PHYSICAL REVIEW D **70,** 114037 (2004)

Neutrino emission from ungapped quark matter

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(Received 20 October 2004; published 30 December 2004)

We study neutrino emission from a normal, ungapped, quark phase in the core of a compact star. Neutrino emission from noninteracting quark matter leads to an emissivity that scales as $\epsilon \sim T^7$. We show that the emissivity is enhanced by a combination of Fermi liquid and non-Fermi liquid effects. Fermi liquid effects lead to an emissivity that scales as $\epsilon \sim \alpha_s T^6$, as originally shown by Iwamoto. We demonstrate that non-Fermi liquid effects further enhance the rate, leading to $\epsilon \sim \alpha_s^3 T^6 \log(m/T)^2$, where m is the electric screening scale and $m \gg T$ under the conditions found in compact stars. We show, however, that combined with non-Fermi liquid effects in the specific heat the enhancement in the emissivity only leads to a modest reduction in the temperature of the star at late times. Our results confirm existing bounds on the presence of ungapped quark matter in compact stars. We also discuss neutrino emission from superconducting phases with ungapped fermionic excitations.

DOI: 10.1103/PhysRevD.70.114037 PACS numbers: 12.38.Mh, 12.38.Cy, 97.60.Jd

I. INTRODUCTION

Compact stars provide a unique opportunity to study cold and dense strongly interacting matter and its rich phase structure. An important source of information about the structure of compact stars is their cooling behavior [1–5]. For the first $\sim 10^5$ years after the star is born neutrino emission from the bulk is the most efficient energy loss mechanism. Since the matter in the interior of the star is almost degenerate the exact nature of the cooling mechanism is very sensitive to the structure of the low energy excitations. As a result the cooling behavior places important constraints on the phase diagram of dense matter.

Cooling mechanisms are generally grouped into fast processes with an emissivity $\epsilon \sim T^6$ and slow processes with $\epsilon \sim T^8$. Slow processes include the modified Urca process [6] and neutrino bremsstrahlung. Examples for fast mechanisms are the direct Urca process [7], neutrino emission from pion or kaon condensates [8,9], and neutrino emission from an ungapped quark phase [10]. In a fully gapped phase of nuclear or quark matter the emissivity is exponentially small. Near the critical temperature for superconductivity the emissivity is dominated by pair breaking and recombination with a temperature behavior that is intermediate between fast and slow mechanisms [11,12].

In this paper we wish to focus on the emissivity of a possible quark matter phase. At asymptotically high baryon density quark matter is in the color-superconducting CFL phase [13]. In this phase all quark excitations are gapped and the emissivity is dominated by exponentially small processes involving massive pseudo-Goldstone modes [14–16]. At densities that are relevant to compact stars distortions of the CFL due to the nonzero strange quark mass cannot be neglected and the phase structure is much more complicated. Most of the phases that have been proposed in the literature involve quarks with vanishing or very small gaps. Examples are the 2SC phase [17–19] in which one of the colors remains ungapped, single flavor spin-one superconductors that have very small gaps [20,21], and gapless CFL or 2SC phases [22,23].

In the following we will concentrate on the neutrino emissivity of ungapped quark matter. This problem is important both in connection with the (almost) gapless phases mentioned above and as a benchmark in order to exclude the presence of ungapped quark matter in compact stars. The emissivity of a normal quark phase was first studied by Iwamoto [10], see also [24–26]. Iwamoto showed that the direct Urca process is strongly suppressed in noninteracting quark matter, but that Fermi liquid corrections in interacting matter lead to a fast rate $\epsilon \sim \alpha_s T^6$, where α_s is the strong coupling constant.

It also is known, however, that unscreened color magnetic interactions lead to a breakdown of the Fermi liquid description at temperatures $T_{nfl} \sim m \exp(-9\pi/(4\alpha_s))$, where $m^2 = N_F \alpha_s \mu^2/\pi$ is the electric screening scale [27,28]. This scale is very small compared to ordinary QCD scales, but it is large compared to the temperature of neutron stars after the first minute or so. For $\mu = 500$ MeV we have $T_{nfl} \sim 500$ keV. It was recently shown that non-Fermi liquid effects lead to large corrections of the specific heat of degenerate quark matter for $T \ll T_{nfl}$ [29]. In the present work we compute non-Fermi liquid corrections to the emissivity and study the cooling behavior of degenerate ungapped quark matter.

