Coupled-channel analysis of the D and D_s mesons

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The shift of the p-wave D_s meson mass due to coupling to the DK channel is calculated using a QCD string model and the chiral Lagrangian. As a result the original $Q\bar{q}$ mass, 2.490 MeV, generically calculated in the relativistic quark models is shifted down to the experimental value 2317 MeV. With the same Lagrangian the shift of the radial excited 1^- level is much smaller, while the total width $\Gamma >$ 100 MeV and the width ratio is in contradiction with the $D^*(2632)$ state observed by SELEX Collaboration.

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I. INTRODUCTION

The heavy-light D and D_s mesons have extensively been investigated experimentally in the last 20 years [1]. Recently a lot of attention has been paid to the 0^+ state of the D_s meson. It was found by the *BABAR* Collaboration [2] at the mass $M = 2318 \pm 1.3$ MeV and confirmed by CLEO and BELLE [3]. This value of mass is $\sim$40 MeV below the KD threshold and the width is very small. On the theoretical side this state was the object of intensive study (see Ref. [4] for reviews and references). There exists some disparity between the theoretical predictions of the $D_s^*(0^+)$ mass in the relativistic quark model (RQM) calculations [5–10] and experimental results. Indeed the RQM predictions vary from 2380 MeV [6,7] to 2487 MeV [10], being however substantially larger than measured experimentally.

Most recently a new D_s state was observed in the SELEX experiment [11] at the mass value $M =$ 2632 MeV. It is argued in Ref. [12] that this state may be associated with the radial excited 1^{-*} level of D_s meson, which is shifted down due to coupling to the ηD_s and KD channels. As one will see both levels $D_s^*(2317)$ and $D_s^*(2632)$ are connected to thresholds via chiral decays and we shall treat them below using the chiral Lagrangian containing only f_π, f_K, f_η as parameters.

It is the purpose of the present paper to study the effect of nearby thresholds on the position of resonances first in the most general setting, and then to calculate numerically using the chiral quark Lagrangian. The resonance structure is described by a QCD string model, containing as parameters the string tension σ, Eq. (10), and the one gluon exchange coupling α_s. The paper is organized as follows. In Sec. II we present a general discussion of channel coupling (CC) and level shift using the (relativistic) Hamiltonian formalism, where we also give a classification of possible S-matrix poles. In Sec. III the chiral quark Lagrangian is written down and used to describe the decay transition $D_s^* \to DK, D_s\eta$. The final equation for the resonance position with account of this decay is explicitly written. In Sec. IV the numerical solution of this equation is described and the final results are presented. The paper closes with a discussion and comparison with other results.

II. RESONANCE STATES IN THE COUPLED-CHANNEL SYSTEM

The relativistic quark model has been remarkably successful in predicting the D and D_s meson spectrum, apart from some exceptions of a few resonances, which are experimentally found at substantially lower masses. Similar results have been found recently in the relativistic Hamiltonian approach [13] derived on the basis of the field correlator method (FCM) [14] and applied to the D, D_s mesons in Ref. [15]. The interaction constructed contains a linear confining interaction parametrized by a string tension σ and the perturbative one-gluon exchange with strength α_s. The results for the masses in the FCM analysis are shown in Tables I and II . (The entries given in the tables are recalculated for parameters given in the table captions).

As is seen from Tables I and II the overall agreement is reasonably good except for a few states. In particular, the $D_s^*(0^+)$ state is one example of such a discrepancy in the prediction of theoretical models which can be associated with the KD threshold at 2366 MeV.

For comparison, in Table III a summary of the results is given of other theoretical quark model predictions for this state. A look at the table tells us that all the theoretical predictions are about $\sim 60 \div 190$ MeV higher. So one needs a shift of about this value to get agreement with the experimental value. A similar discrepancy can be seen from Table II for the $D_s^*(2632)$ resonance. We explore in this paper whether this disparity can be explained due to the presence of coupled channels with nearby thresholds.

TABLE I. Masses of $L = 0, 1$ states of D mesons. Input parameters used in the FCM calculations are: $\alpha_s = 0.46$, $\sigma = 0.17$ GeV^2, $m_c = 1.44$ GeV, $m_n = 7$ MeV.

