Octet, decuplet, and antidecuplet magnetic moments in the chiral quark soliton model reexamined

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We reanalyze the magnetic moments of the baryon octet, decuplet, and antidecuplet within the framework of the chiral quark-soliton model, with SU(3) symmetry-breaking taken into account. We consider the contributions of the mixing of higher representations to the magnetic moment operator arising from the SU(3) symmetry-breaking. Dynamical parameters of the model are fixed by experimental data for the magnetic moments of the baryon octet and from the masses of the octet, decuplet and of Θ^+. The magnetic moment of Θ^+ depends rather strongly on the pion-nucleon sigma term and reads -1.19 nm to -0.33 nm for $\Sigma_{\pi N} = 45$ and 75 MeV, respectively. The recently reported mass of $\Xi_{\overline{10}}^{--}(1862)$ is compatible with $\Sigma_{\pi N} = 73$ MeV. As a by-product the strange magnetic moment of the nucleon is obtained with a value of $\mu_N^{(s)} = +0.39$ nm.

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I. INTRODUCTION

A recent discovery of the exotic pentaquark Θ^+ state ($uudd\bar{s}$) by the LEPS collaboration [1] and its further confirmation by a number of other experiments [2], together with an observation of exotic $\Xi_{\overline{10}}$ states by the NA49 experiment at CERN [3], though it is still under debate, opened somewhat unexpectedly a new chapter in baryon spectroscopy. Experimental searches for these new states were motivated by the theoretical prediction of the chiral quark-soliton model [4], where masses and decay widths of exotic antidecuplet baryons were predicted. In fact, exotic SU(3) representations containing exotic baryonic states are naturally accommodated within the chiral soliton models [5–7], where the quantization condition emerging from the Wess-Zumino-Witten term selects SU(3) representations of triality zero [8].

The findings of the pentaquark baryon Θ^+ and possibly of $\Xi_{\overline{10}}$ have triggered intensive theoretical investigations which are summarized in Refs. [9,10]. In particular the production mechanism of the Θ^+ has been discussed in Refs. [11–14]. It is of great interest to understand the photoproduction of the Θ^+ theoretically, since the LEPS and CLAS collaborations used photons as a probe to measure the Θ^+. In order to describe the mechanism of the pentaquark photoproduction, we have to know the magnetic moment of the Θ^+ and its strong coupling constants. However, information on the static properties such as antidecuplet magnetic moments and their strong coupling constants is absent to date, so we need to estimate them theoretically. Recently, two of the present authors calculated the magnetic moments of the exotic pentaquarks in a *model-independent approach*, within the framework of the chiral quark-soliton model [15] in the chiral limit. Since we were not able to fix all the parameters for the magnetic moments in the chiral limit, we had to rely on the explicit model calculations [16,17].

The *model-independent approach* was introduced for the first time by Adkins and Nappi [18] in the context of the Skyrme model. In this approach, dynamical quantities like moments of inertia or coefficients in the magnetic moment operator that are in principle calculable within the model are not numerically evaluated but treated as free parameters. Adjusting them to the experimentally known magnetic moments, we allow for maximal phenomenological input and minimal model dependence.

The discovery of Θ^+ and possibly of $\Xi_{\overline{10}}$ constrained the parameters of the chiral quark-soliton model that were previously undetermined. This new phenomenological input reduces the residual freedom in the predictions of static baryon properties evaluated in the *model-independent approach.*

In this paper we revise previous results both for nonexotic [16,17] and exotic baryons [15]. We show that magnetic moments of nonexotic baryons (i.e. decuplet, since octet magnetic moments are used as an input) are little changed. On the contrary, antidecuplet magnetic moments are different from our previous analysis done in Ref. [15]. In particular, our present study shows that the magnetic moment of Θ^+ is negative and rather sensitive to the residual freedom which we parametrize in terms of the pion-nucleon sigma term: $\Sigma_{\pi N}$.

The paper is organized as follows. In Sec. II we recapitulate mass formulae within the chiral quark-soliton

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model and discuss in some detail the constraints on the model parameters that come from the measurement of the mass of Θ^+ and, if one wants, of $\Xi_{\overline{10}}$. In Sec. III we give explicit formulae for the antidecuplet magnetic moments and display some useful intermediate results in the *model-independent approach*. Numerical results and comparison with other models are presented in Sec. IV. Finally we summarize in Sec. V.

