Lepton flavor violating τ and B decays and heavy neutrinos

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We study lepton flavor violating (LFV) τ and B decays in models with heavy neutrinos to constrain the mixing matrix parameters $U_{\tau N}$. We find that the best current constraints when the heavy neutrinos are purely left handed come from LFV radiative τ decay modes. To obtain competitive constraints in LFV B decay, it is necessary to probe $b \to X_s \tau^\pm e^\mp$ at the 10^{-7} level. When the heavy neutrinos have both left- and right-handed couplings, the mixing parameters can be constrained by studying LFV B decay modes and LFV τ decay into three charged leptons. We find that the branching ratios $B(\tau^\pm \to \ell_1^\pm \ell_2^\pm \ell_3^\mp)$, $B(B_s \to \tau^\pm e^\mp)$ and $B(b \to X_s \ell_1^\pm \ell_2^\mp)$ need to be probed at the 10^{-8} level in order to constrain the mixing parameters beyond what is known from unitarity.

I. INTRODUCTION

In this paper, we study lepton flavor violating τ and B decays. In the minimal standard model (SM), generation lepton number is conserved. However, the observation of neutrino oscillations implies that family lepton number must be violated [1]. At present, it is not clear if the total lepton number is violated. The neutrino oscillation is due to a mismatch between the weak and mass eigenstates of neutrinos. This mismatch causes mixing between different generations of leptons in the charged current interaction with the W boson. In principle, flavor changing neutral current (FCNC) processes in the lepton sector occur as well. Some examples would be $\tau \to \ell \gamma$, $\tau \to \ell_1 \ell_2 \bar{\ell}_3$, $B \to \ell \bar{\ell}'$, and $B \to \ell \bar{\ell}' X_s$. Although no direct experimental evidence for such FCNC exists, there are experimental constraints [2–11]. The decays $\tau \to \mu(e)\gamma$ have recently been the subject of considerable attention [12]. They have been studied in connection with lepton flavor violating (LFV) occurring through mixing with heavy neutrinos in the context of supersymmetric theories where these modes are found to be a promising tool to constrain the models. In this paper, we investigate the potential rates for these processes in left-right (LR) models with heavy neutrinos.

FCNC in the lepton sector that are solely due to mixing in the charged current interaction with the usual left-handed W boson and light neutrinos are extremely small because they are suppressed by powers of m_ν^2/M_W^2 [13]. One way to increase the FCNC interaction in the lepton sector is to introduce heavy neutrinos so that the suppression factor m_ν^2/M_W^2 is not in effect. This can be done, for example, by introducing a heavy fourth generation. If one insists on having just three light left-handed neutrinos, one needs to give the right-handed neutrinos heavy Majorana

masses. The heavy neutrino can appear through mixing in the charged current interaction and enhance the FCNC interaction in the lepton sector. The introduction of right-handed neutrinos also raises the possibility of having right-handed charged currents by adding to the theory a right-handed W' boson. This new charged current interaction can lead to additional effects in the above decay processes and here we consider such a possibility [14].

A natural model of this type is the LR model based on the $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ gauge group [15]. In the left-right model, FCNC interactions arise from several sources. In this paper we consider the exchange of $W_{L,R}$ bosons at the one-loop level. It is well known that to obtain gauge invariant results one must also include charged Higgs boson effects at the one-loop level as well as tree-level exchange of neutral Higgs bosons [16]. We will comment on these effects but will not discuss them in detail as they depend on several unknown parameters. We concentrate on the effects that depend only on the W' mass and ignore those that depend on Higgs boson masses to illustrate the constraints that can be placed on the mixing with heavy Majorana neutrinos by the processes $\tau \to l\gamma$, $\tau \to \ell_1 \ell_2 \ell_3$, $B \to \ell \bar{\ell}'$, or $B \to \ell \bar{\ell}' X_s$. Our paper complements existing studies of the modes $\mu \to e\gamma$ and $K_L \to \mu e$ [14], and extends them to include the mixing parameters $U_{\tau N}$.

In LR models there are left-handed light neutrinos ν_L and right-handed heavy neutrinos ν_R', and these neutrinos can be Majorana particles. If there are three light and N heavy neutrinos, the general mass term for the neutrinos can be written as

$$L_M = -\frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R'^c) M^\nu \begin{pmatrix} \nu_L^c \\ \nu_R' \end{pmatrix} + \text{H.c.} \qquad (1)$$

M^ν is a symmetric matrix which can be diagonalized

$$\tilde{U}^T M^\nu \tilde{U} = \hat{M}^\nu, \qquad (2)$$

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with the aid of a unitary matrix $\tilde{U}$ resulting in $\hat{M}^\nu = \text{diag}(m_1, m_2, m_3, M_4, M_5, \ldots)$ with m_i and M_i denoting the light and heavy mass eigenvalues, respectively.

If there is no right-handed W-boson interaction, the number of right-handed heavy neutrinos is unrelated to the number of charged leptons. However, when a right-handed W boson is introduced and the heavy neutrinos are required to interact with it, it is natural to have the heavy neutrinos and the right-handed charged leptons form $SU(2)_R$ doublets. In this case there are as many heavy neutrinos as charged leptons (three).

The most general charged current interactions of charged leptons and neutrinos with W bosons can be parametrized in the weak interaction basis, as

$$
\begin{aligned}
\mathcal{L}_{\text{lepton}} &= -\frac{g_L}{\sqrt{2}} W^\mu \bar{L} \gamma_\mu (g_L^\ell P_L + g_R^\ell P_R)\nu \\
&\quad - \frac{g_R}{\sqrt{2}} W'^\mu \bar{L} \gamma_\mu (\tilde{g}_L^\ell P_L + \tilde{g}_R^\ell P_R)\nu'; \\
\mathcal{L}_{\text{quark}} &= -\frac{g_L}{\sqrt{2}} W^\mu \bar{U} \gamma_\mu (g_L^d P_L + g_R^d P_R)D \\
&\quad - \frac{g_R}{\sqrt{2}} W'^\mu \bar{U} \gamma_\mu (\tilde{g}_L^d P_L + \tilde{g}_R^d P_R)D,
\end{aligned}
\tag{3}
$$

where $L = (e, \mu, \tau)^T$, $\nu = (\nu_e, \nu_\mu, \nu_\tau)^T$, $\nu' = (\nu'_e, \nu'_\mu, \nu'_\tau)^T$, $U = (u, c, t)^T$, and $D = (d, s, b)^T$. In the above, W and W' denote the mass eigenstates of W bosons with W being mostly left handed and W' being mostly right handed.

In the mass eigenstate basis, we have [14]

$$
\begin{aligned}
\mathcal{L}_{\text{lepton}} &= -\frac{g_L}{\sqrt{2}} W^\mu \sum_{l=1}^3 \bar{\ell}_l^m \gamma_\mu (g_L^\ell P_L U_{\ell j}^{L*} + g_R^\ell P_R U_{\ell j}^R)\nu_j^m \\
&\quad - \frac{g_R}{\sqrt{2}} W'^\mu \sum_{l=1}^3 \bar{\ell}_l^m \gamma_\mu (\tilde{g}_L^\ell P_L U_{\ell j}^{L*} + \tilde{g}_R^\ell P_R U_{\ell j}^R)\nu_j^m; \\
\mathcal{L}_{\text{quark}} &= -\frac{g_L}{\sqrt{2}} W^\mu \bar{U}^m \gamma_\mu (g_L^d P_L V^L + g_R^d P_R V^R)D^m \\
&\quad - \frac{g_R}{\sqrt{2}} W'^\mu \bar{U}^m \gamma_\mu (\tilde{g}_L^d P_L V^L + \tilde{g}_R^d P_R V_R)D^m.
\end{aligned}
\tag{4}
$$

The left-handed and right-handed charged leptons are diagonalized by the matrices $S^{L,R}$: $\ell_L^m = S^L \ell_L$ and $\ell_R^m = S^R \ell_R$ and we have defined the matrices $U_{\ell j}^{L*} = \sum_{i=1}^3 S_{\ell i}^{\dagger L} \tilde{U}_{ij}^*$ and $U_{\ell j}^R = \sum_{l'=1}^3 S_{\ell \ell'}^{\dagger R} \tilde{U}_{(\ell'+3)j}$ with $\ell = e, \mu,$ and τ.

In what follows we will drop the superscript "m" from the fermion fields and always refer to mass eigenstates. Note that $U^{L,R}$ are 3×6 matrices and we construct the matrix

$$
U' = \begin{pmatrix} U^L \\ U^R \end{pmatrix},
\tag{5}
$$

with

$$
U^L = \begin{pmatrix} U_{e1}^L & U_{e2}^L & U_{e3}^L & U_{e4}^L & U_{e5}^L & U_{e6}^L \\ U_{\mu 1}^L & U_{\mu 2}^L & U_{\mu 3}^L & U_{\mu 4}^L & U_{\mu 5}^L & U_{\mu 6}^L \\ U_{\tau 1}^L & U_{\tau 2}^L & U_{\tau 3}^L & U_{\tau 4}^L & U_{\tau 5}^L & U_{\tau 6}^L \end{pmatrix} \quad \text{and}
$$

$$
U^R = \begin{pmatrix} U_{e1}^R & U_{e2}^R & U_{e3}^R & U_{e4}^R & U_{e5}^R & U_{e6}^R \\ U_{\mu 1}^R & U_{\mu 2}^R & U_{\mu 3}^R & U_{\mu 4}^R & U_{\mu 5}^R & U_{\mu 6}^R \\ U_{\tau 1}^R & U_{\tau 2}^R & U_{\tau 3}^R & U_{\tau 4}^R & U_{\tau 5}^R & U_{\tau 6}^R \end{pmatrix}.
\tag{6}
$$