State J^P	0^-	1^-	0^+	$1^+(l)$	$1^+(h)$	2^+	1^{-*}
Mass (MeV) from [15]	1859	2047	2370	2425	2455	2456	2729
Mass (MeV) experiment	1869	2010	2300 ± 60	2400	2422	2459	2640(?)
Γ (MeV) experiment	$\cdots$	<0.13	280	~250	20	$23 \div 45$	<15

TABLE II. Masses of $L = 0, 1$ states of D_s mesons. Input parameter used in the FCM calculations are: $\alpha_s = 0.46$, $\sigma = 0.17$ GeV^2, $m_c = 1.44$ GeV, $m_s = 0.175$ GeV.

State J^P	0^-	1^-	0^+	$1^+(l)$	$1^+(h)$	2^+	1^{-*}
Mass (MeV) from [15]	1929	2087	2404	2462	2488	2494	2774
Mass (MeV) experiment	1968	2112	2317	2462	2536	2572	2632(?)
Γ (MeV) experiment	$\cdots$	<1.9	<10	<6.6 $8.6 \div 0.4$	<2.3	~15	<17

Resonances in the coupled-channel system were considered in numerous papers both in nonrelativistic nuclear physics and in the relativistic Hamiltonian dynamics; see [16] for a review and references. Assuming that a local or nonlocal relativistic Hamiltonian can be written for each channel H_i, $i = 1, 2, \ldots$ and for the CC V_{ij}, $i, j = 1, 2, \ldots$, the time-independent system of equations can be written as

$$[(H_l - E)\delta_{ll'} + V_{ll'}]G_{l'l''} = 1. \tag{1}$$

For two channels it is

$$\begin{aligned} (H_1 - E)G_{11} + V_{12}G_{21} &= 1, \\ (H_1 - E)G_{12} + V_{12}G_{22} &= 0, \\ (H_2 - E)G_{22} + V_{21}G_{12} &= 1, \\ V_{21}G_{11} + (H_2 - E)G_{21} &= 0. \end{aligned} \tag{2}$$

The system (2) can be reduced to the effective one-channel problem, corresponding to the Feshbach equation [17]

$$(H_1 - E)G_{11} - V_{12}\frac{1}{H_2 - E}V_{21}G_{11} = 1. \tag{3}$$

At this point one can classify all possible poles E of the Green functions G_{ik}. These poles may originate from the bound states or resonances in a given channel i, located at $E_i^{(n)}$, and shifted due to CC to a new position, which we will denote by $E_i^{(n)*}$. Another possibility is that resonance poles appear solely due to the strong CC interaction—the so-called CC poles [16,18]. These extra poles usually originate from distant dynamical poles in the complex plane, which move close to threshold when the CC coupling increases. The quantitative characteristics of the CC interaction is given by the last term on the left-hand side of Eq. (3), which can be called the Feshbach potential,

$$V_{121}(E) \equiv -V_{12}G_2V_{21} = -V_{12}\frac{1}{H_2 - E}V_{21}. \tag{4}$$

Note that $V_{121}(E)$ can support bound states or resonances even in the case when diagonal interaction V_i, $i = 1, 2$ vanishes but $V_{12} = V_{21}^+$ is large enough.

Of special importance for us is the case when in one channel, e.g. $i = 1$, the spectrum is discrete (see Ref. [18] for a more extensive discussion), and one is interested in the shift of the discrete level due to the coupling to channel 2, where states can be unconfined.

A somewhat similar approach was undertaken in recent papers [19,20], where in our notations the scattering channel 2 and the corresponding Feshbach potential V_{212} were modeled to calculate the scattering cross section in channel 2. We shall compare the results of Refs. [19,20] with ours in the concluding section. Equation (3) connects, in general, all states in channel 1 and channel 2. If one separates one state and neglects all other states in channel 1, then one gets the following equation for the position of the pole(s) in the Green function:

$$E = E_1^{(n)} - \left\langle n \left| V_{12}\frac{1}{H_2 - E}V_{21} \right| n \right\rangle, \tag{5}$$

where $E_1^{(n)}$ is the selected unperturbed level in channel 1. Insertion of the complete set of states $|m\rangle\langle m|$ with eigenvalues $E_2^{(m)}$ in channel 2 yields

TABLE III. Theoretical predictions of the mass $D_s^*(0^+)$ in various quark models.

Ref.	[5]	[6]	[7]	[8]	[9]	[10]
Mass (MeV)	2480	2388	2380	2508	2455	2487

$$E = E_1^{(n)} - \sum_m \langle n|V_{12}|m\rangle \frac{1}{E_2^{(m)} - E} \langle m|V_{21}|n\rangle. \quad (6)$$

In what follows we shall be using Eq. (6) to calculate the shift $\Delta E_n = E_1^{(n)*} - E_1^{(n)}$ of the $c\bar{s}$ levels due to the open channel 2: KD or ηD_s scattering states, neglecting interaction in these states. The most important point is how to find the operators V_{12}. In the next section we shall use the chiral Lagrangian which will provide V_{12} explicitly without free parameters.