II. QUARK DISPERSION RELATION

In a Fermi gas of free quarks the direct Urca process is strongly suppressed. As a consequence, the rate is very sensitive to modifications of the quark dispersion relation. Iwamoto noticed that Fermi liquid corrections to the relation between the Fermi energy and the Fermi momentum lead to a significant enhancement of the neutrino emissivity [10]. Non-Fermi liquid effects due to unscreened transverse gauge boson interactions dramatically alter the dispersion relation in the vicinity of the Fermi surface [28–34] and thereby could have a similar effect. In this section we wish to study the interplay of Fermi liquid and non-Fermi liquid effects in dense quark matter and derive the quark dispersion relation.

We are interested in the propagation of quarks in the vicinity of the Fermi surface. Since the momentum $p \sim v\mu$ is large, typical soft scatterings cannot change the momentum by very much and the velocity is approximately conserved. An effective field theory for particles and holes moving with velocity $v = (1, \vec{v}_F)$ is given by [33,35]

$$\mathcal{L} = \psi_v^\dagger (iv \cdot D + \delta\mu)\psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{\rm HDL} + \sum_{v',\Gamma} f_l^\Gamma R^l(v \cdot v')(\psi_v^\dagger \Gamma \psi_v)(\psi_{v'}^\dagger \Gamma \psi_{v'}) + \ldots \tag{1}$$

Here, D_μ is the covariant derivative and $\delta\mu$ is a counterterm for the chemical potential. The effective theory contains two types of four fermion operators, corresponding to forward scattering $v + v' \to v + v'$ and the BCS process $v + (-v) \to v' + (-v')$ [33]. Since we are interested in the normal phase only the forward scattering operators are included in Eq. (1). The matrix Γ determines the spin, color, and flavor structure of the operator and $R^l(v \cdot v')$ is a set of orthogonal polynomials. Equation (1) also contains the hard dense loop (HDL) effective action

$$\mathcal{L}_{\rm HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^a_{\mu\beta}, \tag{2}$$

which accounts for damping and screening caused by particle-hole pairs on the entire Fermi surface.

The parameters in the effective Lagrangian are determined by matching QCD Green functions near the Fermi surface. In order to determine the dispersion relation we have to match forward scattering amplitudes, see Fig. 1. To leading order in the coupling constant the forward scattering amplitude is the sum of a direct and an exchange term. In the effective field theory the direct term is reproduced by the collinear interaction while the exchange term has to be matched against a contact term [33,36]. The spin-color-flavor symmetric part is given by

$$\mathcal{L} = f_0^s (\psi_v^\dagger \psi_v)(\psi_{v'}^\dagger \psi_{v'}), \qquad f_0^s = \frac{C_F}{4N_c N_f} \frac{g^2}{p_F^2}, \tag{3}$$

with $C_F = (N_c^2 - 1)/(2N_c)$ and all other $f_i^s = 0$. The parameters v_F and $\delta\mu$ are determined by computing the contribution to the fermion dispersion relation from states far away from the Fermi surface. This can be done most easily using the hard dense loop approximation. We find

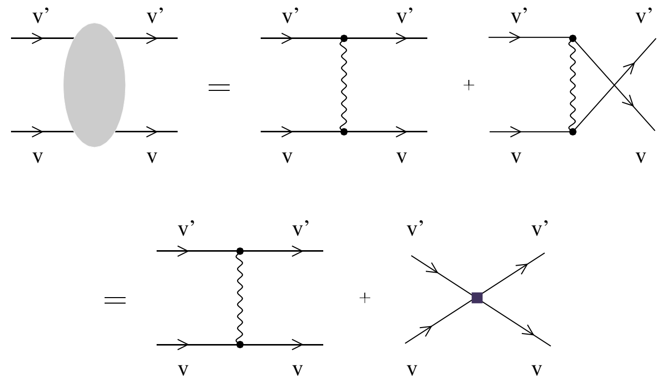

FIG. 1 (color online). Feynman diagrams that appear in the matching procedure for the forward scattering amplitude to leading order in the coupling constant. The upper panel shows the direct and exchange terms in QCD. The lower panel shows the collinear and contact terms in the effective theory.

