PHYSICAL REVIEW D, VOLUME 70, 106006

Quantum theory of tensionless noncommutative p-branes

J. Gamboa,[1,*] M. Loewe,[2,†] and F. Méndez[3,‡]

[1]*Departamento de Física, Universidad de Santiago de Chile, Casilla 307, Santiago 2, Chile*
[2]*Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile*
[3]*INFN, Laboratorio Nazionali del Gran Sasso, SS, 17bis, 67010 Asergi (L'Aquila), Italy*
(Received 12 July 2004; published 19 November 2004)

The quantum theory involving noncommutative tensionless p-branes is studied following path integral methods. Our procedure allows a simple treatment for generally covariant noncommutative extended systems and it contains, as a particular case, the thermodynamics and the quantum tensionless string theory. The effect induced by noncommutativity in the field space is to produce a confinement among pairing of null p-branes.

DOI: 10.1103/PhysRevD.70.106006 PACS numbers: 11.25.Pm, 11.10.Wx, 11.27.+d

I. INTRODUCTION

Tensionless strings are extended objects discovered by Schild many years ago [1] and it corresponds, formally, to a set of infinite massless relativistic particles satisfying the constraint

$$\mathcal{H}_1 = p_\mu x'^\mu,$$

where x^μ transform as a scalar on the volume world and the spacetime index μ runs over $0, 1, 2, \ldots, D-1$ with D, the spacetime dimension.[1]

Physically speaking, this problem is related to the behavior of string theory at very high energy [3], also known as the strong coupling limit. More exactly, when the Regge slope goes to infinite, the spectrum of the string theory is massless; the situation is similar to what occurs in the standard model before the gauge symmetry is broken.

In string theory, however, the situation is quite involved because the "gauge group" corresponds to diffeomorphisms. As in the standard string theory, the critical dimensions for spacetime will be 26 (or 10) depending on the bosonic (or fermionic) character [4] as in the tensionful case.

In the general case (that is p-branes), concerning the critical dimensions, only partial results are known and, probably, they are not definitive [5].

The fact that null strings exist at very high energies, in the sense previously explained, together with the possibility that the Lorentz invariance could be deformed or even broken at such energies [6–9], raises the question of how to study the effect of such deformation in the null string scenario.

There are several proposals to formulate such Lorentz invariance deformations. They can be classified into two groups depending on the existence of a preferred reference frame. For such a case, there is a proposal [10,11] based in the deformation of the commutators between fields which is appropriated to discuss the question previously formulated.

However, one could argue that such deformation should not have any *impact* in the sense of a measurable consequence. That is, the deformation at the level of just one string or p-brane could be washed out. Then, there should be an amplifier mechanism.

Because of this, instead of considering just one p-brane, it turns out to be more interesting to consider a gas of such objects and to study the thermodynamics of such a system. Then, one can introduce the noncommutative fields and explore the *amplified* consequences of it.

The purpose of the present paper is to study the properties of these extended objects described by noncommutative fields as well as some statistical mechanics of null p-brane issues that include the null string as a particular case.

The paper is organized as follows: In Sec. II we consider relativistic particles in a noncommutative space where several quantum statistical mechanics considerations are studied. In Sec. III, the previous results are extended to noncommutative p-dimensional null branes and their quantum statistical mechanics properties described. In Sec. IV, we discuss the previous results as a possible interaction mechanism and we also present our main conclusions. An appendix including the statistical mechanics including the Matsubara modes for a free gas of relativistic particles is also included.

II. NONCOMMUTATIVE RELATIVISTIC QUANTUM MECHANICS

In this section we will construct noncommutative versions of generally covariant systems. We will start considering, first, the relativistic particle on a D-dimensional spacetime and later, in the next section, we will extend our results to tensionless strings and membranes.

*Electronic address: jgamboa@lauca.usach.cl
†Electronic address: mloewe@fis.puc.cl
‡Electronic address: fernando.mendez@lngs.infn.it

[1]For recent results, see [2].