III. COUPLED CHANNELS AND CHIRAL DECAYS

One starts with the Lagrangian for the flavor SU(3) triplet of quarks in the field of the heavy (c or b) quark [21,22]. In the Euclidean notations

$$L = i \int d^4x \psi^+ (\hat{\partial} + m + \hat{M})\psi, \quad (7)$$

where m is the current quark mass and the mass operator $\hat{M}$ is

$$\hat{M} = M\hat{U} = M \exp\left(i\gamma_5 \frac{2\varphi_a t_a}{f_\pi}\right), \quad (8)$$

$t_a = \frac{1}{2}\lambda_a$, λ_a is the Gell-Mann matrix, $a = 1, \ldots 8$, $f_\pi = 0.093$ GeV, and the matrix Nambu-Goldstone SU(3) wave function is

$$\varphi_a \lambda_a = \sqrt{2} \begin{pmatrix} \frac{\eta^0}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}}, & \pi^+, & K^+ \\ \pi^- & \frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & K^0 \\ K^-, & \bar{K}_0, & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \quad (9)$$

M is the effective mass operator, which is, in general, nonlocal in coordinate space. The nonlocality is of the order of 0.25 fm. In the local limit it has the form (see [23,24] for discussion and derivation)

$$M = \sigma|\mathbf{r}|, \quad (10)$$

where $|\mathbf{r}|$ is the distance from the light quark ($u, d, s,$) to the heavy quark (c or b) and σ is the standard string tension. Thus the Lagrangian (7) contains effects of both confinement and chiral symmetry breaking.

From Eqs. (7) and (8), expanding the exponent in Eq. (8), one can derive the meson emission part of the Lagrangian,

$$\Delta L = -\int \psi^+(x)\sigma|\mathbf{x}|\gamma_5 \frac{\varphi_a \lambda_a}{f_\pi} \psi d^4x. \quad (11)$$

This Lagrangian can be expressed as in Ref. [21] in terms of the standard Georgi-Manohar-Weinberg Lagrangian [25]. The latter was used in Ref. [26] to calculate the decay widths of heavy-light mesons with good accuracy.

It is clear that the Lagrangian (7) generates (due to the various Fock components in ψ^+ or ψ), in general, a many-channel system of equations for the Green functions. It contains the main channel (e.g., the D_s channel) and in addition the channel(s) for its virtual decay products like the $(D + K)$ channel or $(D_s + \eta)$ channel.

In what follows we shall be working with Eq. (6) to apply it first of all to the $D_s^*(2317)$ state. In this case $E_1^{(n)}$ refers to the 0^+ level of the D_s^* system, and $E_2^{(m)}$ refers to the (continuous) energy of the system $D + K$ in the orbital S state. One can neglect the DK interaction in the first approximation and write for the wave functions of the Dirac equations. We have

$$|n\rangle = \Psi(D_s^*) = \frac{1}{r}\begin{pmatrix} G_n^{(1)}\Omega_{jlM}^{(1)} \\ iF_n^{(1)}\Omega_{jl'M}^{(1)} \end{pmatrix}, \quad (12)$$

$$\Omega_{jlM}^{(1)} = \Omega_{(1/2)1M_1}, \qquad \Omega_{jl'M}^{(1)} = \Omega_{(1/2)0M_1} \quad (13)$$

$$|m\rangle = \Psi(D)\frac{e^{i\mathbf{pr}}}{\sqrt{2\varepsilon_p V_3}}, \quad (14)$$

$$\Psi(D) = \frac{1}{r}\begin{pmatrix} G^{(2)}\Omega_{jlM}^{(2)} \\ iF^{(2)}\Omega_{jl'M}^{(2)} \end{pmatrix}, \quad (15)$$

$$\Omega_{jlM}^{(2)} = \Omega_{(1/2)0M_2}, \qquad \Omega_{jl'M}^{(2)} = \Omega_{(1/2)1M_2}, \quad (16)$$

where Ω are spinor harmonics (cf. Refs. [23,26]) and V_3 in Eq. (14) is the normalization constant of the plane wave in a box with volume V_3. Using the above expressions the matrix elements in Eq. (6) can be evaluated. As a result we get

