Estimations of heavy neutrino masses and mixing in models with an extended Higgs sector

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Within two-flavor approximation for the left right model, the equations connecting the parameters of the heavy neutrino sector and the charged Higgs sector (CHS) are found. For definition of the CHS parameters the impact of the Higgs bosons on lepton flavor violation decays, low-energy light neutrino scattering, and the muon anomalous magnetic moment are investigated. It is shown, that the heavy neutrino masses $m_{N_{1,2}}$ can be expressed in terms of the triplet Yukawa coupling constants and the mass of the W_2 gauge boson only. Using the data of inverse muon decays, and also the constraints on the masses of the $\tilde{\delta}^{(-)}$, $\Delta_{1,2}^{(-)}$ and W_2 bosons the limits on $m_{N_{1,2}}$ are established in the cases both of the absence and the presence of the heavy neutrino masses (quasi)degeneration. The data on explanation of the $(g-2)_\mu$ anomaly is used for determination of the constraints on $m_{N_{1,2}}$ in the case of the masses (quasi)degeneration only. The suggested scheme for estimations of the heavy neutrino masses and mixing, based on the Higgs sector investigation, can be used for any gauge electroweak theory with an extended Higgs sector and the ‘‘see-saw’’ mechanism.

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I. INTRODUCTION

In 2002 during the SNO experiment with solar neutrinos [1] they obtained a direct evidence of transitions of the ν_e into ν_μ and ν_τ. This result proved the hypothesis of neutrino oscillations, which was used to explain a deficit of solar neutrinos measured in Homestake, SAGE, GALLEX, GNO, and Super-Kamiokande. Thus the solar neutrino problem which was the driving force of developments in neutrino physics during last 35 years has been essentially resolved. The same year, the SNO results were confirmed by the experiments with terrestrial neutrinos, in which a well-controlled initial beam was used, namely, in the experiment with reactor antineutrinos [2]. All this allowed to draw a final conclusion, that the neutrino has mass and there exists mixing in the lepton sector, and allowed to call the year 2002 the ‘‘*annus mirabilis*’’ of the solar neutrino physics. However, it was not the end of the story about the neutrino. Until now the neutrino physics contains a lot of unsolved problems, the most important of which are a smallness of neutrino mass and a connection between oscillation parameters in the neutrino and quark sectors.

From all the data available it follows, that neutrino masses are by many orders of less than masses of other fundamental fermions—charged leptons and quarks. Thus, direct kinematics measurements produce the upper limit on neutrino masses (the mass spectrum is supposed to be degenerated) $m_{\nu_e} \approx (m_i) < 2.2$ eV at 95% C.L. [3]. The cosmological boundary on the masses sum of all the light neutrinos [4] $\sum_i |m_i| < (0.7 \div 2.1)$ eV (95% C.L.) leads to a more strict limit on the neutrino masses $|m_i| < (0.23 \div 0.70)$ eV.

An explanation for such a giant suppression of the neutrino mass is provided by a ‘‘see-saw’’ mechanism which comprises a necessary component of many grand unified theories (GUT’s). Within the GUT’s it is also possible to connect parameters of the quark and lepton sectors, since the GUT’s predict relations between the quark and lepton Yukawa coupling constants at the unification scale. Because of GUT constraints for the fermion mixing matrices, the quark and lepton flavor mixings are related, predicting [5]

$$\theta_{12}^{\rm NMM} + \theta_{12}^{\rm CKM} = \frac{\pi}{4}, \qquad \theta_{23}^{\rm CKM} + \theta_{23}^{\rm NMM} = \frac{\pi}{4}, \tag{1}$$

$$\theta_{13}^{\rm NMM} \sim \theta_{13}^{\rm CKM} \sim O(\theta_C^3). \tag{2}$$

where $\theta_{ij}^{\rm CKM}$ ($\theta_{ij}^{\rm NMM}$) are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) neutrino mixing matrix (NMM) and $\theta_{12}^{\rm CKM} = \theta_C$ is the Cabibbo angle, in a good agreement with the experimental data. The relations (1) and (2) can follow, for example, from SO(10) type GUT constraints for the fermion mixing matrices, and from the structure of the quark and neutrino mixing matrices. No doubt that these predictions can be considered as a new experimental evidence for the idea of grand unification.

An interesting aspect of the SO(10) model lies in the fact that it contains the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group, that is, a group of electroweak symmetry is expanded by the factor SU(2) compared to the standard model (SM). Among the SM extensions $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model (the left-right model) [6] is of special interest. The reasons for this are as follows.

(1) An almost maximum parity violation observed at a low-energy weak interaction can be interpreted within the left-right model (LRM) as appearing spontaneously and is connected with nonzero value of neutrino masses

(2) The Higgs sector in the LRM contains common elements with such popular SM extensions as the modified SM with two Higgs doublets, the MSSM and the model, based on $SU(3)_L \times U(1)_N$ gauge group (N is a number of generations).
(3) In the LRM all the fundamental fermions enter the theory in a symmetric way, that is, they form left and right doublets according to weak isospin.
(4) The LRM belongs to a number of models, in which constants of coupling between Higgs bosons and charged leptons define parameters of neutrino oscillations.

In the LRM the partners in the "see-saw" mechanism are heavy Majorana neutrinos N_a $(a = e, \mu, \tau)$, which enter the right lepton doublet

$$\Psi_{aR} = \begin{pmatrix} N_{aR} \\ l_{aR} \end{pmatrix}.$$

Unfortunately, nowadays, the data concerning the heavy neutrinos in the LRM is very poor. The majority of works, aimed at putting limitations on the heavy neutrinos masses, used the SM modifications, in which the neutrino masses are introduced by hand. In such models almost the only way to obtain limitations on the masses of the heavy neutrinos N_{iR} is to assume, that exactly the N_{iR} causes the appearance of barion asymmetry in the Universe by means of leptogenesis.

Unlike the SM modifications, in the LRM the heavy neutrino sector parameters are not isolated from the remaining part of the model. It allows to define these parameters not only in direct but as well in indirect measurements. The intent of this work is to suggest a method for determination of the mixing angles and masses of the heavy neutrinos N_i from plurality of experiments without direct detection of N_i. In the next section the information concerning the LRM is given which is necessary for our further analysis. In Sec. III lepton flavor violation (LFV) decays and low-energy light neutrino scattering as well as a $(g-2)_\mu$ anomaly are considered from the view point of obtaining the constraints on the Higgs bosons coupling constants α_H. In Sec. IV we put limits on the heavy neutrino sector parameters by using the found values of α_H. Sec. V is dedicated to conclusions and to discussion of the results obtained.

II. THE LINKAGE BETWEEN THE HIGGS AND NEUTRINO SECTORS

We shall consider a symmetric version of the LRM ($g_L = g_R$), which contains in the Higgs sector one bidoublet

$$\Phi\left(\frac{1}{2}, \frac{1}{2}, 0\right) = \begin{pmatrix} \Phi_1^0 & \Phi_2^+ \\ \Phi_1^- & \Phi_2^0 \end{pmatrix}$$

and two triplets

$$\Delta_L(1,0,2) = \begin{pmatrix} \delta_L^{++} \\ \delta_L^{+} \\ \delta_L^{0} \end{pmatrix}, \qquad \Delta_R(0,1,2) = \begin{pmatrix} \delta_R^{++} \\ \delta_R^{+} \\ \delta_R^{0} \end{pmatrix}.$$

Such a choice of the scalar multiplets ensures the Majorana nature of the neutrinos.

For a Higgs potential we use the expression suggested in Ref. [7], while a Yukawa Lagrangian describing a gauge invariant interaction in a lepton sector is defined by the expression

$$\begin{aligned} \mathcal{L}_Y = -\sum_{a,b}\{ & h_{ab}\overline{\Psi}_{aL}\Phi\Psi_{bR} + h'_{ab}\overline{\Psi}_{aL}\tilde{\Phi}\Psi_{bR} \\ & + if_{ab}[\Psi_{aL}^T C\tau_2(\vec{\tau}\cdot\vec{\Delta}_L)\Psi_{bL} \\ & + (L \rightarrow R)] + \text{conj.}\}, \end{aligned} \qquad (3)$$

where $\tau_{1,2,3}$ are the Pauli matrices, $\tilde{\Phi} = \tau_2\Phi^*\tau_2$, h_{ab}, h'_{ab} and $f_{ab} = f_{ba}$ are the bidoublet and triplet Yukawa coupling constants (YCC's).

After a spontaneous symmetry violation we are left with 14 physical Higgs bosons: four doubly charged scalars $\Delta_{1,2}^{(\pm\pm)}$, four singly charged scalars $h^{(\pm)}$ and $\tilde{\delta}^{(\pm)}$, four neutral scalars $S_{1,2,3,4}$ (S_1 is an analog of the Higgs boson in the SM) and two neutral pseudoscalars $P_{1,2}$.

Note, as a result of plurality experiments, masses of many additional compared to the SM particles were moved away from the electroweak scale. However, as far as the Higgs bosons are concerned, the existing experimental data allows some of them to have mass on the electroweak scale. Thus, for example, the low limit on the mass of the singly charged Higgs boson is equal to 75 GeV [8], while the analysis performed with the DO Run II detector at the Fermilab Tevatron [9] sets the mass limits of $m_{\Delta_L} > 118.4$ GeV and $m_{\Delta_R} > 98.2$ GeV for left-handed and right-handed doubly charged Higgs bosons, respectively.