$$p_0 = |\vec{p}| + \frac{m_f^2}{|\vec{p}|}, \qquad m_f^2 = \frac{C_F \alpha_s}{2\pi} \mu^2, \tag{4}$$

which gives

$$v_F = 1 - \frac{C_F \alpha_s}{2\pi}, \qquad \delta\mu = \frac{C_F \alpha_s}{\pi} \mu. \tag{5}$$

These relations can also be derived using the Landau theory of Fermi liquids. Landau showed that Galilei invariance implies a relation between the effective interaction on the Fermi surface, encoded in the Landau parameters f_l^s, and the parameters v_F and $\delta\mu$. These arguments were generalized to the relativistic case by Baym and Chin [37]. They show that

$$v_F = \frac{p_F}{\mu} - \frac{N f_1^s}{3}, \qquad p_F = \mu\left(1 - \frac{N f_0^s}{2}\right), \tag{6}$$

where $N = N_c N_f \mu^2/\pi^2$ is the density of states on the Fermi surface. Using Eq. (3) and (6) gives Eq. (5).

Finally, we can study loop corrections in the effective theory, see Fig. 2. The collinear loop gives

$$\Sigma(\omega) = \frac{C_F \alpha_s}{3\pi} \omega \log\left(\frac{\Lambda}{\omega}\right), \tag{7}$$

where $\omega = p_0 - \mu$. We recently showed that this result does not receive large logarithmic corrections of the form

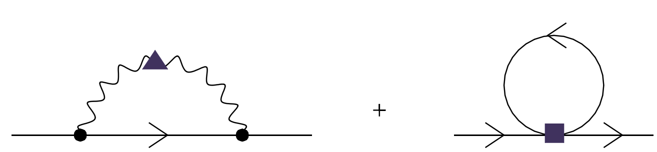

FIG. 2 (color online). Leading order contributions to the fermion self energy in the effective theory. The triangle denotes a gluon self energy insertion and the square is the four fermion operator defined in Fig. 1.

$g^{2n}\log^n(\omega)$ as $\omega \to 0$ [34]. The tadpole diagram is linearly divergent in the effective field theory. A naive estimate can be obtained by cutting the divergence off at $\Lambda \sim \mu$. This gives $\Sigma \sim f_s^0 N\mu$ which agrees with the result for $\delta\mu$ given above. We can now summarize the results obtained in this section. The dispersion relation is

$$\omega + \Sigma(\omega) = v_F l + \delta\mu, \tag{8}$$

where l is the longitudinal momentum, $\Sigma(\omega)$ is given in Eq. (7), and v_F, $\delta\mu$ given in Eq. (5).

III. NEUTRINO EMISSIVITY

The dominant contribution to the emission of neutrinos is given by the quark analogs of β decay (β) and electron capture (ec)

$$d \to u + e^- + \bar{\nu}_e, \tag{9}$$

$$u + e^- \to d + \nu_e. \tag{10}$$

It is straightforward to introduce weak interactions into the effective theory. The charged current interaction is given by

$$\mathcal{L} = \frac{g_2}{\sqrt{2}}\cos\theta_c \psi^\dagger \tau^\pm v \cdot W^\mp \psi, \tag{11}$$

where $\cos\theta_c$ is the Cabbibo angle and g_2 is related to the Fermi coupling by $G_F/\sqrt{2} = g_2^2/(8M_W^2)$. The dependence on the Cabbibo angle suppresses the processes involving the strange quark [10]. Therefore we will neglect the neutrino emission of strange quarks. We have seen in the previous section that almost collinear gluon exchanges can generate large logarithmic corrections to the fermion self energy. This raises the question whether gluon corrections to the weak interaction vertex have to be taken into account. The problem of vertex corrections was studied in [31,33]. It was shown that the vertex receives large logarithmic corrections in the timelike regime but not in the spacelike regime. The coefficient of the logarithm in the timelike regime is exactly equal to the logarithmic term in the fermion self energy. In the fermion self energy problem the vertex is spacelike and does not have to be renormalized. However, since the wave function renormalization is logarithmically divergent the effective coupling constant goes to zero near the Fermi surface. In the neutrino emission problem the vertex is timelike and receives logarithmic corrections. However, since the vertex correction is equal to the field renormalization the effective coupling constant does not change.