$$\begin{aligned}\langle n|V_{12}|m\rangle &= -\int \Psi^+(D_s^*)\sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_\pi}\Psi(D)\frac{e^{i\mathbf{pr}}}{\sqrt{2\varepsilon_p V_3}} d^3r \\ &= \frac{\sqrt{2}\sigma}{if_\pi}\int \frac{d^3\mathbf{r}}{r}(G_n^{(1)+}F^{(2)}\Omega^+_{(1/2)1M_1}\Omega_{(1/2)1M_2} \\ &\quad - F_n^{(1)+}G^{(2)}\Omega^+_{(1/2)0M_1}\Omega_{(1/2)0M_2})\frac{e^{i\mathbf{pr}}}{\sqrt{2\varepsilon_p V_3}} \end{aligned} \quad (17)$$

$$= \frac{\sqrt{2}\sigma\delta_{M_1M_2}}{if_\pi}\int \frac{\sin pr}{p\sqrt{2\varepsilon_p V_3}}(G_n^{(1)+}F^{(2)} - F_n^{(1)+}G^{(2)})dr. \quad (18)$$

Now $G^{(1)}$, $F^{(1)}$ and $G^{(2)}$, $F^{(2)}$ are solutions of the Dirac equation

$$\begin{aligned} \frac{dF_n^{(i)}}{dr} - \frac{\kappa_i}{r}F_n^{(i)} + [\varepsilon_n^{(i)} - V_c(r) - m_i]G_n^{(i)} - MG_n^{(i)} &= 0, \\ \frac{dG_n^{(i)}}{dr} + \frac{\kappa_i}{r}G_n^{(i)} - [\varepsilon_n^{(i)} - V_c(r) + m_i]F_n^{(i)} - MF_n^{(i)} &= 0. \end{aligned} \quad (19)$$

Here $\kappa_1 = 1$, $\kappa_2 = -1$, $M = \sigma r$, $V_c(r) = -(4\alpha_s/3r)$, and

$$m_1 = m_s = 0.15 \div 0.25 \text{ GeV}, \qquad m_2 = 0.$$

The connection between $\varepsilon_n^{(i)}$ and E_i is

$$E_1^{(n)} = \varepsilon_n^{(1)} + m_c; \qquad E_2^{(m)} = \varepsilon_m^{(2)} + m_K + m_c + \frac{\mathbf{p}^2}{2\tilde{m}_K},$$
$$\tilde{m}_K = \frac{m_K m_D}{m_K + m_D}. \tag{20}$$

If one neglects higher states of the D meson, the sum in Eq. (6) can be rewritten as

$$E = E_1^{(n)} - \sum_m \int \frac{V_3 d^3\mathbf{p}}{(2\pi)^3} \frac{|\langle n|V_{12}|m\rangle|^2}{E_2^{(m)}(p) - E}. \tag{21}$$

From Eqs. (17) and (21) it is clear that the free Green function of the KD system has the form (we take into account the fact that only S waves of KD are involved)

$$\begin{aligned} G_0(k,x,x') &= \left(\int \frac{d^3p}{(2\pi)^3} \frac{\exp[i\mathbf{p}.(\mathbf{x}-\mathbf{x}')]}{2\omega(\mathbf{p})(\frac{\mathbf{p}^2}{2\tilde{m}_K} - \delta E)}\right)_{S\text{ waves}} \\ &= \frac{1}{4\pi} \frac{\tilde{m}_K}{\omega(k)} \frac{\sin(kx_<)\exp(ikx_>)}{kxx'}, \end{aligned} \tag{22}$$

where $\delta E = E - \epsilon_m^{(2)} - m_c - m_K$ and

$$k^2 = 2\tilde{m}_K(E - m_D - m_K), \qquad \omega(k) = \sqrt{k^2 + m_K^2}. \tag{23}$$

Finally Eq. (21) can be rewritten as

$$E = E_1^{(n)} - \int V(x)V^+(x')d^3x d^3x' G_0(k,x,x), \tag{24}$$

where $V(x)$ is

$$\begin{aligned} V(x) &= \bar{\psi}_2(x) \frac{\sqrt{2}\sigma|\mathbf{x}|}{f_\pi} \gamma_5 \psi_1(x) \\ &= \frac{\sqrt{2}\sigma}{if_\pi x}(G^{(1)+}F^{(2)} - F^{(1)+}G^{(2)})\Omega^+_{(1/2)0M_1}\Omega_{(1/2)0M_2} \end{aligned} \tag{25}$$