Further on for the sake of simplicity we shall work in the two-flavor approximation. Then, in the basis

$$\Psi = \begin{pmatrix} \nu_{aL} \\ \nu_{bL} \\ N_{aR} \\ N_{bR} \end{pmatrix}$$

the neutrino mass matrix takes the form

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_\nu & \mathcal{M}_D \\ \mathcal{M}_D^T & \mathcal{M}_N \end{pmatrix},$$

where

$$\mathcal{M}_\nu = \begin{pmatrix} f_{aa}v_L & f_{ab}v_L \\ f_{ab}v_L & f_{bb}v_L \end{pmatrix}, \qquad \mathcal{M}_N = \mathcal{M}_\nu(v_L \rightarrow v_R),$$

$$\mathcal{M}_D = \begin{pmatrix} m_D^a & M_D \\ M'_D & m_D^b \end{pmatrix}.$$

$$m_D^a = h_{aa}k_1 + h'_{aa}k_2, \qquad m_D^b = m_D^a(a \to b)$$

$$M_D = h_{ab}k_1 + h'_{ab}k_2, \qquad M'_D = M_D(a \leftrightarrow b),$$

$$v_{L,R} = \langle \delta^0_{L,R} \rangle, \qquad k_{1,2} = \langle \Phi^0_{1,2} \rangle,$$
$$v_L \ll \max(k_1, k_2) \ll v_R.$$

Assuming, that $v_L = 0$ and $\mathcal{M}_N \gg \mathcal{M}_D$, for the light and heavy neutrino mass matrices (M_ν, M_N) we could obtain a "see-saw" relation

$$M_\nu = \mathcal{M}_D^T M_N^{-1} \mathcal{M}_D, \tag{4}$$

where we have put M_N equal to $\mathcal{M}_N$. So to estimate the heavy neutrino masses with help of Eq. (4), it is necessary to determine the obvious form of the matrices M_ν and $\mathcal{M}_D$. The form of the M_ν matrix can be restored with the help of experiments with solar, atmospheric, and reactor neutrinos. As for the $\mathcal{M}_D$ matrix, the situation is not so simple. The LRM itself puts no restrictions on the form of $\mathcal{M}_D$. However, it may be possible to determine the elements of this matrix under study of the reactions with the Higgs bosons participation, namely, under study of interactions described by the Lagrangian

$$\begin{aligned}\mathcal{L}_1 = \sum_{a,b}[&\alpha_{\bar{l}_a\nu_b h}\overline{\nu}_a(x)(1-\gamma_5)l_b(x)h^{(-)*}(x) \\ &+ \alpha_{\bar{l}_a l_b S_1}\bar{l}_a(x)l_b(x)S_1(x) + \alpha_{\bar{l}_a l_b S_2}\bar{l}_a(x)l_b(x)S_2(x) \\ &+ \alpha_{\bar{l}_a l_b P_1}\bar{l}_a(x)\gamma_5 l_b(x)P_1(x) + \text{conj}].\end{aligned} \tag{5}$$

Note, that the S_2 and P_1 bosons have almost the same masses and these masses could lay on the electroweak scale when one uses in the quark sector the Yukawa Lagrangian, which is not inducing a flavor violation at a tree level [10].

In Ref. [11] it was shown, that the constants of interactions between the $h^{(\pm)}$, $S_{1,2}$, P_1 bosons and leptons could be represented as

$$\begin{aligned}\alpha_{\bar{l}_a\nu_a h} &= \frac{1+\tan^2\beta}{2k_+(1-\tan^2\beta)}\left(m_D^a - \frac{2m_{l_a}\tan\beta}{1+\tan^2\beta}\right), \\ \alpha_{\bar{l}_a l_a S_1} &= \sqrt{2}\alpha_{\bar{l}_a\nu_a h}(m_{l_a} \leftrightarrow -m_D^a),\end{aligned} \tag{6}$$

$$\alpha_{\bar{l}_a l_a S_2} = -\frac{m_D^a}{\sqrt{2}k_+}, \qquad \alpha_{\bar{l}_a l_a P_1} = \sqrt{2}\alpha_{\bar{l}_a\nu_a h}, \tag{7}$$

$$\alpha_{\bar{l}_a\nu_b h} = -\frac{M_D}{2k_+}, \qquad \alpha_{\bar{l}_a l_b S_1} = \frac{M_D\tan\beta}{\sqrt{2}k_+(1+\tan^2\beta)}, \qquad a \neq b. \tag{8}$$

$$\alpha_{\bar{l}_a l_b S_2} = \frac{M_D(\tan^2\beta - 1)}{\sqrt{2}k_+(\tan^2\beta + 1)}, \qquad \alpha_{\bar{l}_a l_b P_1} = \sqrt{2}\alpha_{\bar{l}_a\nu_b h}, \qquad a \neq b \tag{9}$$

where $\tan\beta = k_1/k_2$ and $k_\pm^2 = k_1^2 \pm k_2^2$. As it follows from Eqs. (6)–(9), to define four elements of the matrix $\mathcal{M}_D$ the knowledge, at least, five constants of interactions between the $h^{(\pm)}$, $S_{1,2}$, P_1 bosons and leptons are necessary.

However, the approach considered above possesses three disadvantages. The first consists in fact, the angles of mixing between the light and heavy neutrinos in a (φ_a) and b (φ_b) generations, and also the angle of mixing between heavy neutrinos (θ_N) are taken as being equal to 0. Second, the formula (4) is not exact, and errors under its use could be of order of 10 GeV. So, for example, at deriving Eq. (4) we assumed that v_L was equal to 0. It is possible to estimate v_L by means of a quantity ρ, comparing its theoretical value

$$\rho^{\text{theor}} = \frac{m_{Z_1}^2 c_{\theta_W}^2}{m_{W_1}^2} = \frac{1+4x}{1+2x}, \tag{10}$$

($x = (v_L/k_+)^2$, θ_W is the Weinberg angle) with the experiment. Since the current experimental value is

$$\rho^{\text{exp}} = 1.0107 \pm 0.0006, \tag{11}$$

then the value of v_L can reach 13 GeV. As it is reasonable to assume, that f_{aa} and f_{bb} are less than 1, the maximal error connected with the assumption $\mathcal{M}_\nu = 0$ will be $\sim$10 GeV. The same order of the maximal error is expected from the assumption $\mathcal{M}_D \ll \mathcal{M}_N$. The third disadvantage lies in the requirement $\mathcal{M}_N = M_N$ meaning equality to zero of nondiagonal elements of the matrix $\mathcal{M}_N$, i.e. $f_{ab} = 0$. Later we shall show, that if $f_{ab} = 0$, then in the heavy neutrino sector either mass (quasi)degeneration takes place, or the mixing angle θ_N equals to 0. Certainly, it is possible to refuse the assumption $\mathcal{M}_N = M_N$ and instead of Eq. (4) to use the relation

$$M_\nu = \mathcal{M}_D^T \mathcal{M}_N^{-1} \mathcal{M}_D. \tag{12}$$

In this case the angle θ_N is already took into consideration, but the angles φ_a and φ_b are still ignored.

For a determination of the heavy neutrino sector parameters there is one more way connected with the definition of the triplet YCC's and vacuum expectation values (VEV's) of the Higgs triplets. Having diagonalized $\mathcal{M}$, one arrives at the relations [10]

$$\begin{aligned}f_{ab}v_R = {}& s_{\varphi_a}s_{\varphi_b}c_{\theta_\nu}s_{\theta_\nu}(m_{\nu_2} - m_{\nu_1}) \\ &+ c_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_{N_2} - m_{N_1}),\end{aligned} \tag{13}$$

$$\left.\begin{aligned} f_{aa}v_R = (s_{\varphi_a}c_{\theta_\nu})^2 m_{\nu_1} + (c_{\varphi_a}c_{\theta_N})^2 m_{N_1} + (s_{\varphi_a}s_{\theta_\nu})^2 m_{\nu_2} + (c_{\varphi_a}s_{\theta_N})^2 m_{N_2}, \\ f_{bb}v_R = f_{aa}v_R(\varphi_a \to \varphi_b, \theta_{\nu,N} \to \theta_{\nu,N} + \tfrac{\pi}{2}), \end{aligned}\right\} \tag{14}$$

$$f_{ll'}v_L = f_{ll'}v_R\left(\varphi_{l,l'} \to \varphi_{l,l'} + \frac{\pi}{2}\right), \qquad l, l' = a, b, \tag{15}$$

where θ_ν is the angle of mixing between the ν_{aL} and ν_{bL} neutrinos, $c_{\varphi_a} = \cos\varphi_a$, $s_{\varphi_a} = \sin\varphi_a$ and so on. Note, if one demands, that the bidoublet YCC's should be symmetric ($h_{ab} = h_{ba}$, $h'_{ab} = h'_{ba}$), then, as a consequence, $\varphi_a = \varphi_b$. It should be stressed, that the relations (13)–(15), which will make a starting point in our further considerations, are exact.

