Electromagnetic form factors of the nucleons from generalized Skyrme models

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We compare the prediction of Skyrme-like effective Lagrangians with data for electromagnetic form factors of nucleons and consider the possibility of fixing the parameters of these higher-order Lagrangians. Our results indicate that one- or two-parameter models can lead to better agreement with the data but more accurate determination of the effective Lagrangian faces theoretical uncertainties.

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I. INTRODUCTION

The Skyrme model [1], despite its relative successes [2], can only be considered as a prototype of an effective theory of QCD. Indeed, large N_c analysis [3,4] suggests that bosonization of QCD would most likely involve an infinite number of mesons. If this is the case, then taking the appropriate decoupling limits (or large mass limit) for higher spin mesons leads to an all-orders Lagrangian in derivatives of pion fields. For now however, fixing the form of the effective Lagrangian from an exact low-energy limit of QCD seems to be out of our reach and the alternative has been to propose simple effective Lagrangians [5–7] and rely on a few nucleon properties to set the parameters they depend on. From that point of view, a rather stringent test for such models lies in whether they could accurately describe the data for the electromagnetic form factors of nucleons for moderate values of momentum transfer.

The electromagnetic form factors of the semiclassically quantized SU(2) Skyrmion were studied systematically by Braaten *et al.* [8] but this first attempt did not take into account relativistic kinematical corrections which are important for momentum transfers $Q^2 > 1$ GeV^2. These corrections were implemented to the original Skyrme model predictions by Holzwarth [9,10] using the prescription of Ji [11]. Actually, Holzwarth also introduced a second correction to the Skyrme model to account for vector meson effects either by introducing a multiplicative factor to reflect the contributions of the poles coming from these mesons, or by adding the vector mesons as dynamical degrees of freedom in the Lagrangian. Yet, in spite of remarks and suggestions in [10,12,13], an analysis on how higher-order Lagrangians could reproduce the nucleon electromagnetic form factors or, conversely, on how the form factors could help construct a more accurate effective Lagrangian is still lacking.

In this work, we calculate the electromagnetic form factors for a class of higher-order (Skyrme-like) models introduced in [6]. For simplicity, we limit our numerical analysis to one- and two-parameter models with a proper treatment of relativistic effects, and evaluate to what extent the experimental data of the electromagnetic form factors of the nucleons can suggest a form of higher-order Lagrangians or discriminate among possible candidates.

II. THE SKYRME MODEL

Let us first introduce the Lagrangian density for the Skyrme model,

$$\mathcal{L}_S = -\frac{F_\pi^2}{16}\mathrm{Tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{Tr}[L_\mu, L_\nu]^2 \tag{1}$$

where F_π is the pion decay constant, L_μ is the left-handed chiral current $L_\mu = U^\dagger \partial_\mu U$, and the Skyrme constant e is a dimensionless constant. U is a SU(2) field related to the pion field π by $U = \exp(2i\pi \cdot \tau/F_\pi)$. The field configurations with finite energy must satisfy the boundary condition

$$U(\mathbf{r}, t) \to \mathbf{1} \quad \text{for } |\mathbf{r}| \to \infty. \tag{2}$$

These configurations fall into topological sectors characterized by

$$B = \frac{1}{2\pi^2}\int d^3x \det\{L_i^a\} = -\frac{\varepsilon^{ijk}}{48\pi^2}\int \mathrm{d}^3x \mathrm{Tr}(L_i[L_j, L_k]), \tag{3}$$

a topological invariant taking integral values.

Skyrme interpreted this topological invariant as the baryon number. Accordingly, the lowest-energy $B = 1$ sector is identified with the nucleon. In this sector, the lowest-energy field configuration is given by the hedgehog ansatz

$$U(\mathbf{r}) = \exp[i\tau \cdot \hat{\mathbf{r}}F(r)] \tag{4}$$

where $F(r)$ satisfies the boundary conditions $F(0) = \pi$ and $F(\infty) = 0$.