II. CONSTRAINTS FROM THE EXOTIC STATES

The collective Hamiltonian describing baryons in the SU(3) chiral quark-soliton model takes the following form [19]:

$$\hat{H} = \mathcal{M}_{\rm sol} + \frac{J(J+1)}{2I_1} + \frac{C_2[{\rm SU(3)}] - J(J+1) - \frac{N_c^2}{12}}{2I_2} + \hat{H}', \tag{1}$$

where $\mathcal{M}_{\rm sol}$ and $C_2[{\rm SU(3)}]$ denote the classical soliton mass and the SU(3) Casimir operator, respectively. I_1 and I_2 are moments of inertia of the soliton. The symmetry-breaking term in Eq. (1) is expressed by

$$\hat{H}' = \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} D_{8i}^{(8)} \hat{J}_i, \tag{2}$$

where parameters α, β, and γ are of order $\mathcal{O}(m_s)$. Here $D_{ab}^{(\mathcal{R})}(R)$ denotes SU(3) Wigner rotation matrices and $\hat{J}$ is a collective spin operator. The Hamiltonian given in Eq. (2) acts on the space of baryon wave functions $|\mathcal{R}_J, B, J_3\rangle$:

$$\begin{aligned}|\mathcal{R}_J, B, J_3\rangle &= \psi_{(\mathcal{R};Y,T,T_3)(\mathcal{R}^*;-Y',J,J_3)} \\ &= \sqrt{\dim(\mathcal{R})}(-1)^{J_3-Y'/2} D^{(R)*}_{Y,T,T_3;Y',J,-J_3}(R).\end{aligned} \tag{3}$$

Here, $\mathcal{R}$ stands for the allowed irreducible representations of the SU(3) flavor group, i.e., $\mathcal{R} = 8, 10, \overline{10}, \cdots$ and Y, T, T_3 are the corresponding hypercharge, isospin, and its third component, respectively. Right hypercharge $Y' = 1$ is constrained to be unity for the physical spin states for which J and J_3 are spin and its third component. The *model-independent approach* consists now in using Eqs. (1) and (2) (and/or possibly analogous equations for other observables) and determining model parameters such as $I_1, I_2, \alpha, \beta, \gamma$ from experimental data.

Taking into account recent experimental observations of the mass of the Θ^+, the parameters entering Eq. (2) can be conveniently parametrized in terms of the pion-nucleon $\Sigma_{\pi N}$ term (assuming $m_s/(m_u + m_d) = 12.9$) as [20]:

$$\begin{aligned}\alpha = 336.4 - 12.9\Sigma_{\pi N}, \qquad \beta = -336.4 + 4.3\Sigma_{\pi N}, \\ \gamma = -475.94 + 8.6\Sigma_{\pi N}\end{aligned} \tag{4}$$

(in units of MeV). Moreover, the inertia parameters which describe the representation splittings

$$\Delta M_{10-8} = \frac{3}{2I_1}, \qquad \Delta M_{\overline{10}-8} = \frac{3}{2I_2} \tag{5}$$

take the following values (in MeV):

$$\frac{1}{I_1} = 152.4, \qquad \frac{1}{I_2} = 608.7 - 2.9\Sigma_{\pi N}. \tag{6}$$

Eqs. (4) and (6) follow from the fit to the masses of the octet and decuplet baryons as well as that of the Θ^+. If, furthermore, one imposes an additional constraint that $M_{\Xi_{\overline{10}}} = 1860$ MeV, then $\Sigma_{\pi N} = 73$ MeV [20] (see also [21]) in agreement with recent experimental estimates [22].

Since the symmetry-breaking term (2) of the collective Hamiltonian mixes different SU(3) representations, the collective wave functions are given as the following linear combinations [17]:

$$\begin{aligned}|B_8\rangle &= |8_{1/2}, B\rangle + c_{\overline{10}}^B|\overline{10}_{1/2}, B\rangle + c_{27}^B|27_{1/2}, B\rangle, \\ |B_{10}\rangle &= |10_{3/2}, B\rangle + a_{27}^B|27_{3/2}, B\rangle + a_{35}^B|35_{3/2}, B\rangle, \\ |B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d_8^B|8_{1/2}, B\rangle + d_{27}^B|27_{1/2}, B\rangle \\ &\quad + d_{\overline{35}}^B|\overline{35}_{1/2}, B\rangle,\end{aligned} \tag{7}$$

where $|B_{\mathcal{R}}\rangle$ denotes the state which reduces to the SU(3) representation $\mathcal{R}$ in the formal limit $m_s \to 0$ and the spin index J_3 has been suppressed. The m_s-dependent (through the linear m_s dependence of α, β and γ) coefficients in Eq. (7) read:

