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Glueballs, closed fluxtubes, and $\eta(1440)$

Ludvig Faddeev,[1,*] Antti J. Niemi,[2,†] and Ulrich Wiedner[3,‡]
[1]*St. Petersburg Branch of Steklov Mathematical Institute, Russian Academy of Sciences, Fontanka 27, St.Petersburg, Russia*
[2]*Department of Theoretical Physics, Uppsala University, Box 803, S-75 108 Uppsala, Sweden*
[3]*Department of Radiation Sciences, Uppsala University, Box 535, S-75 121 Uppsala, Sweden*
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The $\eta_L(1410)$ component of the $\eta(1440)$ pseudoscalar has strong affinity for glue. But its mass is incompatible with lattice simulations that predict a much higher value for the 0^{-+} glueball. Consequently it has been suggested that $\eta_L(1410)$ signals physics beyond the Standard Model. Here we argue that if glueballs are closed gluonic fluxtubes then $\eta_L(1410)$ is a prime candidate for the 0^{-+} glueball.

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I. INTRODUCTION

Quantum chromodynamics describes the strong interactions of quarks and gluons. At high energies QCD becomes asymptotically free and explains observed phenomena with an impressive accuracy. But at lower energies where the coupling grows and hadrons are formed the structure of QCD remains inexplicable. In this regime strong interactions are ruled by poorly understood nonperturbative phenomena such as color confinement and chiral symmetry breaking [1]. One of the anticipated predictions is the appearance of glueballs as massive and color-neutral bound states of massless and confined gluons. The lowest mass glueball relates to the first excited state in the spectrum of the Yang-Mills theory, thus it is stable within the purely gluonic sector of QCD. Lattice computations predict that in the limit of infinite quark masses the lowest mass glueball is a $J^{PC} = 0^{++}$ state with a mass $1\,611 \pm 163$ MeV [2]. This is within the range of isoscalar $q\bar{q}$ mesons, which can be grouped into SU(3) flavor nonets [3] and glueballs should appear supernumerous to this nonet. Some of the additional experimental signatures of a glueball are:

- (i) Glueballs should be produced in $p\bar{p}$ annihilation processes, as a $q\bar{q}$ annihilation leads to a gluon-rich environment which strongly favors the formation of gluonic degrees of freedom.
- (ii) The central region of various other high-energy hadron-hadron scattering processes should similarly favor the production of glueballs.
- (iii) The radiative decay of quarkonium states and especially the radiative J/ψ decay should be a prime source of glueballs.
- (iv) The decay branching fractions of a glueball should be incompatible with SU(3) predictions for $q\bar{q}$ states.
- (v) Since glueballs have no direct coupling to electromagnetism, they should be absent in any $\gamma\gamma$ collision processes.

Presently, there is a wide consensus that the relatively narrow state $f_0(1500)$ is the best available candidate for the lowest mass 0^{++} glueball [3]. It does fulfill all of the above experimental criteria (i)–(v) and its mass is well within the range of lattice predictions.

Lattice simulations predict further that the next lowest mass glueball should be a 2^{++} state, with a mass of $2\,232 \pm 310$ MeV [2]. This prediction fits wonderfully with the asserted observation of $f_J(2220)$, which has been proposed as a candidate for the 2^{++} glueball. Unfortunately, with new data from $p\bar{p}$ annihilation the existence of a $f_J(2220)$ state seems to fade away [4]. Consequently there does not seem to be any viable candidate for the 2^{++} glueball with a mass below 2.3 GeV, the regime that has been probed by $p\bar{p}$ annihilations.

The lattice prediction of the $0^{++} \sim f_0(1500)$ glueball is almost perfect. But the difficulties in pinpointing a 2^{++} glueball, at least at energies lower than 2.3 GeV suggests that there might be place for additional experimental and theoretical input. Since glueballs relate directly to confinement and the mass gap in Yang-Mills theory, their study is extremely important for our understanding of strong interactions and the origin of mass. Indeed, at the moment our theoretical perception of glueballs is quite lacking and rather marred with puzzling experimental observations. Notorious in this respect is the $J^{PC} = 0^{-+}$ pseudoscalar $\eta(1440)$ which is most likely a mixture of two particles, the $\eta_H(1480)$ and the $\eta_L(1410)$ [3]. The $\eta_H(1480)$ couples strongly to kaons. In fact, it seems to be an almost ideal mix of $s\bar{s}$. This makes it a likely candidate for the $s\bar{s}$ member of the pseudoscalar nonet. The $\eta_L(1410)$ becomes then supernumerous, and consequently it is a candidate for exotics: In a full analogy with the nearby $f_0(1500)$, the $\eta_L(1410)$ is a relatively narrow state that fulfills the above criteria (i)–(v) which are expected to be satisfied by a glueball, with the sole exception that for central production it has been seen in high-energetic πp scattering decaying into $\pi\pi\eta$ [5] and not in the central production of pp