The neutrino emissivity is given by the total energy loss due to neutrino emission averaged over the initial quark helicities and summed over the final state phase space and helicities

$$\begin{aligned}\epsilon \equiv N_c \sum_{\sigma_u,\sigma_d,\sigma_e} \int \frac{d^3p_d}{(2\pi)^3}\frac{1}{2E_d}\int \frac{d^3p_u}{(2\pi)^3}\frac{1}{2E_u}\int \frac{d^3p_e}{(2\pi)^3}\frac{1}{2E_e} \\ \times \int \frac{d^3p_\nu}{(2\pi)^3}\frac{1}{2E_\nu} E_\nu[|M_\beta|^2(2\pi)^4\delta^{(4)}(p_d - p_u - p_e \\ - p_\nu)n(p_d)(1-n(p_u))(1-n(p_e)) + |M_{ec}|^2(2\pi)^4 \\ \times \delta^{(4)}(p_u + p_e - p_d - p_\nu)n(p_u)n(p_e)(1-n(p_d))].\end{aligned} \tag{12}$$

The weak matrix element for the β and ec processes is given by

$$\frac{1}{2}\sum_{\sigma_u,\sigma_d,\sigma_e}|M_{\beta/ec}|^2 = 64G_F^2\cos^2\theta_c p_F^2(v\cdot p_e)(v\cdot p_\nu), \tag{13}$$

where p_e, p_ν are the momenta of the electron and the neutrino. Weak processes establish β equilibrium in the star. In three flavor quark matter with a massive strange quark the resulting electron chemical potential is small. In the following we shall assume that $(T \sim E_\nu) \ll (\mu_e \sim E_e) \ll p_F$. This assumption is appropriate in all cases except during the first few seconds of the proto-neutron star evolution.

In this case we can neglect the neutrino momentum when applying the energy-momentum conservation relation to the matrix element. As a consequence we find $(v \cdot p_\nu) \simeq E_\nu$ after averaging over the direction of the outgoing neutrino. The matrix element is mainly determined by the factor $(v \cdot p_e)$. To leading order in the effective theory the weak decay is exactly collinear and $(v \cdot p_e) = (E_e - v_F l_e) = 0$ up to terms of order $O(T/\mu_e)$, see Fig. 3. If corrections to the dispersion relation are taken into account we get

$$(v\cdot p_e) \simeq (1 - v_F)l_e + \frac{l_\perp^2}{2l_e} \simeq \delta\mu_d - \delta\mu_u \simeq \frac{C_F\alpha_s}{\pi}\mu_e, \tag{14}$$

where $l_\perp$ is the transverse momentum of the electron. This result agrees with Iwamoto's result. Non-Fermi liquid corrections only appear in the phase space integral. To leading order in the T/μ the sum of the rates for electron capture and β decay is given by

$$\begin{aligned}\epsilon = \frac{6G_F^2\cos^2\theta_c}{\pi^5}T^6\int_{-\infty}^{\infty}dx_d\int_{-\infty}^{\infty}dx_u\int_0^{\infty}dx_\nu x_\nu^3 n(x_d) \\ \times n(-x_u)n(x_u - x_d + x_\nu) \\ \times \left[\frac{\partial p(E_d)}{\partial E_d}\frac{\partial p(E_u)}{\partial E_u}\frac{C_F\alpha_s}{\pi}\mu_e E_u E_d\right]_{E_i\to\mu_i+Tx_i}.\end{aligned} \tag{15}$$