A. Relativistic free particle and the proper-time gauge

There are many approaches to discuss relativistic quantum mechanics of a free particle. One of them is the so-called proper-time method, which was used in the early 1950s in connection with quantum electrodynamics [12]. The idea is to consider a particle in a $D+1$-dimensional Euclidean spacetime.

The diffusion equation for such a system is

$$-\frac{1}{2}\Box\varphi(x,s) = \frac{\partial\varphi}{\partial s}, \tag{1}$$

where $\Box$ is the D-dimensional Laplacian.

Then, using the ansatz

$$\varphi(x,s) = e^{-(m^2/2)s}\phi(x), \tag{2}$$

one finds that $\phi(x)$ satisfies the Klein-Gordon equation if m is the mass of the particle.

In this approach, the propagation amplitude is given by the Laplace transform

$$G[x,x';m^2] = \int_0^\infty ds e^{-s(m^2/2)}G[x,x';s], \tag{3}$$

where

$$G[x,x';s] = \int \mathcal{D}x e^{-\int_0^1 d\tau[\dot{x}^2/(2s)]} = s^{-D/2}e^{-[(\Delta x)^2/(2s)]}. \tag{4}$$

From this, one obtains the partition function for a gas of N free relativistic particles[2]

$$Z_s = \{\mathrm{Tr}[e^{-(m^2/2)s}G(x,x';s)]\}^N, \tag{5}$$

or equivalently

$$\ln Z = N\left[-\frac{m^2}{2}s - \frac{D}{2}\ln s + \ln\mathcal{V}\right], \tag{6}$$

where $\mathcal{V} = V\times\mathrm{const}$ is the D-dimensional spacetime, V is the $D-1$-dimensional ordinary spatial volume, and s plays the role of $\beta = 1/kT$.

B. The relativistic particle in a noncommutative space

Equation (1) suggests a simple way to extent the problem to a gas of relativistic particles on a noncommutative space.

Indeed, from (1) we see that the Hamiltonian for a relativistic particle is

$$\hat{H} = \tfrac{1}{2}p_\mu^2. \tag{7}$$

Once (7) is given, noncommutativity is implemented through the deformed algebra

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}, \qquad [p_\mu, p_\nu]iB_{\mu\nu}, \tag{8}$$

$$[x_\mu, p_\nu] = i\delta_{\mu\nu}, \tag{9}$$

where $\theta_{\mu\nu}$ and $B_{\mu\nu}$ are the deformation parameters in the phase space.

For convenience we choose

$$\theta_{i0} = 0, \qquad \theta_{ij} = \epsilon_{ij}\theta, \tag{10}$$

$$B_{i0} = 0, \qquad B_{ij} = \epsilon_{ij}B. \tag{11}$$

Therefore, the equation of motion for this particle is

$$\dot{x}_\mu = p_\mu, \qquad \dot{p}_i = \epsilon_{ij}Bp_j. \tag{12}$$

These equations can be integrated directly by using (10) and (11). Indeed, one of the equations is trivial, namely, the energy conservation condition ($\dot{p}_0 = 0$). Note that the symmetric gauge we have chosen implies that noncommutativity is realized only for the first two momenta and coordinate components. The other components are treated as usual. In principle, we could extend this hypothesis taking also other pairs of momenta and coordinate components, but this is not essential for our discussion.

Keeping this in mind, the remaining equations have the solution

$$\begin{aligned} p_1 &= \frac{1}{2}(\alpha e^{-iBt} + \alpha^\dagger e^{iBt}), \\ p_2 &= \frac{1}{2i}(\alpha e^{-iBt} - \alpha^\dagger e^{iBt}), \end{aligned} \tag{13}$$

where α's are constant operators.