$$\begin{aligned}
c_{\overline{10}}^B &= c_{\overline{10}}\begin{bmatrix}\sqrt{5}\\0\\\sqrt{5}\\0\end{bmatrix}, \qquad c_{27}^B = c_{27}\begin{bmatrix}\sqrt{6}\\3\\2\\\sqrt{6}\end{bmatrix}, \\
a_{27}^B &= a_{27}\begin{bmatrix}\sqrt{15/2}\\2\\\sqrt{3/2}\\0\end{bmatrix}, \qquad a_{35}^B = a_{35}\begin{bmatrix}5/\sqrt{14}\\2\sqrt{5/7}\\3\sqrt{5/14}\\2\sqrt{5/7}\end{bmatrix}, \\
d_8^B &= d_8\begin{bmatrix}0\\\sqrt{5}\\\sqrt{5}\\0\end{bmatrix}, \qquad d_{27}^B = d_{27}\begin{bmatrix}0\\\sqrt{3/10}\\2/\sqrt{5}\\\sqrt{3/2}\end{bmatrix}, \\
d_{\overline{35}}^B &= d_{\overline{35}}\begin{bmatrix}1/\sqrt{7}\\3/(2\sqrt{14})\\1/\sqrt{7}\\\sqrt{5/56}\end{bmatrix},
\end{aligned} \tag{8}$$

respectively in the basis $[N, \Lambda, \Sigma, \Xi]$, $[\Delta, \Sigma^*, \Xi^*, \Omega]$, $[\Theta^+, N_{\overline{10}}, \Sigma_{\overline{10}}, \Xi_{\overline{35}}]$, and analogous states in $\mathcal{R} =$

27, 35, $\overline{35}$, and

$$c_{\overline{10}} = -\frac{I_2}{15}\left(\alpha + \frac{1}{2}\gamma\right), \qquad c_{27} = -\frac{I_2}{25}\left(\alpha - \frac{1}{6}\gamma\right),$$
$$a_{27} = -\frac{I_2}{8}\left(\alpha + \frac{5}{6}\gamma\right), \qquad a_{35} = -\frac{I_2}{24}\left(\alpha - \frac{1}{2}\gamma\right),$$
$$d_8 = \frac{I_2}{15}\left(\alpha + \frac{1}{2}\gamma\right), \qquad d_{27} = -\frac{I_2}{8}\left(\alpha - \frac{7}{6}\gamma\right), \tag{9}$$
$$d_{\overline{35}} = -\frac{I_2}{4}\left(\alpha + \frac{1}{6}\gamma\right).$$

III. MAGNETIC MOMENTS IN THE CHIRAL QUARK-SOLITON MODEL

The collective operator for the magnetic moments can be parametrized by six constants. By definition in the *model-independent approach* they are treated as free [16,17]:

$$\hat{\mu}^{(0)} = w_1 D^{(8)}_{Q3} + w_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{Q8} \hat{J}_3,$$
$$\hat{\mu}^{(1)} = \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{Qp} D^{(8)}_{8q} + w_5 (D^{(8)}_{Q3} D^{(8)}_{88} + D^{(8)}_{Q8} D^{(8)}_{83}) + w_6 (D^{(8)}_{Q3} D^{(8)}_{88} - D^{(8)}_{Q8} D^{(8)}_{83}). \tag{10}$$

The parameters $w_{1,2,3}$ are of order $\mathcal{O}(m_s^0)$, while $w_{4,5,6}$ are of order $\mathcal{O}(m_s)$, m_s being regarded as a small parameter.

The full expression for the magnetic moments can be decomposed as follows:

$$\mu_B = \mu_B^{(0)} + \mu_B^{(\mathrm{op})} + \mu_B^{(\mathrm{wf})}, \tag{11}$$

where the $\mu_B^{(0)}$ is given by the matrix element of the $\hat{\mu}^{(0)}$ between the purely symmetric states $|\mathcal{R}_J, B, J_3\rangle$, and the $\mu_B^{(\mathrm{op})}$ is given as the matrix element of the $\hat{\mu}^{(1)}$ between the symmetry states as well. The wave function correction $\mu_B^{(\mathrm{wf})}$ is given as a sum of the interference matrix elements of the $\mu_B^{(0)}$ between purely symmetric states and admixtures displayed in Eq. (7). These matrix elements were calculated for octet and decuplet baryons in Ref. [17]. It has been shown that the $\mu_B^{(0)}$ for these two representations depend only upon the following combinations:

$$v = \frac{1}{60}\left(w_1 - \frac{w_2}{2}\right) \quad \text{and} \quad w = \frac{w_3}{120}. \tag{12}$$

Therefore, in the leading order in m_s, it is impossible to extract information on w_1 and w_2 separately. In contrast, the wave function corrections $\mu_B^{(\mathrm{wf})}$ depend separately on all three zeroth-order parameters $w_{1,2,3}$. However, prior to the discovery of the Θ^+, both I_2 and one of the parameters entering Eq. (2), which we have chosen to be γ, were unconstrained, since they did not enter the formulae for the nonexotic mass splittings. Therefore, the extraction of w_2 from the $\mu_B^{(\mathrm{wf})}$ was not possible as well.

