_R)2\mathrm{Re}\{O_{ij}^{\prime L}O_{ij}^{\prime R*}\}m_{\chi_i^+}m_{\chi_j^-}], \tag{D5}$$

$$P(Z\tilde{\nu}) = \frac{g^4}{\cos^2\theta_W}L_ec_LE_b^2\mathrm{Re}\{V_{i1}^*V_{j1}\Delta(Z)\Delta(\tilde{\nu})^* \times [O_{ij}^{\prime L}(E_{\chi_i^+}E_{\chi_j^-} - 2E_bq\cos\theta + q^2\cos^2\theta) + O_{ij}^{\prime R}m_{\chi_i^+}m_{\chi_j^-}]\}, \tag{D6}$$

$$P(\tilde{\nu}\,\tilde{\nu}) = \frac{g^4}{4}c_L|V_{i1}|^2|V_{j1}|^2|\Delta(\tilde{\nu})|^2E_b^2(E_{\chi_i^+}E_{\chi_j^-} - 2E_bq\cos\theta + q^2\cos^2\theta). \tag{D7}$$

The propagators are defined by

$$\Delta(\gamma) = \frac{i}{p_\gamma^2}, \qquad \Delta(Z) = \frac{i}{p_Z^2 - m_Z^2 + im_Z\Gamma_Z}, \qquad \Delta(\tilde{\nu}) = \frac{i}{p_{\tilde{\nu}}^2 - m_{\tilde{\nu}}^2}. \tag{D8}$$

The longitudinal beam polarizations are included in the weighting factors

$$c_L = (1 - P_{e^-})(1 + P_{e^+}), \qquad c_R = (1 + P_{e^-})(1 - P_{e^+}). \tag{D9}$$

B. Chargino polarization

The coefficients Σ_P^a, which describe the polarization of the chargino $\tilde{\chi}_i^+$, decompose into

$$\Sigma_P^a = \Sigma_P^a(\gamma\gamma) + \Sigma_P^a(\gamma Z) + \Sigma_P^a(\gamma\tilde{\nu}) + \Sigma_P^a(ZZ) + \Sigma_P^a(Z\tilde{\nu}) + \Sigma_P^a(\tilde{\nu}\,\tilde{\nu}). \tag{D10}$$

The contributions to the transverse $\tilde{\chi}_i^+$ polarization in the production plane are

$$\Sigma_P^1(\gamma\gamma) = \delta_{ij} 2e^4 |\Delta(\gamma)|^2 (c_R - c_L) E_b^2 \sin\theta (m_{\chi_i^+} E_{\chi_j^-} + m_{\chi_j^-} E_{\chi_i^+}), \tag{D11}$$

$$\Sigma_P^1(\gamma Z) = \delta_{ij} 2 \frac{e^2 g^2}{\cos^2\theta_W} E_b^2 \sin\theta \mathrm{Re}\{\Delta(\gamma)\Delta(Z)^* [-(L_e c_L + R_e c_R)(O_{ij}^{\prime R*} - O_{ij}^{\prime L*}) m_{\chi_i^+} q \cos\theta + (R_e c_R - L_e c_L)(O_{ij}^{\prime L*} + O_{ij}^{\prime R*})(m_{\chi_i^+} E_{\chi_j^-} + m_{\chi_j^-} E_{\chi_i^+})]\}, \tag{D12}$$

$$\Sigma_P^1(\gamma\tilde{\nu}) = -\delta_{ij} e^2 g^2 c_L E_b^2 \sin\theta \mathrm{Re}\{V_{i1}^* V_{j1} \Delta(\gamma)\Delta(\tilde{\nu})^*\} \times [m_{\chi_i^+}(E_{\chi_j^-} - q\cos\theta) + m_{\chi_j^-} E_{\chi_i^+}], \tag{D13}$$

$$\Sigma_P^1(ZZ) = \frac{g^4}{\cos^4\theta_W} |\Delta(Z)|^2 E_b^2 \sin\theta [(L_e^2 c_L + R_e^2 c_R)(|O_{ij}^{\prime L}|^2 - |O_{ij}^{\prime R}|^2) m_{\chi_i^+} q \cos\theta + (R_e^2 c_R - L_e^2 c_L) 2\mathrm{Re}\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\} m_{\chi_j^-} E_{\chi_i^+} + (R_e^2 c_R - L_e^2 c_L)(|O_{ij}^{\prime R}|^2 + |O_{ij}^{\prime L}|^2) m_{\chi_i^+} E_{\chi_j^-}], \tag{D14}$$

$$\Sigma_P^1(Z\tilde{\nu}) = -\frac{g^4}{\cos^2\theta_W} L_e c_L E_b^2 \sin\theta \mathrm{Re}\{V_{i1}^* V_{j1} \Delta(Z)\Delta(\tilde{\nu})^* \times [O_{ij}^{\prime L} m_{\chi_i^+}(E_{\chi_j^-} - q\cos\theta) + O_{ij}^{\prime R} m_{\chi_j^-} E_{\chi_i^+}]\}, \tag{D15}$$

$$\Sigma_P^1(\tilde{\nu}\tilde{\nu}) = -\frac{g^4}{4} c_L |V_{i1}|^2 |V_{j1}|^2 |\Delta(\tilde{\nu})|^2 E_b^2 \sin\theta m_{\chi_i^+}(E_{\chi_j^-} - q\cos\theta). \tag{D16}$$