This can be viewed as the unitary matrix which diagonalizes the neutrino mass matrix in the basis where the charged lepton mass matrix is already diagonal. The following relations hold:

$$
\sum_{j=1}^6 U_{\ell j}^{L*} U_{\ell' j}^L = \delta_{\ell\ell'}, \quad \sum_{j=1}^6 U_{\ell j}^{R*} U_{\ell' j}^R = \delta_{\ell\ell'}, \quad \sum_{j=1}^6 U_{\ell j}^L U_{\ell' j}^{R*} = 0,
$$

$$
\sum_{\ell=e,\mu,\tau} U_{\ell j}^{L*} U_{\ell i}^L + \sum_{\ell=e,\mu,\tau} U_{\ell i}^{R*} U_{\ell j}^R = \delta_{ij}.
\tag{7}
$$

There is some information on the matrix elements U_{e2}^L and $U_{\mu 3}^L$ from neutrino oscillation experiments [1], which prefer them to be in the ranges 0.50–0.69 and 0.60–0.80, respectively. For the processes that we discuss in this paper, there can only be large effects if there is substantial mixing with the heavy neutrinos and there are few constraints on these parameters. In seesaw models, the generic size of the matrix elements $U_{\ell,(4,5,6)}^L$ and $U_{\ell,(1,2,3)}^R$ is of order m_D/M_N, where m_D is the Dirac neutrino mass and M_N is the heavy Majorana neutrino mass. In such a scenario, the light neutrino masses are typically m_D^2/M_N. This requires the heavy neutrino masses to be heavier than a few hundred GeV, and results in the above mixing matrix elements being extremely small. In these models, contributions from one W exchange to radiatively induced penguin processes, and from box diagrams with two W's or one W and one W' exchanges are too small to be observed. Even in these models, however, the matrix elements $U_{l,(4,5,6)}^R$ can be of order one. For this type of model the exchange of W''s in both radiative penguin and box processes may produce observable effects. There are also special cases in which the elements $U_{l,(4,5,6)}^L$ and $U_{l,(1,2,3)}^R$ can be sizable. An example has been discussed in Ref. [14], where some of these matrix elements are of order m_D/M_N, but the light neutrino masses, at tree level, are not directly related to them and the ratio m_D/M_N does not need to be very small. Since our aim is not to study specific models, but to provide an estimate of the sensitivity needed in LFV τ and B decay modes in order to constrain the general left-right model mixing beyond the requirement of unitarity, we will treat the matrix elements in U' as arbitrary in this paper.

The couplings $g_{L,R}^{\ell,d}$ and $\tilde{g}_{L,R}^{\ell,d}$ are in general complex numbers. In renormalizable models without left-right gauge boson mixing, $g_L^{\ell,d} = 1$, $g_R^{\ell,d} = 0$, $\tilde{g}_L^{\ell,q} = 0$, and

$\tilde{g}_R^{\ell,q} = 1$. If there is left-right gauge boson mixing with a mixing angle ξ_W ($W = W_L \cos\xi_W + W_R \sin\xi_W$ and $W' = -W_L \sin\xi_W + W_R \cos\xi_W$), then

$$g_L^\ell = g_L^d = \cos\xi_W, \qquad g_R^\ell = g_R^d = \frac{g_R}{g_L} \sin\xi_W;$$

$$\tilde{g}_L^\ell = \tilde{g}_L^d = -\frac{g_L}{g_R} \sin\xi_W, \qquad \tilde{g}_R^\ell = \tilde{g}_R^d = \cos\xi_W. \tag{8}$$

Throughout the paper we will use the notation

$$\lambda_i \equiv \frac{m_i^2}{M_W^2}, \quad \beta \equiv \frac{M_W^2}{M_{W'}^2}, \quad \xi_g = \frac{g_R \xi_W}{g_L}, \quad \beta_g \equiv \frac{g_R^2 \beta}{g_L^2}. \tag{9}$$

For our loop calculations we will assume that all fermions are massless except for the top-quark and the heavy right-handed neutrinos. We will also assume that β is smaller than a few percent in keeping with bounds on W' bosons [17]. Finally, we will assume that $W_L - W_R$ mixing is small as indicated by $b \to s\gamma$. In particular, following [18], we found in [19] at the 2σ level that there are two allowed ranges for ξ_g. They correspond to destructive and constructive interference with the standard model amplitude, respectively, and are

$$-0.032 < \frac{V_{tb}^R}{V_{tb}^L} \xi_g < -0.027,$$
$$-0.0016 < \frac{V_{tb}^R}{V_{tb}^L} \xi_g < 0.0037. \tag{10}$$

Overall, ξ_g is constrained to be very small, and in our analysis we will only keep terms linear in ξ_g in the matrix elements.

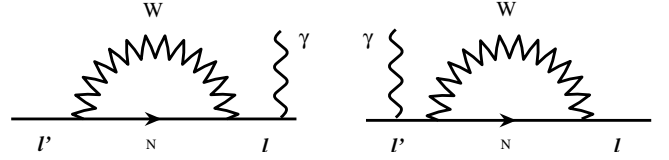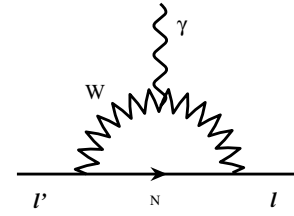We will use this framework to examine LFV induced by neutrino mixing (with new heavy neutrinos). Our purpose

FIG. 1. Diagrams giving rise to $\ell' \to \ell\gamma$ in unitary gauge.

is to provide an estimate of the sensitivity needed in LFV τ and B decay modes in order to constrain this scenario beyond the requirement of unitarity of the matrix U'.

II. LFV RADIATIVE τ DECAY

Beginning with $\mu \to e\gamma$ [20], processes of the form $\ell' \to \ell\gamma$ have been used to constrain new physics, including heavy neutrinos. Here we consider the case of τ decay. The one-loop effective operator can be calculated in unitary gauge from the diagrams in Fig. 1. We first consider the case of a very heavy W' so that only the W is exchanged in the loop. In keeping with current experimental constraints Eq. (10), we work only to first order in the $W_L - W_R$ mixing parameter ξ_W. This results in two operators which we write in the form

$$\mathcal{L} = 4 \frac{G_F}{\sqrt{2}} \frac{e}{16\pi^2} F^{\mu\nu} \sum_N \left[U_{\ell N}^{L*} U_{\ell' N}^{L} F(\lambda_N) m_{\ell'} \bar{\ell} \sigma_{\mu\nu} P_R \ell' + \frac{g_R}{g_L} \xi_W \tilde{F}(\lambda_N) M_N \bar{\ell} \sigma_{\mu\nu} (U_{\ell N}^{R} U_{\ell' N}^{L} P_L + U_{\ell N}^{L*} U_{\ell' N}^{R*} P_R) \ell' \right]. \tag{11}$$

Here $F^{\mu\nu}$ is the electromagnetic field strength tensor and the Inami-Lim functions are given by

$$F(\lambda_N) = \left[\frac{3\lambda_N^3 \log\lambda_N}{4(1-\lambda_N)^4} + \frac{2\lambda_N^3 + 5\lambda_N^2 - \lambda_N}{8(1-\lambda_N)^3} \right], \qquad \tilde{F}(\lambda_N) = \left[\frac{3\lambda_N^2 \log\lambda_N}{2(1-\lambda_N)^3} - \frac{\lambda_N^2 - 11\lambda_N + 4}{4(1-\lambda_N)^2} \right]. \tag{12}$$

The exchange of a W' can be easily included and it leads to similar expressions:

$$\mathcal{L} = 4\beta \frac{G_F}{\sqrt{2}} \frac{e}{16\pi^2} F^{\mu\nu} \sum_N \left[U_{\ell N}^{R} U_{\ell' N}^{R*} F(\beta\lambda_N) m_{\ell'} \bar{\ell} \sigma_{\mu\nu} P_L \ell' - \frac{g_L}{g_R} \xi_W \tilde{F}(\beta\lambda_N) M_N \bar{\ell} \sigma_{\mu\nu} (U_{\ell N}^{R} U_{\ell' N}^{L} P_L + U_{\ell N}^{L*} U_{\ell' N}^{R*} P_R) \ell' \right]. \tag{13}$$

We now calculate the branching ratio for $\ell' \to \ell\gamma$ neglecting m_ℓ. We first consider the case without $W_L - W_R$ mixing, dominated by the first operator in Eq. (11). We find

$$\Gamma(\ell' \to \ell\gamma) = \frac{G_F^2 \alpha}{32\pi^4} m_{\ell'}^5 \left| \sum_N U_{\ell N}^{L*} U_{\ell' N}^{L} F(\lambda_N) \right|^2. \tag{14}$$

For τ decay, it is convenient to express this result as a fraction of the rate

$$\Gamma(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu) = \frac{G_F^2 m_\tau^5}{192\pi^3}, \tag{15}$$

to obtain for $\ell = \mu, e$,

$$R_\ell \equiv \frac{\Gamma(\tau^- \to \ell\gamma)}{\Gamma(\tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu)} = \left(\frac{6\alpha}{\pi}\right) \left| \sum_N U_{\ell N}^{L*} U_{\tau N}^{L} F(\lambda_N) \right|^2. \tag{16}$$

The current experimental bounds $B(\tau \to \mu\gamma) \leq 3.1 \times 10^{-7}$ [2], $B(\tau \to e\gamma) \leq 2.7 \times 10^{-6}$ [3], imply that

$$R_\mu \leq 1.8 \times 10^{-6}, \qquad R_e \leq 1.6 \times 10^{-5}, \qquad (17)$$

and these, in turn, can be used to place the constraints

$$\left| \sum_N U_{\mu N}^{L*} U_{\tau N}^L F(\lambda_N) \right|^2 \leq 1.2 \times 10^{-4},$$

$$\left| \sum_N U_{eN}^{L*} U_{\tau N}^L F(\lambda_N) \right|^2 \leq 1.0 \times 10^{-3}. \qquad (18)$$

If the mixing angles are such that only one heavy neutrino is important, we may use $F(x) \to -1/4$ as $x \to \infty$ to estimate that

$$|U_{\mu N}^{L*} U_{\tau N}^L| \leq 0.044, \qquad |U_{eN}^{L*} U_{\tau N}^L| \leq 0.13. \qquad (19)$$

For comparison, the experimental bound $B(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [17] leads to the constraint

$$|U_{eN}^{L*} U_{\mu N}^L| \leq 1.2 \times 10^{-4}. \qquad (20)$$

Similarly, for W' exchange, one obtains

$$\Gamma(\ell' \to \ell\gamma) = \frac{G_F^2 \alpha}{32\pi^4} m_{\ell'}^5 \left| \sum_N U_{\ell N}^R U_{\ell' N}^{R*} \beta F(\beta\lambda_N) \right|^2. \qquad (21)$$

Considering once again the case where only one heavy neutrino comes into play, and with $\lambda_N \to \infty$, this leads to

$$|U_{\mu N}^R U_{\tau N}^{R*}| \leq \frac{0.044}{\beta}, \qquad |U_{eN}^R U_{\tau N}^{R*}| \leq \frac{0.13}{\beta}. \qquad (22)$$

For a typical $\beta \sim 0.01$ these limits are about an order of magnitude worse than the unitarity constraints. The limits become weaker by a factor of 4 for $M_N \sim M_R$.