With the convenient change of scale, we can use $2\sqrt{2}/eF_\pi$ and $F_\pi/2\sqrt{2}$ as units of length and energy, respectively, and rewrite the Lagrangian density (1) as

$$\mathcal{L}_1 + \frac{1}{2}\mathcal{L}_2 = \left(-\frac{1}{2}\mathrm{Tr}L_\mu L^\mu\right) + \frac{1}{2}\left(\frac{1}{16}\mathrm{Tr}f_{\mu\nu}f^{\mu\nu}\right) \tag{5}$$

where $f_{\mu\nu} = [L_\mu, L_\nu]$. A pion mass term [14] is usually added to account for the chiral symmetry breaking ob-

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served in nature,

$$\mathcal{L}_{\pi} = \frac{m_{\pi}^2 F_{\pi}^2}{8}(\mathrm{Tr}U - 2). \tag{6}$$

This term serves as a regulator for the magnetic radii and form factors (see (36)) of nucleons, otherwise they would diverge [15].

Using the hedgehog ansatz (4), one obtains the mass of the static Skyrmion,

$$\begin{aligned} M &= -\int \mathrm{d}^3\mathrm{x}\mathcal{L}_{\mathrm{S}} \\ &= 4\pi\left(\frac{F_{\pi}}{2\sqrt{2}e}\right)\int_0^{\infty} drr^2\left\{F'^2 + 2\frac{\sin^2 F}{r^2}\right. \\ &\quad \left. + \frac{\sin^2 F}{2r^2}\left[2F'^2 + \frac{\sin^2 F}{r^2}\right] + 2\beta^2(1-\cos F)\right\} \end{aligned} \tag{7}$$

where r has now been rescaled and $\beta = 2\sqrt{2}m_{\pi}/eF_{\pi}$. The stable static soliton is obtained by minimizing the mass and requires solving the chiral equation

$$(1+a)\left[F'' + 2\frac{F'}{r} - 2\frac{sc}{r^2}\right] + a\left[F'^2\frac{c}{s} + \frac{sc}{r^2} - 2\frac{F'}{r}\right] - \beta^2 s = 0 \tag{8}$$

with the boundary conditions $F(0) = \pi$ and $F(\infty) = 0$ for $B = 1$. For simplicity, here we used $a = \sin^2 F/r^2$, $s = \sin F$, $c = \cos F$.

Fluctuations around this static soliton should be quantized. Quantization of the Skyrmion is usually performed with the introduction of a spin and isospin rotation matrix as a collective coordinate [2]. The spin/isospin rotation of Skyrmion takes the form

$$U(\mathbf{r}, t) = A^{\dagger}(t)U(\mathbf{r})A(t) \tag{9}$$

with $A(t)$ an arbitrary time-dependent SU(2) matrix. Substituting (9) in (5), one gets

$$L = -M + \mathcal{I}\mathrm{Tr}[\partial_t A^{\dagger}\partial_t A] = -M + \frac{I(I+1)}{2\mathcal{I}} \tag{10}$$

where M is defined in (7) and

$$\mathcal{I} = \frac{8\pi}{3}\left(\frac{2\sqrt{2}}{e^3 F_{\pi}}\right)\int_0^{\infty} r^2 dr \sin^2 F(r)\left[2 + \frac{\sin^2 F(r)}{r^2} + F'^2\right] \tag{11}$$

is the moment of inertia of the Skyrmion. Here I is the spin or isospin of the nucleon. The parameters F_{π} and e are fixed using two experimental inputs (mass of the nucleons or else). One is then able to reproduce the static properties of baryons within a 30% accuracy [2]. These methods can also be generalized to extensions of the Skyrme model as we will show in the following section.

III. ALL-ORDERS SKYRMIONS

The Skyrme model is more a prototype for low-energy pion interactions than a full effective field theory. Higher-order terms are expected to appear in addition to (5) and (7), but in its most general form the Lagrangian would involve an increasing number of terms at each order in pion field derivatives making the treatment practically intractable. One of us has proposed a special class of models whose energy density, assuming the hedgehog ansatz, is at most linear in F'^2 [6]. This requirement is sufficient to determine a unique term to each order in derivatives and turns out to have deeper geometrical meaning [16].