*Faddeev@euclid.pdmi.ras.ru
†Antti.Niemi@teorfys.uu.se
‡Ulrich.Wiedner@tsl.uu.se

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scattering. However this could simply reflect kinematical suppression. However, since lattice simulations predict that the mass of a 0^{-+} glueball should be in the vicinity of 2.5 GeV, literature [6,7] suggests that $\eta_L(1410)$ is a degree of freedom which is beyond the Standard Model QCD, perhaps a bound state of light gluinos.

Here we shall inspect the possibility that the $\eta_L(1410)$ could actually be interpreted as a glueball within standard QCD. Specifically, we shall suggest that in the intuitively appealing picture where glueballs are viewed as (possibly knotted) closed gluonic fluxtubes, i.e., as closed QCD strings, the $\eta_L(1410)$ is a natural candidate for the 0^{-+} glueball state. Furthermore, we shall argue in a quite model-independent manner that if glueballs are closed fluxtubes there should be a natural degeneracy between the two glueballs $0^{++} \sim f_0(1500)$ and $0^{-+} \sim \eta_L(1410)$. Consequently if $f_0(1500)$ and $\eta_L(1410)$ are indeed glueballs their experimental investigation will directly scrutinize physical depictions of strong interactions, the string interpretation of confinement and the origin of mass in the Universe.

We shall also propose that a scrutiny of the slight 90 MeV mass difference between the $\eta_L(1410)$ and the $f_0(1500)$ might have a deep significance: It could reflect the presence of a new term in the strong interaction Hamiltonian which breaks exact mass degeneracy between eigenstates that are related to each other by parity. However, we also note that experimentally it is well known that interferences with nearby resonances can easily shift masses by as much as 5%. Consequently the quoted values for the $f_0(1500)$, which are in the range of 1445-1560 MeV depending on reaction channels, are not necessarily inconsistent with an exact mass degeneracy with $\eta_L(1410)$. In fact, since $f_0(1500)$ is known to be heavily mixed with nearby meson states, a precise mass comparison with $\eta_L(1410)$ becomes quite delicate.

II. PRODUCTION RATIOS

Besides (essentially) concurrent masses, relative narrowness, and the criteria (i)–(v) which are all satisfied by both $f_0(1500)$ and $\eta_L(1410)$ these two states have also various additional resemblances. In particular their production ratios turn out to be remarkably similar. For example, in $p\bar{p}$ annihilations the production of $f_0(1500)$ has the following measured branching ratios [8],

$$\mathrm{BF}[p\bar{p} \to f_0(1500)/p\bar{p} \to 3\pi^0] = (13 \pm 4\%)$$

and

$$\mathrm{BF}[p\bar{p} \to 3\pi^0] = (5.5 \pm 1.0) \times 10^{-3}.$$

When we take into account that $2\pi^0$ decays represent $(9.3 \pm 2.5\%)$ of all $f_0(1500)$ decays [9] we conclude that

$$\mathrm{BF}[p\bar{p} \to f_0(1500)] = (7.7 \pm 3.8) \times 10^{-3}. \qquad (1)$$

For $\eta_L(1410)$, the $K\bar{K}\pi$ and $\eta\pi\pi$ decay modes are expected to be the dominant [7]. Both have been measured in $p\bar{p}$ annihilations, with results [10]

$$\mathrm{BF}[p\bar{p} \to \eta_L(1410)\pi\pi/\eta_L(1410) \to \eta\pi\pi] = (3.3 \pm 1.0) \times 10^{-3}$$

and [11]

$$\mathrm{BF}[p\bar{p} \to \eta_L(1410)\pi\pi/\eta_L(1410) \to K\bar{K}\pi] = (2.0 \pm 0.2) \times 10^{-3}.$$

When we add these, we find

$$\mathrm{BF}[p\bar{p} \to \eta_L(1410)\pi\pi] = (5.3 \pm 1.7) \times 10^{-3}. \qquad (2)$$

Comparing (1) with (2) and barring for experimental uncertainties, we conclude that the $p\bar{p}$ annihilation production rates of the two glueball candidates $f_0(1500)$ and $\eta_L(1410)$ are remarkably similar.