If the mixing parameter ξ_W is not zero, the second operator in Eq. (11) can dominate the rate because it is not proportional to the light lepton mass. In this case we

obtain

$$\Gamma(\ell' \to \ell\gamma) = \frac{G_F^2 \alpha}{32\pi^4} m_{\ell'}^3 M_W^2 \xi_g^2 \left[\left| \sum_N U_{\ell N}^R U_{\ell' N}^L \sqrt{\lambda_N} \tilde{F}(\lambda_N) \right.\right.$$
$$\left.\left. \times \right|^2 + \left| \sum_N U_{\ell N}^{L*} U_{\ell' N}^{R*} \sqrt{\lambda_N} \tilde{F}(\lambda_N) \right|^2 \right]. \qquad (23)$$

With only one heavy neutrino, and using $\tilde{F}(x) \to -1/4$ as $x \to \infty$, the constraints are

$$\xi_g^2 \lambda_N (|U_{\mu N}^R|^2 |U_{\tau N}^L|^2 + |U_{\mu N}^L|^2 |U_{\tau N}^R|^2) \leq 9.3 \times 10^{-7},$$

$$\xi_g^2 \lambda_N (|U_{eN}^R|^2 |U_{\tau N}^L|^2 + |U_{eN}^L|^2 |U_{\tau N}^R|^2) \leq 8.0 \times 10^{-6},$$

$$\xi_g^2 \lambda_N (|U_{eN}^R|^2 |U_{\mu N}^L|^2 + |U_{eN}^L|^2 |U_{\mu N}^R|^2) \leq 2.4 \times 10^{-14}. \qquad (24)$$

The last result follows from the corresponding analysis for $\mu \to e\gamma$. For LR models with $W_L - W_R$ mixing, LFV B decay modes are proportional to ξ_g^4, making Eq. (24) the most stringent constraint in this case.

III. LFV τ DECAY INTO THREE CHARGED LEPTONS

The pure radiative decays discussed so far do not constrain LR models without $W_L - W_R$ mixing because there is no $W_L^\pm W_R^\mp \gamma$ vertex. Similarly, there is no $W_L^\pm W_R^\mp Z$ vertex, and the modes $\tau^- \to \ell_1^- \ell_2^- \ell_3^+$ proceed through box diagrams at leading order. We now derive the constraints that can be placed on the neutrino mixing matrix from these modes.

The effective operator responsible for these decay modes can be calculated from the diagram in Fig. 2 plus two other diagrams obtained by interchanging $W_L \leftrightarrow W_R$ and by interchanging $\ell_1 \leftrightarrow \ell_2$. Using dimensional regularization we find

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \beta_g \sum_{N_i, N_j} \left\{ E_{LR}^S \left(\lambda_{N_i}, \lambda_{N_j}, \beta\right) \left[U_{\tau N_i}^L U_{\ell_1 N_i}^R U_{\ell_2 N_j}^{L*} U_{\ell_3 N_j}^{R*} \bar{\ell}_2 P_R \ell_3 \bar{\ell}_1 P_L \tau + U_{\tau N_i}^{R*} U_{\ell_1 N_i}^{L*} U_{\ell_2 N_j}^R U_{\ell_3 N_j}^L \bar{\ell}_2 P_L \ell_3 \bar{\ell}_1 P_R \tau \right] \right.$$

$$+ E_{LR}^T(\lambda_{N_i}, \lambda_{N_j}, \beta) \left[U_{\tau N_i}^L U_{\ell_1 N_i}^R U_{\ell_2 N_j}^{L*} U_{\ell_3 N_j}^{R*} \bar{\ell}_2 \gamma_\nu \gamma_\mu P_R \ell_3 \bar{\ell}_1 \gamma_\mu \gamma_\nu P_L \tau + U_{\tau N_i}^{R*} U_{\ell_1 N_i}^{L*} U_{\ell_2 N_j}^R U_{\ell_3 N_j}^L \bar{\ell}_2 \gamma_\nu \gamma_\mu P_L \ell_3 \bar{\ell}_1 \gamma_\mu \gamma_\nu P_R \tau \right]$$

$$+ \left(\ell_1 \leftrightarrow \ell_2 \right) \bigg\}, \qquad (25)$$

where the Inami-Lim functions $E_{LR}^{S,T}$ are given by

$$E^S_{LR}(\lambda_{N_i}, \lambda_{N_j}, \beta) = \sqrt{\lambda_{N_i}\lambda_{N_j}}\left[\frac{1}{\hat{\epsilon}} + \log\left(\frac{\mu^2}{M^2_W}\right) + 1 + \frac{\log\beta}{(1-\beta)(1-\beta\lambda_{N_i})(1-\beta\lambda_{N_j})} + \frac{\beta\lambda^3_{N_i}\log\lambda_{N_i}}{(\lambda_{N_j}-\lambda_{N_i})(1-\lambda_{N_i})(1-\beta\lambda_{N_i})}\right.$$

$$\left. + \frac{\beta\lambda^3_{N_j}\log\lambda_{N_j}}{(\lambda_{N_i}-\lambda_{N_j})(1-\lambda_{N_j})(1-\beta\lambda_{N_j})}\right],$$

$$E^T_{LR}(\lambda_{N_i}, \lambda_{N_j}, \beta) = \sqrt{\lambda_{N_i}\lambda_{N_j}}\left\{\frac{\lambda_{N_i}\log\lambda_{N_i}}{(\lambda_{N_j}-\lambda_{N_i})(1-\lambda_{N_i})(1-\beta\lambda_{N_i})}\left[1 - \frac{\lambda_{N_i}}{4}(1+\beta)\right] + \frac{\lambda_{N_j}\log\lambda_{N_j}}{(\lambda_{N_i}-\lambda_{N_j})(1-\lambda_{N_j})(1-\beta\lambda_{N_j})}\right.$$

$$\left. \times\left[1 - \frac{\lambda_{N_j}}{4}(1+\beta)\right] + \frac{(3\beta-1)\log\beta}{4(1-\beta)(1-\beta\lambda_{N_i})(1-\beta\lambda_{N_j})}\right\}. \tag{26}$$

This result is divergent and we have regulated the divergence by defining

$$\frac{1}{\hat{\epsilon}} = \frac{2}{4-n} + \log 4\pi - \gamma. \tag{27}$$

The divergence arises because in LR models these box diagrams are not the only ones that contribute to this process. In particular, these models require the existence of neutral scalars with tree-level flavor changing couplings [16]. These scalars give rise to a tree-level amplitude for this process as well as to several additional one-loop diagrams. To keep our analysis as model independent and simple as possible, we will not specify the scalar sector of the left-right models. Instead we will use Eq. (26), drop the $1/\hat{\epsilon}$ pole, and take a scale $\mu \sim 1$ TeV. This approach can be considered as a limit in which the scalars that make the left-right model renormalizable are very heavy. To gain some insight into our prescription, we compare our result to the complete calculation of Ref. [14] for $K_L \rightarrow \mu e$ in the appendix.

Taking both the muon and electron to be massless, we can calculate the rates:

$$\Gamma(\tau \rightarrow \ell^-_1\ell^-_2\ell^+_3) = \frac{G^2_F m^5_\tau}{192\pi^3}\left(\frac{\alpha^2}{128\pi^2\sin^4\theta_W}\right)\beta^2_g\left\{\left|\sum_{N_iN_j} U^L_{\tau N_i}U^R_{\ell_1 N_i}U^{L*}_{\ell_2 N_j}U^{R*}_{\ell_3 N_j}E^S_{LR}\right|^2 + 64\left|\sum_{N_iN_j}U^L_{\tau N_i}U^R_{\ell_1 N_i}U^{L*}_{\ell_2 N_j}U^{R*}_{\ell_3 N_j}E^T_{LR}\right|^2\right.$$

$$+ 8\,\text{Re}\left[\left(\sum_{N_iN_j}U^L_{\tau N_i}U^R_{\ell_1 N_i}U^{L*}_{\ell_2 N_j}U^{R*}_{\ell_3 N_j}E^S_{LR}\right)\left(\sum_{N_iN_j}U^{L*}_{\tau N_i}U^{R*}_{\ell_1 N_i}U^L_{\ell_2 N_j}U^R_{\ell_3 N_j}E^T_{LR}\right)\right]$$

$$+ \left|\sum_{N_iN_j}U^{R*}_{\tau N_i}U^{L*}_{\ell_1 N_i}U^R_{\ell_2 N_j}U^L_{\ell_3 N_j}E^S_{LR}\right|^2 + 64\left|\sum_{N_iN_j}U^{R*}_{\tau N_i}U^{L*}_{\ell_1 N_i}U^R_{\ell_2 N_j}U^L_{\ell_3 N_j}E^T_{LR}\right|^2$$

$$+ 8\,\text{Re}\left[\left(\sum_{N_iN_j}U^{R*}_{\tau N_i}U^{L*}_{\ell_1 N_i}U^R_{\ell_2 N_j}U^L_{\ell_3 N_j}E^S_{LR}\right)\left(\sum_{N_iN_j}U^R_{\tau N_i}U^L_{\ell_1 N_i}U^{R*}_{\ell_2 N_j}U^{L*}_{\ell_3 N_j}E^T_{LR}\right)\right] + (\ell_1 \leftrightarrow \ell_2)\right\}. \tag{28}$$