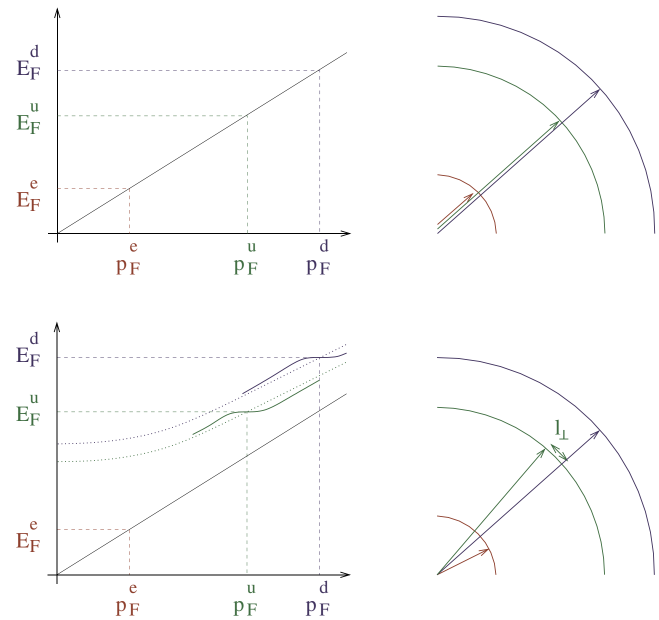

FIG. 3 (color online). Kinematics for the quark direct Urca process in the limit $T \sim E_\nu \ll \mu_e$. In a free quark gas (upper panel) energy-momentum conservation forces the quark and electron momenta to be collinear. If Fermi liquid corrections are taken into account (lower panel) the outgoing quark has a nonzero transverse momentum $l_\perp^2 \sim \alpha_s \mu_e^2$. The dashed dispersion relations give the HDL result whereas the solid lines show the change when non-Fermi liquid corrections are included. These lead to a flattening of the dispersion relation in the vicinity of the Fermi surface.

The expression in the square brackets is determined by the quark dispersion relation given in Eq. (8). Terms of $O[\alpha_s(\alpha_s \log(T))^n]$ with $n = 0, 1, 2$ are independent of $\log(x_i)$ and involve the integral

$$\int_{-\infty}^{\infty} dx_d \int_{-\infty}^{\infty} dx_u \int_0^{\infty} dx_\nu x_\nu^3 n(x_d) n(-x_u) n(x_u - x_d + x_\nu) = \frac{457\pi^6}{5040}. \tag{16}$$

At leading order in T/μ the neutrino emissivity from the quark direct Urca process is given by

$$\epsilon \approx \frac{457}{630} G_F^2 \cos^2\theta_c \alpha_s \mu_q^2 \mu_e T^6 \left[1 + \frac{C_F \alpha_s}{3\pi} \log\left(\frac{\Lambda}{T}\right)\right]^2. \tag{17}$$

The first term is the standard result by Iwamoto [10], and the logarithmic terms are non-Fermi liquid corrections. We note that these terms have to be included because at very low temperature $\alpha_s \log(T)$ becomes large compared to one. We also note that if the scale inside the logarithm is on the order of the screening scale, $\Lambda \sim g\mu$, then $\alpha_s(\mu) \times \log(\Lambda/T)$ stays finite in the limit $\mu \to \infty$ at fixed T.

IV. COMPACT STAR COOLING

In this section we wish to study the impact of non-Fermi liquid effects on the cooling history of an isolated quark phase. Our aim is not to provide a thorough analysis of the cooling behavior of an actual quark or hybrid star but to give a numerical estimate of the size of the non-Fermi liquid corrections. The thermal evolution of the star is governed by the neutrino emissivity, the specific heat, and the thermal conductivity. Non-Fermi liquid corrections to the specific heat were initially considered by Holstein *et al.* [27] in the case of QED. The calculation was recently refined and extended to QCD by Ipp *et al.* [29]. They find

$$c_v = \frac{N_c N_f}{3} \mu_q^2 T \left[1 + \frac{C_F \alpha_s}{3\pi} \log\left(\frac{\Lambda}{T}\right)\right], \tag{18}$$

where the first term is the free gas result and the second term is the non-Fermi liquid correction. Ipp *et al.* also determined the scale inside the logarithm as well as fractional powers of T. From the complete $O(\alpha_s)$ result we find $\Lambda \simeq 0.28m$ where $m^2 = N_f \alpha_s \mu^2/\pi$ is the screening mass.

The thermal conductivity of degenerate quark matter was studied by Heiselberg and Pethick [38]. Their result suggests that equilibration is fast and that the quark phase is isothermal. In this case the cooling behavior is governed by

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial T}\frac{\partial T}{\partial t} = c_v(T)\frac{\partial T}{\partial t} = -\epsilon(T), \tag{19}$$

where u is the internal energy, t is time, and we have assumed that there is no surface emission. Without non-Fermi liquid effects we have $\epsilon \sim T^6$ and $c_v \sim T$. In this case the temperature scales as $T \propto 1/t^{1/4}$. With logarithmic corrections included there is no simple analytic solution, and we have studied Eq. (19) numerically.

