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Chiral gauge models for light sterile neutrinos

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We construct a family of simple gauge models in which three sterile neutrinos become naturally light by virtue of a generalized seesaw mechanism involving a chiral gauge symmetry. Examples where the chiral gauge group is SU(5)′, SU(7)′, SU(3)′ and/or their descendants are presented. A unified model based on SO(10) × SO(10)′ which embeds many of these models is constructed wherein three light sterile neutrinos are just as natural as the three ordinary neutrinos. These gauge models have relevance to current neutrino oscillation data, including the LSND anomaly.

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I. INTRODUCTION

Over the past few years, solar [1], atmospheric [2], reactor [3], and accelerator [4] neutrino experiments have remarkably improved our knowledge of the neutrino mass and mixing parameters. Specifically, solar and atmospheric neutrino data are now excellently understood within a three-neutrino oscillation scheme, where the neutrino mass squared splittings are respectively $\Delta m^2_\odot \simeq 7.5\times 10^{-5}$ eV^2 and $\Delta m^2_{\rm atm} \simeq 2.0 \times 10^{-3}$ eV^2 [5]. However, the evidence for $\overline{\nu}_\mu - \overline{\nu}_e$ oscillations found by the liquid scintillator neutrino detector (LSND) experiment at Los Alamos [6], which will soon be tested by the ongoing MiniBooNE experiment at Fermilab [7], would demand a third mass squared difference $\Delta m^2_{\rm LSND} \gtrsim 10^{-1}$ eV^2, which cannot be accommodated in a three-neutrino oscillation scenario. The LSND anomaly indicates instead the presence of $n \geq 1$ additional neutrinos with masses of order $\sim$1 eV, which give rise to a $(3 + n)$ neutrino oscillation scheme providing additional mass squared splittings. This new species of neutrinos cannot couple to the Z boson, and hence must be sterile with respect to weak interactions. Although a $(3 + 1)$ neutrino mass scheme [8,9] seems now to be almost ruled out [10], a combined fit of the short-baseline experiments Bugey [11], CCFR [12], CDHS [13], CHOOZ [14], KARMEN [15], and LSND shows that the LSND signal can become compatible with the other neutrino oscillation data sets with two (or more) light sterile neutrinos in a $(3 + 2)$ neutrino mass scheme [16].

In models for $(3 + 2)$ neutrino oscillations [17,18], it is important to provide a rationale for the smallness of sterile neutrino masses, since the usual seesaw mechanism [19] does not explain why a sterile neutrino ν' would be light. Actually, if the effective low-energy theory is the standard model (SM), then there is no reason why ν' would not acquire a mass of the order of some high cutoff scale $\Lambda \simeq 10^{13}$–10^{19} GeV. Indeed, there exists a number of suggestions to realize light sterile neutrinos [17,18,20,21]. By copying the criteria which make the seesaw mechanism successful to the sterile sector, we have constructed in Ref. [17] the simplest anomaly-free chiral gauge theory which naturally leads to the $(3 + 2)$ neutrino oscillation scheme. The sterile sector of this model consists of an SU(2)′ gauge group with the ν' transforming as a spin 3/2 multiplet where symmetry breaking is achieved by a single spin 3/2 Higgs field. In this paper, we wish to generalize this idea to a larger class of chiral gauge symmetries in order to arrive at models for $(3 + n)$ neutrino oscillations in which n sterile neutrinos are light.

Our basic approach is here to extend the gauge group of the standard model $G_{\rm SM} = {\rm SU}(3)_c \times {\rm SU}(2)_L \times {\rm U}(1)_Y$ by some "sterile" gauge group G' so that the full gauge symmetry is $G_{\rm SM} \times G'$, with the ν' transforming as a chiral representation[1] of G'. All SM particles have zero G' charges. In all our constructions, we require G' to be a gauge symmetry, rather than a global symmetry, since presumably only gauge symmetries will survive quantum gravity corrections. In this way, the ν' are protected from acquiring large explicit masses of order $M_{\rm Pl} \simeq 10^{19}$ GeV and therefore can serve as candidates for naturally light sterile neutrinos. For $3 + n$ light neutrinos, we will suppose $3 + n$ heavy neutrinos which are total gauge singlets of $G_{\rm SM} \times G'$. This would then lead to a generalization of the seesaw mechanism to the sterile sector. In analogy with the electroweak symmetry breaking in the SM, we assume that G' is spontaneously broken around the TeV scale by a suitable Higgs field S, which has no direct coupling of the type $\nu'\nu'S$. To keep the situation simple, we take S to be a singlet under $G_{\rm SM}$ and require a minimal Higgs sector: A single Higgs S breaks G' to one of its subgroups and provides simultaneously sterile neutrino masses, analogous to the SM Higgs doublet.

Notice that, in the special case when G' becomes a copy of $G_{\rm SM}$, we arrive at the well-studied scenario for "mirror" neutrinos [21]. Here, however, we are interested in a chain of models where $G' = {\rm SU}(N)'$, starting with fermionic fields in the following class:

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[1]By this we mean a fermionic representation for which mass terms are forbidden by gauge invariance.

$$\begin{smallmatrix}\square\\ \square\end{smallmatrix} \oplus (N-4) \times \overline{\square}. \tag{1}$$

This describes an SU$(N)'$ gauge theory with one fermion multiplet in the antisymmetric second rank tensor representation and $N-4$ fermions in the antifundamental representation. For $N \geq 5$, this class of gauge theories is known to be chiral and anomaly-free and has been analyzed extensively in the context of dynamical supersymmetry breaking [22,23]. We will also study, albeit in less detail, the chain of anomaly-free chiral gauge theories arising from fermionic fields transforming under SU$(N)'$ as

$$\square\!\square \oplus (N+4) \times \overline{\square}. \tag{2}$$

Here, one fermion multiplet transforms under the symmetric second rank tensor representation of SU$(N)'$ and $N+4$ fermionic fields transform under the antifundamental representation. For $N \leq 4$, this also gives anomaly-free gauge theories which may, however, be vectorlike. This is indeed the case for $N = 2$, for which the SU$(2)'$ spin-1 representation $\square\!\square$ is vectorial. The simplest such theory is if $N = 3$, which we analyze in some detail.

Starting from the types of gauge theories given in Eq. (1) [and Eq. (2)], we can easily construct for $N \geq 5$ (and $N = 3$) various new anomaly-free chiral models by simply decomposing the chiral multiplets into irreducible representations of the subgroups of SU$(N)'$. Thus, we are able to generate a whole family of chiral gauge models for light sterile neutrinos. We will also see that many of these models can be embedded into a unified theory based on $SO(10) \times SO(10)'$.

The rest of the paper is organized as follows. In Sec. II, we construct gauge models based on the sterile gauge symmetry SU$(5)'$ as well as its various descendants. In Sec. III, we present a fully unified model based on SO$(10) \times$ SO$(10)'$. In Sec. IV, we describe other models arising from SU$(7)'$ as well as SU$(3)'$ with a symmetric tensor representation. Finally, in Sec. V, we give a summary of our main results and list ways of testing these models.

II. $G_{\mathrm{SM}} \times$ SU$(5)'$ MODEL AND ITS DESCENDANTS

In this section, we present the simplest chiral gauge models obtained from the chain shown in Eq. (1). These models are based on an SU$(5)'$ sterile gauge symmetry or one of its descendants. The different patterns of symmetry breaking considered in this section are summarized in Fig. 1.

A. SU$(5)'$ model

The simplest anomaly-free chiral SU$(N)'$ gauge theory which admits fermion representations in the chain of Eq. (1) is SU$(5)'$. As a simple extension of the SM to an anomaly-free chiral gauge theory, we will therefore consider the gauge group $G_{\mathrm{SM}} \times$ SU$(5)'$ with n copies (or

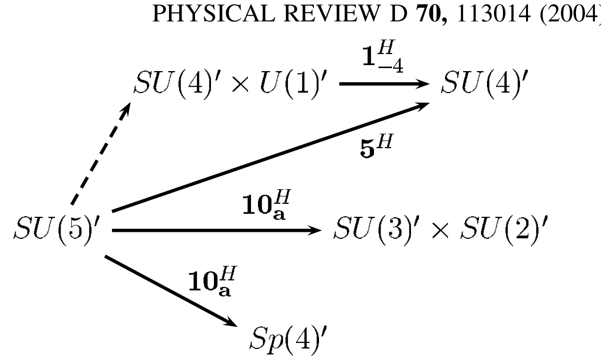

FIG. 1. Symmetry breaking in SU$(5)'$. The solid arrows represent the symmetry breakings by suitable Higgs representations considered in the text. The dashed arrow indicates the embedding of SU$(4)' \times$ U$(1)'$ into SU$(5)'$ as a maximal subgroup.