Before comparing with experiment it is convenient to define the ratio

$$R_{123}(\tau \rightarrow \ell^-_1\ell^-_2\ell^+_3) \equiv \frac{\Gamma(\tau \rightarrow \ell^-_1\ell^-_2\ell^+_3)}{\Gamma(\tau^- \rightarrow \mu^-\nu_\tau\bar{\nu}_\mu)}, \tag{29}$$

and normalize the rates this way. When only one heavy neutrino N is important, this simplifies to

$$R_{123}(\tau \rightarrow \ell^-_1\ell^-_2\ell^+_3) = (|U^L_{\tau N}U^R_{\ell_1 N}|^2|U^{L*}_{\ell_2 N}U^{R*}_{\ell_3 N}|^2$$
$$+ |U^{R*}_{\tau N}U^{L*}_{\ell_1 N}|^2|U^R_{\ell_2 N}U^L_{\ell_3 N}|^2$$
$$+ |U^L_{\tau N}U^R_{\ell_2 N}|^2|U^{L*}_{\ell_1 N}U^{R*}_{\ell_3 N}|^2$$
$$+ |U^{R*}_{\tau N}U^{L*}_{\ell_2 N}|^2|U^R_{\ell_1 N}U^L_{\ell_3 N}|^2)$$
$$\times F_{LR}(\lambda_N, \beta), \tag{30}$$

where we have defined the form factor

$$F_{LR}(\lambda_N, \beta) \equiv \frac{\alpha^2\beta^2_g}{128\pi^2\sin^4\theta_W}[E^S_{LR}(\lambda_N, \lambda_N, \beta)^2$$
$$+ 8E^S_{LR}(\lambda_N, \lambda_N, \beta)E^T_{LR}(\lambda_N, \lambda_N, \beta)$$
$$+ 64E^T_{LR}(\lambda_N, \lambda_N, \beta)^2]. \tag{31}$$

With the experimental limits from Ref. [5,6], we find

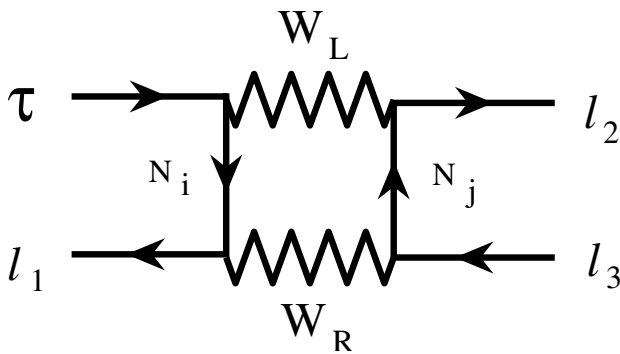FIG. 2.   Box diagram responsible for the decays $\tau^- \rightarrow \ell^-_1\ell^-_2\ell^+_3$ in models with no $W_L - W_R$ mixing.

$$R_{e\mu\mu} = F_{LR}(\lambda_N, \beta)(|U_{\tau N}^L|^2|U_{\mu N}^R|^2 + |U_{\tau N}^R|^2|U_{\mu N}^L|^2)(|U_{\mu N}^L|^2|U_{eN}^R|^2 + |U_{eN}^L|^2|U_{\mu N}^R|^2) < 1.2 \times 10^{-6},$$

$$R_{\mu ee} = F_{LR}(\lambda_N, \beta)(|U_{\tau N}^L|^2|U_{eN}^R|^2 + |U_{\tau N}^R|^2|U_{eN}^L|^2)(|U_{\mu N}^L|^2|U_{eN}^R|^2 + |U_{eN}^L|^2|U_{\mu N}^R|^2) < 1.1 \times 10^{-6},$$

$$R_{\mu\mu e} = 2F_{LR}(\lambda_N, \beta)|U_{\mu N}^L|^2|U_{\mu N}^R|^2(|U_{\tau N}^L|^2|U_{eN}^R|^2 + |U_{\tau N}^R|^2|U_{eN}^L|^2) < 1.2 \times 10^{-6},$$

$$R_{ee\mu} = 2F_{LR}(\lambda_N, \beta)|U_{eN}^L|^2|U_{eN}^R|^2(|U_{\tau N}^L|^2|U_{\mu N}^R|^2 + |U_{\tau N}^R|^2|U_{\mu N}^L|^2) < 1.2 \times 10^{-6},$$

$$R_{\mu\mu\mu} = F_{LR}(\lambda_N, \beta)|U_{\mu N}^L|^2|U_{\mu N}^R|^2(|U_{\tau N}^L|^2|U_{\mu N}^R|^2 + |U_{\tau N}^R|^2|U_{\mu N}^L|^2) < 1.2 \times 10^{-6},$$

$$R_{eee} = F_{LR}(\lambda_N, \beta)|U_{eN}^L|^2|U_{eN}^R|^2(|U_{\tau N}^L|^2|U_{eN}^R|^2 + |U_{\tau N}^R|^2|U_{eN}^L|^2) < 2.0 \times 10^{-6}. \tag{32}$$

In Fig. 3 we show the value of $F_{LR}(\lambda_N, \beta)$ for $g_R = g_L$ as a function of β for selected values of λ_N. This figure indicates that, for a wide range of parameters, $F_{LR}(\lambda_N, \beta)$ is between 10^{-7} and 10^{-6}. With this range in Eq. (32), we can see that the present experimental limits on R_{123} do not yield constraints on the mixing parameters that are significantly better than the unitarity bounds.

A similar exercise for muon decay yields [17]

$$\frac{\Gamma(\mu \to eee)}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)} = F_{LR}(\lambda_N, \beta)|U_{eN}^L|^2|U_{eN}^R|^2(|U_{\mu N}^L|^2|U_{eN}^R|^2$$

$$+ |U_{eN}^L|^2|U_{\mu N}^R|^2) \leq 1.0 \times 10^{-12}. \tag{33}$$

In this case, if we assume that all the angles are of the same order, we obtain the constraint $U_{ij}^{LR} \leq 0.2$. The τ decay modes are still far from achieving this level of sensitivity. If we write, for example,
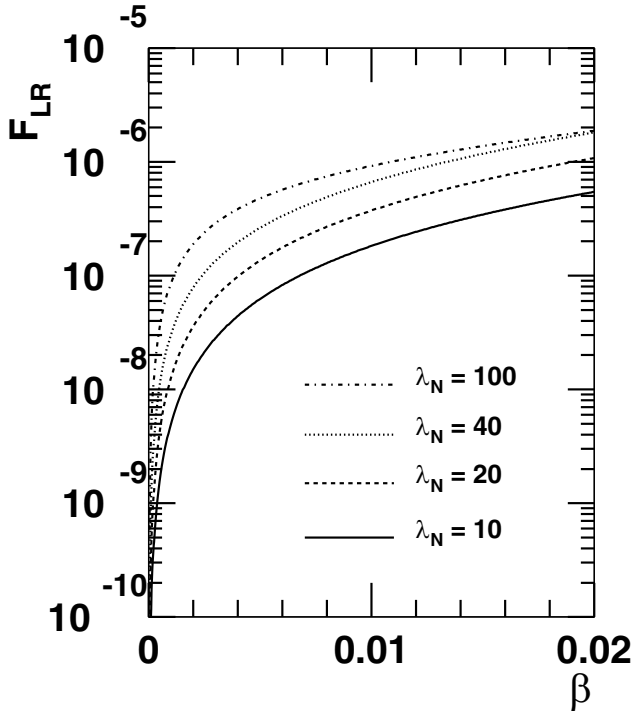
$$R_{\mu ee} = \left(\left| \frac{U_{\tau N}^{L*}}{U_{eN}^{L*}} \right|^2 + \left| \frac{U_{\tau N}^R}{U_{eN}^R} \right|^2 \right)\frac{\Gamma(\mu \to eee)}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)}$$

$$\leq 1.0 \times 10^{-12}\left(\left| \frac{U_{\tau N}^{L*}}{U_{eN}^{L*}} \right|^2 + \left| \frac{U_{\tau N}^R}{U_{eN}^R} \right|^2 \right), \tag{34}$$

we see that the τ decay bounds need to improve by 6 orders of magnitude in order to be competitive with the already available muon decay limit. Of course, the muon decay is not sensitive to all the parameters needed to describe neutrino mixing in general and the τ decay data is complementary.

IV. LFV *B* DECAY: OPERATORS

We now turn our attention to *B* decay modes and start by calculating the basic quark level LFV process $b \to d_j\bar{\ell}\ell'$. In unitary gauge, the process occurs through the box diagram of Fig. 4 . We distinguish several cases as before: left-handed heavy neutrinos; LR models with $W_L - W_R$ mixing; right-handed heavy neutrinos; and LR models without $W_L - W_R$ mixing.