So, in this approach for definition of m_{N_1}, m_{N_2}, φ_a, φ_b and θ_N it is enough to know such quantities as f_{aa}, f_{bb}, f_{ab}, v_L and v_R. To obtain the value of v_R one can use either the relation

$$v_R = \sqrt{\frac{(m_{W_2}^2 - m_{W_1}^2)\cos 2\xi}{g_L^2}}, \tag{16}$$

or the relation

$$v_R = \sqrt{\frac{\cos 2\theta_W[m_{Z_1}^2(\sin^2\zeta - \cos^2\zeta\cos 2\theta_W) + m_{Z_2}^2(\cos^2\zeta - \sin^2\zeta\cos 2\theta_W)]}{2g_L^2\cos^2\theta_W}}, \tag{17}$$

(ξ and $\zeta$are mixing angles in a neutral and charged gauge bosons sector, respectively) which follow from formulas, defining the masses and mixing angles of the gauge bosons.

For the determination of the triplet YCC's we should address to investigation of properties of the $\Delta_1^{(\pm\pm)}$, $\Delta_2^{(\pm\pm)}$ and $\tilde{\delta}^{(\pm)}$ bosons. To be more specific, we should examine processes described by the Lagrangian

$$\begin{aligned} \mathcal{L}_2 = -\sum_{a,b}\Big\{ & \frac{f_{ab}}{2}\bar{l}_a^c(x)(1+\gamma_5)l_b(x)\Delta_1^{(--)*}(x) \\ & + [\Delta_1(x) \to \Delta_2(x), \gamma_5 \to -\gamma_5] \\ & + \frac{f_{ab}}{\sqrt{2}}\bar{l}^c{}_a(x)(1+\gamma_5)\nu_b(x)\tilde{\delta}^{(-)*}(x) + \text{conj.}\Big\}. \end{aligned} \tag{18}$$

Note, that from the expressions

$$m_{\tilde{\delta}}^2 = (\rho_3 - 2\rho_1)v_R^2 - \frac{\beta_1^2 k_+^2}{\alpha + \rho_1 - \rho_3/2}, \tag{19}$$

$$m_{\Delta_1}^2 = 4\rho_2 v_R^2 + \alpha_3 k_-^2 + \frac{k_-^4(\beta_3 k_+^2 + \beta_1 k_1 k_2)^2}{k_1^4(4\rho_2 + \rho_3 - 2\rho_1)v_R^2}, \tag{20}$$

$$m_{\Delta_2}^2 = (\rho_3 - 2\rho_1)v_R^2 + \alpha_3 k_-^2 - \frac{k_-^4(\beta_3 k_+^2 + \beta_1 k_1 k_2)^2}{k_1^4(4\rho_2 + \rho_3 - 2\rho_1)v_R^2}, \tag{21}$$

where α_j, ρ_j, β_j are Higgs potential parameters (we use the same notation as in Ref. [7]), it follows, that the masses of the $\Delta_2^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons could be close to each other.

Further on, let us assume, that the mixing takes place in the $e - \mu$ sector, that is, in the formulas (13)–(15) one should set $a = e$ and $b = \mu$.

It will be useful to express the mixing angles θ_N, $\varphi_{e,\mu}$ as the functions of $f_{\mu\mu}$. The use of the relations (14) and (15), produces the formulas we were searching for

$$s_{\theta_N}^2 = \frac{f_{\mu\mu}(v_R + v_L) - (m_{\nu_1}s_{\theta_\nu}^2 + m_{\nu_2}c_{\theta_\nu}^2) - m_{N_2}}{m_{N_1} - m_{N_2}}, \tag{22}$$

$$\sin 2\varphi_e = \frac{2\sqrt{f_{ee}^2 v_R v_L - [f_{ee}(v_R + v_L) - m_{\nu_1}c_{\theta_\nu}^2 - m_{\nu_2}s_{\theta_\nu}^2](m_{\nu_1}c_{\theta_\nu}^2 + m_{\nu_2}s_{\theta_\nu}^2)}}{f_{ee}(v_R + v_L) - 2(m_{\nu_1}c_{\theta_\nu}^2 + m_{\nu_2}s_{\theta_\nu}^2)}. \tag{23}$$

$$\sin 2\varphi_\mu = \sin 2\varphi_e\left(f_{ee} \to f_{\mu\mu}, \theta_\nu \to \theta_\nu + \frac{\pi}{2}\right). \tag{24}$$

Further on, according to the experimental data, we shall set parameters of the light neutrinos sector equal to

$$\tan^2\theta_\nu = 0.34, \qquad m_{\nu_1} = 0.5 \text{ eV}, \qquad m_{\nu_2} = (0.5 + 7\times 10^{-2}) \text{ eV}.$$

Note, that when one neglects the light neutrinos masses, then for the mixing angles φ_e and φ_μ the relation

$$\sin 2\varphi_e = \sin 2\varphi_\mu = \frac{2\sqrt{v_R v_L}}{v_R + v_L} \tag{25}$$

takes place. Using Eqs. (13) and (15), it is possible to obtain

$$\begin{aligned} f_{e\mu}^2 v_R v_L &= \frac{1}{4}(\sin 2\varphi_e)(\sin 2\varphi_\mu) c_{\theta_N}^2 s_{\theta_N}^2 (m_{N_2} - m_{N_1})^2 \\ &= \frac{1}{4}(\sin 2\varphi_e)(\sin 2\varphi_\mu)[f_{\mu\mu}(v_R + v_L) \\ &\quad - m_{\nu_1} s_{\theta_\nu}^2 - m_{\nu_2} c_{\theta_\nu}^2 - m_{N_2}][f_{ee}(v_R + v_L) \\ &\quad - m_{\nu_1} c_{\theta_\nu}^2 - m_{\nu_2} s_{\theta_\nu}^2 - m_{N_2}], \end{aligned} \tag{26}$$

where we assumed, that the mass spectrum of the light neutrinos is degenerated. With the help of Eq. (26) one can show, when $m_{N_2} \gg m_{N_1}$, then $f_{e\mu}$ reaches its maximal value

$$(f_{e\mu})_{\max} \approx \sqrt{f_{ee} f_{\mu\mu}}, \tag{27}$$

while in case of the heavy neutrino masses (quasi)degeneration $f_{e\mu}$ turns to zero, that is,

$$f_{e\mu} \in \left[0, \sqrt{f_{ee} f_{\mu\mu}}\right]. \tag{28}$$

III. CONSTRAINTS ON THE HIGGS SECTOR PARAMETERS

A. Lepton flavor violation decays

In this Section we consider set of experiments which could give information about the Higgs sector structure. We start with the analysis of the results of the search for the decays with the individual LFV.

Let us examine the muon decay through the channel

$$\mu^- \to e^+ e^- e^-. \tag{29}$$

In the LRM this process in the second order of the perturbation theory is described by the diagrams presented in Fig. 1. In the case of the unpolarized particles in the initial and final states, the decay probability is given by the expression

$$\Gamma_{\mu^- \to e^+ e^- e^-} = \frac{d m_\mu^5}{96(4\pi)^3}, \tag{30}$$

where

$$\begin{aligned} d = {}& (f_{e\mu} f_{ee})^2 [m_{\Delta_1}^{-4} + m_{\Delta_2}^{-4}] + 8\Bigg[\Bigg(\frac{\alpha_{\overline{\mu} e S_1} \alpha_{\overline{e} e S_1}}{m_{S_1}^2} \\ &+ \frac{\alpha_{\overline{\mu} e S_2} \alpha_{\overline{e} e S_2}}{m_{S_2}^2}\Bigg)^2 + \frac{(\alpha_{\overline{\mu} e P_1} \alpha_{\overline{e} e P_1})^2}{m_{P_1}^4}\Bigg] + \Bigg(\frac{\alpha_{\overline{\mu} e S_1} \alpha_{\overline{e} e S_1}}{m_{S_1}^2} \\ &+ \frac{\alpha_{\overline{\mu} e S_2} \alpha_{\overline{e} e S_2}}{m_{S_2}^2} + \frac{\alpha_{\overline{\mu} e P_1} \alpha_{\overline{e} e P_1}}{m_{P_1}^2}\Bigg)^2 \end{aligned} \tag{31}$$

and we have neglected the term being proportional to the electron mass. Then, taking into account, that for the branching of the reaction (29) the experiment produces the limit being equal to 10^{-12}, we find the inequality

$$\sqrt{d} < 6.6 \times 10^{-11}\ \mathrm{GeV}^{-2}. \tag{32}$$

Ignoring the contributions from the neutral Higgs bosons, we could obtain an overestimated limit on the triplet YCC's

$$f_{e\mu} f_{ee} \sqrt{m_{\Delta_1}^{-4} + m_{\Delta_2}^{-4}} < 6.6 \times 10^{-11}\ \mathrm{GeV}^{-2}, \tag{33}$$

which has been presented in Refs. [12].