In order to make numerical estimates, we have assumed in Refs. [16,17] that $\gamma = 0$. This assumption was based on the numerical results of the model calculations as well as on the model value of the $\Sigma_{\pi N}$ being of order of 54 MeV [23]. Moreover, in the nonrelativistic limit of the chiral quark-soliton model $\gamma \equiv 0$. This choice reduced the number of free parameters to seven (six constants w_i and I_2). However, due to an accidental algebraic property, the explicit formulae for the octet magnetic moments depend effectively only on six parameters. On the contrary, the magnetic moments of the decuplet depend on all seven parameters and therefore one could determine them only up to one unknown constant which we called p in Refs. [16,17]. Unfortunately, the dependence on p of the two measured magnetic moments of Ω^- and Δ^{++} is too weak to determine p.

The situation in the $\overline{10}$ multiplet is very different. In this case, the $\mu_B^{(0)}$ depend on a different combination of parameters w_1 and w_2, hence the prediction for μ_{Θ^+} depends on one unknown constant already in the SU(3)-symmetry limit:

$$\mu_B^{\overline{10}(0)} = \left[\frac{5}{2}(-v + w) - \frac{1}{8}w_2\right] Q_B. \tag{13}$$

Since, as explained above, prior to the measurement of the Θ^+ mass, the determination of w_2 from the nonexotic data was not possible, we have assumed in Ref. [15], following explicit model calculations [17], that the parameter w_2 took the value $w_2 \simeq 5$. This assumption led to a small but positive value of the magnetic moment of Θ^+. Surprisingly, we have observed that in the nonrelativistic limit of the chiral quark-soliton model [24] all three parameters w_i can be essentially expressed in terms of one unknown constant K. This feature leads to the remarkable result that the magnetic moment of the positively charged Θ^+ is negative: $\mu_{\Theta^+}^{(0)} < 0$.

The measurement of the Θ^+ mass constrains the parameter space of the model in Eqs. (4) and (6). Recent phenomenological analyses indicate that our previous assumption on γ, i.e., $\gamma = 0$, has to be most likely abandoned. Therefore, our previous results for the magnetic moments of 8, 10, and $\overline{10}$ have to be reanalyzed. In the present work we show that a *model-independent* analysis with this new phenomenological input yields w_2 much larger than initially assumed, which causes $\mu_{\Theta^+}^{(0)}$ for realistic values of $\Sigma_{\pi N}$ to be negative and rather small. We also show that our previous results for the decuplet magnetic moments still hold within the accuracy of the model.

The octet and decuplet magnetic moments were calculated in Refs. [16,17]. For the antidecuplet $\mu_B^{\overline{10}(0)}$ are given in Eq. (13). In order to calculate the $\mu_B^{(\mathrm{op})}$, the following relations, obtained using SU(3) Clebsch-Gordan coefficients [25], hold:

$$\begin{aligned} D^{(8)}_{33}D^{(8)}_{88} &= \frac{1}{5}D^{(8)}_{\Sigma^0\Sigma^0} + \frac{1}{4}D^{(10)}_{\Sigma^0\Sigma^0} + \frac{1}{4}D^{(\overline{10})}_{\Sigma^0\Sigma^0} + \frac{3}{10}D^{(27)}_{\Sigma^0\Sigma^0}, \\ D^{(8)}_{38}D^{(8)}_{83} &= \frac{1}{5}D^{(8)}_{\Sigma^0\Sigma^0} - \frac{1}{4}D^{(10)}_{\Sigma^0\Sigma^0} - \frac{1}{4}D^{(\overline{10})}_{\Sigma^0\Sigma^0} + \frac{3}{10}D^{(27)}_{\Sigma^0\Sigma^0}, \\ D^{(8)}_{83}D^{(8)}_{88} &= -\frac{1}{5}D^{(8)}_{\Lambda\Sigma^0} + \frac{9}{20}D^{(27)}_{\Lambda\Sigma^0} \end{aligned} \tag{14}$$

and

$$\frac{1}{\sqrt{3}}d_{ab3}D^{(8)}_{Qa}D^{(8)}_{8b} = \frac{1}{10}\left[D^{(8)}_{\Sigma^0\Sigma^0} - D^{(27)}_{\Sigma^0\Sigma^0} - \frac{1}{\sqrt{3}}D^{(8)}_{\Lambda\Sigma^0} - \frac{3}{2\sqrt{3}}D^{(27)}_{\Lambda\Sigma^0}\right]. \tag{15}$$

Furthermore, in order to calculate the $\mu_B^{(\mathrm{wf})}$, several off-diagonal matrix elements of the $\hat{\mu}^{(0)}$ are required. These have been calculated in Ref. [20] in the context of the hadronic decay widths of the baryon antidecuplet.