A. Left-handed heavy neutrinos

A straightforward calculation of the diagram in Fig. 4 produces the operator

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{8\pi\sin^2\theta_W}V_{td_j}^{L*}V_{tb}^L U_{\ell N}^{L*}U_{\ell' N}^L E_L(\lambda_t, \lambda_N)$$

$$\times \bar{\ell}\gamma_\mu P_L\ell'\bar{d}_j\gamma^\mu P_L b, \tag{35}$$

where d_j refers to a d or an s quark. With the aid of the

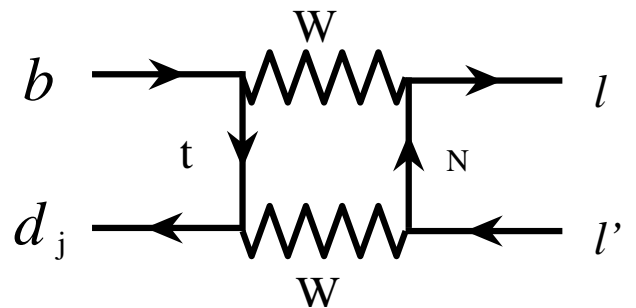FIG. 3. $F_{LR}(\lambda_N, \beta)$ with $g_R = g_L$ as a function of β and λ_N.

FIG. 4.   Box diagram responsible for the process $b \to d_j\bar{\ell}\ell'$.

unitary relations of Eq. (7), we find

$$E_L(\lambda_t, \lambda_N) = \lambda_t \lambda_N \left[\frac{3}{(1 - \lambda_N)(1 - \lambda_t)} \right.$$
$$+ \frac{(4 - 8\lambda_t + \lambda_t^2) \log \lambda_t}{(\lambda_N - \lambda_t)(1 - \lambda_t)^2}$$
$$\left. + \frac{(4 - 8\lambda_N + \lambda_N^2) \log \lambda_N}{(\lambda_t - \lambda_N)(1 - \lambda_N)^2} \right]. \quad (36)$$

This result is in agreement with the existing result for $K_L \to \mu^\pm e^\mp$ [14] when the b quark is replaced by an s quark.

B. Left-right model with $W_L - W_R$ mixing

In LR models with mixing there is a second operator that can be obtained from Fig. 4:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \xi_g^2 [\tilde{E}_L^S(\lambda_t, \lambda_N) \bar{\ell}(U_{\ell N}^L U_{\ell' N}^R P_R$$
$$+ U_{\ell N}^{R*} U_{\ell' N}^{L*} P_L) \ell' \bar{d}_j (V_{td_j}^{L*} V_{tb}^R P_R + V_{td_j}^{R*} V_{tb}^L P_L) b$$
$$+ \tilde{E}_L^T(\lambda_t, \lambda_N) \bar{\ell} \gamma_\nu \gamma_\mu (U_{\ell N}^L U_{\ell' N}^R P_R + U_{\ell N}^{R*} U_{\ell' N}^{L*} P_L)$$
$$\times \ell' \bar{d}_j \gamma^\mu \gamma^\nu (V_{td_j}^{L*} V_{tb}^R P_R + V_{td_j}^{R*} V_{tb}^L P_L) b]. \quad (37)$$

The Inami-Lim functions are the same as those in Eq. (26) with $\beta = 1$. As mentioned above, these operators produce observables proportional to ξ_g^4 and cannot place constraints that are competitive with LFV radiative τ decay.

C. Right-handed heavy neutrino

Models with a mostly right-handed heavy neutrino would proceed through the diagram in Fig. 4 with two W' bosons exchanged. Ignoring the $W_L - W_R$ mixing, this results in an operator

$$\mathcal{L} = \beta \frac{G_F}{\sqrt{2}} \frac{\alpha}{8\pi \sin^2 \theta_W} V_{td_j}^{R*} V_{tb}^R U_{\ell N}^R U_{\ell' N}^{R*} E_L(\beta \lambda_t, \beta \lambda_N)$$
$$\times \bar{\ell} \gamma_\mu P_R \ell' \bar{d}_j \gamma^\mu P_R b. \quad (38)$$

A glance at Eq. (36) reveals that this operator will make contributions to LFV B decay rates that are suppressed by at least a factor of β^4.

D. Left-right models without $W_L - W_R$ mixing

In this scenario, the heavy neutrinos have left- and right-handed couplings and both the W and W' appear. The LFV operator for B decay arises from box diagrams like the one in Fig. 4 with one W and one W'; we find

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \beta_g \left[E_{LR}^S(\lambda_t, \lambda_N, \beta) \right.$$
$$\times (V_{td_j}^{L*} V_{tb}^R U_{\ell N}^R U_{\ell' N}^L \bar{\ell} P_L \ell' \bar{d}_j P_R b$$
$$+ V_{td_j}^{R*} V_{tb}^L U_{\ell N}^{L*} U_{\ell' N}^{R*} \bar{\ell} P_R \ell' \bar{d}_j P_L b) + E_{LR}^T(\lambda_t, \lambda_N, \beta)$$
$$\times (V_{td_j}^{L*} V_{tb}^R U_{\ell N}^R U_{\ell' N}^L \bar{\ell} \gamma_\nu \gamma_\mu P_L \ell' \bar{d}_j \gamma^\mu \gamma^\nu P_R b$$
$$\left. + V_{td_j}^{R*} V_{tb}^L U_{\ell N}^{L*} U_{\ell' N}^{R*} \bar{\ell} \gamma_\nu \gamma_\mu P_R \ell' \bar{d}_j \gamma_\mu \gamma_\nu P_L b) \right], \quad (39)$$

where the Inami-Lim functions were given in Eq. (26).

For the process $B \to \bar{\ell} \ell'$, the two operators in Eq. (39) can be reduced to one by using the relation

$$\gamma_\nu \gamma_\mu \otimes \gamma_\mu \gamma_\nu \to 4(1 \otimes 1) + \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}, \quad (40)$$

and dropping the last term in anticipation of the vanishing of the matrix element

$$\langle 0 | \bar{d}_j \sigma_{\mu\nu} (1 \pm \gamma_5) b | \bar{B}_j^0 \rangle = 0. \quad (41)$$

We obtain

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \beta_g [E_{LR}^S(\lambda_t, \lambda_N, \beta)$$
$$+ 4E_{LR}^T(\lambda_t, \lambda_N, \beta)](V_{td_j}^{L*} V_{tb}^R U_{\ell N}^R U_{\ell' N}^L \bar{\ell} P_L \ell' \bar{d}_j P_R b$$
$$+ V_{td_j}^{R*} V_{tb}^L U_{\ell N}^{L*} U_{\ell' N}^{R*} \bar{\ell} P_R \ell' \bar{d}_j P_L b). \quad (42)$$

V. LFV B DECAY PHENOMENOLOGY

We now use the operators obtained in the previous section to compute their contribution to selected LFV B decay modes.

A. $B_{d_j} \to \tau^\pm \ell^\mp$

We first consider the mode $B \to \tau^\pm \ell^\mp$, the analogue of the $K_L \to \mu^\pm e^\mp$ mode which has been discussed extensively in the literature [14,21].

For a heavy left-handed neutrino, we find after summing the two modes and neglecting the mass of $\ell = \mu, e$,

$$\Gamma(B_j \to \tau^\pm \ell^\mp) = \frac{1}{256} \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi \sin^2 \theta_W} \right)^2 F_B^2 m_\tau^2 M_B$$
$$\times \left(1 - \frac{m_\tau^2}{M_B^2} \right)^2 |V_{tb}^{L\star} V_{tj}^L|^2$$
$$\times \left| \sum_N U_{\ell N}^{L*} U_{\tau N}^L E_L(\lambda_t, \lambda_N) \right|^2. \quad (43)$$

When the heavy neutrino has both left- and right-handed couplings (with an explicit W' and no mixing) we find

$$\Gamma(B_j \to \tau^\pm \ell^\mp) = \frac{1}{32} \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi \sin^2 \theta_W}\right)^2 F_B^2 \frac{M_B^5}{m_b^2} \beta_g^2 \left(1 - \frac{m_\tau^2}{M_B^2}\right)^2 \left[\left| \sum_N V_{tb}^{L\star} V_{tj}^R U_{\ell N}^R U_{\tau N}^L \left(E_{LR}^S(\lambda_t, \lambda_N, \beta) + 4E_{LR}^T(\lambda_t, \lambda_N, \beta)\right) \right. \right.$$

$$\left. \left. \times \right|^2 + \left| \sum_N V_{tb}^{R\star} V_{tj}^L U_{\ell N}^{L*} U_{\tau N}^{R*} (E_{LR}^S(\lambda_t, \lambda_N, \beta) + 4E_{LR}^T(\lambda_t, \lambda_N, \beta)) \right|^2 \right]. \tag{44}$$

For numerical purposes it is natural to compare these rates to the standard model rate for $B_{d_j} \to \tau^+ \tau^-$,

$$\Gamma(B_j \to \tau^+ \tau^-) = \frac{G_F^2}{\pi} \left(\frac{\alpha}{4\pi \sin^2 \theta_W}\right)^2 F_B^2 m_\tau^2 M_B \sqrt{1 - \frac{4m_\tau^2}{M_B^2}}$$
$$\times |V_{tb}^\star V_{tj}|^2 Y^2(\lambda_t), \tag{45}$$

where the Inami-Lim function $Y(\lambda_t) \sim 1.06$ [22]. We define

$$R_{B\tau\ell} \equiv \frac{\Gamma(B_j \to \tau^\pm \ell^\mp)}{\Gamma(B_j \to \tau^+ \tau^-)}, \tag{46}$$

and in terms of this definition we find for left-handed heavy neutrinos

$$R_{B\tau\ell} = 3.7 \times 10^{-3} \left| \sum_N U_{\ell N}^{L*} U_{\tau N}^L E_L(\lambda_t, \lambda_N) \right|^2. \tag{47}$$

To compare the sensitivity of the modes $B \to \tau^\pm \ell^\mp$ and $\tau \to \ell\gamma$ to the neutrino mixing parameters, we plot in Fig. 5 the ratio $(E_L/F)^2$ as a function of λ_N.