We have also compared the observed production rates in radiative J/ψ decays. The branching fraction for the production of the $f_0(1500)$ and its subsequent decay into 4π has been measured to be $(8.2 \pm 1.7) \times 10^{-4}$ [12], with 4π decays accounting for $(61.7 \pm 9.6\%)$ of all $f_0(1500)$ decays. Therefore, we expect a branching fraction of

$$\mathrm{BF}[J/\psi \to \gamma f_0(1500)] = (1.3 \pm 0.3) \times 10^{-3}.$$

The branching fraction for the production of the $\eta_L(1410)$ can be determined from its decays into $K\bar{K}\pi$ [13] and $\eta\pi\pi$ [14]. Adding the measured results leads to the branching fraction

$$\mathrm{BF}[J/\psi \to \gamma\eta_L(1410)] = (1.0 \pm 0.46) \times 10^{-3}.$$

Again, within experimental uncertainties the radiative J/ψ decay production rates of $f_0(1500)$ and $\eta_L(1410)$ are remarkably similar.

We find that these similarities in the production rates together with the relative narrowness of both states and the fact that both satisfy all of the overall criteria (i)–(v), is quite remarkable. Indeed, for us this suggests that the natural interpretation of $\eta_L(1410)$ is in terms of a 0^{-+} glueball, related by parity to the 0^{++} glueball $f_0(1500)$.

III. TWISTED FLUXTUBE

We are certainly aware that our proposal is somewhat unorthodox, and contradicts results from (quenched) lattice simulations. Consequently there is a need for some theoretical backing. For this we proceed to scrutinize whether our two glueball candidates could be interpreted physically as closed gluonic fluxtubes, i.e., closed QCD strings.

The formation of a confining gluonic fluxtube (an open QCD string) between two widely separated quarks is widely accepted [15], and is also supported by a variety of lattice simulations. It has been similarly accepted that glueballs are naturally related to closed QCD strings that can be emitted, for example, by a (relatively) long linear string which connects two widely separated quarks [15]. Glueballs are then closed circular fluxtubes, and they can be modeled by employing the Nambu-Goto action and its extensions. Here we shall consider a somewhat different approach for describing glueballs as closed tubes of flux.

A straight, linear string has an energy which is proportional to its length. This implies that in the absence of other contributions to its energy, a closed string becomes unstable since it can shrink away by minimizing its length. This instability is present, for example, in the Abelian Higgs model with a single complex field; it is well known that a closed toroidal vortex ring in a type-II superconductor is unstable and shrinks away [16]. If glueballs are closed strings, they must be stable against shrinkage within the purely gluonic part of QCD. This is a consequence of mass gap and color confinement, which prevent the glueballs from decaying into massless gluons. Besides the linear string tension there must then be additional contributions to the energy of a closed gluonic string. Lattice simulations indicate that a straight linear gluonic string is only subject to a (classically) linear tension [17]. Thus any additional force which could stabilize a closed toroidal string against shrinkage should have a geometric origin, present in a toroidal configuration but absent when the string is straight. The natural source of this force is in the extrinsic three-dimensional geometry of the string, in the way how the string twists and bends [18]. Indeed, suppose that we bend a finite length linear string into a toroidal ring. If we twist the string once around its core before joining its ends to form the ring, this can lead to a twisting contribution to the energy which prevents the ring from shrinking.

The twisting of a toroidal closed string reveals a natural twofold symmetry in the spectrum: The twist can be either a left-handed (L) or a right-handed (R) rotation around the core. We then have two different but stable configurations, a left-twisted and the right-twisted closed string. If a Yang-Mills theory can indeed differentiate between the left twisting and the right twisting, a parity-invariant quantum Yang-Mills Hamiltonian has then a twofold degeneracy among its stringlike eigenstates, $|L>$ and $|R>$, corresponding to the left-handed and right-handed twisting along the closed gluonic string. The parity operator $\mathtt{P}$ that commutes with the Hamiltonian relates these states by mapping

$$\mathtt{P}|L\rangle = |R\rangle \quad \text{and} \quad \mathtt{P}|R\rangle = |L\rangle.$$

Consequently we can also diagonalize $\mathtt{P}$ by setting

$$|\pm\rangle = 1/\sqrt{2}(|L\rangle \pm |R\rangle),$$

where $|\pm\rangle$ are parity-even and parity-odd eigenstates of the Hamiltonian

$$\mathtt{P}|\pm\rangle = \pm|\pm\rangle.$$

If glueballs correspond to such twisted toroidal states, in a parity-invariant QCD the glueball spectrum must then reflect this degeneracy. In particular, in a parity-invariant QCD we expect the $0^{++} \sim f_0(1500)$ glueball state to naturally relate to a 0^{-+} glueball state, with $\eta_L(1410)$ a wonderful, experimentally observed candidate.