Further we consider the decay

$$\mu^- \to e^- \nu_e \overline{\nu}_\mu. \tag{34}$$

The corresponding diagrams are given in Fig. 2. Assuming that the particles in the initial and final states are unpolarized, we arrive at the result

$$\begin{aligned} \Gamma_{\mu^- \to e^- \nu_e \overline{\nu}_\mu} = {}& \frac{m_\mu^5}{96(2\pi)^3}\Bigg[4\frac{\alpha_{\overline{e}\nu_e h}^2 \alpha_{\overline{\mu}\nu_\mu h}^2 + \alpha_{\overline{\mu}\nu_e h}^4}{m_h^4} \\ &+ \frac{f_{ee}^2 f_{\mu\mu}^2 + f_{e\mu}^4}{m_{\tilde{\delta}}^4}\Bigg]. \end{aligned} \tag{35}$$

The comparison of the obtained probability with the experimental value

$$\mathrm{Br}_{\mu^- \to e^- \nu_e \overline{\nu}_\mu} < 1.2 \times 10^{-2}, \tag{36}$$

results in the inequality

$$\begin{aligned} &4\frac{\alpha_{\overline{e}\nu_e h}^2 \alpha_{\overline{\mu}\nu_\mu h}^2 + \alpha_{\overline{\mu}\nu_e h}^4}{m_h^4} + \\ &\qquad \frac{f_{ee}^2 f_{\mu\mu}^2 + f_{e\mu}^4}{m_{\tilde{\delta}}^4} < 0.65 \times 10^{-11}\ \mathrm{GeV}^{-4}. \end{aligned} \tag{37}$$

Once again, since the inequality (37) contains no inter-

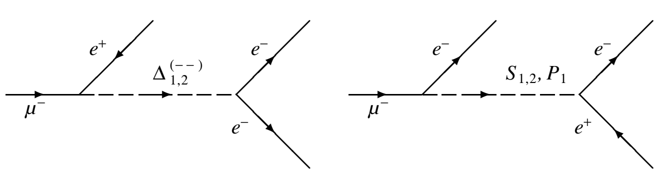

FIG. 1. The Feynman diagrams corresponding to the process $\mu^- \to e^+ e^- e^-$.

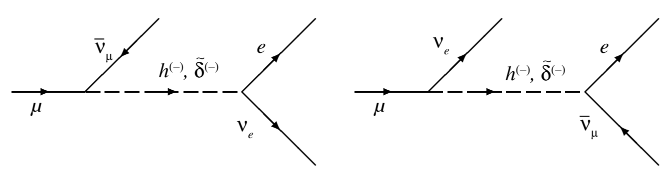

FIG. 2. The Feynman diagrams corresponding to the process $\mu^- \to e^- \nu_e \overline{\nu}_\mu$.

ferential terms, then it could be used to obtain the overestimated limits on the coupling constants both of the $h^{(-)}$ and $\tilde{\delta}^{(-)}$ bosons.

Let us as well discuss the muon decay through the channel

$$\mu^- \to e^- \gamma. \tag{38}$$

Under the analysis of this process in Refs. [12] the following inequality was obtained

$$\frac{f_{ee} f_{\mu\mu}}{m_\Delta^2} < 2 \times 10^{-10}\ \mathrm{GeV}^{-2}, \tag{39}$$

where m_Δ denotes a mass of a lightest doubly charged Higgs boson. This process goes on in the third order and the corresponding Feynman diagrams can be obtained from the diagrams for the muon anomalous magnetic moment (AMM) (see, Ref. [11]) by the replacement of the muon in the final state for the electron. It is obvious, that the authors of Refs. [12] only took into consideration the diagrams with the virtual $\Delta^{(--)}$ boson. On the other hand, the calculations show [11] that the amplitudes describing the diagrams for the muon AMM have different signs and when they are squared, interference takes place. In its turn it means, that the boundaries defined by the inequality (39), are rather arbitrary. The same conclusion can be drawn about the boundaries, obtained as the result of the investigation of the Bhabha scattering cross section in Refs. [12]

$$\frac{f_{ee}^2}{m_\Delta^2} < 9.7 \times 10^{-6}\ \mathrm{GeV}^{-2}. \tag{40}$$

As it was shown in Ref. [11] the contribution to the cross section of this process is also given by the diagrams with the virtual gauge bosons Z_2 and the Higgs bosons S_1, S_2, P_1.

The situation is absolutely the same with the boundaries obtained under the search for the muonium-antimuonium transition [12]

$$\frac{f_{ee} f_{\mu\mu}}{m_\Delta^2} < 5.8 \times 10^{-5}\ \mathrm{GeV}^{-2}. \tag{41}$$

Actually, also in this case, besides the diagrams with the $\Delta_{1,2}^{(--)}$ boson exchange, there are the diagrams with the S_1, S_2, P_1 bosons exchange [10].

B. Low-energy neutrinos scattering by charged leptons

Let us consider the reaction of the elastic antineutrino scattering by the electron

$$\overline{\nu}_e e^- \to \overline{\nu}_e e^-.$$

If one proceeds from the SM, then this reaction goes due to both the neutral and charged currents. It is obvious, that to finally determine the $V - A$ structure of the charged currents Lagrangian $\mathcal{L}_{\overline{e}\nu_e W}$ and the Lagrangian $\mathcal{L}_{\overline{\nu}_e \nu_e Z}$, describing the interaction of the neutrino with the Z boson, it is necessary to detect in the final state all the particles at the same time. Since a modern experiment does not allow to perform such a measurement, then one can not unconditionally state, that the Lagrangians $\mathcal{L}_{\overline{e}\nu_e W}$ and $\mathcal{L}_{\overline{\nu}_e \nu_e Z}$ really have the $V - A$ form. In other words, one can not exclude a possibility, that the neutrino has interactions beyond the SM (nonstandard interactions).

Sometimes under searching for nonstandard interactions (NSI's) of neutrinos in reactions of elastic scattering by electrons they introduce phenomenological parameters ϵ_{aL} and ϵ_{aR} ($a = e, \mu, \tau$), which characterize the intensity of the NSI's with respect to G_F. In this case the neutrino NSI's at low energies are described by the contact four-fermion Lagrangian of the following form

$$-\mathcal{L}_{\mathrm{NSI}} = 2\sqrt{2} G_F \overline{\nu}_a(x)\gamma_\sigma P_L \nu_a(x)[\epsilon_{aR}\overline{e}(x)\gamma^\sigma P_R e(x) + \epsilon_{aL}\overline{e}(x)\gamma^\sigma P_L e(x)], \tag{42}$$

where $P_{L,R} = (1 \pm \gamma_5)/2$. The analysis of the elastic neutrino and antineutrino scattering by the electrons with the help of the Lagrangian (42) gives wide regions for allowed values of the parameters ϵ_{eL} and ϵ_{eR} in which the strength of the extra interactions can reach the values ~ 2 [13]. However, the above mentioned parametrization could not be used for the definition of the limits on the YCC's. The reason is it does not comprise all the possible the neutrino NSI's.

As an example, within the LRM we view the process

$$\nu_e e^- \to \nu_e e^- \tag{43}$$

which is described by the Feynman diagrams presented in Fig. 3. In low-energy limit the amplitude which correspond to the diagrams with the virtual $\tilde{\delta}^{(-)}$ and $h^{(-)}$ bosons has the form

$$\mathcal{A} = \frac{2\alpha_{\overline{e}\nu_e h}^2}{m_h^2}\overline{\nu}_e(x)(1 - \gamma_5)e(x)\overline{e}(x)(1 + \gamma_5)\nu_e(x) + \frac{f_{ee}^2}{m_{\tilde{\delta}}^2}\overline{e}^c(x)(1 + \gamma_5)\nu_e(x)\overline{\nu}_e(x)(1 - \gamma_5)e^c(x). \tag{44}$$

Performing the Fierz transformation by means of the relation

$$\overline{a}(1 \pm \gamma_5)d\overline{c}(1 \mp \gamma_5)b = \frac{1}{2}\overline{a}\gamma_\sigma(1 \pm \gamma_5)b\overline{c}\gamma^\sigma(1 \mp \gamma_5)d, \tag{45}$$

where a, b, c, and d are bispinors, we get

$$\mathcal{A} = \overline{\nu}_e(x)\gamma_\sigma(1 - \gamma_5)\nu_e(x)\left[\frac{2\alpha_{\overline{e}\nu_e h}^2}{m_h^2}\overline{e}(x)\gamma^\sigma(1 + \gamma_5)e(x) + \frac{f_{ee}^2}{m_{\tilde{\delta}}^2}\overline{e}^c(x)\gamma^\sigma(1 + \gamma_5)e^c(x)\right]. \tag{46}$$

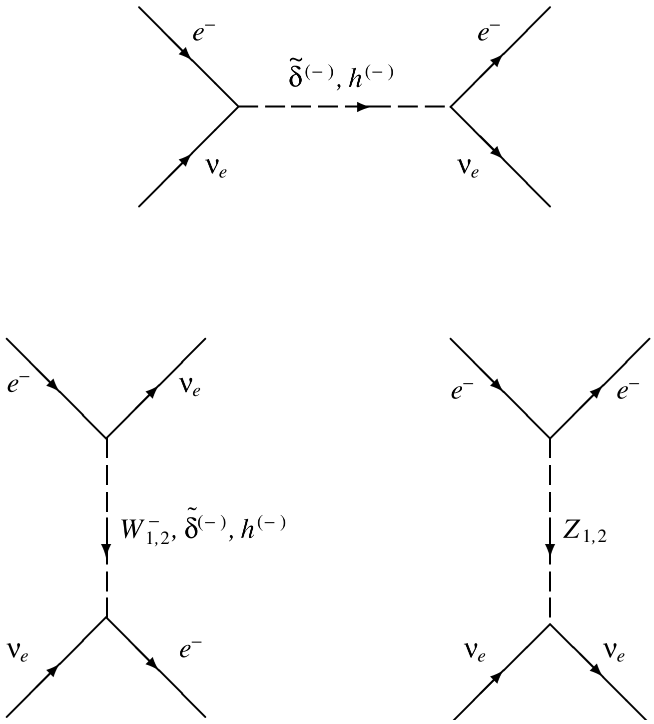

FIG. 3. The Feynman diagrams corresponding to the process $\nu_e e \to \nu_e e$.