Assuming there is only one heavy neutrino and using Eq. (18) as well as $(E_L/F)^2 \sim 7500$ from Fig. 5, we find

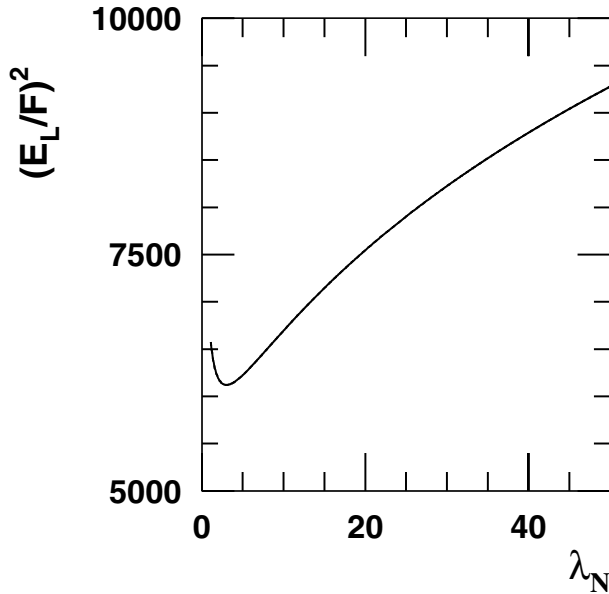$$R_{B\tau\mu} \leq 3.3 \times 10^{-3}, \qquad R_{B\tau e} \leq 2.8 \times 10^{-2}. \tag{48}$$

FIG. 5.   Ratio of form factors $(E_L/F)^2$ as a function of λ_N.

The standard model expectations for $B_{d_j} \to \tau^+ \tau^-$ [22] with the central values for the CKM angles found in [23] are

$$B(B_s \to \tau^+ \tau^-) = 1.1 \times 10^{-6},$$
$$B(B_d \to \tau^+ \tau^-) = 3.3 \times 10^{-8}. \tag{49}$$

Consequently, one would need a single event sensitivity of at least 10^{-8} for $B(B_s \to \tau^\pm e^\mp)$ [10^{-9} for $B(B_s \to \tau^\pm \mu^\mp)$] to improve on the existing constraints from radiative τ decay. A glance at Table I indicates that one would need an order of magnitude improvement over the current best limit from Belle for $B \to e^\pm \mu^\mp$. There is some hope that this sensitivity may be attainable in the future. For example, the estimated single event sensitivity for the $B \to \mu\mu$ modes at the Collider Detector at Fermilab (CDF) with $15fb^{-1}$ is [24]

$$1.3 \times 10^{-9} \quad \text{for } B_s \to \mu^\pm \mu^\mp,$$
$$4.7 \times 10^{-10} \quad \text{for } B_d \to \mu^\pm \mu^\mp. \tag{50}$$

Of course, there are additional experimental difficulties for modes involving a τ lepton, but we may regard Eq. (48) as the benchmark needed to improve upon limits from radiative τ decay. For comparison, the same analysis applied to the limit $B(K_L \to \mu^\pm e^\mp) < 4.7 \times 10^{-12}$ [17] yields the bound

$$|U_{eN}^{L*} U_{\mu N}^L| \leq 0.07, \tag{51}$$

which is also weaker than the corresponding bound from $\mu \to e\gamma$ [Eq. (20)].

For right-handed heavy neutrinos, Eq. (47) becomes

$$R_{B\tau\ell} = 3.7 \times 10^{-3} \left| \sum_N U_{\ell N}^R U_{\tau N}^{R*} \beta E_L(\beta\lambda_t, \beta\lambda_N) \right|^2. \tag{52}$$

The bounds obtained are worse than those for left-handed heavy neutrinos.

TABLE I.   Summary of current experimental bounds for $B \to \ell^+ \ell'^-$.

	Branching ratio
$B \to e^\pm \mu^\mp$	$<1.8 \times 10^{-7}$ Babar [7]
	$<1.7 \times 10^{-7}$ Belle [8]
$B \to e^\pm \tau^\mp$	$<1.1 \times 10^{-4}$ Cleo [9]
$B \to \mu^\pm \tau^\mp$	$<3.8 \times 10^{-5}$ Cleo [9]

Finally, for the case where the heavy neutrino has both left- and right-handed couplings and the $W_L - W_R$ mixing can be ignored, we obtain

$$R_{B\tau\ell} \sim 0.36\beta_g^2 \frac{1}{|V_{tb}^\star V_{tj}|^2}$$

$$\times \left[\left| \sum_N V_{tb}^{L\star} V_{tj}^R U_{\ell N}^R U_{\tau N}^L [E_{LR}^S(\lambda_t, \lambda_N, \beta) \right. \right.$$

$$+ 4E_{LR}^T(\lambda_t, \lambda_N, \beta)]$$

$$\times \left| {}^2 + \left| \sum_N V_{tb}^{R\star} V_{tj}^L U_{\ell N}^{L*} U_{\tau N}^{R*} [E_{LR}^S(\lambda_t, \lambda_N, \beta) \right. \right.$$

$$\left. \left. + 4E_{LR}^T(\lambda_t, \lambda_N, \beta)] \right|^2 \right]. \tag{53}$$

It is harder to interpret this result because there are several unknown parameters. To gain some insight into this result, we consider the simplified case with only one heavy neutrino discussed in the introduction. We further assume that $V_{tj}^R \sim V_{tj}^L$, and that $g_L = g_R$. We define the form factor

$$E_{LR}^{(B)} \equiv 0.36\beta_g^2 [E_{LR}^S(\lambda_t, \lambda_N, \beta) + 4E_{LR}^T(\lambda_t, \lambda_N, \beta)]^2, \tag{54}$$

and plot it in Fig. 6 as a function of β for selected values of the heavy neutrino mass. We see from the figure that $E_{LR}^{(B)}$ can be between 0.01 and 0.1 for a wide range of parameters. Combining this result with Eq. (53), we find

$$R_{B\tau\ell} \sim (0.01 - 0.1)[|U_{\ell N}^R U_{\tau N}^L|^2 + |U_{\ell N}^{L*} U_{\tau N}^{R*}|^2]. \tag{55}$$

This in turn implies that $R_{B\tau\ell}$ has to be probed at the 10^{-3} level to constrain the neutrino mixing parameters beyond
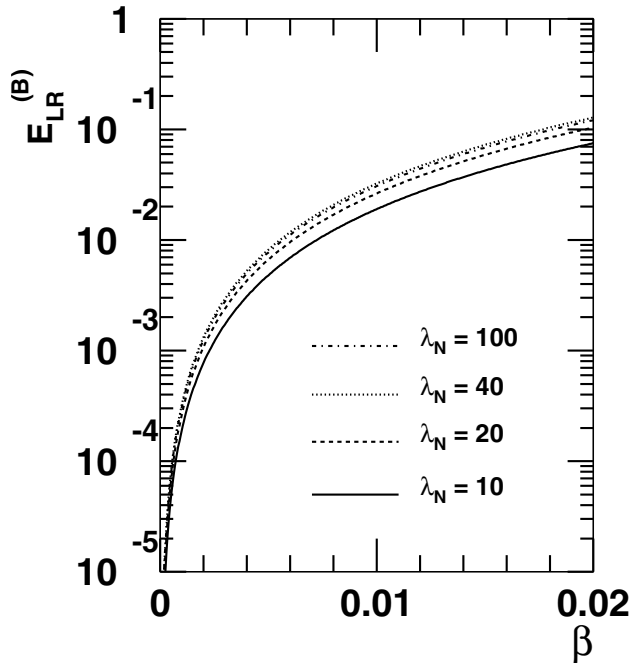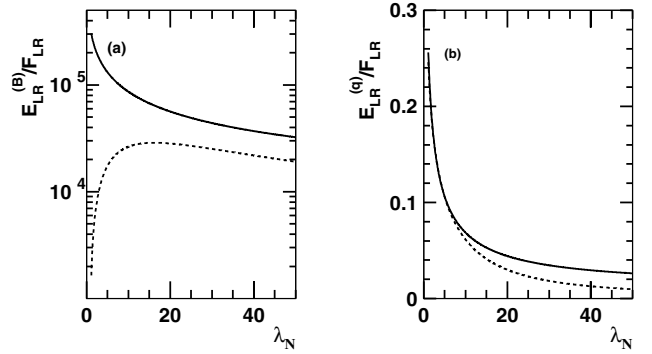

FIG. 7. Squared ratio of form factors for (a) $E_{LR}^{(B)}/F_{LR}$ and (b) $E_{LR}^{(q)}/F_{LR}$ as a function of λ_N for $\beta = 0.0065$.

what is known from unitarity. Considering the SM expectation Eq. (49), this implies a benchmark number for $B(B \to \tau^\pm \ell^\mp)$ at the 10^{-9} level, probably beyond reach for the foreseeable future. The corresponding bound $B(K_L \to \mu^\pm e^\mp) < 4.7 \times 10^{-12}$ [17] leads in turn to

$$[|U_{eN}^R U_{\mu N}^L|^2 + |U_{eN}^{L*} U_{\mu N}^{R*}|^2] < 0.03. \tag{56}$$

It is instructive to compare these modes to the $\tau^- \to \ell_1^- \ell_2^- \ell_3^+$ modes. To this effect we plot in Fig. 7(a) the ratio of form factors $E_{LR}^{(B)}/F_{LR}$ as a function of λ_N for $\beta = 0.0065$ (which corresponds to $M_{W'} \sim 1$ TeV). Using $E_{LR}^{(B)} \sim 10^4 F_{LR}$ from Fig. 7(a), we write

$$B(B_s \to \tau^\pm \ell^\mp) \sim 0.01 F_{LR}[|U_{\ell N}^R U_{\tau N}^L|^2 + |U_{\ell N}^{L*} U_{\tau N}^{R*}|^2], \tag{57}$$

which can be compared directly to Eq. (32).