The static energy density coming from the Lagrangian of order $2m$ in derivatives of the field takes the form

$$\mathcal{E}_m = a^{m-1}[3a + m(b-a)] \tag{12}$$

where $a = \sin^2 F/r^2$ and $b = F'^2$. Using the hedgehog ansatz, the first two terms arise from the nonlinear σ and the Skyrme terms

$$\mathcal{E}_1 = -\mathcal{L}_1 = -\frac{1}{2}\mathrm{Tr}L_i L^i = [2a + b], \tag{13}$$

$$\mathcal{E}_2 = -\mathcal{L}_2 = -\frac{1}{16}\mathrm{Tr}f_{ij}f^{ij} = a[a + 2b], \tag{14}$$

while the third term leads to

$$\mathcal{E}_3 = -\mathcal{L}_3 = \frac{1}{32}\mathrm{Tr}f_{\mu\nu}f^{\nu\lambda}f_{\lambda}^{\mu} = 3a^2 b, \tag{15}$$

as for the term proposed by Jackson *et al.* [5] to allow for the dynamics of the ω meson in the Skyrme model.

Generalizing to all-order, the static energy associated to this class of all-order Lagrangian can be written in a simple form,

$$\mathcal{E} = \sum_{m=1}^{\infty} h_m \mathcal{E}_m = 3\chi(a) + (b-a)\chi'(a) \tag{16}$$

where a specific model is characterized by a choice of the parameters h_m or equivalently of the function $\chi(a) = \sum_{m=1}^{\infty} h_m a^m$ and $\chi'(a) = \frac{d\chi}{da}$. Yet, $\chi(x)$ is not completely arbitrary. Requiring that a unique soliton solution exists sets some constraints on $\chi(x)$ [7]:

$$\begin{gathered} \frac{d}{dx}\chi(x) \geq 0, \qquad \frac{d}{dx}\left(\frac{\chi(x)}{x^3}\right) \leq 0, \\ \frac{d}{dx}\left[\frac{1}{x^2}\frac{d}{dx}\chi(x)\right] \leq 0 \end{gathered} \tag{17}$$

for $x \geq 0$.

The mass of the soliton, including the pion mass term, is

$$\begin{aligned} M = 4\pi\left(\frac{F_{\pi}}{2\sqrt{2}e}\right)\int_0^{\infty} r^2 dr\{3\chi(a) + (b-a)\chi'(a) \\ + 2\beta^2(1-\cos F)\} \end{aligned}$$

and leads to the generalized chiral equation

$$\chi'(a)\left[F'' + 2\frac{F'}{r} - 2\frac{sc}{r^2}\right] + a\chi''(a)\left[F'^2\frac{c}{s} + \frac{sc}{r^2} - 2\frac{F'}{r}\right] - \beta^2 s = 0. \tag{18}$$

In that context, the Skyrme Lagrangian corresponds to $\chi(a) = a + (a^2/2)$. The moment of inertia of the soliton also takes a simple form,

$$\mathcal{I} = \frac{8\pi}{3}\left(\frac{2\sqrt{2}}{e^3 F_\pi}\right)\int_0^\infty r^4 dr\, a[2\chi'(a) + (b-a)\chi''(a)]. \tag{19}$$

Most all-orders models depend on more than two parameters. In the next section, we analyze the behavior of the electromagnetic form factors for a few models in the hope that these could help fix the h_m coefficients and obtain a better agreement with the experimental data in general.

IV. ELECTROMAGNETIC FORM FACTORS

The electromagnetic form factors of the proton (plus sign) and neutron (minus sign) in the Breit frame, for spacelike momentum transfer $q^2 > 0$, are the Fourier transforms of their electric charge and magnetic moment densities:

$$G_E^{p,n}(-q^2) = \frac{1}{2}\int_0^\infty dr\{B_0(r) \pm B_1(r)\} j_0(qr), \tag{20}$$

$$G_M^{p,n}(-q^2) = M_N \int_0^\infty dr\left\{\frac{4r^2 B_0(r)}{e^2 F_\pi^2 \mathcal{I}} \pm \mathcal{I} B_1(r)\right\}\frac{j_1(qr)}{qr} \tag{21}$$

with

$$B_0(r) = \frac{-2}{\pi}\sin^2(F)F', \tag{22}$$

and

$$B_1(r) = \frac{8\pi}{3\mathcal{I}}\left(\frac{2\sqrt{2}}{e^3 F_\pi}\right) r^4 a[2\chi'(a) + (b-a)\chi''(a)] \tag{23}$$

are the baryon density and moment of inertia density, respectively. Here j_n is the spherical Bessel function of order n and M_N is the nucleon's mass.