From the relation obtained it becomes obvious, that the parametrization (42) does not comprise the case of the singly charged Higgs bosons. Note, that the gauge bosons $Z_{1,2}$ also induce the additive to the contact Lagrangian (42).

A more general parametrization to describe processes of low-energy neutrino scattering by charged leptons was introduced in Ref. [14]. With its help the amplitude of the process

$$\nu_a l_b \to l_a \nu_b, \tag{47}$$

can be presented as

$$\mathcal{A} = \frac{4G_F}{\sqrt{2}} \sum g^{\gamma}_{\lambda\lambda'} \langle \bar{l}_{b\lambda} \mid \Gamma_\gamma \mid \nu_{bn} \rangle \langle \bar{\nu}_{am} \mid \Gamma_\gamma \mid l_{a\lambda'} \rangle, \tag{48}$$

where the index γ fixes the type of interaction ($\Gamma_S = 1$ for scalar, $\Gamma_V = \gamma_\mu$ for vector, $\Gamma_T = \sigma_{\mu\nu}/\sqrt{2}$ for tensor), the indices λ, λ' define the helicities of charged leptons, and the helicities of the ν_b and ν_a are denoted by the indices n and m. Nine complex amplitudes $g^{\gamma}_{\lambda\lambda'}$ and G_F constitute the set of 19 independent real parameters, which must be experimentally defined.

In the LRM for the inverse muon decay

$$\nu_\mu e^- \to \mu^- \nu_e \tag{49}$$

the following amplitudes are not equal to zero: g^V_{LL}, g^V_{RR}, g^S_{LL}, g^S_{RR}. Then, using the existing experimental bound on $g^{\gamma}_{\lambda\lambda'}$ [15] one could obtain [16]

$$\frac{f_{ee} f_{\mu\mu}}{m^2_{\tilde{\delta}}} < 0.907 \times 10^{-5}\ \text{GeV}^{-2}, \tag{50}$$

$$\frac{f^2_{e\mu}}{m^2_{\tilde{\delta}}} < 0.109 \times 10^{-5}\ \text{GeV}^{-2}. \tag{51}$$

Note, that the parametrization (48) has not been used for the analysis of the reaction (43).

C. Neutrino oscillations

Now we try to find out what a knowledge about the Higgs sector structure can be obtained under investigation of the solar neutrinos. For the sake of simplicity we assume that a neutrino magnetic dipole moment is so small, that one can neglect an interaction with a solar magnetic field. We also neglect the influence of the heavy neutrino sector on the light neutrino oscillations in a matter. Then the evolution equation for the neutrino flux consisting of the left-handed and the right-handed light neutrinos is decoupled on two independent systems each of which describe the neutrinos with the same helicity.

In vacuum for the weak eigenstates the evolution in time is given in ultrarelativistic limit through the Schrödinger-like equation

$$i\frac{d}{dz}\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \end{pmatrix} = \mathcal{H}^v \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \end{pmatrix} = \mathcal{U}\begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2 \end{pmatrix}\mathcal{U}^\dagger \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \end{pmatrix}, \tag{52}$$

where $\Delta m^2 = (m_1^2 - m_2^2)/2E$, E is the neutrino energy and

$$\mathcal{U} = \begin{pmatrix} c_{\theta_\nu} & s_{\theta_\nu} \\ -s_{\theta_\nu} & c_{\theta_\nu} \end{pmatrix}$$

In a low-energy limit an interaction between neutrinos and a matter can reduce to an appearance of an effective potential (matter potential) in the Hamiltonian $\mathcal{H}^v$. In the SM the matter potential (MP) has the form $V^{CM} = \sqrt{2} G_F n_e(r)$, where $n_e(r)$ is the electron density. In the LRM the interactions between leptons and singly charged Higgs bosons modify the MP compared to the SM [16,17] and the evolution equation takes the form

$$i\frac{d}{dz}\Psi = \mathcal{H}^m \Psi, \tag{53}$$

where

$$\mathcal{H}^m_{11} = \mathcal{H}^v_{11} + V^{\text{SM}} + V^H_{ee}, \qquad \mathcal{H}^m_{12} = \mathcal{H}^v_{12} + V^H_{e\mu},$$

$$\mathcal{H}^m_{22} = \mathcal{H}^v_{22} + V^H_{\mu\mu},$$

$$V^H_{ab} = \left(\frac{\alpha_{\bar{l}_a \nu_e h} \alpha_{\bar{l}_b \nu_e h}}{m_h^2} - \frac{f_{ea} f_{eb}}{m^2_{\tilde{\delta}}}\right) n_e(r).$$

In the most simple case when the adiabatic effects are small the daytime survival probability of the left-handed electron neutrino $\mathcal{P}^{2\nu}_{\nu_e\to\nu_e}$ is defined by the expression

$$\mathcal{P}^{2\nu}_{\nu_e\to\nu_e}(\Delta m^2, \theta_\nu; n_e) = \frac{1}{2}[1 + \cos 2\tilde{\theta}_\nu \cos 2\theta_\nu], \quad (54)$$

where $\tilde{\theta}_\nu$ is the effective mixing angle at the point of the neutrino production, given by

$$\tan 2\tilde{\theta}_\nu = \frac{\Delta m^2 \sin 2\theta_\nu + 4EV^H_{e\mu}}{[\Delta m^2 \cos 2\theta_\nu - 2EV^{SM}] + 2E[V^H_{\mu\mu} - V^H_{ee}]}. \quad (55)$$

Since in the LRM the motion of the neutrino flux in a matter is described within the hybrid three-neutrino scheme (neutrino oscillations + neutrino NSI's), then the investigations could go in two different directions. The first approach, which is more traditional, uses the current best-fit points for the neutrino oscillation parameters obtained within the SM in order to obtain the bounds on the neutrino NSI's. The second approuch is aimed at defining the neutrino oscillation parameters within the chosen SM extension employing the modified MP. Further on we shall hold the first viewpoint.

Measured in all the solar neutrinos experiments the rates of the events set turn out to be less than it is predicted by the standard solar model SSM BP00 [18]. The resonance Mikheyev-Smirnov-Wolfenstein (MSW)-transitions in a matter naturally explain this phenomenon. Let us, for example, consider the Super-Kamiokande experiment (SK) [19] in which the solar neutrinos were detected by the registration of the elastic scattering on electrons (ES)

$$\nu_a e^- \to \nu_a e^-. \quad (56)$$

The total rate of the ES-events set is defined by the expression

$$R^{ES}_{\nu_e} = \langle \sigma_{\nu_e e\to\nu_e e}\rangle \Phi^{ES}_{\nu_e}, \quad (57)$$

where $\langle \sigma_{\nu_e e\to\nu_e e}\rangle$ is the cross section averaged over the neutrino initial spectrum from ^8B and $\Phi^{ES}_{\nu_e}$ is the ν_e flux on the Earth (remind, that in the SK experiment mainly the ν_e are detected). The flux $\Phi^{ES}_{\nu_e}$ is set by the relation

$$\Phi^{ES}_{\nu_e} = \langle \mathcal{P}^{2\nu}_{\nu_e\to\nu_e}\rangle \Phi^0_{\nu_e}, \quad (58)$$

where $\Phi^0_{\nu_e}$ is the total flux of the initial solar neutrinos. Analyzing the SK data within the SM one could find the oscillation parameters values corresponding to the best fit, namely $(\Delta m^2)_{bf}$ and $(\sin^2 2\theta_\nu)_{bf}$. Then, to obtain limits on the Higgs sector parameters in the LRM by means of the SK experiment one must act as follows.

One obtains the cross section of the process (56) in the LRM

$$\sigma_{\nu_e e\to\nu_e e} = \sigma^{SM}_{\nu_e e\to\nu_e e} + \Delta\sigma\left(\frac{\alpha^2_{\overline{\nu}_e eh}}{m_h^2}, \frac{f_{ee}^2}{m^2_{\tilde{\delta}}}\right) \quad (59)$$

and averages it over the ^8B spectrum. One substitutes values of $(\Delta m^2)_{bf}$ and $(\sin^2 2\theta_\nu)_{bf}$ in the expression for the averaged electron neutrino survival probability (54) and defines $\Phi^{ES}_{\nu_e}$. Further, using the experimental value of $R^{ES}_{\nu_e}$ it is possible to get information about the Higgs sector structure. From the explicit form of $\tan 2\tilde{\theta}_\nu$ and $\sigma_{\nu_e e\to\nu_e e}$ it follows, that in so doing the limit on the function having the form

$$f\left(\frac{\alpha^2_{\overline{\mu}\nu_e h}}{m_h^2}, \frac{\alpha_{\overline{\mu}\nu_e h}\alpha_{\overline{e}\nu_e h}}{m_h^2}, \frac{\alpha^2_{\overline{e}\nu_e h}}{m_h^2}, \frac{f_{ee}^2}{m^2_{\tilde{\delta}}}, \frac{f_{ee}f_{e\mu}}{m^2_{\tilde{\delta}}}, \frac{f_{e\mu}^2}{m^2_{\tilde{\delta}}}\right)$$

will be obtained. Consequently, in this case one can not manage to obtain the limit on just one quantity α_H^2/m_H^2 without using additional information.