B. Inclusive modes $b \to s\tau^\pm \ell^\mp$

We turn our attention to the inclusive process $b \to s\tau^\pm \ell^\mp$. This choice is motivated by several factors. A semileptonic mode removes the helicity suppression present in the $B \to \tau^\pm \ell^\mp$ modes for the case of left-handed heavy neutrinos. [1] Requiring strangeness in the final state removes the CKM suppression factor V_{td}/V_{cb}. Finally, considering an inclusive process removes the QCD suppression factor F_B/M_B.

For left-handed heavy neutrinos, we find

$$\Gamma(b \to d_j \tau^\pm \ell^\mp) = \frac{G_F^2 m_b^5}{192\pi^3} \left(\frac{\alpha^2}{512\pi^2 \sin^4\theta_W} \right) I\left(\frac{m_\tau}{m_b} \right) |V_{tb}^{L\star} V_{tj}^L|^2$$

$$\times \left| \sum_N U_{\ell N}^{L*} U_{\tau N}^L E_L(\lambda_t, \lambda_N) \right|^2, \tag{58}$$

where

FIG. 6. $E_{LR}^{(B)}$ as a function of β for selected values of λ_N.

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[1] In the kaon sector, the helicity suppression may be removed by considering instead $K \to \pi\mu e$ modes. The best bound in that case, $|U_{eN}^{L*} U_{\mu N}^L| \lesssim 2$, arises from $B(K^+ \to \pi^+ \mu^+ e^-) < 2.8 \times 10^{-11}$ [17] but is worse than the unitarity constraint.

$$I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log(x) \qquad (59)$$

is the usual kinematic factor for a nonzero τ mass and $I(m_\tau/m_b) \sim 0.33$ with $m_b \sim 4.5$ GeV. Using a b lifetime given by

$$\Gamma(b) = 5.8 \frac{G_F^2 m_b^5}{192\pi^3} I\left(\frac{m_c}{m_b}\right) |V_{cb}|^2, \qquad (60)$$

with $I(m_c/m_b) \sim 0.5$ (for $m_c \sim 1.4$ GeV), this gives

$$B(b \to d_j \tau^\pm \ell^\mp) \approx 2.6 \times 10^{-8} \left| \frac{V_{tj}}{V_{cb}} \right|^2$$
$$\times \left| \sum_N U_{\ell N}^{L*} U_{\tau N}^L E_L(\lambda_t, \lambda_N) \right|^2. \qquad (61)$$

To compare with the radiative τ decay modes, we assume that there is only one heavy neutrino, use Eq. (18) as well as $|E_L/F|^2 \sim 7500$ from Fig. 5 to obtain

$$B(b \to s\tau^\pm \mu^\mp) \leq 2.3 \times 10^{-8},$$
$$B(b \to s\tau^\pm e^\mp) \leq 2.0 \times 10^{-7},$$
$$B(b \to d\tau^\pm \mu^\mp) \leq 9.1 \times 10^{-10}, \qquad (62)$$
$$B(b \to d\tau^\pm e^\mp) \leq 7.7 \times 10^{-9}.$$

These results imply that one needs at least a 10^{-7} sensitivity in $B(b \to s\tau^\pm e^\mp)$ to obtain constraints competitive with the radiative τ decay for left-handed heavy neutrinos. Table II indicates that current B-factory results are close to this benchmark, although there are significant experimental hurdles for detection of τ leptons.

As an estimate for the reach of future experiments, we start from the Tevatron studies indicating that, with $2fb^{-1}$, CDF could detect 61 $B_d \to K^{\star 0} \mu\mu$ events assuming a branching ratio of 1.5×10^{-6} [24]. We can turn this number into an approximate single event sensitivity with $15fb^{-1}$ of 3.2×10^{-9} for this mode. This in turn indicates that improved constraints are possible, at least from the $b \to s\tau e$ mode.

For heavy neutrinos with left- and right-handed couplings but vanishing $W_L - W_R$ mixing, we find

TABLE II. Summary of current experimental bounds for $B \to X\ell^+ \ell'^-$ modes.

	Branching ratio
$b \to se^\pm \mu^\mp$	$<2.2 \times 10^{-5}$ CLEO [10]
$B \to \pi e^\pm \mu^\mp$	$<1.6 \times 10^{-6}$ CLEO [25]
$B \to K^0 e^\pm \mu^\mp$	$<4.0 \times 10^{-6}$ BaBar [11]
$B \to K e^\pm \mu^\mp$	$<1.6 \times 10^{-6}$ CLEO [25]
$B \to \rho e^\pm \mu^\mp$	$<3.2 \times 10^{-6}$ CLEO [25]
$B \to K^* e^\pm \mu^\mp$	$<3.4 \times 10^{-6}$ BaBar [11]
$B \to K^* e^\pm \mu^\mp$	$<6.2 \times 10^{-6}$ CLEO [25]

$$\Gamma(b \to d_j \tau^\pm \ell^\mp) = \frac{G_F^2 m_b^5}{192\pi^3} \left(\frac{\alpha^2}{128\pi^2 \sin^4\theta_W} \right) \beta_g^2 I\left(\frac{m_\tau}{m_b}\right)$$
$$\times \left\{ \left| \sum_N V_{tb}^{L\star} V_{tj}^R U_{\ell N}^R U_{\tau N}^L E_{LR}^S \right|^2 \right.$$
$$+ 64 \left| \sum_N V_{tb}^{L\star} V_{tj}^R U_{\ell N}^R U_{\tau N}^L E_{LR}^T \right|^2$$
$$+ 8\text{Re}\left[\left(\sum_N V_{tb}^{R\star} V_{tj}^L U_{\ell N}^{L*} U_{\tau N}^{R*} E_{LR}^S \right) \right.$$
$$\left. \left. \times \left(\sum_N V_{tb}^R V_{tj}^{L\star} U_{\ell N}^L U_{\tau N}^R E_{LR}^T \right) \right] \right\}. \qquad (63)$$

If we assume that only one heavy neutrino is important, that $g_R \sim g_L$, and that $V_{tj}^R \sim V_{tj}^L$, we can write

$$B(b \to d_j \tau^\pm \ell^\mp) = \left| \frac{V_{tj}}{V_{cb}} \right|^2 (|U_{\ell N}^R U_{\tau N}^L|^2$$
$$+ |U_{\ell N}^{L*} U_{\tau N}^{R*}|^2) E_{LR}^{(q)}(\lambda_t, \lambda_N, \beta), \qquad (64)$$

where we have defined

$$E_{LR}^{(q)}(\lambda_t, \lambda_N, \beta) \equiv 1.04 \times 10^{-7} \beta_g^2 [(E_{LR}^S)^2 + 8 E_{LR}^S E_{LR}^T$$
$$+ 64(E_{LR}^T)^2]. \qquad (65)$$

From Fig. 8 we see that $E_{LR}^{(q)} \sim 10^{-8}$ for a wide range of parameters. This allows us to write
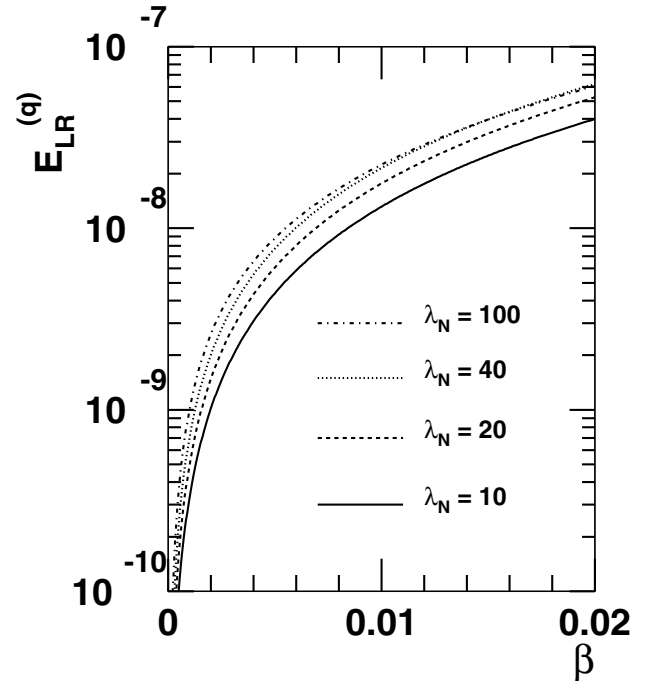

FIG. 8. $E_{LR}^{(q)}$ as a function of β for selected values of λ_N with $g_R = g_L$.

$$B(b \to s\tau^\pm \ell^\mp) \sim 1 \times 10^{-8}(|U_{\ell N}^R U_{\tau N}^L|^2 + |U_{\ell N}^{L*} U_{\tau N}^{R*}|^2). \tag{66}$$

A sensitivity of at least 10^{-8} is thus required for the mode $b \to s\tau^\pm \ell^\mp$ to place significant constraints on the neutrino mixing parameters.

To compare these modes to the $\tau^- \to \ell_1^- \ell_2^- \ell_3^+$ modes, we plot in Fig. 7(b) the ratio of form factors $E_{LR}^{(q)}/F_{LR}$ as a function of λ_N for $\beta = 0.0065$. Using $E_{LR}^{(q)} \sim 0.02 F_{LR}$, we write

$$B(b \to s\tau^\pm \ell^\mp) \sim 0.02 F_{LR}(|U_{\ell N}^R U_{\tau N}^L|^2 + |U_{\ell N}^{L*} U_{\tau N}^{R*}|^2), \tag{67}$$

which can be compared directly to Eq. (32).