Both densities (22) and (23) are normalized,

$$\int_0^\infty dr B_0(r) = \int_0^\infty dr B_1(r) = 1, \tag{24}$$

while the electromagnetic form factors satisfy the normalization condition

$$G_E^{p,n}(0) = 1, \tag{25}$$

$$G_M^{p,n}(0) = \mu_{p,n} = \frac{M_N}{3}\left(\frac{r_B^2}{2\mathcal{I}} \pm \mathcal{I}\right) \tag{26}$$

where $\mu_{p,n}$ is the magnetic moment of the proton and neutron, respectively, and r_B^2 is the baryonic square radius

$$r_B^2 = \frac{8}{e^2 F_\pi^2}\int_0^\infty dr\, r^2 B_0(r). \tag{27}$$

However, the definitions (20) and (21) only hold in the Breit frame moving at velocity v with respect to the nucleon rest frame where the chiral profile $F(r)$ is computed. A correction for this Lorentz boost must be applied. Ji [11] has proposed a simple prescription to circumvent this difficulty for electromagnetic form factors:

$$G_E(q^2) = G_E^{nr}\left(\frac{q^2}{\gamma^2}\right), \tag{28}$$

$$G_M(q^2) = \gamma^{-2} G_M^{nr}\left(\frac{q^2}{\gamma^2}\right), \tag{29}$$

where G_E^{nr} and G_M^{nr} are given, respectively, by (23) and (24) and γ is the Lorentz factor

$$\gamma^2 = (1 - v^2)^{-1} = 1 + \frac{q^2}{4M^2} \tag{30}$$

with the nucleon mass M.

Unfortunately, the boost transformations (28) and (29) violate the so-called superconvergence rule

$$q^2 G_{E,M}(q^2) \to 0, \quad \text{for } q^2 \to \infty \tag{31}$$

which is expected to hold for electromagnetic form factors. Indeed the limit $q^2 \to \infty$ in the Breit frame corresponds to $q^2 = 4M^2$ in the rest frame and generally $G_{E,M}^{nr}(4M^2)$ does not vanish.

A possible approach to restore superconvergence is to relax the condition that M must take the value of the nucleon mass and instead allow M to vary in (30) in order to get the best agreement with the data of $G_M^p/(\mu_p G_D)$ at the highest available values of q^2 [17]. However, for the models under study here, this procedure turned out to be unsatisfactory. The value of M ensuring superconvergence caused the ratio of the electromagnetic form factors to become too suppressed in the large-q^2 limit. On the other hand, the models which we propose as candidates for the description of QCD at low energy are not expected to hold for large values of q^2. Since the superconvergence rule seems to be too restrictive, it will not be applied here. We will instead promote M as a parameter and adjust its value to provide a better fit of high-q^2 data.

V. RESULTS AND DISCUSSION

The parameters of the Skyrme model, e and F_π, are usually set with either of the following two methods: (i) adjust e and F_π to obtain the mass of the nucleons (939 MeV) and of the Δ resonance (1232 MeV) or (ii) set F_π according to its experimental value (186 MeV) and adjust e to reproduce the nucleon-Δ mass split (295 MeV).

For comparison purposes, we adopt the second method, also used in [9,10], and assume that the pion mass takes its physical value.

We consider here two simple extensions of the Skyrme model that fall into the class of models described in Sec. III:

$$\text{Model A:}\quad \chi_{\mathrm{A}}(a) = a + \frac{a^2}{2} + ca^3, \tag{32}$$

$$\text{Model B:}\quad \chi_{\mathrm{B}}(a) = a + \frac{c_1 a^3}{(1 + c_2 a)}. \tag{33}$$

The first extension, Model A, consists of adding a term proportional to $\mathcal{L}_3$ to the Skyrme model to implement the ω meson dynamics. On the other hand, Model B is a more elaborate attempt to reproduce poles due to vector mesons with a rational form. This latter model is a generalization of a model introduced by Jackson *et al.* [7] with $c_1 = \frac{1}{3}$ and $c_2 = \frac{2}{3}$.