The coordinates $x_{1,2}$ are obtained in a similar way using (12), i.e.,

$$\begin{aligned} x_1 &= \frac{1}{2iB}(\alpha^\dagger e^{iBt} - \alpha e^{-iBt}) + x_{01}, \\ x_2 &= \frac{1}{2B}(\alpha e^{-iBt} + \alpha^\dagger e^{iBt}) + x_{02}. \end{aligned} \tag{14}$$

From the commutation relation of p's, we see that it is possible to define operators a and $a^\dagger$ satisfying the algebra

$$[a,a] = 0 = [a^\dagger, a^\dagger], \qquad [a, a^\dagger] = 1, \tag{15}$$

where

$$\alpha \to \sqrt{B}a, \qquad \alpha^\dagger \to \sqrt{B}a^\dagger.$$

The equations of motion, as a second order equation system, are

$$\ddot{x}_\mu = B_{\mu\nu}\dot{x}_\nu,$$

which can be solved by the Ansatz $x_\mu = a_\mu e^{i\omega s}$.

[2]Throughout this paper, we assume that the particles are spinless. The reader should note also that we are assuming the Maxwell-Boltzmann statistics; for a justification about this; see the appendix.

The last equation is

$$(i\omega\delta_{\mu\nu} - B_{\mu\nu})a_\nu = 0.$$

Therefore, the dispersion relation for this system is

$$\omega_\pm = \begin{cases} \pm B \\ 0, \end{cases} \tag{16}$$

and, since one of the eigenvalues vanishes, the Hamiltonian spectrum is degenerated.

Thus, the Hamiltonian for a relativistic particle living on a noncommutative space is

$$H = \frac{B}{2}\left(a^\dagger a + \frac{1}{2}\right) + \frac{1}{2}\sum_{n=1}^{D-3}(p_\mu^2)_n. \tag{17}$$

Finally, the statistical mechanics for a gas of N relativistic particles on a noncommutative space, in the symmetric gauge, is obtained from the partition function

$$\begin{aligned} Z_s &= \left(s^{-(D-3)/2}e^{-(m^2/2)s}\sum_{n=0}^{\infty}\mathcal{G}_0 e^{-s(B/2)[n+(1/2)]}\right)^N \\ &= \left[\frac{\mathcal{G}_0 e^{-(m^2/2)s}s^{-(D-3)/2}}{\sinh(\frac{B}{2}s)}\right]^N, \end{aligned} \tag{18}$$

where $\mathcal{G}_0$ is the degeneracy factor due to the zero eigenvalue of the Hamiltonian.[3]

The thermodynamic properties of this system can be computed directly from (18).

III. THE STRONG COUPLING REGIME FOR MEMBRANES IN NONCOMMUTATIVE SPACES

In this section, we will discuss the extension of the previous problem to membranes moving on a noncommutative space in the strong coupling regime.

A relativistic membrane is a p-dimensional object embedded on a D-dimensional flat spacetime and described by the Lagrangian density

$$\mathcal{L} = \frac{T}{2}\sqrt{g^{(p+1)}}[g_{\alpha\beta}G^{\mu\nu}\partial^\alpha x_\mu \partial^\beta x_\nu - (p-1)],$$

where $g_{\alpha\beta}^{(p+1)}$ ($\alpha, \beta = 0, 1, 2, \ldots, p$) is the metric tensor on the world volume, $G^{\mu\nu}$ is the metric tensor where the p-brane is embedded with $\mu, \nu = 0, 1, 2, \ldots, D$, and T is the superficial tension.

The Hamiltonian analysis yields to the following constraints:

$$H_\perp = \tfrac{1}{2}(p^2 + T^2 g^{(p)}), \tag{19}$$

$$H_i = p_\mu \partial_i x^\mu, \tag{20}$$

[3] Although this factor can be computed by using a regularization prescription, here this factor is absorbed as a normalization constant.

where $g^{(p)}$ is the spatial metric determinant and T is the superficial tension.

The strong coupling regime corresponds to $T \longrightarrow 0$, and in this limit the constraints are

$$H_\perp = \tfrac{1}{2}p^2, \tag{21}$$

$$H_i = p_\mu \partial_i x^\mu, \tag{22}$$

and the membrane becomes an infinite set of free massless relativistic particles moving perpendicularly to the p-dimensional surface.

In the special case of the tensionless string ($p = 1$), each point of the string is associated with a massless relativistic particle and, as a consequence, all the points of the string are causally disconnected.