"generations") of SM singlet fermions which transform under $G_{\mathrm{SM}} \times$ SU$(5)'$ as

$$(\mathbf{1}, \mathbf{10})^i + (\mathbf{1}, \bar{\mathbf{5}})^i, \tag{3}$$

where $i = 1, \ldots, n$ is the generation index of the SM singlet fermions. In analogy with the SM, we will, in what follows, choose for definiteness $n = 3$. Notice in Eq. (3) that the sterile sector then actually becomes a copy of the usual SU(5) model. This model can thus be realized as a descendant of a unified SU$(5) \times$ SU$(5)'$ model. While $(\mathbf{1}, \mathbf{10})^i$ and $(\mathbf{1}, \bar{\mathbf{5}})^i$ in Eq. (3) are sterile with respect to G_{SM}, the SM particles are singlets under SU$(5)'$. We denote the SM Higgs by H and the SM lepton doublets by ℓ_i, where $i = 1, 2, 3$ is the generation index. To generate small neutrino masses via the seesaw mechanism, we assume six right-handed neutrinos ν_k^c, where $k = 1, \ldots, 6$, which are total singlets under $G_{\mathrm{SM}} \times$ SU$(5)'$. We suppose that SU$(5)'$ is spontaneously broken at the TeV scale by a single SM singlet Higgs representation S which transforms as $S \sim (\mathbf{1}, \mathbf{5})^H$ under $G_{\mathrm{SM}} \times$ SU$(5)'$. When S acquires a vacuum expectation value (VEV), $\langle S \rangle \simeq \mathcal{O}$(TeV), SU$(5)'$ is broken down to SU$(4)'$, thereby eating nine Nambu Goldstone bosons from S via the Higgs mechanism. This is, e.g., immediately seen in the unitary gauge, where $\langle S \rangle$ can always be written as $\langle S \rangle = (0, 0, 0, 0, |s|)^T$. Since we impose minimality of the Higgs sector, SU$(4)'$ will remain unbroken.

The most general renormalizable Lagrangian relevant for neutrino masses in this model is given by

$$\mathcal{L}_Y = a_{ik} H \ell_i \nu_k^c + b_{ik} S (\mathbf{1}, \bar{\mathbf{5}})^i \nu_k^c + c_{ij} S^* (\mathbf{1}, \mathbf{10})^i (\mathbf{1}, \bar{\mathbf{5}})^j + d_{ij} S (\mathbf{1}, \mathbf{10})^i (\mathbf{1}, \mathbf{10})^j + M_{kl} \nu_k^c \nu_l^c + \text{H.c.}, \tag{4}$$

where $i, j = 1, 2, 3$ and $k, l = 1, \ldots, 6$. In Eq. (4), the coefficients a_{ik}, b_{ik}, c_{ij}, and d_{ij} denote complex order one Yukawa couplings and $M_{kl} \simeq \mathcal{O}(\Lambda)$, where $\Lambda \simeq 10^{13}$–10^{19} GeV is some high cutoff scale of the theory. When S acquires a VEV along its fifth component, the Yukawa interactions in Eq. (4) with coefficients c_{ij} and d_{ij} generate Dirac masses $\sim\mathcal{O}$(TeV) for 14 of the 15 fermions in each generation of $\mathrm{SU}(5)'$, which thus decouple from the low-energy theory below $\langle S\rangle \simeq \mathcal{O}(\mathrm{TeV})$. This is analogous to the embedding of the SM in SU(5) where all particles except ν_i from $(\bar{\mathbf{5}}, \mathbf{1})^i$ acquire masses from analogous Yukawa couplings. The remaining Weyl fermions, one per family (denoted as ν_i'), on the other hand, mix with the right-handed neutrinos through the interaction $\sim b_{ik} S(\mathbf{1}, \bar{\mathbf{5}})^i \nu_k^c$. The ν_k^c fields have Majorana masses of order Λ, and therefore generate in the effective theory for the ν' fields small masses from nonrenormalizable operators obtained after integrating out the heavy states ν_k^c. The effective Lagrangian for neutrino masses becomes

$$\mathcal{L}_{\mathrm{eff}} = \frac{Y_{ij}^a}{\Lambda} H^2 \ell_i \ell_j + \frac{Y_{ij}^b}{\Lambda} H S \ell_i (\mathbf{1}, \bar{\mathbf{5}})^j + \frac{Y_{ij}^c}{\Lambda} S^2 (\mathbf{1}, \bar{\mathbf{5}})^i (\mathbf{1}, \bar{\mathbf{5}})^j + \mathrm{H.c.}, \tag{5}$$

where Y_{ij}^a, Y_{ij}^b, and Y_{ij}^c are complex order one Yukawa couplings which are related to the parameters a_{ij}, b_{ik}, and M_{kl} in Eq. (4). Inserting the VEV $\langle S\rangle$ into Eq. (5), we thus observe that $\mathcal{L}_{\mathrm{eff}}$ gives rise to a seesaw mass operator which generates per generation one active and one sterile neutrino mass in the $\sim 10^{-2}$ eV range. Since the active and sterile neutrinos exhibit a nonzero mixing through the second term in Eq. (5), we hence obtain a $(3+3)$ scheme for sterile neutrino oscillations.

B. "Flipped" $\mathrm{SU}(5)'$ model

Let us now consider a variation of the model in Sec. II A where only the Higgs content in the sterile sector is modified. As a gauge group we therefore have again $G_{\mathrm{SM}} \times \mathrm{SU}(5)'$ with three extra generations of SM singlet fermions transforming according to the $G_{\mathrm{SM}} \times \mathrm{SU}(5)'$ representations as given in Eq. (3). As in the model in Sec. II A, we require six right-handed neutrinos ν_k^c, where $k = 1, \ldots, 6$, which are total singlets of $G_{\mathrm{SM}} \times \mathrm{SU}(5)'$. We now assume that the scalar sector is augmented by a single Higgs field S which transforms as $S \sim (\mathbf{1}, \mathbf{10})^H$ under $G_{\mathrm{SM}} \times \mathrm{SU}(5)'$ and acquires a VEV $\langle S\rangle \simeq \mathcal{O}(\mathrm{TeV})$. The renormalizable neutrino mass and mixing terms of this model then read

$$\mathcal{L}_Y = a_{ik} H \ell_i \nu_k^c + b_{ik} S^* (\mathbf{1}, \mathbf{10})^i \nu_k^c + c_{ij} S (\mathbf{1}, \bar{\mathbf{5}})^i (\mathbf{1}, \bar{\mathbf{5}})^j + M_{kl} \nu_k^c \nu_l^c + \mathrm{H.c.}, \tag{6}$$

where $i, j = 1, 2, 3$ and $k, l = 1, \ldots, 6$. In Eq. (6), the coefficients a_{ik}, b_{ik}, and c_{ij} denote complex order one Yukawa couplings and M_{kl} the cutoff scale. For a range of parameters, S will acquire a VEV of the skew-symmetric form $\langle S\rangle \sim \mathrm{diag}(0, 0, 0, 1 \otimes \mathrm{i}\sigma^2)$, thereby breaking $\mathrm{SU}(5)'$ down to $\mathrm{SU}(3)' \times \mathrm{SU}(2)'$ [24]. This model resembles the flipped SU(5) model for the SM sector [25], and hence we use the terminology flipped $\mathrm{SU}(5)'$ model. Under $\mathrm{SU}(5)' \supset \mathrm{SU}(3)' \times \mathrm{SU}(2)'$, the representations in Eq. (3) decompose as

$$\begin{aligned}(\mathbf{1}, \mathbf{10})^i &= (\mathbf{1}, \mathbf{3}, \mathbf{2})^i + (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{1})^i, \\ (\mathbf{1}, \bar{\mathbf{5}})^i &= (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})^i + (\mathbf{1}, \mathbf{1}, \mathbf{2})^i.\end{aligned} \tag{7}$$

If we define, as in the usual SU(5) model, the components $(\mathbf{1}, \mathbf{1}, \mathbf{2})^i \equiv (e_i', \nu_i')^T$, we observe that the Yukawa interaction $c_{ij} S (\mathbf{1}, \bar{\mathbf{5}})^i (\mathbf{1}, \bar{\mathbf{5}})^j$ in Eq. (6) will finally generate a mass term of the form $\sim f_{ij}(\nu_i' e_j' - \nu_j' e_j')$, where $f_{ij} = -f_{ji}$ due to Fermi statistics. Since the matrix f_{ij} has rank two, this interaction will give masses $\sim \mathcal{O}(\mathrm{TeV})$ to four out of six states in the multiplets $(e_i', \nu_i')^T$ which consequently decouple from the theory. Only one linear combination of e_i' and one linear combination of ν_i' remain massless. Absence of $\mathrm{SU}(2)'$ Witten anomaly [26] also requires that one $\mathrm{SU}(2)'$ doublet must remain massless. Moreover, all the states from $(\mathbf{1}, \mathbf{10})^i$ and the $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{1})^i$ state from $(\mathbf{1}, \bar{\mathbf{5}})^i$ will also remain massless.