Denoting the set of the model parameters by

$$\vec{w} = (w_1, \ldots, w_6) \tag{16}$$

the model formulae for the set of the magnetic moments in representation $\mathcal{R}$ (of dimension R)

$$\vec{\mu}^{\,\mathcal{R}} = (\mu_{B_1}, \ldots, \mu_{B_R}) \tag{17}$$

can be conveniently cast into the form of the matrix equations:

$$\vec{\mu}^{\,\mathcal{R}} = A^{\mathcal{R}}[\Sigma_{\pi N}] \cdot \vec{w}, \tag{18}$$

where rectangular matrices A^8 and A^{10} can be found in Refs. [16,17]. Note their dependence on the pion-nucleon $\Sigma_{\pi N}$ term. As for the antidecuplet, we find $A^{\overline{10}}$ in the following form:

$$\begin{bmatrix}
-\frac{1}{24}+\frac{d_{\overline{35}}}{84} & -\frac{5}{48}-\frac{d_{\overline{35}}}{168} & \frac{1}{48}+\frac{d_{\overline{35}}}{56} & \frac{1}{56} & -\frac{1}{84} & 0 \\
-\frac{1}{24}-\frac{7d_{27}}{72}+\frac{d_{\overline{35}}}{112} & -\frac{5}{48}+\frac{11d_{27}}{144}-\frac{d_{\overline{35}}}{224} & \frac{1}{48}+\frac{d_{27}}{48}+\frac{3d_{\overline{35}}}{224} & \frac{1}{189} & -\frac{1}{63} & 0 \\
-\frac{c_{\overline{10}}}{3}+\frac{7d_{27}}{180}+\frac{d_{\overline{35}}}{56} & -\frac{c_{\overline{10}}}{3}-\frac{11d_{27}}{360}-\frac{d_{\overline{35}}}{112} & -\frac{c_{\overline{10}}}{6}-\frac{d_{27}}{120}+\frac{3d_{\overline{35}}}{112} & -\frac{5}{1512} & \frac{13}{252} & 0 \\
-\frac{1}{24}-\frac{7d_{27}}{36}+\frac{d_{\overline{35}}}{168} & -\frac{5}{48}+\frac{11d_{27}}{72}-\frac{d_{\overline{35}}}{336} & \frac{1}{48}+\frac{d_{27}}{24}+\frac{d_{\overline{35}}}{112} & -\frac{11}{1512} & -\frac{5}{252} & 0 \\
-\frac{c_{\overline{10}}}{6}-\frac{7d_{27}}{90}+\frac{d_{\overline{35}}}{84} & -\frac{c_{\overline{10}}}{6}+\frac{11d_{27}}{180}-\frac{d_{\overline{35}}}{168} & -\frac{c_{\overline{10}}}{12}+\frac{d_{27}}{60}+\frac{d_{\overline{35}}}{56} & -\frac{1}{189} & \frac{1}{63} & 0 \\
\frac{1}{24}-\frac{c_{\overline{10}}}{3}+\frac{7d_{27}}{180}+\frac{d_{\overline{35}}}{56} & \frac{5}{48}-\frac{c_{\overline{10}}}{3}-\frac{11d_{27}}{360}-\frac{d_{\overline{35}}}{112} & -\frac{1}{48}-\frac{c_{\overline{10}}}{6}-\frac{d_{27}}{120}+\frac{3d_{\overline{35}}}{112} & -\frac{5}{1512} & \frac{13}{252} & 0 \\
-\frac{1}{24}-\frac{7d_{27}}{24}+\frac{d_{\overline{35}}}{336} & -\frac{5}{48}+\frac{11d_{27}}{48}-\frac{d_{\overline{35}}}{672} & \frac{1}{48}+\frac{d_{27}}{16}+\frac{d_{\overline{35}}}{224} & -\frac{5}{252} & -\frac{1}{42} & 0 \\
-\frac{7d_{27}}{36}+\frac{d_{\overline{35}}}{168} & \frac{11d_{27}}{72}-\frac{d_{\overline{35}}}{336} & \frac{d_{27}}{24}+\frac{d_{\overline{35}}}{112} & -\frac{11}{1512} & -\frac{5}{252} & 0 \\
\frac{1}{24}-\frac{7d_{27}}{72}+\frac{d_{\overline{35}}}{112} & \frac{5}{48}+\frac{11d_{27}}{144}-\frac{d_{\overline{35}}}{224} & -\frac{1}{48}+\frac{d_{27}}{48}+\frac{3d_{\overline{35}}}{224} & \frac{1}{189} & -\frac{1}{63} & 0 \\
\frac{1}{12}+\frac{d_{\overline{35}}}{84} & \frac{5}{24}-\frac{d_{\overline{35}}}{168} & -\frac{1}{24}+\frac{d_{\overline{35}}}{56} & \frac{1}{56} & -\frac{1}{84} & 0
\end{bmatrix} \tag{19}$$