In this tensionless string approach, the field $x^\mu(\sigma, \tau)$ is replaced by $x_i^\mu(\tau)$, where $i = 1, 2, \ldots$ is an infinite countable set labeling each point of the tensionless string.

Using this philosophy, we will start constructing tensionless strings.

A. Tensionless strings from particles

Let us start by noticing that a tensionless string [4] is made up of infinite massless relativistic particles causally disconnected and, therefore, instead of (1), one has

$$\begin{aligned} -\frac{1}{2}\Box\varphi_1(x, s_1) &= \frac{\partial\varphi_1}{\partial s_1}, \\ -\frac{1}{2}\Box\varphi_2(x, s_2) &= \frac{\partial\varphi_2}{\partial s_2}, \\ &\vdots \\ -\frac{1}{2}\Box\varphi_k(x, s_k) &= \frac{\partial\varphi_k}{\partial s_k}. \end{aligned} \tag{23}$$

These equations can be solved by generalizing the Ansatz (2), i.e.,

$$\varphi(x_1, \ldots, x_k, \ldots; s_1 \ldots, s_k, \ldots) = \prod_{i=1}^{\infty} e^{-(m^2/2)s_i}\phi(x_i), \tag{24}$$

where m^2 is an infrared regulator that will vanish at the end of the calculation.

The limit of an infinite number of particles is delicate but here, formally, one can take this limit, simply, assuming that in the continuous limit one can replace the set $\{i\}$ by an integral in σ and, as a consequence, the propagation amplitude can be written as

$$\begin{aligned} G[x(\sigma), x'(\sigma)] = \int_0^\infty \mathcal{D}s(\sigma)e^{-(m^2/2)\int d\sigma s(\sigma)} \\ \times G[x(\sigma), x'(\sigma); s(\sigma)], \end{aligned} \tag{25}$$

where $G[x(\sigma), x'(\sigma); s(\sigma)]$ is given by

$$G[x(\sigma), x'(\sigma); s(\sigma)] = s^{-D/2}(\sigma)e^{-\int d\sigma\{[\Delta x(\sigma)]^2/[2s(\sigma)]\}}. \tag{26}$$

The formula (25) generalizes the proper-time method to the tensionless string case. Probably, this approach to string theory was first used by Eguchi in [4].

Using (25) and (26), the partition function of an N tensionless string gas is

$$Z[s(\sigma)] = \left[\int \mathcal{D}x(\sigma)G[x(\sigma), x(\sigma); s(\sigma)]\right]^N = (s^{-D/2}e^{-\int d\sigma(m^2/2)s(\sigma)})^N. \tag{27}$$

This partition function reproduces correctly the results for the thermodynamics of a tensionless string gas [13].

Indeed, from (27), the Helmholtz free energy is

$$F[s] = \frac{N}{s(\sigma)}\left[\frac{D}{2}\ln[s(\sigma)] + \frac{m^2}{2}\int d\sigma s(\sigma) + \ln(\mathcal{V})\right].$$

As $1/s$ is the temperature, then from the limit $m^2 \rightarrow 0$ we see that $F/T \sim \ln(T)$, again in agreement with other null string calculations [13,14].

From the last equation, one obtains that

$$P[s(\sigma)]V = \frac{N}{s(\sigma)} \tag{28}$$

is the state equation for an ideal tensionless string gas.

B. Tensionless membranes from tensionless strings

In order to construct tensionless membranes, we begin by considering a membrane as an infinite collection of tensionless strings. Thus, if the membrane is a p-dimensional object, with local coordinates $(\sigma_1, , \sigma_p)$, then the propagation amplitude, formally, corresponds to (25), with the substitution

$$\sigma \rightarrow (\sigma_1, \ldots, \sigma_p).$$

Therefore, the partition function for a gas of N tensionless membranes is

$$Z[s(\sigma)] = (\lim_{n\rightarrow\infty}\{[s(\sigma)]^{-D/2}e^{-(m^2/2)\int d^p\sigma s(\sigma)}\mathcal{V}\}^n)^N, \tag{29}$$

where n is the number of tensionless strings.