These particles will, however, acquire masses from the dynamics of the unbroken $\mathrm{SU}(3)'$ and $\mathrm{SU}(2)'$. The one-loop beta function coefficients [27] for $\mathrm{SU}(3)'$ and $\mathrm{SU}(2)'$ are, respectively, $-7g_3'^3/(16\pi^2)$ and $-g_2'^3/(4\pi^2)$. In computing these coefficients, we included contributions from all the light fermionic states, and assumed that none of the scalar Higgs components from S have masses below a TeV. We see that both $\mathrm{SU}(3)'$ and $\mathrm{SU}(2)'$ are asymptotically free. All massless nontrivial $\mathrm{SU}(3)' \times \mathrm{SU}(2)'$ representations can, hence, decouple from the low-energy theory by acquiring masses through chiral symmetry breaking condensates. Note, in addition, that these states have zero mixing with the active neutrinos.

After integrating out the heavy states ν_k^c, the effective Lagrangian relevant for neutrino masses becomes therefore similar to the Lagrangian in Eq. (5) with $(\mathbf{1}, \mathbf{5})^H$ replaced by $(\mathbf{1}, \overline{\mathbf{10}})^H$ and $(\mathbf{1}, \bar{\mathbf{5}})^i$ replaced by $(\mathbf{1}, \mathbf{10})^i$. At low energies, we hence identify the $G_{\mathrm{SM}} \times \mathrm{SU}(3)' \times \mathrm{SU}(2)'$ singlets $(\mathbf{1}, \mathbf{1}, \mathbf{1})^i$ in Eq. (7) as three light sterile neutrinos giving in total a $(3+3)$ model of neutrino oscillations.

C. $Sp(4)'$ model

Let us now suppose the same gauge group and particle content as in the flipped $\mathrm{SU}(5)'$ model in Sec. II B, but consider a different symmetry breaking of $\mathrm{SU}(5)'$. In particular, we assume now a range of parameters in the scalar potential, for which $S \sim (\mathbf{1}, \mathbf{10})^H$ acquires a VEV $\langle S\rangle \simeq \mathcal{O}(\mathrm{TeV})$ and is of the skew-symmetric form $\langle S\rangle \sim \mathrm{diag}(1 \otimes \mathrm{i}\sigma^2, 1 \otimes \mathrm{i}\sigma^2, 0)$. This VEV breaks $\mathrm{SU}(5)' \to Sp(4)' \sim \mathrm{SO}(5)$ [24] such that the representations in Eq. (3) decompose under $\mathrm{SU}(5)' \supset Sp(4)'$ as

$$\begin{aligned}(\mathbf{1}, \mathbf{10})^i &= (\mathbf{1}, \mathbf{5})^i + (\mathbf{1}, \mathbf{4})^i + (\mathbf{1}, \mathbf{1})^i, \\ (\mathbf{1}, \bar{\mathbf{5}})^i &= (\mathbf{1}, \mathbf{4})^i + (\mathbf{1}, \mathbf{1})^i.\end{aligned} \tag{8}$$

Here, the Higgs field $(\mathbf{1}, \mathbf{10})^H$ decomposes under $\mathrm{SU}(5)' \supset Sp(4)'$ according to the first equation in Eq. (8) with the index i replaced by H. The Yukawa Lagrangian relevant for neutrino masses is given by Eq. (6). Inserting the representations in Eq. (8) into Eq. (6), we thus obtain for this model in the language of the unbroken $G_{\rm SM}\times Sp(4)'$ subgroup the renormalizable Lagrangian for neutrino masses

$$\begin{aligned}\mathcal{L}_Y &= a_{ik}H\ell_i\nu_k^c + b_{ik}(\mathbf{1},\mathbf{1})^H(\mathbf{1},\mathbf{1})^i\nu_k^c \\ &\quad + c_{ij}(\mathbf{1},\mathbf{1})^H(\mathbf{1},\mathbf{4})^i(\mathbf{1},\mathbf{4})^j + c_{ij}(\mathbf{1},\mathbf{1})^H(\mathbf{1},\mathbf{1})^i(\mathbf{1},\mathbf{1})^j \\ &\quad + M_{kl}\nu_k^c\nu_l^c + \text{H.c.},\end{aligned} \tag{9}$$

where the states $(\mathbf{1},\mathbf{1})^i$ and $(\mathbf{1},\mathbf{4})^i$ belong to the decomposition of the SU(5)$'$ representations $(\mathbf{1},\mathbf{10})^i$ and $(\mathbf{1},\overline{\mathbf{5}})^i$ in Eq. (8), respectively. Defining $(\mathbf{1},\mathbf{4})^i$ appearing in Eq. (9) in component form as $(\mathbf{1},\mathbf{4})^i\equiv(\psi_1^i,\psi_2^i,\psi_3^i,\psi_4^i)^T$, we see that the third term in Eq. (9) will generate a mass term of the form $\sim c_{ij}(\psi_1^i\psi_2^j-\psi_1^j\psi_2^i+\psi_3^i\psi_4^j-\psi_3^j\psi_4^i)$, where $c_{ij}=-c_{ji}$ due to Fermi statistics. Since the matrix c_{ij} has rank two, this term will give masses to eight out of twelve components in the representations $(\mathbf{1},\mathbf{4})^i$, which then decouple. Moreover, we can assume that after spontaneous symmetry breaking (SSB) the $G_{\rm SM}\times Sp(4)'$ representations $(\mathbf{1},\mathbf{5})^H$ and $(\mathbf{1},\mathbf{4})^H$ acquire masses $\sim\mathcal{O}(\text{TeV})$ and thus also decouple from the theory. As a consequence, the one-loop beta function coefficient of $Sp(4)'$ is given by[2] $-23g'^3/(48\pi^2)$. Therefore, $Sp(4)'$ is asymptotically free and the three fundamental and four spinor representations of $Sp(4)'$, which do not acquire masses from SSB, decouple through confinement. Hence, after integrating out the heavy states ν_k^c, the effective Lagrangian for neutrino masses becomes similar to the Lagrangian in Eq. (5) with $(\mathbf{1},\mathbf{5})^H$ replaced by $(\mathbf{1},\mathbf{1})^H$ and $(\mathbf{1},\overline{\mathbf{5}})^i$ replaced by $(\mathbf{1},\mathbf{1})^i$ [from $(\mathbf{10},\mathbf{1})^i$ in Eq. (8)] of $G_{\rm SM}\times Sp(4)'$. We are thus left with one light sterile neutrino per generation, which mixes with the active neutrinos, thereby leading in total to a (3 + 3) neutrino oscillation scheme. Note that one linear combination of $(\mathbf{1},\mathbf{1})^i$ from $(\mathbf{1},\overline{\mathbf{5}})^i$ will also remain light since c_{ij} has rank two, but this state has zero mixing with the other light neutrinos.