in the basis

$$\vec{\mu}^{\,\overline{10}} = (\mu_{\Theta^+}, \mu_{p^*}, \mu_{n^*}, \mu_{\Sigma^+}, \mu_{\Sigma^0}, \mu_{\Sigma^-}, \mu_{\Xi^+}, \mu_{\Xi^0}, \mu_{\Xi^-}, \mu_{\Xi^{--}}). \tag{20}$$

IV. RESULTS AND DISCUSSION

In order to find the set of parameters $w_i[\Sigma_{\pi N}]$, we minimize the mean square deviation for the octet magnetic moments:

$$\Delta\mu^8 = \frac{1}{7}\sqrt{\sum_B (\mu^8_{B,th}[\Sigma_{\pi N}] - \mu^8_{B,\mathrm{exp}})^2}, \tag{21}$$

where the sum extends over all octet magnetic moments, but the Σ^0. The value $\Delta\mu^8 \simeq 0.01$ is in practice not sensitive to the $\Sigma_{\pi N}$ in the physically interesting range 45–75 MeV. Therefore, the values of the $\mu^8_{B,th}[\Sigma_{\pi N}]$ are also not sensitive to $\Sigma_{\pi N}$. Table I lists the results of the magnetic moments of the baryon octet.

Similarly, the value of the nucleon strange magnetic moment is not sensitive to $\Sigma_{\pi N}$ and reads $\mu_N^{(s)} = 0.39$ nm in fair agreement with our previous analysis of Ref. [17]. Parameters w_i, however, do depend on $\Sigma_{\pi N}$. This is shown in Table II. Note that parameters $w_{2,3}$ are formally $\mathcal{O}(1/N_c)$ with respect to w_1. For smaller $\Sigma_{\pi N}$, this N_c counting is not borne by explicit fits. Interestingly, the chiral-limit parameters v and w defined in Eq. (12) do not depend on $\Sigma_{\pi N}$ and read:

TABLE I. Magnetic moments of the baryon octet.

	p	n	Λ^0	Σ^+	Σ^-	Ξ^0	Ξ^-
Th.	2.814	−1.901	−0.592	2.419	−1.172	−1.291	−0.656
Exp.	2.793	−1.913	−0.613	2.458	−1.16	−1.25	−0.651

TABLE II. Dependence of the parameters w_i on $\Sigma_{\pi N}$.

$\Sigma_{\pi N}$ [MeV]	w_1	w_2	w_3	w_4	w_5	w_6
45	−8.564	14.983	7.574	−10.024	−3.742	−2.443
60	−10.174	11.764	7.574	−9.359	−3.742	−2.443
75	−11.783	8.545	7.574	−6.440	−3.742	−2.443

$$v = -0.268, \qquad w = 0.063. \tag{22}$$

The values of v and w in Eq. (22) almost exactly coincide with the parameters extracted from the linear combinations

$$v = (2\mu_n - \mu_p + 3\mu_{\Xi^0} + \mu_{\Xi^-} - 2\mu_{\Sigma^-} - 3\mu_{\Sigma^+})/60 = -0.268,$$
$$w = (3\mu_p + 4\mu_n + \mu_{\Xi^0} - 3\mu_{\Xi^-} - 4\mu_{\Sigma^-} - \mu_{\Sigma^+})/60 = 0.060. \tag{23}$$

which are free of linear m_s corrections [17]. This is a remarkable feature of the present fit, since when the m_s corrections are included, the m_s-independent parameters need not be refitted. This property will be used in the following when we restore the linear dependence of the $\mu_B^{\overline{10}}$ on m_s.

The magnetic moments of the baryon decuplet and antidecuplet depend on the $\Sigma_{\pi N}$. However, the dependence of the decuplet is very weak. The results are summarized in Table III, where we also display the theoretical predictions from Ref. [16] for $p = 0.25$. Let us note that the m_s corrections are not large for the decuplet and the approximate proportionality of the μ_B^{10} to the baryon charge Q_B still holds.

Finally, for antidecuplet we have a strong dependence on $\Sigma_{\pi N}$, yielding the numbers of Table IV. The results listed in Table IV are further depicted in Fig. 1.