VI. SUMMARY AND CONCLUSIONS

We have studied LFV τ and B decay modes within the context of neutrino mixing with additional heavy neutrinos. We have considered generic left-right models and distinguished three scenarios. In the first scenario we consider, the W' has a negligible effect and the lepton interactions are purely left handed. In this case the best constraints on the parameters describing the neutrino mixing arise from radiative τ decay. Assuming one very heavy neutrino dominates, the current best bound is placed by $\tau \to \mu\gamma$,

$$|U_{\mu N}^{L*} U_{\tau N}^L| \leq 0.044. \tag{68}$$

To obtain a competitive constraint from $B_s \to \tau^\pm \mu^\mp$, this mode would have to be probed at the 10^{-9} level as indicated by Eqs. (48) and (49). Similarly, to obtain a comparable constraint in any of the inclusive modes, $b \to s\tau^\pm e^\mp$ needs to be probed at the 10^{-7} level as indicated in Eq. (62).

The second scenario we considered involved a very heavy W'. In this case, the effect of the right-handed interaction can only be felt at low energies through $W_L - W_R$ mixing. In this scenario the best constraints arise from

radiative τ decay and lead to unobservably small rates in LFV B decay modes.

The third and final scenario we considered is one in which both the W and the W' play a role in the lepton charged currents, but there is no $W_L - W_R$ mixing. In this more general case, the (6×6) mixing matrix in the neutrino sector has many unknown parameters and as a practical matter different decay modes will in general probe different combinations of these parameters. As a benchmark for the sensitivity needed to probe this scenario we have considered three simplified cases with results summarized in Table III. The salient features are

(i) Radiative τ decay modes do not probe this scenario in the limit of no $W_L - W_R$ mixing.

(ii) Unitarity implies that the matrix elements of the neutrino mixing matrix satisfy $|U_{\ell,N}^{L,R}| \leq 1$. To place significant constraints (better than the unitarity limit), R_{123} has to be measured with a sensitivity of at least 10^{-8}, 2 orders of magnitude better than current limits.

(iii) The benchmark for significant constraints from B decay is a sensitivity of 10^{-9} for $B(B_s \to \tau^\pm \ell^\mp)$ and of 10^{-8} for $B(b \to s\tau^\pm \ell^\mp)$.

In all cases, the sensitivity to the neutrino mixing parameters is much smaller than what already exists from the study of $\mu \to e\gamma$ and $K_L \to \mu e$ modes. However, LFV τ and B decay modes offer an opportunity to complement those results by providing constraints on the $U_{\tau N}$ mixing angles.

APPENDIX: COMPARISON WITH THE $M_H \to \infty$ LIMIT OF REF. [14]

In this appendix, we compare the form factor in $M \to \ell_1^+ \ell_2^-$ (for a spinless meson M) as obtained from Eq. (26) and the complete one-loop calculation of Ref. [14]. In [14], a LR model with a simple scalar sector is considered and a complete (gauge independent and finite) result for the form factor is obtained. In addition to the gauge boson contribution that we consider in this paper, the complete result

TABLE III. Summary of results for a heavy neutrino with left- and right-handed couplings. We consider three cases with only one nonzero mixing with a heavy right-handed neutrino: (a) $U_{eN}^R \neq 0$, (b) $U_{\mu N}^R \neq 0$, and (c) $U_{\tau N}^R \neq 0$.

	$U_{eN}^R \neq 0$	$U_{\mu N}^R \neq 0$	$U_{\tau N}^R \neq 0$
$R_{e\mu\mu}/F_{LR}$	0	$|U_{\tau N}^L|^2|U_{eN}^L|^2|U_{\mu N}^R|^4$	0
$R_{\mu ee}/F_{LR}$	$|U_{\tau N}^L|^2|U_{\mu N}^L|^2|U_{eN}^R|^4$	0	0
$R_{\mu\mu\mu}/F_{LR}$	0	$|U_{\mu N}^L|^2|U_{\tau N}^L|^2|U_{\mu N}^R|^4$	0
R_{eee}/F_{LR}	$|U_{eN}^L|^2|U_{\tau N}^L|^2|U_{eN}^R|^4$	0	0
$R_{B\tau\mu}/E_{LR}^{(B)}$	0	$|U_{\mu N}^R|^2|U_{\tau N}^{L*}|^2$	$2|U_{\tau N}^R|^2|U_{\tau N}^{L*}|^2$
$R_{B\tau e}/E_{LR}^{(B)}$	$|U_{eN}^R|^2|U_{\tau N}^{L*}|^2$	0	$2|U_{\tau N}^R|^2|U_{\tau N}^{L*}|^2$
$B(b \to s\tau_\pm\mu_\mp)/E_{LR}^{(q)}$	0	$|U_{\mu N}^R|^2|U_{\tau N}^{L*}|^2$	$2|U_{\tau N}^R|^2|U_{\tau N}^{L*}|^2$
$B(b \to s\tau_\pm e_\mp)/E_{LR}^{(q)}$	$|U_{\mu N}^R|^2|U_{\tau N}^{L*}|^2$	0	$2|U_{\tau N}^R|^2|U_{\tau N}^{L*}|^2$

depends on the parameters of the scalar sector, namely, at least three scalar masses and scalar sector couplings. To compare this to our result, we take the Higgs masses to infinity in the result of Ref. [14].

In what follows, we use the notation in Appendix B of [14]. We drop a common factor containing the mixing angles, the coupling constants, and the factor $\sqrt{\lambda_i \lambda_N}$. The diagrams that contribute to the form factor in the limit when the Higgs masses are very heavy are

(i) 3a + 3b.—Box diagrams with one W and one W' (these are the ones we consider in unitary gauge)

$$(FF)_{ab} = \beta \left[\left(1 + \frac{\beta \lambda_t \lambda_N}{4} \right) J_2(\lambda_t, \lambda_N, \beta) \right.$$
$$\left. - \frac{1 + \beta}{4} F_1(\lambda_t, \lambda_N, \beta) \right], \quad \text{(A1)}$$

where the functions J_2 and F_1 are defined in [14].

(ii) 3c + 3d + 3e + 3f.—Vertex corrections involving the exchange of the neutral scalars with tree-level flavor changing couplings:

$$(FF)_{cdef} = -\frac{\beta^2}{4} \left[\frac{\lambda_t^2 \log \lambda_t}{(1 - \lambda_t)(1 - \beta \lambda_t)} \right.$$
$$+ \frac{\lambda_N^2 \log \lambda_N}{(1 - \lambda_N)(1 - \beta \lambda_N)}$$
$$\left. + \frac{(2\beta \lambda_t \lambda_N - \lambda_t - \lambda_N) \log \beta}{(1 - \beta)(1 - \beta \lambda_t)(1 - \beta \lambda_N)} \right]$$
$$+ \mathcal{O}\left(\frac{1}{M_H^2} \right). \quad \text{(A2)}$$

(iii) 1 + 3g + 3h.—Tree-level exchange of scalars with flavor changing couplings as well as self-energy corrections to this:

$$(FF)_{gh} = \frac{\beta}{4} \left(\frac{\log \beta}{1 - \beta} + \log \lambda_\phi - 2 \right) + \mathcal{O}\left(\frac{1}{M_H^2} \right), \quad \text{(A3)}$$

where $\lambda_\phi = M_\phi^2 / M_W^2$ and M_ϕ is the mass of the neutral scalars (assumed degenerate in [14]) with tree-level flavor changing couplings.

(iv) All other diagrams considered in Ref. [14] yield contributions to the form factor that vanish as inverse powers of M_H in the limit of infinite mass for the physical Higgs's that occur.

The final result in the limit $M_H \to 1$ is then

$$(FF) \to \frac{\beta}{4} \left[\log \lambda_\phi - 2 + \frac{\lambda_t \log \lambda_t (\beta \lambda_t^2 - \beta \lambda_t - \lambda_t + 4)}{(\lambda_N - \lambda_t)(1 - \lambda_t)(1 + \beta \lambda_t)} \right.$$
$$+ \frac{\lambda_N \log \lambda_N (\beta \lambda_N^2 - \beta \lambda_N - \lambda_N + 4)}{(\lambda_t - \lambda_N)(1 - \lambda_N)(1 - \beta \lambda_N)}$$
$$\left. + \frac{3\beta \log \beta}{(1 - \beta)(1 - \beta \lambda_t)(1 - \beta \lambda_N)} \right]. \quad \text{(A4)}$$

This is to be compared to our result for the same form factor [which appears in Eq. (44)] as obtained from Eq. (26): $(FF)_{us} \sim [E_{LR}^S(\lambda_t, \lambda_N, \beta) + 4E_{LR}^T(\lambda_t, \lambda_N, \beta)]$ and normalized in the same way as the result of Ref. [14]:

$$(FF) = \frac{\beta}{4} \left[\frac{1}{\hat{\epsilon}} + \log\left(\frac{\mu^2}{M_W^2} \right) \right.$$
$$+ 1 + \frac{\lambda_t \log \lambda_t (\beta \lambda_t^2 - \beta \lambda_t - \lambda_t + 4)}{(\lambda_N - \lambda_t)(1 - \lambda_t)(1 - \beta \lambda_t)}$$
$$+ \frac{\lambda_N \log \lambda_N (\beta \lambda_N^2 - \beta \lambda_N - \lambda_N + 4)}{(\lambda_t - \lambda_N)(1 - \lambda_N)(1 - \beta \lambda_N)}$$
$$\left. + \frac{3\beta \log \beta}{(1 - \beta)(1 - \beta \lambda_t)(1 - \beta \lambda_N)} \right]. \quad \text{(A5)}$$

Comparing these two results after using our prescription shows that we reproduce the complete calculation in the limit $M_H \to \infty$ up to the numerical factor 3 for the choice $\mu = M_H$. This number depends on the renormalization scheme used and is of the same order as the logarithm $\log(\mu^2 / M_W^2) \sim 5$ for the scale $\mu \sim 1$ TeV we choose.

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