

# Non-BPS brane dynamics and duality

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The Green-Schwarz action for a non-BPS  $p = 2$  brane embedded in a  $N = 1$ ,  $D = 4$  target superspace is shown to be equivalent to the Nambu-Goto-Akulov-Volkov action obtained via the nonlinear realization of the associated target space super-Poincaré symmetries. Introducing a U(1) gauge field strength as a Lagrange multiplier, this  $p = 2$  brane action is recast into its equivalent dual form non-BPS D2-brane Born-Infeld action. Following the procedure given by Sen, the Green-Schwarz action for a non-BPS D2-brane is determined directly. From it, conversely, the dual form non-BPS  $p = 2$  brane action is derived. The  $p = 2$  brane and the D2-brane actions obtained by these two approaches are different in form. Through explicitly determined field redefinitions, these actions are shown to be equivalent.

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## I INTRODUCTION

Super  $p$ -brane dynamics can be obtained by means of the Green-Schwarz method [1] for constructing the brane action. In the BPS case the action consists of two sets of separately super-Poincaré invariant terms which are related to each other by  $\kappa$  symmetry [2]. Exploiting this local fermionic symmetry, half of the fermionic world volume fields can be set to zero along with the Wess-Zumino set of action terms in order to achieve the final form of the BPS brane action. The super  $Dp$ -brane actions can be obtained by means of duality transformations of the super  $p$ -brane actions [3,4]. Indeed the action for a BPS super D2-brane embedded in  $D = 10$  superspace was first obtained by a duality transformation on the action for a BPS super  $p = 2$  brane action embedded in  $D = 11$  superspace [5]. In addition, non-BPS branes occur in field theory [6] as well as string theory [7–9]. Excluding the Wess-Zumino action terms from the total Green-Schwarz action *ab initio* eliminates the  $\kappa$  symmetry of the action and so prevents the gauging away of half of the fermionic fields. With all fermionic fields present the resulting super-Poincaré invariant action describes a non-BPS brane action. This approach can be applied to the construction of non-BPS  $p$ -brane actions as well as non-BPS  $Dp$ -brane actions.

Besides the above Green-Schwarz approaches to brane dynamics, the coset method [11,12] has been used to construct actions for static gauge BPS super  $p$ -branes [13,14] and BPS super  $Dp$ -branes [15]. Nonlinearly realized supersymmetry in non-BPS D-branes was empha-

sized in [16]. In Ref. [17] the action for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace was constructed via the nonlinear realization of the spontaneously broken super-Poincaré symmetries of the target superspace. The action described the motion of the brane in  $N = 1$ ,  $D = 4$  superspace through the brane localized Nambu-Goldstone boson field  $\phi$  associated with motions in space directions transverse to the brane, hence in the direction of the broken space translation symmetry. It also involved brane localized  $D = 3$  Majorana Goldstino fields  $\theta_i$  and  $\lambda_i$ ,  $i = 1, 2$ , describing brane oscillations in Grassmann directions of superspace which are associated with the completely broken  $N = 1$ ,  $D = 4$  supersymmetry (SUSY). The action, after application of the “inverse Higgs mechanism” [18], is the  $N = 1$ ,  $D = 4$  super-Poincaré invariant synthesis of the Akulov-Volkov [11] and Nambu-Goto [19] actions

$$\Gamma = -\sigma \int d^3x \det \hat{e} \sqrt{1 - [\hat{e}_a^{-1m}(\partial_m \phi + \bar{\theta} \vec{\partial}_m \lambda)]^2}, \quad (1)$$

with  $\hat{e}_m^a$  the Akulov-Volkov dreibein in the static gauge  $\hat{e}_m^a = \delta_m^a + i\bar{\theta} \gamma^a \partial_m \theta + i\lambda \gamma^a \partial_m \lambda$ . Exploiting the coset method further, the action was shown to be dual to the action for a space-filling non-BPS D2-brane whose supersymmetric Born-Infeld [20] action was given by

$$\Gamma = -\sigma \int d^3x \left[ \sqrt{\det(\hat{g}_{mn} + F_{mn})} - F^m(\bar{\theta} \vec{\partial}_m \lambda) \right], \quad (2)$$

where the Akulov-Volkov metric is given by  $\hat{g}_{mn} = \hat{e}_m^a \eta_{ab} \hat{e}_n^b$  and the field strength vector and tensor are related as  $F^m = \frac{1}{2} \epsilon^{mnr} F_{nr}$  where  $F_{mn} = \partial_m A_n - \partial_n A_m$  with  $A_m$  the Born-Infeld U(1) gauge field.