In the chiral limit, the antidecuplet magnetic moments are proportional to the corresponding charges, see Eq. (13), but with opposite sign, and they read numerically

$$\mu_B^{\overline{10}(0)} = -(1.05 \sim 0.24)Q_B \tag{24}$$

for $\Sigma_{\pi N} = 45$ and 75 MeV, respectively. The inclusion of the m_s corrections introduces splittings and proportionality to the charge is violated. The magnitude of the splittings increases with $\Sigma_{\pi N}$. This is depicted in Fig. 2, where linear dependence on m_s is reproduced from the knowledge of two points: $\mu_B^{\overline{10}}$ in the chiral limit for $m_s = 0$ (24) and for physical $m_s = 1$ in arbitrary units as given in Table IV. We see that for small $\Sigma_{\pi N}$ corrections due to the nonzero m_s are moderate and the perturbative approach is reliable. On the contrary, for large $\Sigma_{\pi N}$, corrections are large. This is due to the wave function corrections, since the dependence of the operator part on the $\Sigma_{\pi N}$ given in terms of the coefficients $w_{4,5,6}$ is small as in Table II. The wave function corrections cancel for the nonexotic baryons and add constructively for the baryon antidecuplet. In particular, for $\Sigma_{\pi N} = 75$ MeV we have large admixture coefficient of 27-plet: d_{27}^B tends to dominate otherwise small magnetic moments of the antidecuplet. At this point, the reliability of the perturbative expansion for the antidecuplet magnetic moments may be questioned. On the other hand, as remarked above, the N_c counting for the w_i coefficients works much better for large $\Sigma_{\pi N}$. One notices for reasonable values of $\Sigma_{\pi N}$

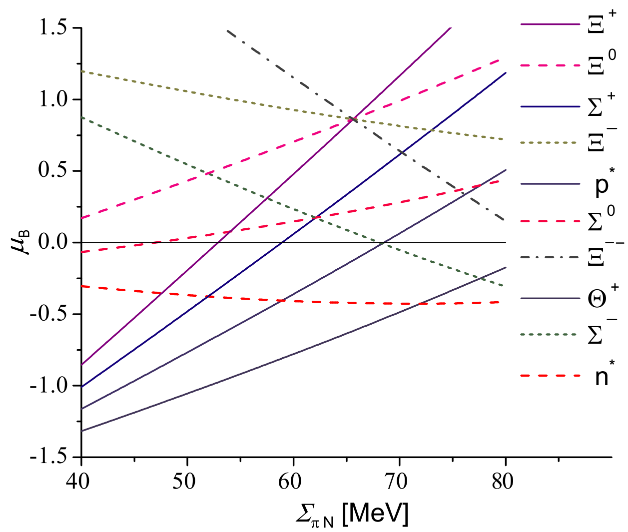

FIG. 1 (color online). Magnetic moments of antidecuplet as functions of $\Sigma_{\pi N}$.

TABLE III. Magnetic moments of the baryon decuplet.

$\Sigma_{\pi N}$ [MeV]	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^{-}
45	5.40	2.65	−0.09	−2.83	2.82	0.13	−2.57	0.34	−2.31	−2.05
60	5.39	2.66	−0.08	−2.82	2.82	0.13	−2.56	0.34	−2.30	−2.05
75	5.39	2.66	−0.07	−2.80	2.81	0.13	−2.55	0.33	−2.30	−2.05
Ref.[16]	5.34	2.67	−0.01	−2.68	3.10	0.32	−2.47	0.64	−2.25	−2.04

TABLE IV. Magnetic moments of the baryon antidecuplet.

$\Sigma_{\pi N}$ [MeV]	Θ^+	p^*	n^*	$\Sigma_{\overline{10}}^+$	$\Sigma_{\overline{10}}^0$	$\Sigma_{\overline{10}}^-$	$\Xi_{\overline{10}}^+$	$\Xi_{\overline{10}}^0$	$\Xi_{\overline{10}}^-$	$\Xi_{\overline{10}}^{--}$
45	−1.19	−0.97	−0.34	−0.75	−0.02	0.71	−0.53	0.30	1.13	1.95
60	−0.78	−0.36	−0.41	0.06	0.15	0.23	0.48	0.70	0.93	1.15
75	−0.33	0.28	−0.43	0.90	0.36	−0.19	1.51	1.14	0.77	0.39