We take the quark chemical potential to be $\mu_q = 500$ MeV corresponding to densities $\rho_B \approx 6\rho_0$ where ρ_0 is nuclear matter saturation density. We note that both c_v and ϵ are proportional to μ^2 and the main dependence of the cooling behavior on μ cancels. We evaluate the strong coupling constant using the one loop renormalization group solution at a scale μ. We take the scale parameter to be $\Lambda_{QCD} = 250$ MeV which gives $\alpha_s \simeq 1$ at $\mu = 500$ MeV. It is clear that the naive use of perturbation theory is in doubt if the coupling is this large. In practice we estimate the uncertainty by varying α_s between 1 and 0.4 which is the value used by Iwamoto [10]. We take the scale of non-Fermi liquid effects to be $\Lambda = 0.28m$ as explained above and assess the uncertainty by varying Λ

within a factor of 2. Finally, we took the initial temperature to be $T_0 = 15$ MeV.

The electron chemical potential is determined by the requirements of charge neutrality and β equilibrium. In a noninteracting quark gas we find $\mu_e \simeq m_s^2/(4p_F)$. With a strange quark mass $m_s = 150$ MeV this relation gives $\mu_e \approx 11$ MeV. This result, however, is very sensitive to interactions. To first order in α_s the chemical potential for a massive strange quark is [39,40]

$$\mu_s = E_{Fs}^0 + \frac{2\alpha_s}{3\pi}\left[p_{Fs} - \frac{3m_s^2}{E_{Fs}^0}\log\left(\frac{p_{Fs}+E_{Fs}^0}{m_s}\right)\right], \quad (20)$$

where $E_{Fs}^0 = \sqrt{p_{Fs}^2 + m_s^2}$. The important point is that the $O(\alpha_s m_s^2)$ term is negative and enhanced by a large logarithm $\log(p_F/m_s)$. The sign is related to the fact that the correlation energy changes sign in going from the relativistic to the nonrelativistic limit.

Equation (20) implies that the strange quark chemical potential can become equal to or even smaller than the up quark chemical potential. To leading order in m_s^2/p_F^2 the electron chemical potential is given by

$$\mu_e \simeq \frac{m_s^2}{4p_F}\left[1 - \frac{4\alpha_s}{\pi}\log\left(\frac{2p_F}{m_s}\right)\right]. \quad (21)$$

For the values of the parameters given above this equation gives a negative electron chemical potential $\mu_e \approx -16$ MeV. In this case the quark phase contains a Fermi sea of positrons and the quark direct Urca process is

$$u \to d + e^+ + \nu, \qquad d + e^+ \to u + \bar{\nu}. \quad (22)$$

The neutrino emissivity is again governed by Eq. (17) where μ_e has to be replaced by $-\mu_e$. We observe that despite the large correction to μ_e the emissivity is not strongly affected. The large variation in μ_e when perturbative corrections are included implies, however, that the electron chemical potential is very uncertain. In particular, there is a possibility that μ_e is much smaller than $m_s^2/(4p_F)$. If $\alpha_s\mu_e < T$ then the neutrino emissivity is no longer proportional to $\alpha_s\mu_e T^6$ but to T^7 [25].

In the following we shall use the value $\mu_e = 16$ MeV corresponding to interacting quarks. In Fig. 4 we show the temperature dependence of both the neutrino emissivity and the specific heat (solid) compared to the Fermi liquid result (dashed). The gray band shows an estimate of the uncertainties which are dominated by the uncertainty in the value of the strong coupling. For both c_v and ϵ the anomalous logarithmic terms (dotted) dominate in the relevant temperature range and exceed the Fermi liquid result considerably. The cooling behavior is controlled by the ratio ϵ/c_v. Since $\epsilon \sim \log^2(T)$ and $c_v \sim \log(T)$ this ratio is logarithmically enhanced. However, because the temperature at late times scales roughly as the fourth root of the numerical coefficient in ϵ/c_v this logarithmic enhancement only translates into a modest reduction of the temperature. This can be seen in more detail in Fig. 5. We observe that compared to the Fermi liquid result (dashed) the non-Fermi liquid effects (solid) lead to a reduction of the temperature at late times which is nearly independent of time. The magnitude of the effect is on the order of 20%. For comparison, we also show the cooling behavior of normal nuclear matter via the modified Urca process $n +$