D. $(g - 2)_\mu$ anomaly

Now we consider the experiment on measurement of the muon AMM which is carried out at the Brookhaven National Laboratory (BNL). The BNL'00 and BNL'01 results lead to the following averaged value of a_μ^{exp}

$$a_\mu^{exp} = (116592080 \pm 60) \times 10^{-11}\mu_0, \quad (60)$$

where μ_0 is the muon magnetic moment predicted by the Dirac theory. It is obvious, that to successfully compare experimental and theoretical data it is necessary to achieve the same high accuracy in calculations of the muon AMM value. First of all, it means, that contributions from all the sectors of the model, within which the calculations are being done, must be taken into account. In other words, the expression for the muon AMM must consist of three terms

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}. \quad (61)$$

The largest theoretical ambiguities in a_μ are governed by quantity a_μ^{had} which is mainly defined by contributions of virtual hadrons to a photon propagator in the 4th and 6th orders. Lower corrections cause vacuum polarization by hadrons a_μ^{had}(VP1), while corrections of the 6th order along with vacuum polarization by hadrons a_μ^{had}(VP2) include in itself light-by-light scattering a_μ^{had}(L by L) as well. Since a_μ^{had} contains integrals over virtual particles energies including a region of soft energies, a_μ^{had} can not be reliably calculated within perturbative QCD. Only the existing value of the quantity a_μ^{had}(VP2) causes no doubts. As for a_μ^{had}(L by L) that all its estimations made so far are model dependent. The value of a_μ^{had}(VP1) can be calculated on the basis of experimental data concerning the investigations of a cross section $e^+e^- \to$ hadron and/or on the basis of the analysis of data concerning a τ decay into hadrons. However, the inclusion of the latter introduces systematic uncertainties originating from isospin symmetry breaking effects which are difficult to estimate. A recent analysis, performed in Ref. [20] showed, that $a_\mu^{\rm had}$(VP1), obtained exclusively on the basis of e^+e^- data differs a lot from $a_\mu^{\rm had}$(VP1), which calculation is based on the data, concerning $\tau \to$ hadrons.

For our purpose we shall use the recent evaluations of $a_\mu^{\rm had}$ obtained on the basis of the latest data of the CMD-2 Collaboration on the analysis of a cross section $\sigma_{e^+e^-\to{\rm hadrons}}$ [21]. Then the difference between the SM prediction and the experimental value is given by

$$\delta a_\mu = a_\mu^{\rm exp} - (a_\mu)^{\rm SM} = (245 \pm 90) \times 10^{-11}\mu_0, \quad (62)$$

which corresponds to 2.4σ. Further on we shall consider, that the deviation δa_μ is connected with contributions from additional (compared to the SM) particles. In the LRM only Higgs bosons can apply for the role of such particles [11,22]. Since under investigation of the $(g-2)_\mu$ anomaly one usually assumes, that effects of the new physics are hidden in the electroweak corrections only, we shall reason that

$$(a_\mu^{\rm QED} + a_\mu^{\rm had})^{\rm SM} = (a_\mu^{\rm QED} + a_\mu^{\rm had})^{\rm LRM}, \quad \text{but} \quad (a_\mu^{\rm EW})^{\rm SM} \neq (a_\mu^{\rm EW})^{\rm LRM}. \quad (63)$$

Then, at 90% C.L., $\delta a_\mu/\mu_0$ must lie within the interval

$$130 \times 10^{-11} \leq \frac{\delta a_\mu}{\mu_0} \leq 360 \times 10^{-11}. \quad (64)$$

Let us use some consequences of Refs. [11,22] in which the BNL'99 and BNL'00 results have been explained by the contributions coming from $\Delta_{1,2}^{(--)}$, $\tilde{\delta}^{(-)}$, $h^{(-)}$ and S_1 bosons. As only the contributions from $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons contain the triplet YCC's we shall concentrate the attention on these bosons. We also note, that there are some variants of a choice of the LRM parameters in which one may completely neglect the contributions coming from the $h^{(-)}$, S_1, S_2 and P_1 bosons. The first variant is obvious, namely, it is enough to assume, that the masses of these bosons are beyond the electroweak scale.

The expression determining the contributions to the muon AMM from the $h^{(-)}$ boson contains the terms which might have opposite signs at the particular parameters values [22]. Then one can show that in the case of the opposite signs these contributions are so small that they do not explain the observed value of the muon AMM. Further, it is easy to show that the contribution to muon AMM from the P_1 boson is negative, and at the same values of varied parameters that is almost equal in absolute value to the contribution from the S_1 boson. As far as the S_2 boson is concerned, its contribution to the muon AMM is positive, but negligibly small. All this allows us to suggest the second variant, namely, the contributions from the $h^{(-)}$, S_1, S_2 and P_1 bosons to the muon AMM practically compensate each other.

Since the contributions from the $h^{(-)}$, S_1 and P_1 bosons gets comparable with $\delta a_\mu/\mu_0$ only in the case when $\tan\beta$ is close to 1 [22], in the third variant $\tan\beta$ is rather far from 1.

So, we shall discuss the LRM version in which the observed value of the muon AMM is explained by corrections from the $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons. In this case we shall also need the Lagrangian which describes the interactions of these bosons with photons and heavy neutrinos

$$\mathcal{L}_3 = ie\Big\{2\sum_{i=1,2}[\partial_\mu \Delta_i^{(--)*}(x)\Delta_i^{(--)}(x) - \Delta_i^{(--)*}(x)\partial_\mu\Delta_i^{(--)}(x)] + ie\beta_1 m_{W_1}[s_\xi W_{1\mu}(x) + c_\xi W_{2\mu}(x)]\tilde{\delta}^{(-)*}(x)\}A^\mu(x) - \sum_{a,b}\{\alpha_{\bar{l}_a N_b \tilde{\delta}}\bar{l}^c{}_b(x)(1-\gamma_5)N_a(x)\tilde{\delta}^{(-)*}(x)\Big\} + \text{conj.}, \quad (65)$$

where

$$\alpha_{\bar{l}_a N_b \tilde{\delta}} = \frac{f_{ab}\beta_1 k_+}{\sqrt{2}(\alpha_3 k_+^2/k_-^2 + 2\rho_1 - \rho_3)v_R}.$$

The diagrams, which induce the corrections from the $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons to the muon AMM are given in Fig. 4. The dominant contribution to the muon AMM is defined by the expression

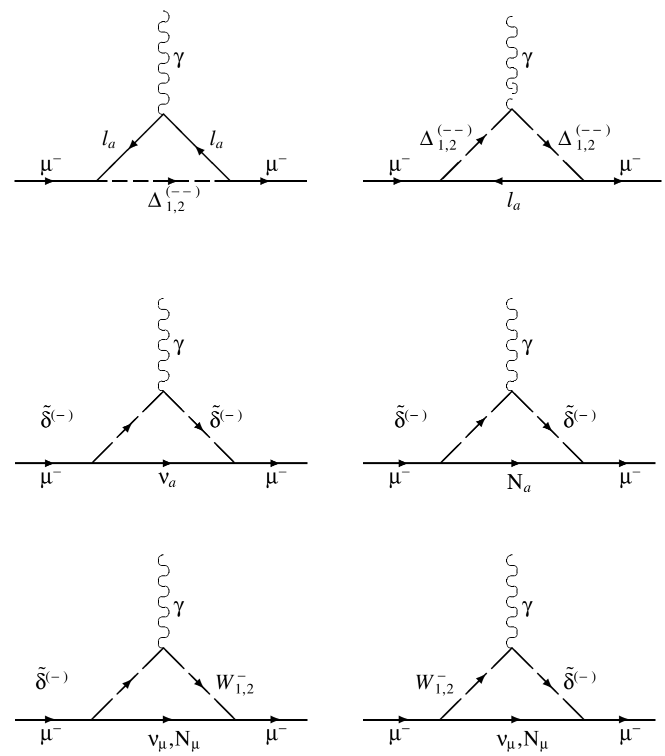

FIG. 4. One-loop diagrams contribute to the muon AMM due to the $\Delta_{1,2}^{(--)}$ and $\tilde{\delta}^{(-)}$ bosons.