and we have used the relation $\Omega_{(1/2)1M} = \sigma.\mathbf{n}\Omega_{(1/2)0M}$, where σ are the Pauli spin matrices and $\mathbf{n}$ is the unit vector in the $\mathbf{r}$ direction. As a first approximation one can use the fact that functions $G^{(i)}, F^{(i)}$ are concentrated around the middle point $x \cong b$ and write

$$V(x) \cong C\delta(x - b); \qquad C = \int V(x)dx, \tag{26}$$

$$b = \frac{\int xV(x)dx}{\int V(x)dx}.$$

As a result one obtains

$$E = E_1^{(n)} - C^2 \frac{4\pi b^2 \tilde{m}_K \sin(kb)e^{ikb}}{\omega(k)k}, \tag{27}$$

where $k = \sqrt{2\tilde{m}_K(E - M_D - m_K)}$ is the relative momentum of the K meson.

In the vicinity of the DK threshold one can replace $\omega(k) \cong m_k$ (this is implied by the form of Eq. (22)). Equation (27) is a transcendental equation for the position of the pole E. Since C does not depend on k Eq. (27) has a simple square-root threshold at $E = m_D + m_K$. The starting position of $E = E_{\text{pole}}$ is at $E_1^{(n)}$. When one takes into account the second term in Eq. (27) with gradually increasing C^2 the pole moves to the final value in the upper physical k-sheet. One expects that the trajectory will go down in mass, possibly near the final value of $m(D_s^*) = 2317$ MeV. In Fig. 1 we display the trajectory of the pole solution of Eq. (27) in k-plane parametrized by C. One can see that for strong enough channel coupling the resonance pole moves down under the DK threshold, which will be substantiated by the exact calculation of V_{12}, given by Eq. (18).

We now turn to the case of the $D_s^*(2632)$ [11] assuming, after Ref. [12], that it can be associated with the radially excited $D_s^*(1^-)$ state. From Table II one can see that the expected shift should be around 100–150 MeV downwards, and from the channels $DK, D_s\eta$ the decay is in the p wave. As before we shall use Eqs. (12) and (17) where now instead of $D_s^*(0^+)$ one should write the $D_s^*(1^-)$ state, i.e.,

$$\Omega_{jlM}^{(1)} = \Omega_{(1/2)0M_1}, \qquad \Omega_{jl'M_1}^{(1)} = \Omega_{(1/2)1M_1}. \tag{28}$$

The wave function in channel 2 is either the same as in Eqs. (14)–(16), or in the case of the η channel, one should replace D by D_s, and the K meson with momentum $\mathbf{p}$ by the η meson with momentum $\mathbf{p}'$. With these assignments of the various states Eq. (17) retains its form, but Eq. (18)

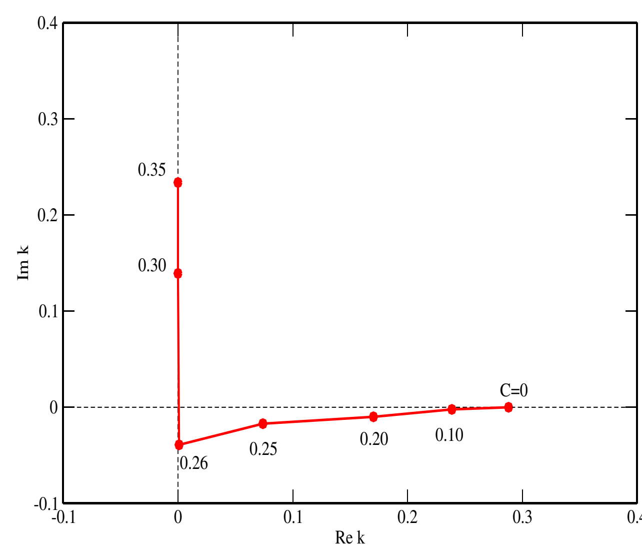

FIG. 1 (color online). The trajectory of the D_s^* pole in the complex k plane parametrized by C. For b we have used $b = 0.4\ \text{GeV}^{-1}$.

becomes

$$\langle n|V_{12}|m\rangle = \frac{\sqrt{2}\sigma}{if_\pi}\int \frac{d^3\mathbf{r}}{r}(G_n^{(1)+}F^{(2)}\Omega^+_{(1/2)0M_1}\Omega_{(1/2)1M_2} - F_n^{(1)+}G^{(2)}\Omega^+_{(1/2)1M_1}\Omega_{(1/2)0M_2})\frac{e^{i\mathbf{pr}}}{\sqrt{2\varepsilon_p V_3}} \tag{29}$$

$$= -\frac{\sqrt{2}\sigma}{f_\pi p\sqrt{2\varepsilon_p V_3}}\int_0^\infty dr\left(\frac{\sin pr}{pr} - \cos pr\right)(G_n^{(1)+}F^{(2)} - F_n^{(1)+}G^{(2)}),$$

where p is the momentum of the K or η meson.