D. SU(4)$'$ × U(1)$'$ model

Under the subgroup SU(4)$'\times$U(1)$'$ of SU(5)$'$ with U(1) generator $T=(1,1,1,1,-4)$, the representations in Eq. (3) decompose as

$$\begin{aligned}(\mathbf{1},\mathbf{10})^i &= (\mathbf{1},\mathbf{6}_2)^i + (\mathbf{1},\mathbf{4}_{-3})^i, \\ (\mathbf{1},\overline{\mathbf{5}})^i &= (\mathbf{1},\overline{\mathbf{4}}_{-1})^i + (\mathbf{1},\mathbf{1}_4)^i,\end{aligned} \tag{10}$$

where in the parentheses $(\mathbf{1},\mathbf{x}_y)$, the subscript y denotes the U(1)$'$ charge of the states in $(\mathbf{1},\mathbf{x}_y)$ and $i=1,2,3$. Let us now assume for the gauge symmetry of our model $G_{\rm SM}\times$ SU(4)$'\times$U(1)$'$ with three generations of sterile fermions transforming according to Eq. (10). Note that this gauge theory is automatically anomaly-free since it is obtained from the model in Sec. II A by restriction to a subgroup, while the U(1)$'$ charge ensures that the model is chiral. As in Sec. II A, we furthermore assume six right-handed neutrinos ν_k^c, where $k=1,\ldots,6$, which are total singlets of $G_{\rm SM}\times$ SU(4)$'\times$U(1)$'$. To generate small sterile neutrino masses, we add to the SM scalar sector a single Higgs field S which transforms under $G_{\rm SM}\times$ SU(4)$'\times$U(1)$'$ as $S\sim(\mathbf{1},\mathbf{1}_{-4})^H$ and breaks SU(4)$'\times$U(1)$'$ down to SU(4)$'$ by acquiring a VEV $\langle S\rangle\simeq\mathcal{O}(\text{TeV})$. The most general renormalizable Lagrangian for neutrino masses then reads

$$\begin{aligned}\mathcal{L}_Y &= a_{ik}H\ell_i\nu_k^c + b_{ik}S(\mathbf{1},\mathbf{1}_4^i)\nu_k^c + c_{ij}S(\mathbf{1},\mathbf{6}_2)^i(\mathbf{1},\mathbf{6}_2)^j \\ &\quad + d_{ij}S^*(\mathbf{1},\mathbf{4}_{-3})^i(\mathbf{1},\overline{\mathbf{4}}_{-1})^j + M_{kl}\nu_k^c\nu_l^c + \text{H.c.},\end{aligned} \tag{11}$$

where $i,j=1,2,3$ and $k,l=1,\ldots,6$. In Eq. (4), the coefficients a_{ik}, b_{ik}, c_{ij}, and d_{ij} denote complex $\mathcal{O}(1)$ Yukawa couplings and $M_{kl}\simeq\mathcal{O}(\Lambda)$. When S assumes its VEV, all nontrivial representations of SU(4)$'$ acquire masses of order TeV at tree level and, hence, decouple from the low-energy theory.[3] After integrating out the heavy states ν_k^c, we are thus left with an effective neutrino mass Lagrangian which is similar to the Lagrangian given in Eq. (5) with $(\mathbf{1},\mathbf{5})^H$ replaced by $(\mathbf{1},\mathbf{1}_{-4})^H$ and $(\mathbf{1},\overline{\mathbf{5}})^i$ replaced by $(\mathbf{1},\mathbf{1}_4)^i$. We therefore obtain one light sterile neutrino per generation, leading to a (3 + 3) model of sterile neutrino oscillations.

It is instructive to examine the effect of the condensates on the U(1)$'$ symmetry breaking. Although S is an SU(4)$'$ singlet, we expect U(1)$'$ to be broken below the confining scale through loop corrections to the scalar potential. In the presence of the condensate $\langle(\mathbf{1},\mathbf{4}_{-3})(\mathbf{1},\overline{\mathbf{4}}_{-1})\rangle\simeq\Lambda^3_{\rm SU(4)}$, where $\Lambda_{\rm SU(4)}$ denotes the confining scale of SU(4)$'$, the scalar potential $V(S)$ of S reads

$$V(S)=\lambda\frac{\Lambda^3_{\rm SU(4)}}{16\pi^2}S+M^2|S|^2+\lambda|S|^4+\text{H.c.}, \tag{12}$$

where λ is an order one coupling related to c_{ij} and d_{ij} in Eq. (11), λ' is an order one quartic coupling, and the first term is induced by the tadpole diagram shown in Fig. 2. Minimization of $V(S)$ leads to $\langle S\rangle\simeq\lambda\Lambda^3_{\rm SU(4)}/(16\pi^2M^2)$. For $\langle S\rangle\simeq10^3$ GeV and a typical confining scale $\Lambda_{\rm SU(4)}\simeq10^3$ GeV, we hence obtain for the mass $M\simeq10$ GeV. Therefore, even if $M^2>0$, the U(1)$'$ symmetry may be spontaneously broken by tadpole terms.

[2] The index $\frac{1}{2}$ of the $Sp(4)'$ spinor representation $\mathbf{4}$ can be found from the regular embedding $Sp'(4)\supset$ SU(2)$'\times$SU(2)$'$ with branching rule $\mathbf{4}=(\mathbf{2},\mathbf{1})+(\mathbf{1},\mathbf{2})$. Note also, that a $\mathbf{5}$ decomposes as $\mathbf{5}=(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$.

[3] Note that the singlet in the SU(4)$'$ tensor product $\mathbf{6}\times\mathbf{6}$ is in the symmetric representation.

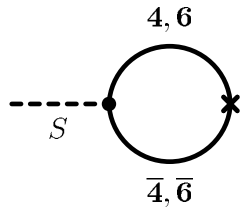

FIG. 2. Tadpole diagram generating a nonzero VEV for the $\mathrm{SU}(4)'$ singlet $S \sim (\mathbf{1}, \mathbf{1}_{-4})^H$ in the presence of the condensates $\langle(\mathbf{1}, \mathbf{4}_{-3})^i(\mathbf{1}, \overline{\mathbf{4}}_{-1})^j\rangle$ and/or $\langle(\mathbf{1}, \mathbf{6}_2)^i(\mathbf{1}, \overline{\mathbf{6}}_{-2})^j\rangle$.

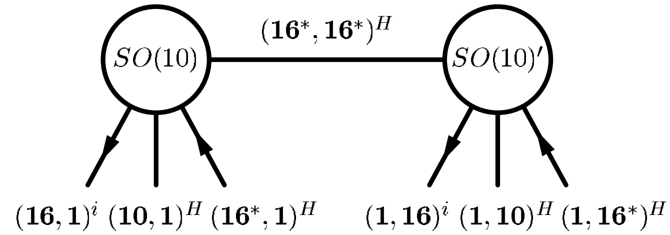

FIG. 3. Moose or quiver diagram for the $\mathrm{SO}(10) \times \mathrm{SO}(10)'$ model. The effective scalar operator $M_{\mathrm{Pl}}^{-1} S_1 S_2 \sim (\mathbf{16}^*, \mathbf{16}^*)^H$ links (with respect to the Yukawa interactions) as a bispinor the neighboring gauge groups and thus introduces a nonzero active-sterile neutrino mixing through the right-handed Majorana sector.

E. $G_{\mathrm{SM}} \times \mathrm{SU}(3)' \times \mathrm{SU}(2)' \times \mathrm{U}(1)'$ model

The chiral and anomaly-free $G_{\mathrm{SM}} \times \mathrm{SU}(3)' \times \mathrm{SU}(2)' \times \mathrm{U}(1)'$ model of Ref. [21] is simply obtained from the $G_{\mathrm{SM}} \times \mathrm{SU}(5)'$ model in Sec. II A by restriction to a subgroup. To arrive at this gauge group from $G_{\mathrm{SM}} \times \mathrm{SU}(5)'$ through SSB, however, one would require an adjoint Higgs $(\mathbf{1}, \mathbf{24})^H$ which does not couple to any fermions. According to our requirement of having a minimal Higgs content admitting only a single Higgs representation in the sterile sector, such a model would give massless sterile neutrinos with zero mixing between active and sterile neutrinos. A model consistent with our minimality assumption can arise if we start with the $\mathrm{SU}(3)' \times \mathrm{SU}(2)' \times \mathrm{U}(1)'$ subgroup of $\mathrm{SU}(5)'$ as in Ref. [21]. Such a model has already been developed, and we have nothing new to add to this case.