$$\frac{\delta a_\mu}{\mu_0} = \frac{1}{8\pi^2}\bigg[4f_{\mu e}^2\sum_{i=1}^{2} I_e^{\Delta_i} + f_{\mu\mu}^2\sum_{i=1}^{2} I_\mu^{\Delta_i} + \sum_{b=e,\mu}\bigg(\frac{1}{2} f_{\mu b}^2 I_{\nu_b}^{\tilde\delta\tilde\delta} + \alpha^2_{\overline{\mu}N_b\tilde\delta} I_{N_b}^{\tilde\delta\tilde\delta}\bigg) + f_{\mu\mu}\bigg(-\frac{\beta_1 s_\xi m_{W_1}}{4} I^{W_1\tilde\delta} + \frac{\beta_1^2 m_{W_1}^2 c_\xi(\tan^2\beta - 1)}{2\alpha_3 g_L(\tan^2\beta + 1)v_R} I^{W_2\tilde\delta}\bigg)\bigg] \tag{66}$$

where

$$I_{l_a}^{\Delta_i} = \int_0^1\bigg[\frac{2m_\mu^2(z^2 - z^3)}{m_\mu^2(z^2 - z) + m_{\Delta_i}^2 z + m_{l_a}^2(1 - z)} + \frac{m_\mu^2(z^2 - z^3)}{m_\mu^2(z^2 - z) + m_{\Delta_i}^2(1 - z) + m_{l_a}^2 z}\bigg]dz,$$

$$I_{l_a}^{\Delta_i} > 0, \qquad I_i^{\tilde\delta\tilde\delta} = \int_0^1 \frac{m_\mu^2(z^3 - z^2)dz}{m_\mu^2 z^2 + (m_{\tilde\delta}^2 - m_i^2 - m_\mu^2)z + m_i^2},$$

$$i = \nu_a, N_a, \qquad I_i^{\tilde\delta\tilde\delta} < 0,$$

$$I^{W_1\tilde\delta} = \frac{m_\mu}{m_{W_1}^2 - m_{\tilde\delta}^2}\bigg\{\ln\bigg(\frac{m_{W_1}^2}{m_{\tilde\delta}^2}\bigg) - \int_0^1 \frac{z^2[m_\mu^2(2z - 1) + m_{W_1}^2 - m_{\nu_\mu}^2]dz}{m_\mu^2 z^2 + (m_{W_1}^2 - m_{\nu_\mu}^2 - m_\mu^2)z + m_{\nu_\mu}^2} + (m_{W_1} \to m_{\tilde\delta})\bigg\},$$

$$I^{W_2\tilde\delta} = I^{W_1\tilde\delta}(m_{W_1} \to m_{W_2}, m_{\nu_\mu} \to m_{N_\mu}), \qquad I^{W_k\tilde\delta} > 0.$$

Since in the case under study we have a large number of parameters (among them m_{N_1} and m_{N_2} are present too), then it is impossible to obtain definite information concerning the triplet YCC's and the Higgs boson masses. The information about the model parameters becomes more unambiguous, when the contribution to the muon AMM is induced only by the diagrams with the virtual $\Delta_1^{(--)}$ bosons. However, even so, the value of the muon AMM is the function of three parameters, namely, m_{Δ_1}, $f_{\mu\mu}$ and $f_{e\mu}$. Then, one could assume, that the nondiagonal YCC $f_{e\mu}$ turns to zero. From Eq. (13) follows, that this is true when $m_{N_1} = m_{N_2}$ or $\theta_N = 0$. Setting $m_{W_2} =$ 2500 GeV and $\xi = 10^{-2}$, in Fig. 5 we display the solid and dotted curves, which correspond to the values of $\delta a_\mu/\mu_0$ equal to 360×10^{-11} and 130×10^{-11}, respectively.

Again there are few LRM versions in which we may neglect the contributions to the muon AMM coming from the $\Delta_2^{(--)}$ and $\tilde\delta^{(-)}$ bosons. The first version is realized when the masses of the $\Delta_2^{(--)}$ and $\tilde\delta^{(-)}$ lay beyond the electroweak scale.

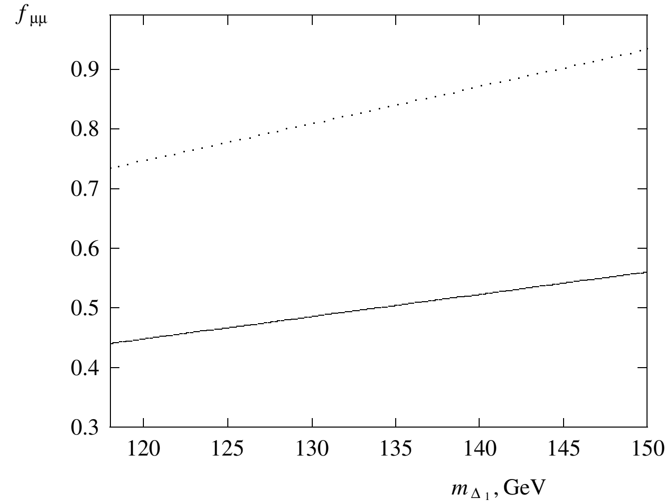

FIG. 5. The curves limiting the allowed region of the $f_{\mu\mu}$ and m_{Δ_1} values in the case when the contribution to the muon AMM is caused by the $\Delta_1^{(--)}$ boson and the mass degeneration takes place.

Since $I^{W_k\tilde\delta} > 0$, then the contribution to the muon AMM from the last two terms in the expression (66) is larger than zero if the coefficients, preceding them, turn out to be positive. Since

$$m_h^2 \approx \frac{\alpha_3(1 + \tan^2\beta)}{(\tan^2\beta - 1)} v_R^2$$

(see, Ref. [10]), then the last term in Eq. (67) is always positive. The coefficient in front of $I^{W_1\tilde\delta}$ would be bigger than zero, provided s_ξ and β_1 have the opposite signs. Therefore, such LRM version could occur in which the sum of contributions to δa_μ from the $\Delta_2^{(--)}$ and $\tilde\delta^{(-)}$ bosons is negligibly small.

It is also possible to assume existence of the LRM where the contributions to the muon AMM from $\Delta_2^{(--)}$, $\tilde\delta^{(-)}$, $h^{(-)}$, S_1, S_2 and P_1 bosons compensate each other.

IV. OSCILLATION PARAMETERS OF HEAVY NEUTRINOS

The analysis of the LFV decays and the low-energy light neutrino scattering bring us to the conclusion, that the most unambiguous limits on the triplet YCC's are given by the inequalities (33), (50), and (51). If in them one uses the upper limits, then two scenarios are possible.

In the first scenario the $\tilde\delta^{(-)}$ boson mass belongs to the electroweak scale while the $\Delta_{1,2}^{(--)}$ bosons masses do not belong to it. As it follows from Eqs. (19)–(21), to make the $\Delta_{1,2}^{(--)}$ bosons heavy, it is sufficient to demand

$$\rho_3/2 - \rho_1 \sim 1, \qquad \rho_2 \sim 1. \tag{67}$$

However, in so doing, to keep the $\tilde{\delta}^{(-)}$ boson on the electroweak scale is only possible in the case of the fine-tuning of the potential parameters, namely, if the relation

$$\frac{\rho_1 - \rho_3/2 + \alpha_3 k_+^2/(2k_-^2)}{\beta_1^2} \approx 10^{-2} \tag{68}$$

takes place. Since in this scenario a simultaneous fulfillment of the inequalities (33) and (51) can be ensured at rather large values of $f_{e\mu}$ then to determine the heavy neutrino masses one should use Eq. (26). Two roots of this equation are defined by the expression

$$(m_{N_1})_{1,2} = \frac{1}{2}[(f_{ee} + f_{\mu\mu})(v_R + v_L) - (m_{\nu_1} + m_{\nu_2})] \pm \Omega, \tag{69}$$

where

$$\Omega = \sqrt{\frac{1}{4}[(f_{ee} + f_{\mu\mu})(v_R + v_L) - m_{\nu_1} - m_{\nu_2}]^2 - f_{ee}f_{\mu\mu}(v_R + v_L)^2 + \frac{4f_{e\mu}^2 v_R v_L}{\sin 2\varphi_e \sin 2\varphi_\mu}}. \tag{70}$$

When $m_{\nu_1} = m_{\nu_2} = 0$, Eq. (70) is significantly simplified

$$\Omega = \frac{v_R + v_L}{2}\sqrt{(f_{\mu\mu} - f_{ee})^2 + 4f_{e\mu}^2}. \tag{71}$$

Using the expressions (13) and (14), it easy to obtain

$$f_{ee} + f_{\mu\mu} = \frac{m_{\nu_1} + m_{\nu_2} + m_{N_1} + m_{N_2}}{v_R + v_L}. \tag{72}$$

Combining this relation with Eq. (69), one gets

$$(m_{N_2})_{1,2} = \frac{1}{2}[(f_{ee} + f_{\mu\mu})(v_R + v_L) - (m_{\nu_1} + m_{\nu_2})] \mp \Omega. \tag{73}$$

So, in this scenario there are the two sets of the symmetric values for m_{N_1} and m_{N_2}. It means that both the direct ($m_{N_1} > m_{N_2}$) and inverse ($m_{N_1} < m_{N_2}$) hierarchies are possible. On the other hand the case of the heavy neutrino masses (quasi)degeneration is excluded because $f_{e\mu}$ is not a small quantity. Owing to Eq. (72) the neutrino masses sum is constant for every set of f_{ab}. Note, that the heavy neutrino masses are mainly defined by f_{ab} and v_R. The corrections to $m_{N_{1,2}}$ due to v_L could be reach of few $\times$ GeV while those caused by the light neutrino masses have the order of eV.