Fixing the model parameters e, c, c_1, and c_2 as well as the scale parameter M is a tedious procedure which requires a few steps: First, we choose a set of parameters (c for Model A or c_1 and c_2 for Model B) and solve the differential equation (18) for massive pions and use the method described above (inputs are F_π and nucleon-Δ mass split) to fit for the appropriate value of e. In the second step, we compute the form factors, compare them with data, and adjust M to minimize the χ^2 of the ratio of the electromagnetic form factors of the proton. The whole procedure is repeated with different sets of parameters until we get the configuration e, c, M or e, c_1, c_2, M with lowest χ^2. Note that the neutron form factors are plagued with large experimental uncertainties. Indeed, the fit is almost insensitive to the nucleon form factor data and for computational reasons, we choose to use only the proton data as input.

The results for the electromagnetic form factor of the proton are presented in Fig. 1. We get $e = 5.03$, $c \simeq \frac{1}{27}$, and $M = 1.20$ GeV for Model A, and $e = 3.40$, $c_1 \simeq \frac{1}{3}$, $c_2 \simeq 1$, and $M = 1.20$ GeV for Model B. The results for the Skyrme model are also shown for comparison ($e = 4.25$ and $M = 1.66$ GeV). According to Fig. 1, both models present improvements over the Skyrme model, especially for the ratio $G_E/\mu_p G_M^p$ of the form factors. Despite its simplicity, Model A seems to overcome Model B. However both models exhibits a rapid divergence of the magnetic form factor of proton.

In view of these results, we must conclude that although the models examined here provide clear improvements, they cannot reproduce the data for the magnetic form factor of the proton adequately. The problem originates with their inability to mimic the pole coming from the vector mesons. Computing these predictions for an arbitrary number of Skyrme model extensions being prohibitive, we resort to another approach [9,10]. The ρ-meson effects are incorporated by multiplying the form factors (20) and (21) by

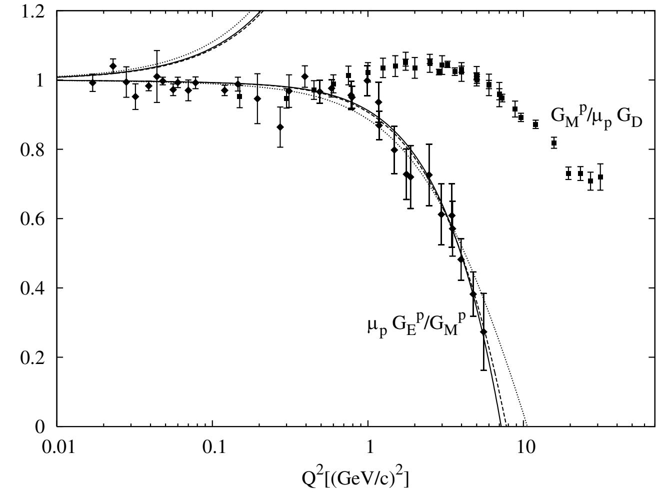

FIG. 1. Electromagnetic form factors of the proton from Model A: $c = \frac{1}{27}$, $e = 5.03$ (solid lines); Model B: $c_1 = \frac{1}{3}$, $c_2 = 1$, $e = 3.40$ (dashed lines); and the Skyrme model: $e = 4.25$ (dotted lines). The data are from [21–23] (diamonds) and [17,24,25] (squares).

$$\Lambda(q^2) = \lambda\left(\frac{M_\rho^2}{M_\rho^2 + q^2}\right) + (1 - \lambda) \tag{34}$$

with $M_\rho = 770$ MeV. The electromagnetic mean square radii of the proton (plus sign) and neutron (minus sign) then become

$$\begin{aligned}\langle r^2\rangle_E^{p,n} &= -\frac{6}{G(0)}\frac{dG_E^{p,n}(-q^2)}{dq^2}\bigg|_{q^2=0}\\ &= \frac{6\lambda}{M_\rho^2}\frac{(1 \pm 1)}{2} + \frac{4}{e^2F_\pi^2}\int_0^\infty dr\, r^2\{B_0(r) \pm B_1(r)\},\end{aligned} \tag{35}$$