III. $\mathrm{SO}(10) \times \mathrm{SO}(10)'$ MODEL

In the models presented above, we have always treated the right-handed neutrinos ν_k^c, necessary for the sterile neutrino seesaw mechanism, as total gauge singlets of the gauge group $G_{\mathrm{SM}} \times G'$. We will now slightly deviate from our general discussion of the $\mathrm{SU}(N)$ chains and consider instead the attractive possibility of unifying all particles, including the right-handed neutrinos, into an $\mathrm{SO}(10) \times \mathrm{SO}(10)'$ product gauge group. Here, we assume that the SM is embedded into the first group, i.e., $\mathrm{SO}(10) \supset G_{\mathrm{SM}}$. As the fermionic particle content of this model we choose the $\mathrm{SO}(10) \times \mathrm{SO}(10)'$ representations in a symmetrical way as

$$(\mathbf{16}, \mathbf{1})^i + (\mathbf{1}, \mathbf{16})^i, \tag{13}$$

where $i = 1, 2, 3$. Like the usual SO(10) models, this gauge theory is chiral and anomaly free. Since all right-handed neutrinos have now been unified into the SO(10) multiplets, we require for the generation of light active and sterile neutrino masses that the Higgs sector contains two scalars $S_1 \sim (\mathbf{16}^*, \mathbf{1})^H$ and $S_2 \sim (\mathbf{1}, \mathbf{16}^*)^H$. These scalars can generate at the nonrenormalizable level Planck-scale suppressed effective operators $M_{\mathrm{Pl}}^{-1} S_1 S_1 \sim (\mathbf{126}^*, \mathbf{1})^H$ and $M_{\mathrm{Pl}}^{-1} S_2 S_2 \sim (\mathbf{1}, \mathbf{126}^*)^H$ which can supply large Majorana masses of order $\sim 10^{14}$ GeV to the right-handed neutrinos. Similarly, a nonrenormalizable operator $M_{\mathrm{Pl}}^{-1} S_1 S_2 \sim (\mathbf{16}^*, \mathbf{16}^*)^H$ is generated, which transforms as a bispinor under $\mathrm{SO}(10) \times \mathrm{SO}(10)'$ and appears (with respect to the Yukawa sector) as an effective scalar linking the two SO(10) gauge groups. The representation content giving rise to neutrino masses can then be summarized in a "moose" [28] or "quiver" [29] notation in Fig. 3. Under $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)$ and $\mathrm{SO}(10)' \supset \mathrm{SU}(5)' \times \mathrm{U}(1)'$, the representations in Eq. (13) decompose as[4]

$$\begin{aligned}(\mathbf{16}, \mathbf{1})^i &= (\mathbf{10}_1, \mathbf{1})^i + (\overline{\mathbf{5}}_{-3}, \mathbf{1})^i + (\mathbf{1}_5, \mathbf{1})^i,\\ (\mathbf{1}, \mathbf{16})^i &= (\mathbf{1}, \mathbf{10}_1)^i + (\mathbf{1}, \overline{\mathbf{5}}_{-3})^i + (\mathbf{1}, \mathbf{1}_5)^i.\end{aligned} \tag{14}$$

Here, the scalars S_1 and S_2 decompose according to the conjugate of the first and the second branching rule, respectively. To generate Dirac masses in both sectors, we assume two fundamental Higgs representations $(\mathbf{10}, \mathbf{1})^H$ and $(\mathbf{1}, \mathbf{10})^H$ which have under the decomposition in Eq. (14) the branching rules

$$\begin{aligned}(\mathbf{10}, \mathbf{1})^H &= (\mathbf{5}_{-2}, \mathbf{1})^H + (\overline{\mathbf{5}}_2, \mathbf{1})^H,\\ (\mathbf{1}, \mathbf{10})^H &= (\mathbf{1}, \mathbf{5}_{-2})^H + (\mathbf{1}, \overline{\mathbf{5}}_2)^H.\end{aligned} \tag{15}$$

In Eq. (14), the light active and sterile neutrinos are contained in the $(\overline{\mathbf{5}}_{-3}, \mathbf{1})^i$ and $(\mathbf{1}, \overline{\mathbf{5}}_{-3})^i$ multiplets, whereas the heavy right-handed neutrinos are identified with the $\mathrm{SU}(5) \times \mathrm{SU}(5)'$ singlets $(\mathbf{1}_5, \mathbf{1})^i$ and $(\mathbf{1}, \mathbf{1}_5)^i$. Up to mass dimension five, the most general Lagrangian relevant for neutrino masses is found to be

$$\begin{aligned}\mathcal{L}_Y = {}& a_{ij}(\mathbf{10}, \mathbf{1})^H(\mathbf{16}, \mathbf{1})^i(\mathbf{16}, \mathbf{1})^j\\ &+ a'_{ij}(\mathbf{1}, \mathbf{10})^H(\mathbf{1}, \mathbf{16})^i(\mathbf{1}, \mathbf{16})^j\\ &+ b_{ij}\frac{S_1^2}{M_{\mathrm{Pl}}}(\mathbf{16}, \mathbf{1})^i(\mathbf{16}, \mathbf{1})^j + b'_{ij}\frac{S_2^2}{M_{\mathrm{Pl}}}(\mathbf{1}, \mathbf{16})^i(\mathbf{1}, \mathbf{16})^j\\ &+ c_{ij}\frac{S_1 S_2}{M_{\mathrm{Pl}}}(\mathbf{16}, \mathbf{1})^i(\mathbf{1}, \mathbf{16})^j + \text{H.c.},\end{aligned} \tag{16}$$

where the coefficients a_{ij}, a'_{ij}, b_{ij}, b'_{ij}, and c_{ij} are order one Yukawa couplings. The two first operators in Eq. (16) give

[4] The symmetry could be broken along this direction, e.g., by two extra Higgs fields $(\mathbf{45}, \mathbf{1})^H$ and $(\mathbf{1}, \mathbf{45})^H$.

rise to Dirac masses of the type $\sim\nu_i\nu_k^c$ and $\sim\nu_i'\nu_k'^c$ for the active and the sterile neutrinos. In the language of the decompositions in Eq. (14), these Dirac masses arise in $\mathcal{L}_Y$ from the terms $a_{ij}(\mathbf{5}_{-2},\mathbf{1})^H(\overline{\mathbf{5}}_{-3},\mathbf{1})^i(\mathbf{1}_5,\mathbf{1})^j$ and $a'_{ij}(\mathbf{1},\mathbf{5}_{-2})^H(\mathbf{1},\overline{\mathbf{5}}_{-3})^i(\mathbf{1},\mathbf{1}_5)^j$, respectively. When S_1 and S_2 acquire their VEV's $\langle S_1\rangle \simeq \langle S_2\rangle \simeq 10^{16}$ GeV, S_1 breaks $SO(10)\to SU(5)$ and S_2 breaks $SO(10)'\to SU(5)'$ [the effective bispinor $(\mathbf{16}^*,\mathbf{16}^*)^H$ acquires its VEV along the $(\mathbf{1},\mathbf{1})$ component under $SU(5)\times SU(5)'$]. Consequently, the third and fourth terms in Eq. (16) will generate at the nonrenormalizable level masses for the right-handed neutrinos of the order $\sim 10^{14}$ GeV. A nonzero mixing between the active and sterile neutrinos is introduced only in terms of the effective bispinor $(\mathbf{16}^*,\mathbf{16}^*)^H$, which couples in the last term in Eq. (16) the right-handed neutrinos belonging to SO(10) with the right-handed neutrinos belonging to $SO(10)'$ by generating a mixed Majorana mass term of order $\sim 10^{14}$ GeV. After electroweak symmetry breaking and integrating out the right-handed neutrinos, we thus arrive at three light active and three light sterile neutrinos with masses in the (sub)eV range, which exhibit a nonzero mixing and thus lead to a $(3+3)$ scenario for sterile neutrino oscillations.

Additional Higgs fields, such as $(\mathbf{45},\mathbf{1})^H$ and $(\mathbf{1},\mathbf{45})^H$, are needed for breaking the $SU(5)\times SU(5)'$ symmetry down to $G_{SM}\times G'$. If the $(\mathbf{45},\mathbf{1})^H$ acquires a VEV along its SM singlet direction $\langle(\mathbf{45},\mathbf{1})^H\rangle \sim \mathrm{diag}(a,a,a,b,b)\otimes i\sigma^2$, while $\langle(\mathbf{1},\mathbf{45})^H\rangle = 0$, we obtain $G_{SM}\times SU(5)'$ of Sec. II. If, instead, $\langle(\mathbf{1},\mathbf{45})^H\rangle \sim \mathrm{diag}(a',a',a',b',b')\otimes i\sigma^2$ [along the $SU(3)'\times SU(2)'\times U(1)'$ direction], we have the model of Ref. [21]. If, on the other hand, $\langle(\mathbf{1},\mathbf{45})^H\rangle \sim \mathrm{diag}(a',a',a',a',b')\otimes i\sigma^2$, we obtain $G_{SM}\times SU(4)'\times U(1)'$ of Sec. II D. Thus, we see that all the models described in Sec. II have a natural origin within the $SO(10)\times SO(10)'$ framework.