But we emphasize that parity invariance does not necessarily imply equality of masses, the QCD Hamiltonian can contain parity-invariant terms which remove an exact mass degeneracy. For example with $\mathtt{P}$ a Hermitian parity operator a perturbation

$$H \to H + \epsilon \mathtt{P}$$

leads to a mass difference $\Delta m = 2\epsilon$ between the parity-related $|\pm\rangle$ states. As a consequence a small observed mass difference between $f_0(1500)$ and $\eta_L(1410)$ could mean that an exact parity degeneracy between the Yang-Mills string degrees of freedom become broken in QCD.

IV. TWISTING YANG-MILLS

To complete our argument we need to explain how the twisting degree of freedom can be realized in a pure Yang-Mills theory. Indeed, it is not entirely unnatural to expect a L-R symmetry to be present, in some form: The high-energy limit QCD involves massless gluons that can be prepared in two polarization states which can be chosen to be either left-handed or right-handed. Consequently one can expect that some kind of L-R symmetry could also be present in the low-energy theory, such as a L-R twisting symmetry of the closed string. Indeed, this twisting degree of freedom can be identified in the Yang-Mills theory, by employing an appropriate decomposition of the gauge field A_μ^a [19,20]. For notational simplicity but without any loss of generality [21] we consider the decomposition of A_μ^a in a SU(2) Yang-Mills theory. Following [19] we interpret the Cartan component A_μ^3 as a $U(1) \in SU(2)$ gauge field. The

$$A_\mu^+ = A_\mu^1 + iA_\mu^2$$

together with its complex conjugate then transform as charged vector fields under the ensuing diagonal SU(2) gauge rotations. The two vectors A_μ^1 and A_μ^2 lie in a plane of a four-dimensional space. This plane can be parametrized by a *zweibein* $e^a{}_\mu$ ($a = 1, 2$) with

$$e^a{}_\mu e^b{}_\nu = \delta^{ab}.$$

With

$$\mathbf{e}_\mu = 1/\sqrt{2}(e^1{}_\mu + ie^2{}_\mu)$$

we can then represent the most general A_μ^+ as

$$A_\mu^1 + iA_\mu^2 = i\psi_1 \mathbf{e}_\mu + i\psi_2 \mathbf{e}_\mu^\star,$$

where ψ_1 and ψ_2 are two complex fields; see [19] for details. We set

$$\rho^2 = |\psi_1|^2 + |\psi_2|^2$$

and define the three-component unit vector $\vec{n}$ by

$$\vec{n} = \frac{1}{\rho^2}(\psi_1^\star \psi_2^\star)\vec{\sigma}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \tag{3}$$

where $\vec{\sigma}$ are the standard Pauli matrices. We substitute the decomposed fields in the Yang-Mills Lagrangian, and keep *only* terms which involve ρ and $\vec{n}$. The result is [19]

$$-\frac{1}{4}F_{\mu\nu}^2 = (\partial_\mu \rho)^2 + \rho^2(\partial_\mu n^a)^2 + \frac{1}{4}(\epsilon_{abc}n^a\partial_\mu n^b\partial_\nu n^c)^2 + V(\rho^2, \vec{h}\cdot\vec{n}) + \ldots \tag{4}$$

Here V is a potential term. It involves some of the additional terms that we have deleted. This potential term leads to the breaking of the global O(3) invariance of the action under rigid rotations of $\vec{n}$, giving a mass to the two Goldstone bosons [19]. The function ρ relates to the average density of the two scalars ψ_1 and ψ_2,

$$\int (A_\mu^1 A_\mu^1 + A_\mu^2 A_\mu^2) = \int (|\psi_1|^2 + |\psi_2|^2) = \int \rho^2. \tag{5}$$

The minimization of (5) along gauge orbits selects the background gauge condition

$$D_\mu^{ab}(A^3)A_\mu^b = 0,$$

w.r.t. the Cartan A_μ^3 and, in particular, the minimum $\rho_{\rm min}$ of (5) is gauge invariant [22]. A one-loop computation suggests that the average $\langle \rho_{\rm min}\rangle$ is nonvanishing [23]. The unit vector $\vec{n}$ can detect whether a gluonic fluxtube is left-handed or right-handed. In fact, when $\langle \rho_{\rm min}^2\rangle = 0$ the action (4) is known to support stable knotted solitons [24,25] and the simplest soliton describes either a left-handed or a right-handed unknot, i.e., it is a twisted, *L-R* degenerate toroidal state as we desired. Provided these knotted solitons indeed survive in the full Yang-Mills quantum theory, we then have natural candidates for the glueballs as closed and knotted *L-R* degenerate fluxtubes.