The contributions to the transverse $\tilde{\chi}_i^+$ polarization perpendicular to the production plane are

$$\Sigma_P^2(\gamma\gamma) = \Sigma_P^2(\tilde{\nu}\tilde{\nu}) = 0, \tag{D17}$$

$$\Sigma_P^2(\gamma Z) = \delta_{ij} 2 \frac{e^2 g^2}{\cos^2\theta_W} (R_e c_R - L_e c_L) \mathrm{Im}\{\Delta(\gamma)\Delta(Z)^* \times (O_{ij}^{\prime R*} - O_{ij}^{\prime L*})\} E_b^2 m_{\chi_j^-} q \sin\theta, \tag{D18}$$

$$\Sigma_P^2(\gamma\tilde{\nu}) = \delta_{ij} e^2 g^2 c_L \mathrm{Im}\{V_{i1}^* V_{j1} \Delta(\gamma)\Delta(\tilde{\nu})^*\} E_b^2 m_{\chi_j^-} q \sin\theta, \tag{D19}$$

$$\Sigma_P^2(ZZ) = 2\frac{g^4}{\cos^4\theta_W} |\Delta(Z)|^2 (R_e^2 c_R - L_e^2 c_L) \mathrm{Im}\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\} E_b^2 m_{\chi_j^-} q \sin\theta, \tag{D20}$$

$$\Sigma_P^2(Z\tilde{\nu}) = \frac{g^4}{\cos^2\theta_W} L_e c_L \mathrm{Im}\{V_{i1}^* V_{j1} O_{ij}^{\prime R} \Delta(Z)\Delta(\tilde{\nu})^*\} \times E_b^2 m_{\chi_j^-} q \sin\theta. \tag{D21}$$

The contributions to the longitudinal $\tilde{\chi}_i^+$ polarization are

$$\Sigma_P^3(\gamma\gamma) = \delta_{ij} 2e^4 |\Delta(\gamma)|^2 (c_L - c_R) E_b^2 \cos\theta (q^2 + E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-}), \tag{D22}$$

$$\Sigma_P^3(\gamma Z) = \delta_{ij} 2\frac{e^2 g^2}{\cos^2\theta_W} E_b^2 \mathrm{Re}\{\Delta(\gamma)\Delta(Z)^* [(L_e c_L - R_e c_R) \times (O_{ij}^{\prime R*} + O_{ij}^{\prime L*})(q^2 + E_{\chi_i^+} E_{\chi_j^-} + m_{\chi_i^+} m_{\chi_j^-}) \times \cos\theta + (L_e c_L + R_e c_R)(O_{ij}^{\prime R*} - O_{ij}^{\prime L*}) q (E_{\chi_j^-} + E_{\chi_i^+} \cos^2\theta)]\}, \tag{D23}$$

$$\Sigma_P^3(\gamma\tilde{\nu}) = -\delta_{ij} e^2 g^2 c_L E_b^2 \mathrm{Re}\{V_{i1}^* V_{j1} \Delta(\gamma)\Delta(\tilde{\nu})^*\}[q E_{\chi_j^-} - (q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta + q E_{\chi_i^+} \cos^2\theta - m_{\chi_i^+} m_{\chi_j^-} \cos\theta], \tag{D24}$$

$$\Sigma_P^3(ZZ) = \frac{g^4}{\cos^4\theta_W} |\Delta(Z)|^2 E_b^2 [(L_e^2 c_L + R_e^2 c_R)(|O_{ij}^{\prime R}|^2 - |O_{ij}^{\prime L}|^2) q (E_{\chi_j^-} + E_{\chi_i^+}\cos^2\theta) + (L_e^2 c_L - R_e^2 c_R) 2\mathrm{Re}\{O_{ij}^{\prime L} O_{ij}^{\prime R*}\} m_{\chi_i^+} m_{\chi_j^-} \cos\theta + (L_e^2 c_L - R_e^2 c_R)(|O_{ij}^{\prime L}|^2 + |O_{ij}^{\prime R}|^2)(q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta], \tag{D25}$$

$$\Sigma_P^3(Z\tilde{\nu}) = \frac{g^4}{\cos^2\theta_W} L_e c_L E_b^2 \mathrm{Re}(V_{i1}^* V_{j1} \Delta(Z)\Delta(\tilde{\nu})^* \times \{O_{ij}^{\prime R} m_{\chi_i^+} m_{\chi_j^-} \cos\theta - O_{ij}^{\prime L}[q E_{\chi_j^-} - (q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta + q E_{\chi_i^+} \cos^2\theta]\}), \tag{D26}$$

$$\Sigma_P^3(\tilde{\nu}\tilde{\nu}) = -\frac{g^4}{4} c_L |V_{i1}|^2 |V_{j1}|^2 |\Delta(\tilde{\nu})|^2 E_b^2 [q E_{\chi_j^-} - (q^2 + E_{\chi_i^+} E_{\chi_j^-})\cos\theta + q E_{\chi_i^+} \cos^2\theta]. \tag{D27}$$

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