IV. OTHER MODELS

A. $G_{SM}\times SU(7)'$ model and its descendants

In this section, we examine chiral gauge models, where the chain in Eq. (1) provides three generations of antifundamental representations. The sterile gauge symmetry of these models is $SU(7)'$, for which we consider different symmetry breakings as summarized in Fig. 4.

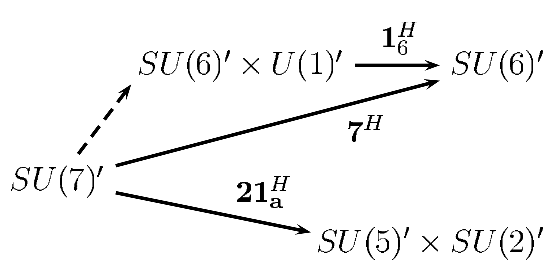

FIG. 4. Symmetry breaking in $SU(7)'$. The solid arrows represent the symmetry breaking by suitable Higgs representations considered in the text. The dashed arrow indicates the embedding of $SU(6)'\times U(1)'$ into $SU(7)'$ as a maximal subgroup.

1. $SU(7)'$ model

Thus far, we have obtained three generations of sterile fermions by taking three copies of the fermion representations in Eq. (1) for $N=5$. As another possibility, we shall now consider the case $N=7$, where the particle content in Eq. (1) already provides three antifundamental representations of $SU(7)'$, which we identify with three sterile fermion generations. As the gauge group of this model, we therefore have $G_{SM}\times SU(7)'$ with SM singlet fermions transforming as

$$(\mathbf{1},\mathbf{21}) + (\mathbf{1},\overline{\mathbf{7}})^i, \tag{17}$$

where $i=1,2,3$ is the generation index of the fermions in the antifundamental representation of $SU(7)'$. As in the other models discussed above, we assume in addition to the fermions in Eq. (17) six right-handed neutrinos ν_k^c ($k=1,\dots,6$) which carry zero $G_{SM}\times SU(7)'$ quantum numbers. To break the $SU(7)'$ symmetry, we suppose a single Higgs representation S which transforms as $S\sim(\mathbf{1},\mathbf{7})^H$ under $G_{SM}\times SU(7)'$ and acquires its VEV at the TeV scale, i.e., $\langle S\rangle \simeq \mathcal{O}(\mathrm{TeV})$. The Lagrangian for neutrino masses of this model reads

$$\mathcal{L}_Y = a_{ik}H\ell_i\nu_k^c + b_{ik}S(\mathbf{1},\overline{\mathbf{7}})^i\nu_k^c + c_iS^*(\mathbf{1},\mathbf{21})(\mathbf{1},\overline{\mathbf{7}})^i + M_{kl}\nu_k^c\nu_l^c + \mathrm{H.c.}, \tag{18}$$

where $i,j=1,2,3$ and $k,l=1,\dots,6$. In Eq. (18), the coefficients a_{ik}, b_{ik}, and c_i denote complex $\sim\mathcal{O}(1)$ Yukawa couplings and $M_{kl}\simeq\mathcal{O}(\Lambda)$. When S acquires its VEV, $SU(7)'$ is broken $SU(7)'\to SU(6)'$. For $SU(7)'\supset SU(6)'$, the representations in Eq. (17) and S decompose as

$$(\mathbf{1},\mathbf{21}) = (\mathbf{1},\mathbf{15}) + (\mathbf{1},\mathbf{6}), \qquad (\mathbf{1},\overline{\mathbf{7}})^i = (\mathbf{1},\overline{\mathbf{6}})^i + (\mathbf{1},\mathbf{1})^i, \tag{19}$$

where the scalar $(\mathbf{1},\mathbf{7})^H$ decomposes according to the second equation with i replaced by H. After SSB, the interaction $\sim c_iS^*(\mathbf{1},\mathbf{21})(\mathbf{1},\overline{\mathbf{7}})^i$ in Eq. (18) generates masses $\sim\mathcal{O}(\mathrm{TeV})$ for $(\mathbf{1},\mathbf{6})$ and one linear combination of the states $(\mathbf{1},\overline{\mathbf{6}})^i$ in Eq. (19) which then decouple. Assuming that also the $G_{SM}\times SU(6)'$ representation $(\mathbf{1},\mathbf{6})^H$ acquires a mass of order ~ 1 TeV and thus decouples from the low-energy theory, the leading order beta function coefficient for $SU(6)'$ is given by $-5g'^3/(4\pi^2)$ implying that $SU(6)'$ is asymptotically free. As a result, all nontrivial fermionic $SU(6)'$ representations will confine by building condensates of the types $\langle(\mathbf{1},\overline{\mathbf{15}})(\mathbf{1},\mathbf{15})\rangle$ and $\langle(\mathbf{1},\overline{\mathbf{6}})(\mathbf{1},\mathbf{6})\rangle$ and decouple at low energies. The $(\mathbf{1},\mathbf{1})^i$ from the $(\mathbf{1},\overline{\mathbf{7}})^i$, identified with ν_i', will not acquire any confinement mass, being a singlet of $SU(6)'$. After integrating out the right-handed neutrinos ν_k^c, the relevant effective Lagrangian for neutrino masses therefore reads as in Eq. (5), with $(\mathbf{1},\mathbf{5})^H$

and $(\mathbf{1},\overline{\mathbf{5}})^i$, respectively, replaced by the representations $(\mathbf{1},\mathbf{21})^H$ and $(\mathbf{1},\overline{\mathbf{7}})^i$. As a consequence, we obtain one sterile neutrino per generation and therefore a (3 + 3) neutrino scheme.

2. Flipped SU(7)′ *model*

We will now consider a model with gauge group $G_{\rm SM}\times {\rm SU}(7)'$, which has a fermion sector identical with the model described in Sec. IVA 1. In contrast to the previous model, however, we assume that the Higgs sector is extended by a Higgs field S, which transforms as $S\sim(\mathbf{1},\mathbf{21})^H$ under $G_{\rm SM}\times{\rm SU}(7)'$ and acquires a VEV $\langle S\rangle\simeq\mathcal{O}({\rm TeV})$. We term this model as flipped SU(7)′, in analogy with the flipped SU(5)′ model, where an antisymmetric second rank tensorial Higgs was used. The renormalizable Lagrangian for neutrino masses now reads

$$\mathcal{L}_Y = a_{ik}H\ell_i\nu_k^c + b_kS^*(\mathbf{1},\mathbf{21})\nu_k^c + c_{ij}S(\mathbf{1},\overline{\mathbf{7}})^i(\mathbf{1},\overline{\mathbf{7}})^j + M_{kl}\nu_k^c\nu_l^c + \text{H.c.}, \tag{20}$$

where $i,j=1,2,3$ and $k,l=1,\ldots,6$. In Eq. (20), the coefficients a_{ik}, b_k, and $c_{ij}=-c_{ji}$ denote complex order one Yukawa couplings and $M_{kl}\simeq\mathcal{O}(\Lambda)$. For a range of parameters, S will acquire a VEV of the skew-symmetric form $\langle S\rangle\sim{\rm diag}(0,0,0,0,0,1\otimes {\rm i}\sigma^2)$, thereby breaking ${\rm SU}(7)'\rightarrow{\rm SU}(5)'\times{\rm SU}(2)'$ [24]. Under ${\rm SU}(7)'\rightarrow{\rm SU}(5)'\times{\rm SU}(2)'$, the representations in Eq. (17) decompose as

$$\begin{aligned}(\mathbf{1},\mathbf{21}) &= (\mathbf{1},\mathbf{10},\mathbf{1})+(\mathbf{1},\mathbf{5},\mathbf{2})+(\mathbf{1},\mathbf{1},\mathbf{1}),\\ (\mathbf{1},\overline{\mathbf{7}})^i &= (\mathbf{1},\overline{\mathbf{5}},\mathbf{1})^i+(\mathbf{1},\mathbf{1},\mathbf{2})^i,\end{aligned} \tag{21}$$