Here the radial quantum number in Eq. (29) is the first radial excited state $n = 1$. To compute the transition potential matrix elements for the $D_s^* \leftrightarrow DK$ coupling one can use the coordinate representation as in Eqs. (18)–(25) including Eq. (28), or directly calculate the $d^3\mathbf{p}$ integral in Eq. (21). In the latter way one needs to compute Eq. (21), $\omega(p) \equiv \varepsilon_p = \sqrt{p^2 + m_K^2}$,

$$E = E_1^{(n)} - \frac{1}{(2\pi)^3}\int \frac{d^3\mathbf{p}|v_{12}^{(K)}(p)|^2}{2\omega(p)[E_2(\mathbf{p}) - E]}, \tag{30}$$

and $v_{12}(\mathbf{p})$ is

$$v_{12}^{(K)}(\mathbf{p}) = V_{12}\cdot\sqrt{2\varepsilon_p}. \tag{31}$$

Equation (30) has to be extended to also include the $D_s\eta$-channel contribution. Observation of Eqs. (9) and (11) shows that in Eq. (9) only the lowest diagonal term enters in the transition potential matrix element. Hence we get for $\langle n|V_{12}|m\rangle$ an additional factor $(-\frac{2}{\sqrt{6}})$. As a result we have

$$v_{12}^{(\eta)}(p) = -\frac{2}{\sqrt{6}}v_{12}^{(K)}(p).$$

The modification of V_{12}, Eq. (29), for the case of the η channel is straightforward. Clearly, the assignment of the D_s^* state in Eq. (28) remains the same and we have to replace in Eq. (20) $m_K \to m_\eta$ and $m_D \to m_{D_s}$. Moreover, the state $\Psi(D)$ in Eq. (17) has to be replaced by $\Psi(D_s)$, where $F^{(2)}$, $G^{(2)}$ now refer to the D_s state.

IV. MASS SHIFT IN THE CHIRAL LAGRANGIAN FORMALISM

We may determine the energy shift ΔE using the Dirac wave function as found by solving Eq. (19) for the case of an effective quark mass operator in the field correlator method [24]. In the leading order it has a nonlocal form and can be parametrized as

$$M(x,y) \approx \frac{1}{2T_g\sqrt{\pi}}\sigma\left|\frac{\mathbf{x}+\mathbf{y}}{2}\right|\exp\left(-\frac{(\mathbf{x}-\mathbf{y})^2}{4T_g^2}\right), \tag{32}$$

where σ is the string tension and T_g the gluon correlation length, characterizing the scale of nonlocality. Note that for $T_g \to 0$ one obtains from Eq. (32) the local limit (6). The physical value of T_g found analytically and in lattice simulations is small, $T_g = 0.25$ fm [14,23].

For the case of a single coupled channel $D_s^* \leftrightarrow DK$ we may write Eq. (32) as

$$\Delta E = -\frac{1}{(2\pi)^3}\int \frac{d^3\mathbf{p}|v_{12}^{(K)}(p)|^2}{2\omega(p)[E(\mathbf{p}) - E_0 - \Delta E]}, \tag{33}$$

where $\Delta E = E - E_0$ with $E_0 = m_{D_s^*} - m_D - m_K$ is the mass shift and $E(\mathbf{p})$ the kinetic operator $E(\mathbf{p}) = p^2/2\tilde{m}_K$.

In the considered QCD string model we have taken $T_g = 0.25$ fm, in accordance with lattice gauge simulations [14]. For a given string tension σ the wave functions of the D and D_s system can be found as solutions of the Schwinger-Dyson-Dirac equation [24] for the light-heavy quark system. It is given by Eq. (19) with the quark mass operator (32). The latter is found from the self-consistent solution of nonlinear equations, (Eqs. (15), (16) in [23]). It exhibits the properties of both confinement and chiral symmetry breaking. The states are, in general, characterized by the quantum numbers j, l, κ. In particular, D and the orbitally excited state D_s^* correspond to the solution of the ground state in the $j, l, \kappa = 1/2, 0, -1$ and $j, l, \kappa = 1/2, 1, +1$ channels, respectively.