The purpose of this paper is to determine the action for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace and the action for a space-filling  $N = 2$ ,  $D = 3$  non-BPS D2-brane directly by means of the Green-

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Schwarz [1] approach as modified by Sen [7] for the non-BPS case. The actions dual to these actions are then determined. In the cases that the actions obtained are of a different form than the corresponding brane actions obtained by means of the coset method their equivalence is established through explicit field redefinitions. Specifically in Sec. II, the non-BPS Green-Schwarz method is used to obtain the action for a non-BPS  $p = 2$  brane. Its form, in the static gauge, is the same as that obtained via the coset method, Eq. (1). Introducing a U(1) gauge field strength [13] as a Lagrange multiplier [3], the dual form of this action is found to have the same Born-Infeld form as obtained using the coset method, Eq. (2). In Sec. III, the procedure of Sen is applied to the space-filling  $N = 2$ ,  $D = 3$  non-BPS D2-brane case in order to obtain its Green-Schwarz action directly. This non-BPS D2-brane action in an arbitrary gauge is found to be

$$\Gamma = -\sigma \int d^3\xi \sqrt{\det(\hat{g}_{mn} + \mathcal{F}_{mn})}, \quad (3)$$

where  $\mathcal{F}_{mn} = F_{mn} - b_{mn}$  with the two-form  $b_{mn}$  given in terms of the brane degrees of freedom so that  $\mathcal{F}_{mn}$  is supersymmetric. Further this action is shown to be dual to the action for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace with the form

$$\Gamma = -\sigma \int d^3\xi \left[ \det \hat{e} \sqrt{1 - (\hat{e}_a^{-1m} \partial_m \phi)^2} - \frac{1}{2} \epsilon^{mnr} b_{mn} \partial_r \phi \right], \quad (4)$$

where now the two-form  $b_{mn}$  couples directly to the trivially conserved current  $\epsilon^{mnr} \partial_r \phi$ . The non-BPS D2-brane action and its dual  $p = 2$  brane action obtained by the modified Green-Schwarz method of Sen, Eqs. (3) and (4), respectively, have different forms than the corresponding non-BPS brane actions derived via the coset method (and Green-Schwarz  $p$ -brane method), Eqs. (1) and (2), respectively. In Sec. IV, the explicit field redefinitions are given that render the two forms of the actions equivalent.

## II. THE NON-BPS $P = 2$ BRANE ACTION AND DUALITY

The  $N = 1$ ,  $D = 4$  superspace coordinates of an embedded  $p = 2$  brane are denoted by the bosonic space-time coordinates,  $x^\mu(\xi^m)$ , with  $\mu = 0, 1, 2, 3$ , and the fermionic Weyl spinor coordinates,  $\theta_\alpha(\xi^m)$  and  $\bar{\theta}_{\dot{\alpha}}(\xi^m)$ , where  $\xi^m$ ,  $m = 0, 1, 2$  express the world volume space-time coordinates of the embedded brane. The linear representation of the  $N = 1$ ,  $D = 4$  super-Poincaré symmetries on the superspace coordinates is most easily obtained from the motion that group multiplication induces on the parameter space of the SUSY graded Lie group. The  $N = 1$ ,  $D = 4$  supertranslation subgroup elements can be written as

$$\Omega(x, \theta, \bar{\theta}) = e^{i(x^\mu P_\mu + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})}. \quad (5)$$

Multiplication by the group elements and use of the Baker-Campbell-Hausdorff formula yields the induced motion in superspace. For translations and SUSY transformations it is obtained that the group multiplication law

$$\Omega(a, \eta, \bar{\eta}) \Omega(x, \theta, \bar{\theta}) = \Omega(x + a + i[\eta\sigma\bar{\theta} - \theta\sigma\bar{\eta}], \theta + \eta, \bar{\theta} + \bar{\eta}) \quad (6)$$

induces a movement in superspace  $(x, \theta, \bar{\theta}) \rightarrow (x', \theta', \bar{\theta}') = (x + a + i[\eta\sigma\bar{\theta} - \theta\sigma\bar{\eta}], \theta + \eta, \bar{\theta} + \bar{\eta})$ . Proceeding similarly, all the  $N = 1$ ,  $D = 4$  super-Poincaré symmetry transformations, including  $R$  symmetry [cf. Eq. (18)], are obtained

$$\begin{aligned} \delta^P(a)x^\mu &= a^\mu, & \delta^Q(\eta, \bar{\eta})x^\mu &= i(\eta\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\eta}), \\ \delta^P(a)\theta_\alpha &= 0, & \delta^Q(\eta, \bar{\eta})\theta_\alpha &= \eta_\alpha, \\ \delta^P(a)\bar{\theta}_{\dot{\alpha}} &= 0, & \delta^Q(\eta, \bar{\eta})\bar{\theta}_{\dot{\alpha}} &= \bar{\eta}_{\dot{\alpha}}, \end{aligned} \quad (7)$$

$$\begin{aligned} \delta^M(\omega)x^\mu &= \omega^{\mu\nu}x_\nu, & \delta^R(\rho)x^\mu &= 0, \\ \delta^M(\omega)\theta_\alpha &= -\frac{i}{4}\omega^{\mu\nu}(\theta\sigma_{\mu\nu})_\alpha, & \delta^R(\rho)\theta_\alpha &= +i\rho\theta_\alpha, \\ \delta^M(\omega)\bar{\theta}_{\dot{\alpha}} &= -\frac{i}{4}\omega^{\mu\nu}(\bar{\theta}\bar{\sigma}_{\mu\nu})_{\dot{\alpha}}, & \delta^R(\rho)\bar{\theta}_{\dot{\alpha}} &= -i\rho\bar{\theta}_{\dot{\alpha}}, \end{aligned}$$

where  $\delta x^\mu = x'^\mu - x^\mu$ , and so on. These variations represent the SUSY charge algebra, for example,

$$[\delta^Q(\eta, \bar{\eta}), \delta^Q(\zeta, \bar{\zeta})] = +2\delta^P(i[\zeta\sigma\bar{\eta} - \eta\sigma\bar{\zeta}]). \quad (8)$$