while $(\mathbf{1},\mathbf{21})^H$ decomposes according to the first equation. The interaction $\sim c_{ij}S(\mathbf{1},\overline{\mathbf{7}})^i(\mathbf{1},\overline{\mathbf{7}})^j$ in Eq. (20) generates masses of order ~1 TeV for two linear combinations of the SU(2)′ doublets $(\mathbf{1},\mathbf{1},\mathbf{2})^i$ in Eq. (21). Assuming that the $G_{\rm SM}\times{\rm SU}(5)'\times{\rm SU}(2)'$ representations $(\mathbf{1},\mathbf{10},\mathbf{1})^H$ and $(\mathbf{1},\mathbf{5},\mathbf{2})^H$ have masses of order ~1 TeV, the leading order coefficients of the beta functions for SU(5)′ and SU(2)′ are, respectively, given by $-47g_5'^3/(48\pi^2)$ and $-g_2'^3/(3\pi^2)$, i.e., SU(5)′ and SU(2)′ are asymptotically free. As a result, all nontrivial fermionic representations of ${\rm SU}(5)'\times{\rm SU}(2)'$ will confine and decouple from the low-energy theory, while the trivial $(\mathbf{1},\mathbf{1},\mathbf{1})$ representation from $(\mathbf{1},\mathbf{21})$ remains light. After integrating out the heavy right-handed neutrinos ν_k^c, the relevant effective Lagrangian for neutrino masses is then on a form similar to the one given in Eq. (5), with $(\mathbf{1},\mathbf{5})^H$ and $(\mathbf{1},\overline{\mathbf{5}})^i$, respectively, replaced by the representations $(\mathbf{1},\overline{\mathbf{21}})^H$ and the single field $(\mathbf{1},\mathbf{21})$ of $G_{\rm SM}\times{\rm SU}(7)'$. From Eq. (21) we then conclude that this model gives in total one light sterile neutrino leading to a (3 + 1) neutrino oscillation scheme.

3. SU(6)′ × U(1)′ *model*

Let us now examine a model which is obtained from the model in Sec. IVA 1 by restricting to the (maximal) subgroup ${\rm SU}(6)'\times{\rm U}(1)'$ of SU(7)′ (see Fig. 4). All sterile fermion representations of this model thus follow from breaking up the fermion representations in Sec. IVA 1 into the representations of $G_{\rm SM}\times{\rm SU}(6)'\times{\rm U}(1)'$. Here, the representations in Eq. (17) decompose under ${\rm SU}(7)'\supset{\rm SU}(6)'\times{\rm U}(1)'$ as

$$\begin{aligned}(\mathbf{1},\mathbf{21}) &= (\mathbf{1},\mathbf{15}_2)+(\mathbf{1},\mathbf{6}_{-5}),\\ (\mathbf{1},\overline{\mathbf{7}})^i &= (\mathbf{1},\overline{\mathbf{6}}_{-1})^i+(\mathbf{1},\mathbf{1}_{-6})^i,\end{aligned} \tag{22}$$

where $i=1,2,3$. We assume that the SM Higgs sector is extended by a single Higgs field S, which transforms under $G_{\rm SM}\times{\rm SU}(6)'\times{\rm U}(1)'$ as $S\sim(\mathbf{1},\mathbf{1}_6)^H$ and acquires a VEV $\langle S\rangle\simeq\mathcal{O}({\rm TeV})$. The most general renormalizable Lagrangian for neutrino masses of this model is

$$\mathcal{L}_Y = a_{ik}H\ell_i\nu_k^c + b_{ik}S(\mathbf{1},\mathbf{1}_{-6})^i\nu_k^c + c_iS(\mathbf{1},\mathbf{6}_{-5})(\mathbf{1},\overline{\mathbf{6}}_{-1})^i + M_{kl}\nu_k^c\nu_l^c + \text{H.c.}, \tag{23}$$

where $i=1,2,3$, and $k,l=1,\ldots,6$. In Eq. (23), the quantities a_{ik}, b_{ik}, and c_i denote complex order one Yukawa couplings and M_{kl} the cutoff. When S acquires its VEV at the TeV scale, the gauge group is broken ${\rm SU}(6)'\times{\rm U}(1)'\rightarrow{\rm SU}(6)'$. Moreover, $(\mathbf{1},\mathbf{6}_5)$ and one linear combination of the states $(\mathbf{1},\overline{\mathbf{6}}_{-1})^i$ acquire through SSB a mass ~1 TeV and decouple from the low-energy theory. Then, the leading order coefficient of the beta function for SU(6)′ is $-5g'^3/(4\pi^2)$, i.e., SU(6)′ is asymptotically free. Therefore, all nonsinglet SU(6)′ representations will decouple by forming the condensates $\langle(\mathbf{1},\overline{\mathbf{6}}_{-1})^i(\mathbf{1},\mathbf{6}_{+1})^j\rangle$, $\langle(\mathbf{1},\overline{\mathbf{6}}_5)(\mathbf{1},\mathbf{6}_{-5})\rangle$, and $\langle(\mathbf{1},\overline{\mathbf{6}}_{-1})^i(\mathbf{1},\overline{\mathbf{6}}_{-1})^j(\mathbf{1},\mathbf{15}_2)\rangle$. After integrating out the right-handed neutrino singlets ν_k^c, the effective Lagrangian for neutrino masses becomes similar to the Lagrangian in Eq. (5) with $(\mathbf{1},\mathbf{5})^H$ and $(\mathbf{1},\overline{\mathbf{5}})^i$, respectively, replaced by $(\mathbf{1},\mathbf{1}_6)^H$ and $(\mathbf{1},\mathbf{1}_{-6})^i$. In this model, we therefore obtain one light sterile neutrino per generation leading to a (3 + 3) neutrino oscillation scheme.

B. Models in the $G_{\rm SM}\times$ SU(3)′ chain

1. SU(3)′ *model*

The smallest gauge group which allows a chiral and anomaly-free gauge theory in the class shown in Eq. (2) is SU(3)′, which has seven fields in the antifundamental representation and one in the symmetric second rank tensor representation. As a total gauge group, let us now take $G_{\rm SM}\times{\rm SU}(3)'$ with the extra fermion representations transforming as

$$(\mathbf{1},\mathbf{6})+(\mathbf{1},\overline{\mathbf{3}})^i, \tag{24}$$

where $i=1,\ldots,7$. In order to break SU(3)′, we assume a Higgs field S which transforms as $S\sim(\mathbf{1},\mathbf{3})^H$ under $G_{\rm SM}\times{\rm SU}(3)'$ and acquires a VEV $\langle S\rangle\simeq\mathcal{O}({\rm TeV})$, which can always be written as $\langle S\rangle = (0, 0, |s|)^T$. We furthermore add six right-handed neutrinos ν_k^c ($k = 1, \ldots, 6$) which are total singlets of $G_{\rm SM} \times {\rm SU}(3)'$. The most general renormalizable Lagrangian for neutrino masses is then given by

$$\mathcal{L}_Y = a_{\alpha k} H \ell_\alpha \nu_k^c + b_{ik} S(\mathbf{1}, \overline{\mathbf{3}})^i \nu_k^c + c_i S^*(\mathbf{1}, \mathbf{6})(\mathbf{1}, \overline{\mathbf{3}})^i + M_{kl} \nu_k^c \nu_l^c + \text{H.c.}, \tag{25}$$

where $\alpha = 1, 2, 3$ and $i = 1, \ldots, 7$. In Eq. (25), the coefficients $a_{\alpha k}$, b_{ik}, and c_i are $\sim\mathcal{O}(1)$ Yukawa couplings and M_{kl} is of the order of the cutoff scale. When S acquires its VEV a the TeV scale, the gauge group is broken as ${\rm SU}(3)' \supset {\rm SU}(2)'$. With this embedding, the decomposition of the fermion representations in Eq. (24) for ${\rm SU}(3)' \supset {\rm SU}(2)'$ reads

$$\begin{aligned} (\mathbf{1}, \mathbf{6}) &= (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{2}) + (\mathbf{1}, \mathbf{1}), \\ (\mathbf{1}, \overline{\mathbf{3}})^i &= (\mathbf{1}, \mathbf{2})^i + (\mathbf{1}, \mathbf{1})^i, \end{aligned} \tag{26}$$