In this study values of $\sigma = 0.18$ GeV^2 and $\alpha_s = 0.35$ are adopted. For convenience, a zero mass is used for the u, d quark, while for the s quark we have taken $m_s = 200$ MeV. Having constructed these wave functions with these parameters we determine the matrix elements $v_{12}^{(K)}$. From this we may then solve the resulting Eq. (33) iteratively. To determine the actual position of the pole we in general have to analytically continue the integral into the second sheet in the case that E is above the KD threshold. Although this can be done, we will assume in this study that the imaginary part of the pole position does not affect the solution substantially, which is certainly true when the pole dives under the DK threshold, and was checked in other situations. Confining ourself to real values of E and taking the principal value of the integral in Eq. (33) when E is above threshold the energy shift is determined as a solution of the resulting equation. In case of a solution above threshold, the width of the resonance can be obtained by calculating the discontinuity of the integral at this energy.

In the calculations we have used for the threshold mass of the KD system $m_K + m_D = 2.366$ GeV. A typical value of 2.49 GeV is adopted for the unperturbed D_s^* meson mass. As is seen in Table III this is in accordance with the predictions of FCM and of many quark-constituent quark models.

In Fig. 2 the prediction is shown of the shifted mass of the D_s^* meson due to the channel coupling to the KD

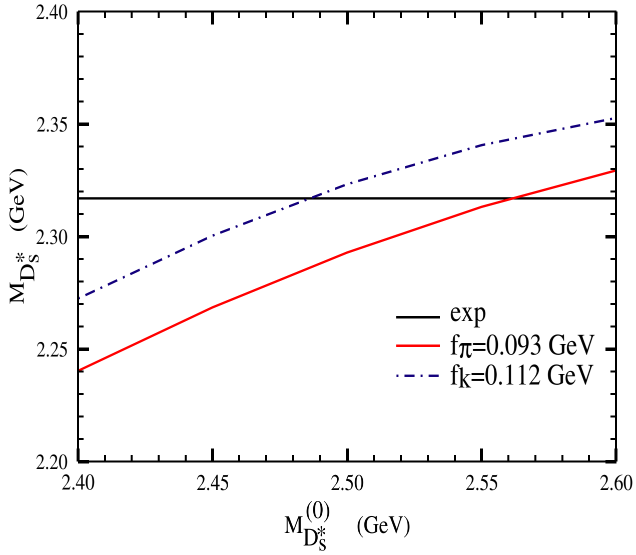

FIG. 2 (color online). The shifted D_s^* meson mass as a function of the unperturbed $M^{(0)}_{D_s^*}$ for two values of the decay constant f. The experimentally observed mass is given by the dashed horizontal line.

system as a function of the unperturbed mass of the D_s^*. The flavored symmetric and broken value of $f_\pi = 93$ MeV and $f_K = 112$ MeV have been used [1]. The shifted mass is found using

$$M_{D_s^*} = M^*_{D_s^{(0)}} + \Delta E, \tag{34}$$

where ΔE satisfies Eq. (33). Because of the coupling to the KD system the D_s^* meson can either become unstable or stable, depending on the sign and magnitude of the mass shift. We find in our model that it can be substantial and of the order of hundred MeV. As is seen from the Fig. 2 we find the experimentally observed value of 2.317 GeV for an unperturbed mass of approximately 2.49 GeV. Clearly the above value of the unperturbed mass depends somewhat on the choice of the parameters used in the QCD string model. Small variations of the string tension and α_s yield small variations in the required unperturbed mass.

In the string model studied we find that the position of the pole moves well down below the KD threshold, yielding a stable state. The channel coupling is found to yield attraction, so that the position of the mass pole shifts downwards.

Similarly as for the $D_s^*(2317)$ meson we may estimate the mass shift of the $D_s^*(2632)$ due to channel coupling. The calculations proceed in the same way. The bare $D_s^*(2632)$ is assumed to correspond to the first radial excited state with $j, l, \kappa = 1/2, 0, -1$ and to have a mass of 2.76 GeV as found in the FCM [15]. Clearly a mass shift of about -140 MeV is needed to obtain agreement with the observed mass. As discussed, coupling can occur in this case to KD and ηD_s channels. For the D meson we assume the experimental observed mass of 1.869 GeV. There are two candidates for the D_s state—the 0^- and 1^- states with masses of 1.968 GeV and 2.120 GeV, respectively. Using the various wave functions obtained from the QCD string model, the interaction matrix elements are calculated from Eq. (29). The two D_s states are degenerate in the considered model and as a result have the same wave function, but differ in the kinematics of the momenta in view of the adopted mass difference, which occurs due to the hyperfine interaction neglected in our heavy-quark approximation. For simplicity we have considered only one D_s state with mass 2.0 GeV. With the obtained potential matrix elements we solve numerically the eigenvalue equation for ΔE. It has essentially the same form as Eq. (33), but now has two terms due to the contributions from the two coupled channels.