Obviously it would be too naive to expect the solitons of (4) to provide a *quantitatively* accurate description of QCD fluxtubes. For this we must account for the roughening which is due to quantum fluctuations in the additional fields that appear in the decomposition of A_μ^a in the full SU(3) Yang-Mills theory. However, it is interesting to consider the predictions of these solitons; maybe some of their properties are sufficiently universal to survive a more comprehensive analysis in the full SU(3) theory.

The energy spectrum E_Q of the solitons in (4) follows a rational curve in their self-linking number (Hopf invariant) Q_H; see [24] for details:

$$E_Q \geq c \times |Q_H|^{3/4}.$$

If $f_0(1500)$ and $\eta_L(1410)$ are indeed the lowest mass states, we then have $c \approx 1500$ MeV suggesting the mass spectrum

$$M_Q \approx 1500 \times |Q_H|^{3/4} \text{ MeV}.$$

This predicts that the next ($|Q_H| = 2$) glueball has a mass in the vicinity of 2500-2600 MeV, well within the range of the planned radiative charmonium decay experiments at CLEO-II and BES. Obviously this estimate is very crude, but we note that there are general topological, model-independent arguments [26] which suggest that the 3/4-scaling law should be universal and reliable at least for larger values of $|Q_H|$. If this persists at lower values of $|Q_H|$, there could be very interesting physics around 6.4-6.5 GeV which is near the upper reach of energy at the recently approved antiproton facility at GSI. This corresponds to $Q_H = \pm 7$, and numerical simulations suggest that the ensuing soliton is a trefoil which is a nontrivial, chiral knot.

In three spatial dimensions the self-linking number of a knot is a topological invariant, it remains intact under continuous deformations of the knot. But if a knot is embedded in a space with more than three spatial dimensions its self-linking number ceases to be a topological invariant and the knot can disentangle. This suggests that the stability and decay properties of glueballs could be employed to explore the dimensionality of space-time and how the strong interaction couples to possible extra dimensions. In particular, the very presence of a (relatively) stable knotted gluonic fluxtube is indicative that strong interactions live in three spatial dimensions.

Obviously, a crucial test of our proposal comes from comparisons with detailed lattice simulations. If correct, the pseudoscalar configurations employed until now in lattice simulations can only have a tiny *Umklapp* with the lowest mass 0^{-+} glueball state.

We do expect that there should be some relation between our approach and the conventional description of glueballs in terms of closed strings, introduced in [15]. We propose that a relation could be obtained by deriving equations that describe the motion of the guiding center of a knotted soliton. This could also lead to an interesting description of long distance QCD in terms of more conventional string variables, and we plan to present the relations in future publications.

Finally, in full QCD we expect that (virtual) light $q\bar{q}$ pairs unstabilize a closed and knotted gluonic string. A closed string can open itself and become disentangled into another closed string but with a different self-linking number, through the formation and subsequent annihilation of a light quark-antiquark pair. This leads to an intuitively very attractive picture of interactions between guarks and glueballs, where quarks act much like certain enzymes act in the process of DNA replication by allowing one strand of the gluonic fluxtube to pass through another, thus changing its self-linking number and eventually leading to its decay into mesons. In particular, when quark loops are suppressed like in the limit of large quark masses or large-N, we expect the knotted fluxtubes to become stable provided the space-time is four-dimensional.

V. CONCLUSIONS

In conclusion, we have inspected the known meson spectrum up to energies around 2.3 GeV, which is the upper limit that has been experimentally probed by the $p\bar{p}$ annihilation processes. By assuming that glueballs can be viewed as closed gluonic fluxtubes we have concluded that the mysterious $\eta_L(1410)$ has a natural interpretation as the 0^{-+} glueball, parity related to the 0^{++} glueball $f_0(1500)$. This double degeneracy of the glueball spectrum reflects the left-right-twisting symmetry of a closed toroidal fluxtube. By employing a decomposed version of the gauge field we have also explained how this twisting degree of freedom can be realized in a pure Yang-Mills theory. Our arguments are quite general and model-independent, suggesting that the interpretation of $\eta_L(1410)$ as the 0^{-+} glueball provides a test of various qualitative aspects of strong interactions. These include the properties of QCD string and confinement, the formation of a mass gap, a detailed study of the QCD Hamiltonian, and the dimensionality of space-time as seen by the strong interaction. Our rough estimate of the glueball mass spectrum suggests that all these could be studied by the recently approved antiproton facility at GSI in Darmstadt.

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