One should note here that the expression

$$\{[s(\sigma)]^{-D/2}e^{-(m^2/2)\int d^p\sigma s(\sigma)}\}^n,$$

formally emphasizes that a tensionless p-brane is made up of n tensionless strings.

However, this last expression was computed in (26) and in our case is

$$\prod_{i=1}^p [s(\sigma_i)]^{-D/2}e^{-(m_i^2/2)\int d\sigma_i s(\sigma_i)}.$$

Then, the total partition function for an ideal gas of N tensionless p-branes is given by

$$Z = \prod_{i=1}^p \{[s(\sigma_i)]^{-D/2}e^{-(1/2)m_i^2\int d\sigma_i s(\sigma_i)}\}^N.$$

In order to compute the state equation we proceed as follows: First, one chooses $s(\sigma_1) = s(\sigma_2)\cdots = s(\sigma)$ and one puts also $m_1 = m_2 = \cdots = m$, then

$$P[s(\sigma)]V = \frac{N}{s(\sigma)}. \tag{30}$$

The Helmholtz free energy, compared to the tensionless string case, has a different behavior. Indeed, the Helmholtz free energy becomes

$$F[s] = \frac{pN}{s(\sigma)}\left[\frac{D}{2}\ln[ps(\sigma)] + \frac{m^2}{2}\int d\sigma s(\sigma) + \ln(\mathcal{V})\right],$$

and for $s \rightarrow \infty$ one has the quantity $sF \sim \frac{D}{2}\ln[ps]$ similar to the string case, but in this case p could smooth out the behavior of sF.

C. Including noncommutativity in tensionless p-branes

Using the previous results, we can generalize our arguments in order to include noncommutativity in tensionless p-branes. In order to do that, one starts considering a tensionless p-brane described by the field $x_i^\mu(\tau)$ with i labeling the dependence in $(\sigma_1, \sigma_2, \ldots, \sigma_p)$. This field transforms as a scalar on the world-volume but as a vector in the space where the p-brane is embedded.

Let us suppose that the components—we say x_i^{D-1} and x_i^D—do not commute, then, in such case, the Green function can be written as

$$G[x(\sigma), x'(\sigma); s(\sigma)] = \int_0^\infty ds e^{-(m^2/2)s}\prod_{k=0}^{D-3}\left[\int \mathcal{D}x_i^k e^{-\int_0^1 d\tau[1/(2s)](\dot{x}_i^k)^2}\right]\int \mathcal{D}x_i^{(D-2)}\mathcal{D}x_i^{(D-1)}e^{-\int_0^1 d\tau[1/(2s)][(\dot{x}_i^{(D-2)})^2+(\dot{x}_i^{(D-1)})^2]}. \tag{31}$$

The integral in the second line on the right-hand side (rhs), corresponds formally to a nonrelativistic particle with mass (s^{-1}) moving in plane in the presence of a constant perpendicular magnetic field B. In the first line on the rhs, however, the integral formally corresponds to the Green function for a set of p-free relativistic particles moving in $(D-3)$-dimensional spacetime.

Thus, the calculation of these integral is straightforward. Indeed,

$$G[x(\sigma), x'(\sigma)] = \int_0^\infty ds[s(\sigma)]^{-[(D-3)/2]} \times s(\sigma) e^{-[1/(2s)](\Delta x_i^k)^2 - (p/2)m^2 \int d\sigma s(\sigma)} \mathrm{HO},$$

where HO means the harmonic oscillator calculation for the two-dimensional relativistic Landau problem.

The partition function for this gas of N-tensionless p branes

$$\begin{aligned} Z[s(\sigma)] &= \mathrm{Tr}\{G[x(\sigma), x'(\sigma); s(\sigma)]\} \\ &= \Big([s(\sigma)]^{-[(D-3)/2]} e^{-[(pm^2)/2] \int d\sigma s(\sigma)} \\ &\quad \times \sum_{n=0}^{\infty} \mathcal{G}_0 e^{-(B/2)[n+(1/2) \int d^p \sigma s(\sigma)]}\Big)^N \\ &= \left[\frac{\mathcal{G}_0 [s(\sigma)]^{-[(D-3)/2]}}{\sinh[\frac{pB}{2} \int d\sigma s(\sigma)]}\right]^N. \end{aligned} \tag{32}$$

Therefore, if we assume pairing interaction, then noncommutativity induces a motion for a tensionless p-branes confined via a harmonic potential oscillator.