$$\begin{aligned}\langle r^2\rangle_M^{p,n} &= -\frac{6}{G(0)}\frac{dG_M^{p,n}(-q^2)}{dq^2}\bigg|_{q^2=0}\\ &= \frac{6\lambda}{M_\rho^2} + \frac{8}{e^2F_\pi^2}\frac{M_N}{3\mu_{p,n}}\int_0^\infty dr\, r^2\left\{\frac{4r^2B_0(r)}{e^2F_\pi^2\mathcal{I}} \pm \mathcal{I}B_1(r)\right\}.\end{aligned} \tag{36}$$

This correction introduces an additional parameter whose allowed values go from the purely pionic model ($\lambda = 0$) to the mesonic dominance ($\lambda = 1$). It should be noted that neither the ratio of the electromagnetic form factors of the proton nor the electric mean square radius of the neutron depend on the parameter λ. A similar effect could also include taking into account the ω-meson effects, but a corresponding factor would involve a second λ parameter and our calculations indicate that this additional factor does not leads to noticeable amelioration of the behavior.

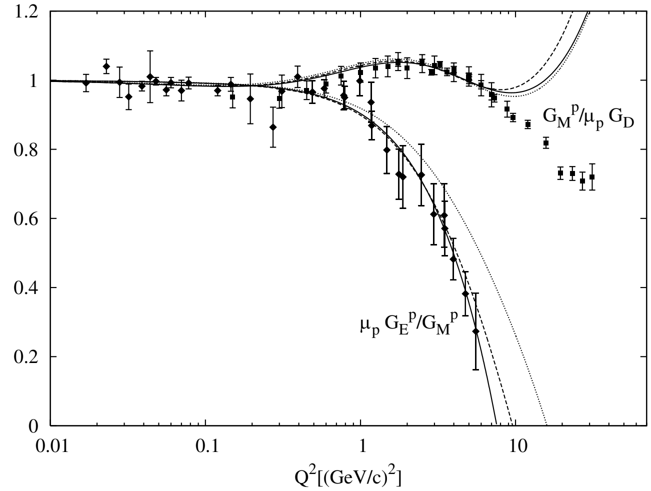

FIG. 2. Electromagnetic form factors of the proton from Model A: $c = \frac{1}{54}$, $e = 4.73$ (solid lines); Model B: $c_1 = \frac{1}{3}$, $c_2 = 1$, $e = 3.39$ (dashed lines); and the Skyrme model: $e = 4.25$ (dotted lines). The data are the same as for Fig. 1.

The electromagnetic form factors of nucleons obtained from Model A and B including pole effects (34) are presented in Figs. 2–4. All the models' parameters as well as the parameters M and now λ are fitted using the procedure described above to minimize the χ^2 of the magnetic form factor. The best agreement with data is reached for values of the parameters $e = 4.73$, $c \simeq \frac{1}{54}$ with $M = 1.31$ GeV and $\lambda = 0.73$ for Model A, and $e = 3.39$, $c_1 \simeq \frac{1}{3}$, and $c_2 \simeq 1$ with $M = 1.26$ GeV and $\lambda = 0.72$ for Model B. The results for the Skyrme model are found to be consistent with those of Refs. [9,10] with $M = 1.42$ GeV and $\lambda = 0.75$. Clearly, the inclusion of vector meson effects produce a significant improvement $G_M^p/\mu_p G_D$ over the results of Fig. 1. Again both extensions of the Skyrme model, Model A and B, do a better job of reproducing the data, the former

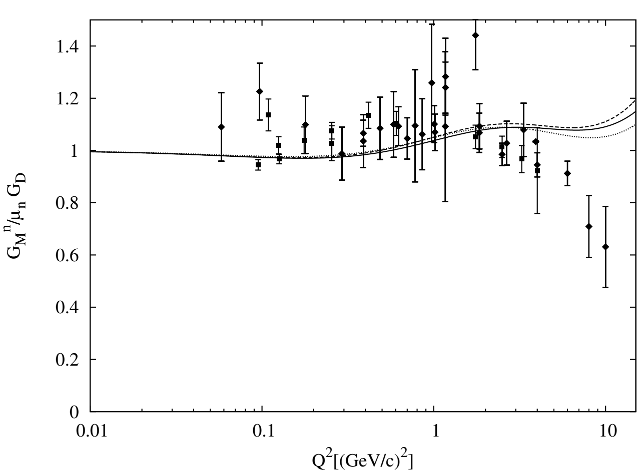

FIG. 3. Magnetic form factor of neutron for models of Fig. 2. The data are from [26–29] (squares) and [30–33] (diamonds).