According to the Green-Schwarz construction [1], the SUSY (and translation) invariant one-forms  $\partial_m\theta$ ,  $\partial_m\bar{\theta}$ , and

$$\Pi_m^\mu = \partial_m x^\mu + i\partial_m\theta\sigma^\mu\bar{\theta} - i\theta\sigma^\mu\partial_m\bar{\theta} \quad (9)$$

can be used to construct the translation and SUSY invariant induced world volume metric  $g_{mn}$ :

$$g_{mn} = \Pi_m^\mu \eta_{\mu\nu} \Pi_n^\nu. \quad (10)$$

Thus, the action for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace is given by

$$\Gamma = -\sigma \int d^3\xi \sqrt{\det g_{mn}}, \quad (11)$$

where  $\sigma$  is the brane tension. Besides being  $N = 1$ ,  $D = 4$  super-Poincaré invariant, the action is world volume reparametrization invariant. Consider the reparametrization  $\xi = \xi(\xi')$ , the world volume element is rescaled by the Jacobian  $d^3\xi = (\det \partial\xi/\partial\xi')d^3\xi'$ , while the world volume one-form transforms as a world volume vector  $\Pi_m^\mu(\xi) = (\partial\xi'^n/\partial\xi^m)\Pi_n^\mu(\xi')$ . Hence the metric is indeed a world volume two tensor and  $\sqrt{\det g}$  a world volume density,

$\sqrt{\det g(\xi)} = (\det \partial \xi' / \partial \xi) \sqrt{\det g(\xi')}$ . Thus the action is left invariant.

The reparametrization invariance can be used to choose the static gauge in which the world volume coordinates are chosen to be the first three space-time coordinates,  $\xi^m = x^m$  while the remaining space point is relabeled as the  $p = 2$  brane degree of freedom  $\phi(x^m) = x^3(x^m)$ , a world volume field. Having chosen the space-time coordinates thusly, it is important to choose a commensurate linear combination of fermion fields to describe the brane oscillations into Grassmann directions. More specifically, this choice of gauge masks the initial explicit four dimensional symmetry; indeed it leaves explicit a three dimensional symmetry. Exploiting this manifest covariance, the  $N = 1, D = 4$  super-Poincaré algebra can be reexpressed as a centrally extended  $N = 2, D = 3$  super-Poincaré algebra including the  $D = 4$  Lorentz symmetry generators as additional  $N = 2, D = 3$  automorphism charges. The components of the Weyl fermion supersymmetry charges  $Q_\alpha$  and  $\bar{Q}_{\dot{\alpha}}$  will then be combined to form the components of the two  $N = 2, D = 3$  Majorana spinor supersymmetry charges, denoted  $q_i$  and  $s_i, i = 1, 2$  such that they anticommute with themselves to yield the translation generator  $p^m = P^m$  and with each other to yield the central charge  $Z = P_3$ , the direction of brane oscillations in this gauge. This rearrangement of components is also reflected in the fermion fields. The Weyl fields will now be written in terms of two  $D = 3$  Majorana fermion world volume fields,  $\theta_i(x^m)$  and  $\lambda_i(x^m)$ . The corresponding relations between the charges and fields are obtained from the equality of the SUSY transformations in each basis; that is  $\theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}} = \bar{\theta}_i q_i + \bar{\lambda}_i s_i$ .

The action (11) can be written in terms of these new field variables,  $(\phi, \theta, \lambda)$ , or equivalently, one can start with the  $N = 2, D = 3$  SUSY transformations and construct the action according to the Green-Schwarz method directly. Implementation of the static gauge will then be straightforward. Both approaches yield the same expressions for  $\Pi_m^\mu$  and hence the action. Their interrelation will be noted below. To find the new fields and charges consider the space-time translation generator  $P^\mu$ , which transforms as a vector  $(\frac{1}{2}, \frac{1}{2})$  representation of the  $D = 4$  Lorentz group; it consists of a  $D = 3$  Lorentz group vector,  $p^m = P^m$ , with  $m = 0, 1, 2$ , and a  $D = 3$  scalar,  $Z \equiv P_3$ . Likewise, the Lorentz transformation charges  $M^{\mu\nu}$  are in the  $D = 4 (1, 1)_A$  representation which consists of two  $D = 3$  vector representations:  $M^{mn} = \epsilon^{mnr} M_r$  and  $K^m \equiv M^{m3}$ . The  $R$  charge is a singlet from both points of view. Finally the  $D = 4$  SUSY  $(\frac{1}{2}, 0)$  spinor  $Q_\alpha$  and the  $(0, \frac{1}{2})$  spinor  $\bar{Q}_{\dot{\alpha}}$  consist of two  $D = 3$  two-component Majorana spinors:  $q_i$  and  $s_i$ , with  $i = 1, 2$  comprising the charges for  $N = 2, D = 3$  SUSY. These spinors are given as linear combinations of  $Q_\alpha$  and  $\bar{Q}_{\dot{\alpha}}$  according to