IV. INTERACTIONS VIA NONCOMMUTATIVITY IN THE PHASE SPACE

In the previous section, we argued how to construct noncommutative extended objects. In this section, we would like to give insight in a different physical context and to investigate the possibility of a possible interaction by means of noncommutativity. This procedure is a simple extension of the noncommutative field.

From the nonrelativistic point of view, apparently there is no problem with nonlocal communication [15]. Indeed, let us suppose two nonrelativistic particles in one dimension, labeled by coordinates x_1 and y_1 and canonical momenta p_1 and p_2, respectively. Note that the index refers now to the particles involved.

The Hamiltonian for this system is

$$H = \tfrac{1}{2} p_1^2 + \tfrac{1}{2} p_2^2. \tag{33}$$

Although naively the particles in (33) are free, they can interact if we posit the commutator

$$[p_1, p_2] = iB, \tag{34}$$

where B measures the strength of this interaction which can play (or not) the role of a magnetic field.

The exact equivalence between this system and the Landau problem is a subtle point because, by considering only a noncommutative phase space with noncommutative parameters θ and B, one can show that noncommutative quantum mechanics and the Landau problem coincide if the relation $\theta = 1/B$ is fulfilled, i.e., if we have just the magnetic length [16]. From this example, one extracts as a conclusion that the equivalence between a physical system such as the Landau problem and noncommutative quantum mechanics occurs only for the critical point $\theta B = 1$. For other values of θB, noncommutative quantum mechanics describes a physics completely different from the Landau problem.

The above example can be generalized for more particles; for instance, let us consider two free particles moving in a commutative plane.

The Hamiltonian is

$$H = \tfrac{1}{2}(p_{1x}^2 + p_{1y}^2) + \tfrac{1}{2}(p_{2x}^2 + p_{2y}^2). \tag{35}$$

Then, let us assume that the interaction is given by[4]

$$[p_{1x}, p_{2x}] = iB, \qquad [p_{1y}, p_{2y}] = iB, \tag{36}$$

then, as in the previous case, the Hamiltonian is

$$H = \tfrac{1}{2}(p_{1x}^2 + p_{2x}^2) + \tfrac{1}{2}(p_{1y}^2 + p_{2y}^2). \tag{37}$$

Thus, the commutator (36) and the Hamiltonian (37) describe a couple of particles living on a plane and interacting formally with a magnetic field perpendicular to the plane.

We would like to remark that our procedure, of course, has generated a nonlocal interaction between both particles.

In the general case for N particles moving on a D dimensional commutative space, the generalization is straightforward.

Indeed, the Hamiltonian is

$$H = \tfrac{1}{2}(p_{1x}^2 + p_{1y}^2 + \cdots) + \tfrac{1}{2}(p_{2x}^2 + p_{2y}^2 + \cdots) + \cdots, \tag{38}$$

then the interaction can be written as

$$[p_i^a, p_j^b] = i\delta_{ij}\epsilon^{ab} B, \tag{39}$$

where a, b run on $1, \ldots, N$ labeling the different species of particles and the indexes $i, j, \ldots$ select the vectorial component of $\mathbf{x}$.[5]

We rewrite the Hamiltonian as

$$H = \tfrac{1}{2}(p_{1x}^2 + p_{1x}^2 + \cdots) + \tfrac{1}{2}(p_{1y}^2 + p_{1y}^2 + \cdots) + \cdots. \tag{40}$$

Thus, in the critical point, this generalized system is related to the quantum Hall effect as has been proposed using a different argument by [17].

Thus, in our context, one could conclude that, if two particles interact via nonlocal communication, the phase space could be noncommutative. However, this fact does

[4] Of course this is a simplification because we are assuming that the noncommutative parameters are the same.