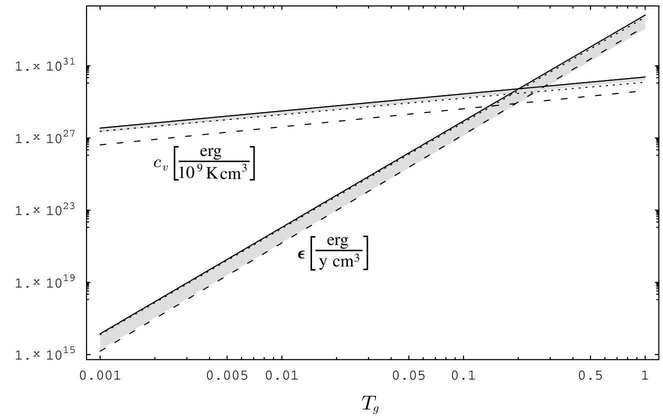

FIG. 4. Neutrino emissivity ϵ and specific heat c_v of quark matter. The dashed lines show the Fermi liquid results and the dotted lines show the anomalous corrections. The solid lines give the sum of the two contributions and the gray band shows an estimate of the uncertainties.

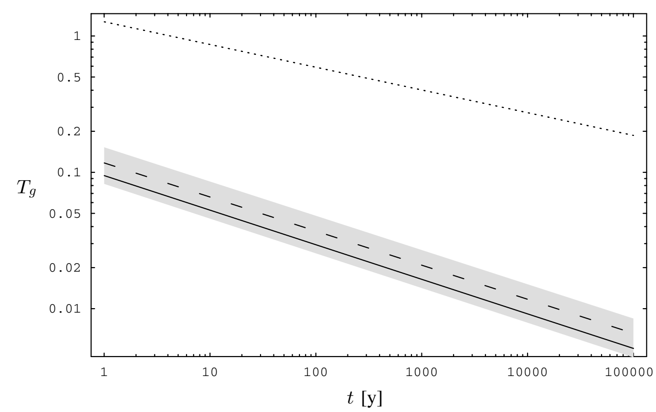

FIG. 5. Cooling behavior of ungapped quark matter. We show the temperature T_9 in units of 10^9 K as a function of the age of the star in years. The dashed line shows the Fermi liquid result whereas the solid line gives the result including non-Fermi liquid effects with the estimated uncertainty range. Although the non-Fermi liquid corrections to both the specific heat and the neutrino emissivity are significant, there is only a modest reduction in the temperature at late times. However, for both cases the cooling of quark matter is considerably faster than the cooling of neutron matter via the modified Urca process given by the dotted line.

$n \rightarrow n + p + e^- + \bar{\nu}$ [6]. We have chosen the same density and initial temperature and the effective baryon masses given in [5]. We clearly see the difference between the fast $\sim T^6$ quark direct Urca process and the slow $\sim T^8$ modified Urca process.

V. SUMMARY AND DISCUSSION

In this work, we have discussed the influence of non-Fermi liquid effects on the cooling behavior of compact stars containing quark matter in the normal phase. Non-Fermi liquid effects lead to a logarithmic enhancement in both the neutrino emissivity and the specific heat. The net result of these two effects is a mild logarithmic enhancement in the cooling rate. As our rate is even larger than the Iwamoto rate we confirm and sharpen earlier bounds on the existence of ungapped quark matter in neutron stars [41–44]. More quantitative statements will require detailed studies of realistic models in which the quark core is in contact with a hadronic phase or an atmosphere. This is beyond the scope of our investigation.

Let us now discuss the importance of non-Fermi liquid effects in partially gapped color-superconducting phases, see Table I. The simplest case is the 2SC phase. The 2SC phase can arise when the difference between the Fermi momenta of the strange quark and the up and down quarks is too large for strange-nonstrange pairing to occur [46,47]. There also are regions in the phase diagram where *us* (dSC) or *ds* (uSC) pairing might occur [48–50]. The 2SC phase is characterized by a partial Higgs mechanism. Color SU(3) is broken according to SU(3) $\rightarrow$ SU(2) and five out of eight gluons acquire a mass. The gapless fermions of the third color interact via screened gauge bosons. As a consequence, there are no non-Fermi liquid corrections in either the specific heat or the neutrino emissivity and the standard result of Iwamoto applies [10].