$$\begin{pmatrix} q_i \\ s_i \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{pmatrix} \sigma & i\sigma\sigma_z \\ -i\sigma & -\sigma\sigma_z \end{pmatrix} \begin{pmatrix} Q_\alpha \\ \bar{Q}_{\dot{\alpha}} \end{pmatrix}, \quad (12)$$

where  $(\sigma_x, \sigma_y, \sigma_z)$  are the Pauli matrices and

$$\sigma \equiv \begin{pmatrix} 1 & -1 \\ -i & -i \end{pmatrix}. \quad (13)$$

The  $N = 1, D = 4$  super-Poincaré algebra can be written in terms of these  $N = 2, D = 3$  representation charges as

$$\begin{aligned} [p^m, p^n] &= 0, & [M^m, M^n] &= -i\epsilon^{mnr} M_r, \\ [p^m, Z] &= 0, & [M^m, K^n] &= -i\epsilon^{mnr} K_r, \\ & & [K^m, K^n] &= +i\epsilon^{mnr} M_r, \\ [M^m, p^n] &= -i\epsilon^{mnr} p_r, & [K^m, p^n] &= +i\eta^{mn} Z, \\ [M^m, Z] &= 0, & [K^m, Z] &= +ip^m, \\ [M^{mn}, q_i] &= -\frac{1}{2}\gamma_{ij}^{mn} q_j, & [K^m, q_i] &= +\frac{1}{2}\gamma_{ij}^m s_j, \\ [M^{mn}, s_i] &= -\frac{1}{2}\gamma_{ij}^{mn} s_j, & [K^m, s_i] &= -\frac{1}{2}\gamma_{ij}^m q_j, \\ [R, q_i] &= +is_i, & \{q_i, q_j\} &= +2(\gamma^m C)_{ij} p_m, \\ [R, s_i] &= -iq_i, & \{s_i, s_j\} &= +2(\gamma^m C)_{ij} p_m, \\ & & \{q_i, s_j\} &= -2iC_{ij} Z. \end{aligned} \quad (14)$$

Note, the notation used in this paper is that of Ref. [17], in particular, the charge conjugation matrix and the  $2 + 1$  ( $D = 3$ ) dimensional gamma matrices in the appropriate associated representation are given there.

The group element  $\Omega$  is now parametrized by the coordinates of the centrally extended  $N = 2, D = 3$  superspace,  $x^m(\xi), \phi(\xi), \theta_i(\xi)$ , and  $\lambda_i(\xi)$ . The above transformation algebra induces a motion in  $N = 2$  superspace,

$$\begin{aligned} x'^m &= x^m + a^m - i(\bar{\zeta}\gamma^m\theta + \bar{\eta}\gamma^m\lambda) - \phi b^m + \epsilon^{mnr}\alpha_n x_r, \\ \phi' &= \phi + z + (\zeta\gamma^0\lambda - \theta\gamma^0\eta) - b^m x_m, \\ \theta'_i &= \theta_i + \zeta_i - \frac{i}{2}b_m(\gamma^m\lambda)_i - i\rho\lambda_i - \frac{i}{2}\alpha_m(\gamma^m\theta)_i, \\ \lambda'_i &= \lambda_i + \eta_i + \frac{i}{2}b_m(\gamma^m\theta)_i + i\rho\theta_i - \frac{i}{2}\alpha_m(\gamma^m\lambda)_i, \end{aligned} \quad (15)$$

resulting in the linear representation of the  $N = 2, D = 3$  transformation generators

$$\begin{aligned}\delta^p(a)x^m &= a^m, & \delta^Z(z)x^m &= 0, \\ \delta^p(a)\phi &= 0, & \delta^Z(z)\phi &= z, \\ \delta^p(a)\theta &= 0, & \delta^Z(z)\theta &= 0, \\ \delta^p(a)\lambda &= 0, & \delta^Z(z)\lambda &= 0,\end{aligned}$$

$$\begin{aligned}\delta^q(\zeta)x^m &= -i\bar{\zeta}\gamma^m\theta, & \delta^s(\eta)x^m &= -i\bar{\eta}\gamma^m\lambda, \\ \delta^q(\zeta)\phi &= -\bar{\zeta}\lambda, & \delta^s(\eta)\phi &= +\bar{\eta}\theta, \\ \delta^q(\zeta)\theta_i &= \zeta_i, & \delta^s(\eta)\theta_i &= 0, \\ \delta^q(\zeta)\lambda_i &= 0, & \delta^s(\eta)\lambda_i &= \eta_i,\end{aligned}\tag{16}$$