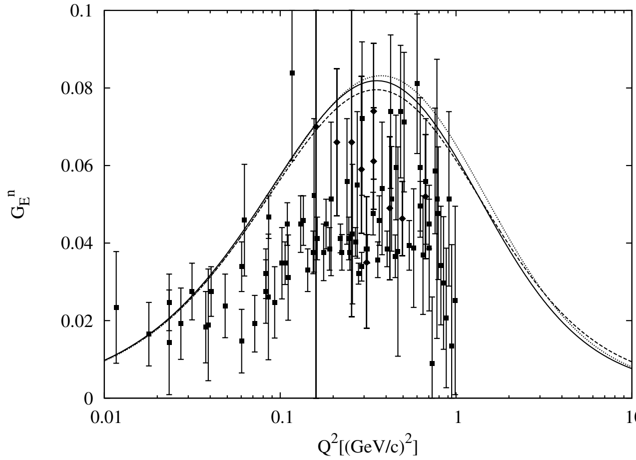

FIG. 4. Electric form factor of neutron for models of Fig. 2. The data are from [34,35] (squares) and [36–42] (diamonds).

model being slightly superior despite its relative simplicity. We note also that a sharp rise of $G_M^p/\mu_p G_D$ prediction still appears but this time only at the end of the spectrum, i.e., for Q^2 beyond 10 GeV^2. Any attempts to separately correct the rise in this Q^2 region for the proton or to lower the overall magnitude of G_E^n in Fig. 4 has failed. The increase of M or lowering of λ, respectively, which is necessary to carry out these corrections, completely jeopardizes the remainder of the fit which otherwise accurately reproduces most of the proton data. Furthermore, as shown in Table I, the addition of higher-order terms has little effect on other static properties of the nucleons with the optimized values of the parameters (Note that the magnetic moments of nucleons predicted by the models are all lower than their experimental values but one has to remember that these properties are subject to quantum corrections [20]). Note that higher values of λ, as result from our calculations, indicate a strong dominance of the ρ meson as in [9,10].

Our results therefore indicate that the data are best reproduced by a higher-order effective Lagrangian in the low-energy limit of QCD. Unfortunately, as we have no-

TABLE I. Static nucleon properties and parameters for models of Fig. 2. The experimental data come from [18,19].

	Skyrme	Model A	Model B	Exp.
M (GeV)	1.43	1.31	1.26	···
λ	0.75	0.73	0.72	···
μ_p (μ_N)	1.78	1.80	1.81	2.79
μ_n (μ_N)	− 1.42	− 1.41	− 1.40	− 1.91
r_E^p (fm)	0.685	0.702	0.707	0.870 ± 0.008
r_M^p (fm)	0.826	0.842	0.844	0.858 ± 0.056
$\langle r_E^n \rangle^2$ (fm^2)	−0.2364	−0.2424	−0.2399	−0.1161 ± 0.0022
r_M^n (fm)	0.857	0.877	0.879	0.876 ± 0.070

ticed, the determination of a more accurate effective Lagrangian faces theoretical uncertainties which remain to be addressed. First, the boost prescription (28) and (29), not being compatible with superconvergence, prevents us from reproducing results for $q^2 > 10(\mathrm{GeV/c})^2$. An adequate boost prescription holding account of superconvergence should allow extension of the analysis to higher momentum transfer. Secondly, we have only explored the possibilities of two types of Skyrme model extensions. These could not mimic the vector meson effects satisfactorily without the artifact of Eq. (34). Finding a form of Skyrme-like Lagrangian that would allow one to bypass the approach in (34) and avoid the introduction of the λ parameter remains a challenge.

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