where $i = 1, \ldots, 7$ and S decomposes here as $(\mathbf{1}, \mathbf{3})^H = (\mathbf{1}, \mathbf{2})^H + (\mathbf{1}, \mathbf{1})^H$. In Eq. (26), the representations $(\mathbf{1}, \mathbf{2})$, $(\mathbf{1}, \mathbf{1})$, one linear combination of the states $(\mathbf{1}, \mathbf{2})^i$, and one linear combination of the singlets $(\mathbf{1}, \mathbf{1})^i$ will acquire masses of order TeV through the interaction $c_i S(\mathbf{1}, \mathbf{6})(\mathbf{1}, \overline{\mathbf{5}})^i$ in Eq. (25). Assuming that the $G_{\rm SM} \times {\rm SU}(2)'$ representation $(\mathbf{1}, \mathbf{2})^H$ decouples by obtaining a mass $\sim\mathcal{O}(\text{TeV})$, the leading order coefficient of the beta function for ${\rm SU}(2)'$ becomes $-g_2'^3/(4\pi^2)$, i.e., ${\rm SU}(2)'$ is asymptotically free. Consequently, in Eq. (26), the two massless linear combinations of the states $(\mathbf{1}, \mathbf{2})^i$ will decouple by forming condensates of the types $\langle(\mathbf{1}, \mathbf{2})(\mathbf{1}, \mathbf{2})\rangle$. After integrating out the right-handed neutrinos ν_k^c, the effective Lagrangian generating neutrino masses can therefore be written as in Eq. (5), with $(\mathbf{1}, \mathbf{5})^H$ replaced by $(\mathbf{1}, \mathbf{3})^H$ and $(\mathbf{1}, \overline{\mathbf{5}})^i$ replaced by $(\mathbf{1}, \overline{\mathbf{3}})^i$. In total, this model therefore leads to six light sterile neutrinos identified in Eq. (26) with six linear combinations of the singlets $(\mathbf{1}, \mathbf{1})^i$ and, hence, we obtain a (3 + 6) model for sterile neutrino oscillations.

2. SU(2)′ × U(1)′ *model*

We consider now the anomaly-free and chiral gauge theory which is obtained from the previous model in Sec. IV B 1 by restricting to the subgroup $G_{\rm SM} \times {\rm SU}(2)' \times {\rm U}(1)'$. The fermion content of this model then results from breaking up the fermion representations in Sec. IV B 1 into the representations of $G_{\rm SM} \times {\rm SU}(2)' \times {\rm U}(1)'$. In particular, the multiplets in Eq. (24) decompose under ${\rm SU}(3)' \supset {\rm SU}(2)' \times {\rm U}(1)'$ as

$$\begin{aligned} (\mathbf{1}, \mathbf{6}) &= (\mathbf{1}, \mathbf{3}_2) + (\mathbf{1}, \mathbf{2}_{-1}) + (\mathbf{1}, \mathbf{1}_{-4}), \\ (\mathbf{1}, \overline{\mathbf{3}})^i &= (\mathbf{1}, \mathbf{2}_{-1})^i + (\mathbf{1}, \mathbf{1}_{+2})^i, \end{aligned} \tag{27}$$

where $i = 1, \ldots, 7$ and the subscript denotes the ${\rm U}(1)'$ charge. Moreover, we assume seven right-handed neutrinos ν_k^c ($k = 1, \ldots, 7$) which are total singlets under $G_{\rm SM} \times {\rm SU}(2)' \times {\rm U}(1)'$. We suppose that ${\rm U}(1)'$ is spontaneously broken by a single Higgs fields S which transforms under $G_{\rm SM} \times {\rm SU}(2)' \times {\rm U}(1)'$ as $S \sim (\mathbf{1}, \mathbf{1}_{-2})$ and acquires a VEV $\langle S\rangle \simeq \mathcal{O}(\text{TeV})$. Here, the Lagrangian relevant for neutrino masses reads

$$\begin{aligned} \mathcal{L}_Y = {} & a_{\alpha k} H \ell_\alpha \nu_k^c + b_{ik} S(\mathbf{1}, \mathbf{1}_{+2})^i \nu_k^c \\ & + c_i S^*(\mathbf{1}, \mathbf{1}_{-4})(\mathbf{1}, \mathbf{1}_{+2})^i + d_{ij} S^*(\mathbf{1}, \mathbf{2}_{-1})^i(\mathbf{1}, \mathbf{2}_{-1})^j \\ & + d_i' S^*(\mathbf{1}, \mathbf{2}_{-1})(\mathbf{1}, \mathbf{2}_{-1})^i + M_{kl} \nu_k^c \nu_l^c + \text{H.c.}, \end{aligned} \tag{28}$$

where $\alpha = 1, 2, 3$ and $i, j, k, l = 1, \ldots, 7$. In Eq. (28), the coefficients $a_{\alpha k}$, b_{ik}, b_i', c_{ij}, and d_i denote $\sim\mathcal{O}(1)$ Yukawa couplings and $M_{kl} \simeq \mathcal{O}(\Lambda)$. When the scalar S acquires its VEV at the TeV scale, the gauge group is broken as ${\rm SU}(2)' \times {\rm U}(1)' \to {\rm SU}(2)'$. The Yukawa interactions with S will therefore generate masses of order $\sim$1 TeV for all eight ${\rm SU}(2)'$ doublets, $(\mathbf{1}, \mathbf{1}_{-4})$, and one linear combination of the states $(\mathbf{1}, \mathbf{1}_2)^i$ in Eq. (27), which hence decouple from the low-energy theory. The leading order coefficient of the beta function for ${\rm SU}(2)'$ then becomes $-g'^3/(4\pi^2)$ and ${\rm SU}(2)'$ is asymptotically free. After integrating out the heavy right-handed neutrinos ν_k^c, we therefore obtain two light sterile neutrinos per generation, leading to a (3 + 6) scheme for sterile neutrino oscillations.

V. SUMMARY AND CONCLUSIONS

In this paper, we have presented a family of chiral gauge models which would protect the masses of sterile neutrinos. A sterile gauge symmetry enables us to realize a seesaw mechanism for the SM singlet neutrinos. Our main motivation for studying this class of models is to provide an explanation for the LSND neutrino oscillation result, in conjunction with the solar and atmospheric neutrino data. The naturally light sterile neutrinos of our models can find application as candidates for warm dark matter, which could also provide an understanding of the observed anomalously large radio pulsar velocities exceeding $\sim$500 km/s (“pulsar kicks”) [30]. There is yet another possible application. With the recent revision in the value of the boron production rate $S_{17}(O)$, the total solar neutrino flux observed by the SNO experiment seems to indicate a deficit of active neutrinos by $\sim$12%. This deficit may be understood via a small admixture of light sterile neutrinos [31] which would be easily provided by our models.

The class of models we have constructed has a natural embedding in an ${\rm SO}(10) \times {\rm SO}(10)'$ unified theory as discussed in Sec. III. The ${\rm SU}(5)'$ model of Sec. II A as well as its descendants of Secs. II B, II C, and II D can all be neatly embedded into ${\rm SO}(10) \times {\rm SO}(10)'$. We have also constructed models based on an ${\rm SU}(7)'$ gauge symmetry, which provides *three* light sterile neutrinos naturally, and models based on an ${\rm SU}(3)'$ gauge symmetry with fermions in the sextet representation.

The most direct test of our models will be a confirmation of the LSND oscillation data by the ongoing MiniBooNE experiment. Reactor neutrino disappearance experiments, as well as neutrinoless double beta decay experiments, should be sensitive to the existence of sterile neutrinos [17]. It should be noted that the standard big bang nucleosynthesis [32] will be affected by the presence of ν'; however, there are ways around it, such as by assuming primordial lepton asymmetry [33] or with low reheating temperature [34]. The bound on neutrino masses from recent cosmological data [35] may also be alleviated by such a lepton asymmetry [36]. Although the scale of new physics is of order TeV in our models, testing them at colliders will be challenging, since the extended gauge sector has no direct couplings to the SM sector. One possible signature is the invisible decay of the SM Higgs boson H, as its mixing with the Higgs field S used for sterile gauge symmetry breaking can be substantial. Invisible decays such as $H \rightarrow W'W'$ can then occur, with W', the gauge bosons of the sterile gauge symmetry, decaying into sterile fermions.

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