$$\begin{aligned}\delta^M(\alpha)x^m &= \epsilon^{mnr}\alpha_n x_r, & \delta^K(b)x^m &= -b^m\phi, \\ \delta^M(\alpha)\phi &= 0, & \delta^K(b)\phi &= -b^m x_m, \\ \delta^M(\alpha)\theta_i &= -\frac{i}{2}\alpha_m(\gamma^m\theta)_i, & \delta^K(b)\theta_i &= +\frac{i}{2}b_m(\gamma^m\lambda)_i, \\ \delta^M(\alpha)\lambda_i &= -\frac{i}{2}\alpha_m(\gamma^m\lambda)_i, & \delta^K(b)\lambda_i &= -\frac{i}{2}b_m(\gamma^m\theta)_i,\end{aligned}$$

$$\begin{aligned}\delta^R(\rho)x^m &= 0, & \delta^R(\rho)\theta_i &= -i\rho\lambda_i, \\ \delta^R(\rho)\phi &= 0, & \delta^R(\rho)\lambda_i &= +i\rho\theta_i.\end{aligned}$$

Once again these variations obey the  $N = 2$ ,  $D = 3$  SUSY transformation algebra, for example,

$$\begin{aligned}[\delta^q(\zeta), \delta^q(\zeta')] &= 2\delta^p(i\bar{\zeta}\gamma\zeta'), \\ [\delta^s(\eta), \delta^s(\eta')] &= 2\delta^p(i\bar{\eta}\gamma\eta'), \\ [\delta^q(\zeta), \delta^s(\eta)] &= 2\delta^Z(\bar{\zeta}\eta).\end{aligned}\tag{17}$$

These transformations are precisely of the same form for the total variations of the fields that are obtained by means of the coset method. This is because the same exponential representation of the subgroup elements is used as in Ref. [17].

The  $N = 2$  SUSY invariant one-forms now become

$$\begin{aligned}\Pi_m^a &= \partial_m x^a + i\bar{\theta}\gamma^a\partial_m\theta + i\bar{\lambda}\gamma^a\partial_m\lambda, \\ \Pi_m^3 &= \partial_m\phi + \bar{\theta}\partial_m\lambda - \bar{\lambda}\partial_m\theta.\end{aligned}\tag{18}$$

The induced metric on the world volume, requiring  $D = 4$  Lorentz invariance (specifically under the broken  $K^m$  transformations), now is given by

$$g_{mn} = \Pi_m^a \eta_{ab} \Pi_n^b - \Pi_m^3 \Pi_n^3.\tag{19}$$

Equivalently, Eqs. (18) and (19) are obtained by substituting the expressions for  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  in terms of  $\theta_i$  and  $\lambda_i$  (i.e.,  $\theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}} = \bar{\theta}_i q_i + \bar{\lambda}_i s_i$ ),

$$\begin{pmatrix} \theta^\alpha \\ \bar{\theta}_{\dot{\alpha}} \end{pmatrix} = \frac{1}{2} e^{i(\pi/4)} \begin{pmatrix} \sigma_z \sigma^T & -i\sigma_z \sigma^T \\ -i\sigma^T & -\sigma^T \end{pmatrix} \begin{pmatrix} \theta_i \\ \lambda_i \end{pmatrix},\tag{20}$$

into Eqs. (9) and (10). The  $N = 1$ ,  $D = 4$  super-Poincaré

invariant action is as previously

$$\Gamma = -\sigma \int d^3\xi \sqrt{\det g_{mn}}.\tag{21}$$

Introducing the Akulov-Volkov induced dreibein in this general gauge

$$\hat{e}_m^a = \Pi_m^a = \partial_m x^a + i\bar{\theta}\gamma^a\partial_m\theta + i\bar{\lambda}\gamma^a\partial_m\lambda,\tag{22}$$

the induced metric then takes the form

$$g_{mn} = \hat{e}_m^a n_{ab} \hat{e}_n^b,\tag{23}$$

with the Nambu-Goto metric  $n_{ab}$  given by

$$\begin{aligned}n_{ab} &= \eta_{ab} - \hat{e}_a^{-1m} \Pi_m^3 \hat{e}_b^{-1n} \Pi_n^3 \\ &= \eta_{ab} - [\hat{D}_a\phi - \hat{D}_a\bar{\theta}\lambda + \bar{\theta}\hat{D}_a\lambda] \\ &\quad \times [\hat{D}_b\phi - \hat{D}_b\bar{\theta}\lambda + \bar{\theta}\hat{D}_b\lambda],\end{aligned}\tag{24}$$

where the Akulov-Volkov partially covariant derivative,  $\hat{D}_a$ , and the partially covariant derivative of  $\phi$ ,  $\hat{\nabla}_a\phi$ , are, respectively,

$$\begin{aligned}\hat{D}_a &\equiv \hat{e}_a^{-1m} \partial_m, \\ \hat{\nabla}_a\phi &\equiv \hat{e}_a^{-1p} \Pi_p^3 = \hat{D}_a\phi + \bar{\theta}\hat{D}_a\lambda - \hat{D}_a\bar{\theta}\lambda \\ &= \hat{e}_a^{-1m} (\partial_m\phi + \bar{\theta}\vec{\partial}_m\lambda).\end{aligned}\tag{25}$$