We find that the bare mass is shifted downwards by 51 MeV. Furthermore, the contributions from the various inelastic channels are given by

$$\Delta E(KD) = -35 \text{ MeV}, \qquad \Delta E(\eta D_s) = -16 \text{ MeV}.$$

The magnitude of the total mass shift is clearly smaller here than that found in the first considered case of the $D_s^*(2317)$ meson as can be explained by the presence of P wave, rather than S wave for the case of $D_s^*(2317)$. It is clearly not sufficient to explain the experimental observed mass. We can also calculate the width, which is the discontinuity of the right-hand side of Eq. (33). We find $\Gamma = 174$ MeV, decaying predominantly into the KD channel. The corresponding partial decay widths to the various channels are found to be

$$\Gamma(KD) = 139 \text{ MeV}, \qquad \Gamma(\eta D_s) = 35 \text{ MeV}.$$

Clearly the predicted width is considerably larger than that found in the SELEX experiment. We have also solved Eq. (33) for the case of the flavor symmetric value $f_K = f_\eta = .093$ GeV, which yields again a large value for the width. The above results suggest in this case that the resonance becomes very broad and does not support the SELEX observation; also, the ratio of KD to ηD_s channels quantitatively disagrees with the experiment.

In general, the size of the mass shift clearly depends on the structure of the quark wave functions and hence it should be expected to be model dependent. Our study demonstrates that the size of the shift due to channel coupling is in general large, but that it also can lead to very large widths in the case that the resonance is above threshold of the coupled channels. As a result it can accommodate for the discrepancy between the predictions of dynamical quark models and the observed $D_s^*(2317)$ resonance, but there may exist situations where the resonance can become very broad due to inelastic channel coupling, as it is in the case of radial excited D_s^*.

V. DISCUSSION

Let us compare our results to the existing results in the literature. The explanation of $D_s^*(2317)$ as the $c\bar{s}$ p wave level shifted down by the coupling to the decay channel $D + K$ was considered in a series of papers [19,20], where the authors have used a simple phenomenological model similar to our Eqs. (26) and (27) to describe the p-wave mesons.

Another type of phenomenological model for the channel coupling, namely, the model of Eichten *et al.* [10] was used in Ref. [27] to calculate the shift of the $D_s(0^+)$ level and it was shown that the desired mass shift is obtained for a reasonable choice of parameters.

Our results obtained with the parameter-free chiral Lagrangian containing full x dependence qualitatively agree with those in Refs. [19,28] and exactly reproduce the experimentally found mass $D_s^*(2318)$.

We now turn to the state $D_s^*(2632)$, found in Ref. [11], but not yet confirmed by other groups [29]. The theoretical prediction for the 2^3S_1 state varies from 2774 MeV in Ref. [15] to 2737 MeV made using the relativistic Salpeter equation in Ref. [30] and 2716 in Ref. [8], and in principle are subject to the correction due to the global string breaking effect occurring for states of large size [31] (mean square radius of the 2^3S_1 state in Ref. [30] is around 1 fm). The expected correction is around -20 MeV, which brings the theoretical mass of the $D_s(2^3S_1)$ state to 2700–2720 MeV.

Our calculation for the initial mass $D_s^*(1^-), m = 2710$ MeV, using Eq. (33), yields the shift $\Delta E = -76$ MeV, with the total width $\Gamma = 131$ MeV, $\Gamma_{KD} = 111$ MeV, $\Gamma_{D_s\eta} = 20$ MeV. Hence, also in this case the total width and width ratio contradicts experimental data, implying that $D_s^*(2632)$ cannot be explained as the shifted 1^- level. In a recent paper [20] the $D_s^*(2632)$ has been studied using a many-coupled-channel model including the Okubo-Zweig-Iizuka rule. Because of the presence of many closed channels they succeed to obtain a considerably smaller width and the partial decay width is found to be in agreement with the SELEX experiment. It should be noted that in the considered QCD string model the predicted mass shift would become larger when more channels are included and the width does not decrease significantly as the mass pole is gets closer to 2623 MeV.

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