[5] The component of the antisymmetric density tensor ϵ^{ab} is defined as $+1$ if $a > b$.

not exclude other possible mechanisms as a source of nonlocal interactions.

V. CONCLUSIONS

In conclusion, we have constructed the statistical mechanics of generally covariant systems such as p-branes assuming that for each point of the world volume one define a noncommutative field. From these results, we have studied the quantum statistical mechanics of tensionless p-brane gas, which is a qualitatively different system in comparison to the commutative one.

In addition, we have discussed a possible mechanism to implement no-local interactions by means of noncommutativity that could be useful in the quantum Hall effect or other systems.

The possible cosmological implications of these results as well as other results are also studied in [18,19].

ACKNOWLEDGMENTS

We would like to thank J. L. Cortés, A. P. Polychronakos, and J. C. Retamal for discussions. This work has been partially supported by Grants No. 1010596 and No. 1010976 from Fondecyt. F. M. was partially supported by INFN (Italy).

APPENDIX: STATISTICAL MECHANICS FOR A FREE RELATIVISTICS PARTICLES GAS

In this appendix, we will study the statistical mechanics for a free relativistic particle gas. One starts by considering the partition function,

$$Z = \mathrm{Tr}G[x_2, x_1], \tag{A1}$$

where $G[x_2, x_1; m^2]$ is the Green function for a free particle in the sense discussed in Sec. II and it is given by

$$G[x_2, x_1; m^2] = s^{-(D/2)} e^{-[(\Delta x)^2/(2s)]}, \tag{A2}$$

where D is the dimension of the spacetime.

In the Euclidean space (A1) is computed using periodic boundary conditions but instead one uses

$$x_1^0(0) = x_2^{(0)}(T) + 2n\pi R, \tag{A3}$$

$$x_1^{D-1} = x_2^{D-1}(T). \tag{A4}$$

where $i = 1, 2, \ldots, D-1$, $n = 0, \pm 1, \pm 2, \ldots$ are the Matsubara frequencies, and R is the compactification radius.

Using this fact, one finds that the total partition function is

$$Z = \sum_{n=1}^{\infty} Z^{(n)} \tag{A5}$$

$$= \sum_{n=1}^{\infty} e^{in\theta} Z^{(n)} \tag{A6}$$

$$= s^{-D/2} \vartheta_3(e^{-[(2\pi^2 R^2)/s]}) e^{i\theta}, \tag{A7}$$

where ϑ_3 is the Jacobi function and θ is the phase factor that plays the analogous role of the magnetic flux in the Aharonov-Bohm effect. Since s plays an analog role of β in statistical mechanics, in the high temperatures limit $\vartheta_3 \to 1$ and the logarithm of the partition function in this case is

$$\ln Z = N\left[-\frac{m^2}{2}s - \frac{D}{2}\ln s + \ln \mathcal{V}\right]. \tag{A8}$$

Thus, one finds that, in the spinless case and in the high temperature region, there are no Matsubara modes and the statistics is, of course, the Maxwell-Boltzmann one.

[1] A. Schild, Phys. Rev. D **16**, 1722 (1977); A. Karlhede and U. Lindström, Classical Quantum Gravity **3**, L73 (1986).

[2] A. Bredthauer, U. Lindstrom, J. Persson, and L. Wulff, J. High Energy Phys. 02 (2004) 051; G. K. Savvidy, Int. J. Mod. Phys. A **19**, 3171 (2004).

[3] See, e.g., M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987).

[4] See, e.g., T. Eguchi, Phys. Rev. Lett. **44**, 126 (1980); F. Lizzi, B. Rai, G. Sparano, and A. Srivastava, Phys. Lett. **182**, 326 (1986); J. Gamboa, C. Ramírez, and M. Ruiz-Altaba, Phys. Lett. **225**, 335 (1989); Nucl. Phys. **338**, 143 (1990); Phys. Lett. **231**, 57 (1989).

[5] U. Marquard and M. Scholl, Phys. Lett. B **227**, 227 (1989).