The determinant of  $n_{ab}$  is simply

$$\det n = 1 - (\hat{\nabla}_a\phi)^2 = 1 - [\hat{D}_a\phi - \hat{D}_a\bar{\theta}\lambda + \bar{\theta}\hat{D}_a\lambda]^2.\tag{26}$$

Hence, taking the square root of this determinant, the action for the non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace is obtained

$$\Gamma = -\sigma \int d^3\xi \det \hat{e} \sqrt{1 - (\hat{D}_a\phi + \bar{\theta}\hat{D}_a\lambda - \hat{D}_a\bar{\theta}\lambda)^2}.\tag{27}$$

Exploiting the world volume reparametrization invariance in order to choose the static gauge,  $\xi^m = x^m$ , all the world volume fields become functions of  $x^m$  and the Akulov-Volkov dreibein becomes that of [17]

$$\hat{e}_m^a = \Pi_m^a = \delta_m^a + i\bar{\theta}\gamma^a\partial_m\theta + i\bar{\lambda}\gamma^a\partial_m\lambda.\tag{28}$$

The action reproduces that found in [17], Eq. (1) above, by means of the coset method.

Once in the static gauge the action is no longer invariant under the original symmetry transformations since now  $x^m = \xi^m$ . A compensating general coordinate transformation must be made on the fields so that when applied to the  $x^m$  coordinates they are invariant and so remain in the static gauge. Hence intrinsic variations of the fields,  $\delta\varphi$ , are defined with a compensating coordinate transforming factor, denoted as  $C^m$ ,

$$\begin{aligned}\bar{\delta}x^m &= \delta x^m - C^l \partial_l x^m, & \bar{\delta}\phi &= \delta\phi - C^l \partial_l \phi, \\ \bar{\delta}\theta &= \delta\theta - C^l \partial_l \theta, & \bar{\delta}\lambda &= \delta\lambda - C^l \partial_l \lambda.\end{aligned}\quad (29)$$

In particular  $x^m$  must remain in the static gauge, hence it is required that  $\bar{\delta}x^m = 0$ , thus the compensating factor is determined for each super-Poincaré transformation  $C^m = \delta x^m$ . Consequently it is obtained that

$$\begin{aligned}C^m(a) &= a^m, & C^m(z) &= 0, & C^m(\zeta) &= -i\bar{\zeta}\gamma^m\theta, \\ C^m(\eta) &= -i\bar{\eta}\gamma^m\lambda, & C^m(\alpha) &= \epsilon^{mnr}\alpha_n x_r, \\ C^m(b) &= -b^m\phi, & C^m(\rho) &= 0.\end{aligned}\quad (30)$$

$$\begin{aligned}\delta^P(a)\phi &= -a^m \partial_m \phi(x), \\ \delta^P(a)\theta &= -a^m \partial_m \theta(x), \\ \delta^P(a)\lambda &= -a^m \partial_m \lambda(x),\end{aligned}$$

$$\begin{aligned}\delta^Q(\zeta)\phi &= -\bar{\zeta}\lambda + i\bar{\zeta}\gamma^m\theta\partial_m\phi(x), \\ \delta^Q(\zeta)\theta_i &= \zeta_i + i\bar{\zeta}\gamma^m\theta\partial_m\theta_i(x), \\ \delta^Q(\zeta)\lambda_i &= +i\bar{\zeta}\gamma^m\theta\partial_m\lambda_i(x),\end{aligned}$$

$$\begin{aligned}\delta^M(\alpha) &= -\epsilon^{mnr}\alpha_n x_r \partial_m \phi(x), \\ \delta^M(\alpha)\theta_i &= -\frac{i}{2}\alpha_m(\gamma^m\theta)_i - \epsilon^{mnr}\alpha_n x_r \partial_m \theta_i(x), \\ \delta^M(\alpha)\lambda_i &= -\frac{i}{2}\alpha_m(\gamma^m\lambda)_i - \epsilon^{mnr}\alpha_n x_r \partial_m \lambda_i(x),\end{aligned}$$

$$\begin{aligned}\delta^R(\rho)\phi &= 0, \\ \delta^R(\rho)\theta_i &= -i\rho\lambda_i, \\ \delta^R(\rho)\lambda_i &= +i\rho\theta_i.\end{aligned}$$

In general in  $D = 3$  a scalar field and a U(1) gauge field provide equivalent descriptions for the single field degree of freedom. Returning to the action in a general gauge, Eq. (27), this relation can be made explicit by first introducing a Lagrange multiplier field  $L^a$  [3] (this corresponds to the broken Lorentz Nambu-Goldstone field in the coset approach [13]) in order to define  $k_a = \hat{\nabla}_a \phi$  so that the action becomes

$$\Gamma = -\sigma \int d^3\xi \det\hat{e} \left\{ \sqrt{(1-k^2)} + L^a(k_a - \hat{\nabla}_a \phi) \right\}. \quad (32)$$