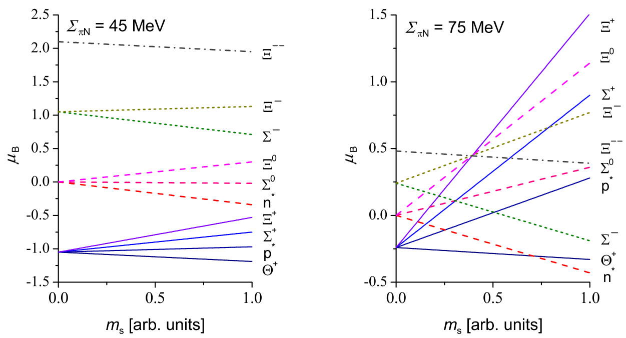

FIG. 2 (color online). Dependence of magnetic moments on m_s for $\Sigma_{\pi N} = 45$ and 75 MeV.

some interesting facts, which were partially reported already in Ref. [15]: The magnetic moments of the antidecuplet baryons are rather small in absolute value. For Θ^+ and p^* one obtains negative values although the charges are positive. For $\Xi_{\overline{10}}^-$ and $\Xi_{\overline{10}}^{--}$ one obtains positive values although the signs of the charges are negative.

V. CONCLUSION AND SUMMARY

The magnetic moments of the positive-parity pentaquarks have been studied by a number of authors in different models [26–30]. The results are displayed in Table V. We see that in all quark models the magnetic moment of the Θ^+ is rather small and positive. On the contrary, our present analysis shows that $\mu_{\Theta^+} < 0$, although the magnitude depends strongly on the model parameters. The measurement of μ_{Θ^+} could therefore discriminate between different models. This also may add to reduce the ambiguities in the pion-nucleon sigma term $\Sigma_{\pi N}$.

The measurement of the antidecuplet magnetic moments by ordinary precession techniques is not possible. However, it is crucial to know the magnetic moment of the Θ^+ in order to study its production via photoreactions. One can use the measured cross section to determine the magnetic moment of the Θ^+. The cross sections for the Θ^+ production from nucleons induced by photons [12] have been already described theoretically. A similar approach was used to determine the magnetic moments of the Δ^{++} [35,36] and Δ^+ [37], which are much broader than the Θ^+. The measurements of the Δ^{++} magnetic moment comes from the reaction such as $\pi^+ p \to \pi^+ \gamma' p$ [35,35], while that of the Δ^+ was measured in $\gamma p \to \pi^0 \gamma' p$ [37]. This shows that the measurement of the magnetic moments of resonances is in principle possible, despite the fact that it is difficult and is hampered by large uncertainties which mainly come from the systematic error of cross section calculations.

In the present work, we determined the magnetic moments of the baryon antidecuplet in the *model-independent* analysis within the chiral quark-soliton model, i.e., using the rigid-rotor quantization with the linear m_s corrections included. Starting from the collective operators with dynamical parameters fixed by experimental data, we obtained the magnetic moments of the baryon antidecuplet (19). The expression for the magnetic moments of the baryon antidecuplet is different from those of the baryon decuplet. We found that the magnetic moment μ_{Θ^+} is negative and rather strongly dependent on the value of the $\Sigma_{\pi N}$. Indeed, the μ_{Θ^+} ranges from -1.19 nm to -0.33 nm for $\Sigma_{\pi N} = 45$ and 75 MeV, respectively. This is in contrast with our previous estimate of the μ_{Θ^+} in the chiral limit [15], where we have used $w_2 \sim 5$ motivated by the explicit model calculations. Indeed, Eq. (13) yields in this case $\mu_B^{\overline{10}} \sim 0.20 Q_B$.

One should note that the magnetic moments of the decuplet do not differ from our previous estimates [16].

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TABLE V. Magnetic moments of Θ^+, Ξ^{--} and Ξ^+ in nuclear magnetons from different papers in different models. (JW) stands for Jaffe and Wilczek [31], (SZ) for Shuryak and Zahed [32], (KL) for Karliner and Lipkin [33] and (S) for Strottman [34].

Ref. first author	model	remarks	Θ^+	Ξ^{--}	Ξ^+
[26] Q. Zhao	diquarks (JW)		0.08	···	···
[27] P.Z. Huang	sum rules	abs. value	0.12 ± 0.06	···	···
[28] Y.-R. Liu	diquarks (JW)		0.08	0.12	−0.06
	clusters (SZ)		0.23	−0.17	0.33
	triquarks (KL)		0.37	0.43	0.13
	MIT bag (S)		0.37	−0.42	0.45
[29] R. Bijker	QM, harm.osc.		0.38	−0.44	0.50
[30] D.K. Hong	chiral eff. th.	$m_s = 400$ MeV	0.71	···	···
	with diquarks	$m_s = 450$ MeV	0.56	···	···
present work	chiral soliton	$\Sigma_{\pi N} = 45$ MeV	−1.19	1.95	−0.53
	model	$\Sigma_{\pi N} = 75$ MeV	−0.33	0.39	1.51

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