[6] The problem of Lorentz invariance violation has been extensively studied during the last years by Kostelecký and collaborators. Some references are D. Colladay and V. A. Kostelecký, Phys. Lett. B **511**, 209 (2001); V. A. Kostelecký and R. Lehnert, Phys. Rev. D **63**, 065008 (2001); R. Bluhm and V. A. Kostelecký, Phys. Rev. Lett. **84**, 1381 (2000); V. A. Kostelecký and Charles D. Lane, Phys. Rev. D**60**, 116010 (1999); R. Jackiw and V. A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999); D. Colladay and V. A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

[7] G. Amelino-Camelia, J. Ellis, N. E. Mavromatos, D. V. Nanopoulos, and S. Sarkhar, Nature (London) **393**, 763 (1998); L. Smolin and J. Maguejo, Phys. Rev. Lett. **88**, 190403 (2002); R. Gambini and J. Pullin, Phys. Rev. **59**, 124021 (1999); J. Alfaro, H. A. Morales-Técotl, and L. F. Urrutia, Phys. Rev. Lett. **84**, 2318 (2000); J. Alfaro, H. A. Morales-Técotl, and L. F. Urrutia, Phys. Rev. D **65**, 103509 (2002); J. Alfaro and G. Palma, Phys. Rev. D

65, 103516 (2002); R. Aloisio, P. Blasi, P. Ghia, and A. Grillo, Phys. Rev. D **62**, 053010 (200); R. Aloisio, P. Blasi, A. Galante, P. L. Ghia, and A. F. Grillo, astro-ph/0210402; O. Bertolami, Gen. Relativ. Gravit. **34**, 707 (2002).
[8] G. Amelino-Camelia, Int. J. Mod. Phys. D **11**, 35 (2002); Phys. Lett. B **510**, 255 (2001).
[9] T. Kifune, Astrophys. J. Lett. **518**, L21 (1999). See also G. Amelino-Camelia and T. Piran, Phys. Rev. D **64**, 036005 (2001), where a somewhat more systematic approach is developed.
[10] J. Carmona, J. L. Cortés, J. Gamboa, and F. Méndez, Phys. Lett. B **565**, 222 (2003).
[11] J. Carmona, J. L. Cortés, J. Gamboa, and F. Méndez, J. High Energy Phys. 03 (2003) 058.
[12] See, e.g., R. P. Feynman, Phys. Rev. **80**, 440 (1950).
[13] F. Lizzi and G. Sparano, Phys. Lett. B **232**, 311 (1989).
[14] J. J. Atick and E. Witten, Nucl. Phys. **B310**, 291 (1989).
[15] See, e.g., A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer Academic Publishers, Dordrecht, 1993).
[16] For discussions on noncommutative quantum mechanics in the context of this paper see, e.g., V. P. Nair and A. P. Polychronakos, Phys. Lett. B **505**, 267 (2001); J. Gamboa, M. Loewe, and J. C. Rojas, Phys. Rev. D **66**, 045018 (2002); J. Gamboa, M. Loewe, F. Méndez, and J. C. Rojas, Int. J. Mod. Phys. A **17**, 2555 (2002); Mod. Lett. Phys. A **16**, 2075 (2001); S. Bellucci, A. Nersessian, C. Sochichiu, Phys. Lett. B **522**, 345 (2001); C. Duval and P. A. Horvathy, J. Phys. A **34**, 10097 (2001); C. Acatrinei, J. High Energy Phys. 09 (2001) 007; A. E. F. Djemai, Commun. Theor. Phys. **41**, 837 (2004); K. Bolonek and Piotr Kosinski, Phys. Lett. B **547**, 51 (2002); M. Lubo, Phys. Rev. D **65**, 066003 (2002).
[17] A. P. Polychronakos, J. High Energy Phys. 04 (2001) 011; 06 (2001) 070; L. Susskind, hep-th/0101029.
[18] A. Das, J. Gamboa, J. Lopéz-Sarrión, and F. Méndez, J. High Energy Phys. 05 (2004) 022.
[19] J. Gamboa, F. Méndez, and M. Loewe, hep-th/0311014.