The action is now a functional of the independent fields  $\Gamma = \Gamma[\theta, \lambda, \phi, k, L]$ . The  $\phi$  equation of motion will imply the  $D = 3$  Bianchi identity for the U(1) field strength Lagrange multiplier field  $L^a$

$$\frac{\delta\Gamma}{\delta\phi} = -\sigma \partial_m (\det\hat{e} L^a \hat{e}_a^{-1m}) = 0. \quad (33)$$

These compensating terms reproduce the intrinsic variations of the fields obtained in Ref. [17] by means of the coset method. The compensating factors are the Taylor expansion factors necessary to go from the linearly represented total variations of the fields to the nonlinear realizations of the intrinsic (Lie) transformations. The static gauge action is invariant under the nonlinearly realized  $N = 1, D = 4$  super-Poincaré variations of the  $\phi, \theta, \lambda$  fields (dropping the bar on the intrinsic variations  $\bar{\delta} \rightarrow \delta$ )

$$\begin{aligned}\delta^Z(z)\phi &= z, \\ \delta^Z(z)\theta &= 0, \\ \delta^Z(z)\lambda &= 0,\end{aligned}$$

$$\begin{aligned}\delta^S(\eta)\phi &= +\bar{\eta}\theta + i\bar{\eta}\gamma^m\lambda\partial_m\phi(x), \\ \delta^S(\eta)\theta_i &= +i\bar{\eta}\gamma^m\lambda\partial_m\theta_i(x), \\ \delta^S(\eta)\lambda_i &= \eta_i + i\bar{\eta}\gamma^m\lambda\partial_m\lambda_i(x),\end{aligned}$$

(31)

$$\begin{aligned}\delta^K(b)\phi &= -b^m x_m + \phi b^m \partial_m \phi(x), \\ \delta^K(b)\theta_i &= +\frac{i}{2}b_m(\gamma^m\lambda)_i + \phi b^m \partial_m \theta_i(x), \\ \delta^K(b)\lambda_i &= -\frac{i}{2}b_m(\gamma^m\theta)_i + \phi b^m \partial_m \lambda_i(x),\end{aligned}$$

Hence the field strength  $F^m \equiv \det\hat{e} L^a \hat{e}_a^{-1m}$  obeys the  $D = 3$  Bianchi identity  $\partial_m F^m = 0$ , which has the U(1) gauge potential,  $A_m$ , solution

$$F^m = \epsilon^{mnr} \partial_n A_r, \quad (34)$$

that is  $F_{mn} = \epsilon_{mnr} F^r = \partial_m A_n - \partial_n A_m$ . At the same time the Bianchi identity allows  $\phi$  to be eliminated from the action by an integration by parts as seen below.

The  $k^a$  field equation is algebraic and allows  $k^a$  to be directly eliminated from the action in favor of  $L^a$  and hence  $F^m$

$$\frac{\delta\Gamma}{\delta k_a} = -\sigma \det\hat{e} \left( L^a - \frac{k^a}{\sqrt{1-k^2}} \right) = 0. \quad (35)$$

Thus it is obtained that

$$\frac{k^a}{\sqrt{1-k^2}} = L^a. \quad (36)$$

Contracting with  $L^a$  yields

$$L^a k_a = L^2 \sqrt{1 - k^2}, \quad (37)$$

while squaring yields the equation

$$\sqrt{1 - k^2} = \frac{1}{\sqrt{1 + L^2}}. \quad (38)$$

Substituting these into the  $p = 2$  brane action (32), the dual D2-brane Born-Infeld action is secured

$$\Gamma = -\sigma \int d^3 \xi \left\{ \det \hat{e} \sqrt{1 + L^2} - F^m (\partial_m \phi + \bar{\theta} \vec{\partial}_m \lambda) \right\}. \quad (39)$$

Substituting the U(1) field strength for  $L^a$

$$L^2 = \frac{1}{(\det \hat{e})^2} (F^m \hat{e}_m^a) \eta_{ab} (F^n \hat{e}_n^b) = F^a \eta_{ab} F^b = F^a F_a, \quad (40)$$

the action is obtained in the general gauge

$$\begin{aligned} \Gamma &= -\sigma \int d^3 \xi \left( \det \hat{e} \sqrt{1 + F^a F_a} \right. \\ &\quad \left. - F^m [\partial_m \phi + \bar{\theta} \vec{\partial}_m \lambda] \right) \\ &= -\sigma \int d^3 \xi \left[ \sqrt{\det(\hat{g}_{mn} + F_{mn})} - F^m (\bar{\theta} \vec{\partial}_m \lambda) \right], \end{aligned} \quad (41)$$

where the  $\phi$  term has been eliminated by an integration by parts and the use of the Bianchi identity  $\partial_m F^m = 0$ , as alluded to above. Once again going to the static gauge, the action of [17], Eq. (2) above, is obtained. Hence the non-BPS  $p = 2$  brane Nambu-Goto-Akulov-Volkov action (27) obtained by the Green-Schwarz method and the coset method is dual to the space-filling  $N = 2, D = 3$  non-BPS D2-brane supersymmetric Born-Infeld action first obtained in [13,17] by the coset method [13] and here by Lagrange multiplier and field equation techniques [3,4].