TABLE I. Dominant cooling rates in different phases of quark matter. This table is a summary of the discussion in Sec. V. In the entry $[1SC]^3$ we have assumed that ungapped modes exist and the Urca process is allowed. This is not true in the color spin locking phase. In the gCFL phase we have used the estimate of Kouvaris *et al.* and in the gCFLK phase we have assumed that only slow processes such as neutrino bremsstrahlung are allowed. In the g2SC phase we have assumed that the Fermi liquid direct Urca rate can be used, but this is not consistent with the calculation of the gluon polarization in [45].

phase	emissivity	specific heat
normal phase	$\epsilon \sim \alpha_s^3 T^6 \log^2(T)$	$c_v \sim \alpha_s \mu^2 T \log(T)$
CFL	$\epsilon \sim T^{3/2} \exp(-m_K/T)$	$c_v \sim T^3$
2SC	$\epsilon \sim \alpha_s T^6$	$c_v \sim \mu^2 T$
2SC + 1SC	$\epsilon \sim \alpha_s T^6$	$c_v \sim \mu^2 T$
$[1SC]^3$	$\epsilon \sim \alpha_s T^6$	$c_v \sim \mu^2 T$
gCFL	$\epsilon \sim \alpha_s T^{5.5}$	$c_v \sim \mu^2 \sqrt{\Delta T}$
gCFLK	$\epsilon \sim \alpha_s T^8$	$c_v \sim \mu^2 T$
g2SC	$\epsilon \sim \alpha_s T^6$	$c_v \sim \mu^2 T$

If the 2SC phase occurs in quark matter with three flavors then the unpaired quark flavor can form a spin-one condensate. Spin-one condensates are interesting from the point of view of cooling because the typical gaps are much smaller than the spin zero gap. In particular, we can have $T_c \simeq T_{nfl}$ and non-Fermi liquid effects in the specific heat are important. Whether or not gapless modes occur below T_c depends on the exact nature of the spin-one condensate [20,21]. In a phase with color spin locked pairing in both LL and LR chirality channels the order parameter is isotropic and no gapless modes exist. In all other phases not all fermions are gapped and the order parameter has nodes on Fermi surface. If spin-one pairing takes place in all flavor sectors (referred to as $[1SC]^3$ in Table I) these modes will dominate the cooling behavior.

The cooling behavior of the recently proposed gapless CFL [22] and 2SC [23] phases is a more difficult question. In the gCFL phase there are gapless charged and neutral modes and the direct Urca process is possible. Alford *et al.* [51] have argued that the (almost) quadratic dispersion relation of one of the gapless modes leads to an enhancement of the cooling rate by a factor $\sqrt{T}$. A similar enhancement occurs in the specific heat [22]. Kryjevski and Schäfer studied quark modes in a kaon condensed CFL phase (CFLK) [52]. They find at most one gapless mode. As a consequence, the dominant cooling mechanism is expected to be a slow process such as neutrino bremsstrahlung. However, there are several light modes that will contribute to neutrino transport at MeV temperatures. In the g2SC phase there are gapless up and down quark excitations and the direct Urca process is possible. We should stress that our discussion of neutrino emission from the g2SC and gCFL phase ignores potential instabilities related to a negative current-current correlation function [53–55]. Wu and Yip as well as Giannakis and Ren [53,56] have argued that these instabilities indicate that the correct ground state is an inhomogeneous superconductor of the type discussed by Larkin, Ovchninkov, Fulde, and Ferrell [57–59]. The LOFF phase also contains gapless fermions and the calculation of the neutrino emissivity from this phase is an important problem for future studies.

ACKNOWLEDGMENTS

We would like to thank C. Kouvaris, M. Prakash, K. Rajagopal, and S. Reddy for useful discussions and Q. Wang for pointing out an error in Eq. (12) of the first version of this manuscript. This work was begun at the Institute for Nuclear Theory during the workshop on "QCD and Dense Matter: From Lattices Stars." We thank the INT for hospitality. This work was supported in part by U.S. DOE Grant No. DE-FG-88ER40388.

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