Now we set the masses of the W_2 and $\tilde{\delta}^{(-)}$ bosons to their low boundaries

$$m_{W_2} = 786 \text{ GeV}, \qquad m_{\tilde{\delta}} = 72 \text{ GeV} \tag{74}$$

and assume the mass of the lightest $\Delta^{(--)}$ boson being equal to 10 TeV. Then, by means of the relations (33), (50), and (51), for upper limits on the triplet YCC's we obtain

$$(f_{e\mu})_{\max} = 0.0748, \qquad (f_{ee})_{\max} = 0.0881, \qquad (f_{\mu\mu})_{\max} = 0.533.$$

Setting $v_L = 13$ GeV, $\xi = 10^{-2}$ and assuming the direct hierarchy we find

$$m_{N_1} = 91.7 \text{ GeV}, \qquad m_{N_2} = 659.6 \text{ GeV}. \tag{75}$$

By virtue of the relation (72) there are two possibilities

$$m_{N_1} > 91.7 \text{ GeV}, \qquad m_{N_2} < 659.6 \text{ GeV}, \tag{76}$$

or

$$m_{N_1} < 91.7 \text{ GeV}, \qquad m_{N_2} > 659.6 \text{ GeV} \tag{77}$$

Substituting (75) into Eqs. (22)–(24) one could obtain the mixing angles

$$\varphi_e = \varphi_\mu = 0.103, \qquad \theta_N = 0.162. \tag{78}$$

Now we switch over to the second scenario in which both $m_{\tilde{\delta}}$ and m_{Δ_2} belong to the electroweak scale (the value of m_{Δ_1} plays no significant role). As it follows from the inequality (33), in this case $f_{e\mu}$ must be negligibly small. This is so indeed, when either the mixing between heavy neutrinos is absent or the heavy neutrino masses are (quasi)degenerated. In the former case we obtain

$$m_{N_1} = f_{ee}(v_R + v_L), \qquad m_{N_2} = f_{\mu\mu}(v_R + v_L). \tag{79}$$

However, now we are left with the relation (50) which defines $f_{\mu\mu}f_{ee}$ only. As a result, in this case it is not possible to get unambiguous knowledge about m_{N_1} and m_{N_2}. Therefore, we should consider the latter case exclusively. With the help of Eqs. (14) and (15) we obtain

$$m_{N_1} \approx m_{N_2} \approx f_{\mu\mu}(v_R + v_L). \tag{80}$$

Note, that in this case no limits on θ_N can be found. Substituting $m_{\tilde{\delta}} = 72$ GeV in Eq. (50) for $f_{\mu\mu}$ we obtain the upper limit equal to 0.215. Then putting $v_L = 13$ and $m_{W_2} = 786$ GeV, we arrive at the following result

$$(m_{N_1})_{\max} = 260, \qquad (\varphi_e)_{\min} = (\varphi_\mu)_{\min} = 0.103. \tag{81}$$

From the analysis of the $(g-2)_\mu$ anomaly follows that the fixed limits on $f_{\mu\mu}$ can be obtained, provided, that the (quasi)degeneration of the heavy neutrinos masses takes place and the dominating contribution to muon AMM is caused by the $\Delta_1^{(--)}$ boson. Let us define the quantity

$$(f_{\mu\mu})_m = \frac{1}{2}[(f_{\mu\mu})_{\max} + (f_{\mu\mu})_{\min}],$$

where $(f_{\mu\mu})_{\max}$ and $(f_{\mu\mu})_{\min}$ correspond to the upper and lower curves in Fig. 5. Then, setting m_{Δ_1} to its low experimental boundary 118.4 GeV, we have $(f_{\mu\mu})_m = 0.587$. Then, for $m_{W_2} = 786$ GeV, $v_L = 13$ GeV with the help of Eq. (80), we obtain

$$m_{N_1} = 710.5 \text{ GeV}, \qquad \varphi_e = \varphi_\mu = 0.103. \tag{82}$$

V. CONCLUSIONS

The aim of the work under consideration was to set up a scheme to define the parameters of the sector of the heavy neutrinos N_i without their direct detecting. Within the two-flavor approximation the equations were obtained, which connect the neutrino oscillation parameters with the YCC's and VEV's of the Higgs triplets. The triplet YCC's in their turn, define constants of interaction between the $\Delta_{1,2}^{(--)}$, $\tilde{\delta}^{(-)}$ Higgs bosons and leptons. Thus, to find the triplet YCC's it is necessary to investigate the processes in which these bosons participate.

We considered the contributions from the Higgs bosons to the LFV decays, to the low-energy light neutrino scattering and to the muon AMM. As the analysis demonstrated, the constraints on the function like

$$F\left(\frac{f_{ee}f_{\mu\mu}}{m_{\tilde{\delta}}^2}, \frac{f_{e\mu}^2}{m_{\tilde{\delta}}^2}, \frac{f_{e\mu}f_{ee}}{m_{\Delta_1}^2}, \frac{\alpha_{l_a\nu_b h}^2}{m_h^2}, \frac{\alpha_{l_a l_b h}^2}{m_{S_1}^2}, \cdots\right)$$

can be most often obtained. It is obvious, that in such cases it is not possible to get unambiguous information about just one quantity like $\alpha_{H_i}/m_{H_i}^2$. However, in some instances a situation appears to be somewhat more favorable.

It was shown, that upper limits on just one quantity like $\alpha_{H_i}^2/m_{H_i}^2$ can be obtained under investigations both of the processes

$$\begin{aligned} &\mu^- \to e^+e^-e^-, \qquad \mu^- \to e^-\nu_e\overline{\nu}_\mu, \\ &\nu_\mu e^- \to \mu^-\nu_e. \end{aligned} \tag{83}$$

and the $(g-2)_\mu$ anomaly. However, in the latter case we had to assume that the corrections to the muon AMM are caused by the $\Delta_1^{(--)}$ boson only and the heavy neutrino masses (quasi)degeneration takes place. Therefore, to obtain more exact knowledge about the heavy neutrino parameters under investigation of the above mentioned processes one should increase the measurements precision and reduce the calculations ambiguities. In addition, one must examine the Lorentz structure of the amplitudes concerning the low-energy neutrinos scattering

$$e^-\overline{\nu}_a \to e^-\overline{\nu}_b, \qquad e^-\nu_a \to e^-\nu_b,$$

within the most general parametrization of Ref. [14]. In this respect the Borexino experiment, which is the laboratory-style experiment, gives a good chance for investigating the $\nu_e e$ elastic scattering. The Borexino detector can probe the monoenergetic Beryllium neutrinos from the Sun, with $E_{\nu_e} = 0.86$ MeV what allows to measure the energy spectrum of the recoil electron with very high precision.

The exact formulas which define the heavy neutrino masses in terms of $f_{ee}, f_{e\mu}, f_{\mu\mu}, v_L$ and v_R were obtained. It was disclosed, that using the data concerning the processes (83) as well as the constraints on the masses of the $\tilde{\delta}^{(-)}$, $\Delta_{1,2}^{(--)}$ and W_2 bosons, the limits on the heavy neutrinos masses could be set both in the case of the (quasi)degeneration and in the case when the (quasi)degeneration is absent. On the other hand, the data to explain the $(g-2)_\mu$ anomaly could be used to obtain limits on the heavy neutrino masses only in the case of the (quasi)degeneration.

It is reasonable to assume, that $f_{ee}, f_{\mu\mu} \leq 1$. Then, in the (quasi)degeneration case it is possible to obtain the heavy neutrino mass limit, connected with just one unknown parameter

$$m_{N_1} < \sqrt{\frac{m_{W_2}^2 - m_{W_1}^2}{g_L^2}}. \tag{84}$$

In the absence of (quasi)degeneration the similar formula can be obtained only for $m_{N_1} \ll m_{N_2}$, i.e., when $f_{e\mu}$ takes its maximal value $\sqrt{f_{ee}f_{\mu\mu}}$

$$m_{N_2} < 2\sqrt{\frac{m_{W_2}^2 - m_{W_1}^2}{g_L^2}}. \tag{85}$$

Then, for the current limit $m_{W_2} > 786$ GeV [8], Eqs. (84) and (85) result in

$$m_{N_1} \approx m_{N_2} < 1197 \text{ GeV} \tag{86}$$

and

$$m_{N_1} \ll m_{N_2} < 2394 \text{ GeV}. \tag{87}$$

The formalism, developed in this work, can be easily generalized for the case of the mixing of neutrinos of all three flavors. It should be also stressed, that the scheme of the heavy neutrino masses estimations, based on the investigation of the Higgs sector parameters, can be used for any gauge electroweak theory with an extended Higgs sector and the "see-saw" mechanism.

So, in the LRM the suggested scheme will have produced the exact $m_{N_{1,2}}$ values under the direct measurement of the $\Delta_{1,2}^{(--)}$, $\tilde{\delta}^{(-)}$ bosons parameters and the W_2 (or Z_2) gauge boson mass. This, for example, can be done under the registration of the $\Delta_{1,2}^{(--)}$ bosons in the reactions

$$e^- e^- \to e^- e^-, \mu^- \mu^-, \qquad e^- \mu^- \to e^- \mu^-$$

and after the discovery of the W_2 boson.

Thus, the information about the heavy neutrinos can be obtained under investigation of the processes without their direct participation. The most important thing is that now the subject of investigation is the charged, not neutral particles, and this makes the experimental part of the task much more easy. Definition of the heavy neutrinos masses will allow to start searching for them in the established energy interval. This, for example, can be done at the Fermilab Tevatron or at the $e^-\gamma$ colliders under investigations of the reactions

$$p\overline{p} \to Z^*_{1,2} \to N_l \overline{N}_l, \qquad e^- \gamma \to W_1^{-*} \to W_1^- N_e,$$

where the heavy neutrinos will be identified through the decay channels

$$N_l \to lW_1^+, lW_2^+, l\tilde{\delta}^{(+)}, lh^{(+)}.$$

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