### III. THE NON-BPS D2-BRANE ACTION AND DUALITY

Consider a space-filling non-BPS D2-brane in  $N = 2, D = 3$  superspace directly. The  $N = 2, D = 3$  superspace coordinates of the brane are  $x^m(\xi)$ ,  $\theta_i(\xi)$ , and  $\lambda_i(\xi)$  with  $\xi^m$  the world volume coordinates of the brane. The isometries of the target space are now those of the noncentrally extended  $N = 2, D = 3$  super-Poincaré group. Their algebra is that given in Eq. (14) with  $Z = 0$  and  $K^m$  absent. Likewise the transformations of the fields are just those in (15) and (16) without the equations for  $\phi$  and with  $b^m = 0$ . Consequently the SUSY invariant building block is the Akulov-Volkov dreibein as in Eq. (22)

$$\hat{e}_m^a = \Pi_m^a = \partial_m x^a + i \bar{\theta} \gamma^a \partial_m \theta + i \bar{\lambda} \gamma^a \partial_m \lambda \quad (42)$$

with the associated invariant Akulov-Volkov metric made with the Minkowski metric  $\eta_{ab}$  [not the Nambu-Goto metric  $n_{ab}$  as in Eq. (23)]

$$\hat{g}_{mn} = \Pi_m^a \eta_{ab} \Pi_n^b = \hat{e}_m^a \eta_{ab} \hat{e}_n^b. \quad (43)$$

In addition a non-BPS D2-brane is described by a gauge potential  $A_m$  through a generalized SUSY invariant field strength  $\mathcal{F}_{mn} = F_{mn} - b_{mn}$  where  $F_{mn} = \partial_m A_n - \partial_n A_m$  and  $b_{mn}$  is a world volume two-form. The  $b_{mn}$  can be found by its required property that its SUSY variations are exact,  $\delta^{q,s} b = d\beta$ . Following the construction of [4,7],  $b_{mn}$  is given by

$$\begin{aligned} b_{mn} &= i [\bar{\theta} \gamma^l \partial_m \theta - \bar{\lambda} \gamma^l \partial_m \lambda] \left[ \partial_n x_l + \frac{i}{2} \bar{\theta} \gamma_l \partial_n \theta \right. \\ &\quad \left. + \frac{i}{2} \bar{\lambda} \gamma_l \partial_n \lambda \right] - (m \leftrightarrow n). \end{aligned} \quad (44)$$

Applying the SUSY variations to  $b_{mn}$  it is found that

$$\begin{aligned} \delta^q(\zeta) b_{mn} &= \partial_m [i \bar{\zeta} \gamma^l \theta \partial_n x_l + \frac{1}{2} (\bar{\zeta} \partial_n \theta) \bar{\theta} \theta] \\ &\quad - \partial_n [i \bar{\zeta} \gamma^l \theta \partial_m x_l + \frac{1}{2} (\bar{\zeta} \partial_m \theta) \bar{\theta} \theta], \\ \delta^s(\eta) b_{mn} &= -\partial_m [i \bar{\eta} \gamma^l \lambda \partial_n x_l + \frac{1}{2} (\bar{\eta} \partial_n \lambda) \bar{\lambda} \lambda] \\ &\quad + \partial_n [i \bar{\eta} \gamma^l \lambda \partial_m x_l + \frac{1}{2} (\bar{\eta} \partial_m \lambda) \bar{\lambda} \lambda]. \end{aligned} \quad (45)$$

Since the variations are exact,  $\mathcal{F}_{mn}$  can be made SUSY invariant,  $\delta^{q,s} \mathcal{F}_{mn} = 0$ , by canceling the  $b_{mn}$  variations with those of the gauge field  $A_m$

$$\begin{aligned} \delta^q(\zeta) A_m &= [i \bar{\zeta} \gamma^l \theta \partial_m x_l + \frac{1}{2} (\bar{\zeta} \partial_m \theta) \bar{\theta} \theta], \\ \delta^s(\eta) A_m &= -[i \bar{\eta} \gamma^l \lambda \partial_m x_l + \frac{1}{2} (\bar{\eta} \partial_m \lambda) \bar{\lambda} \lambda]. \end{aligned} \quad (46)$$

The  $N = 2, D = 3$  super-Poincaré invariant non-BPS D2-brane Born-Infeld action is given by, Eq. (3),

$$\Gamma = -\sigma \int d^3 \xi \sqrt{\det(\hat{g}_{mn} + \mathcal{F}_{mn})}. \quad (47)$$

Recalling Eq. (43) for the metric, the action becomes, including a Lagrange multiplier field,  $\Lambda^{mn}$ , defining the field strength tensor [3,13],

$$\begin{aligned} \Gamma &= -\sigma \int d^3 \xi \det \hat{e} \left\{ \sqrt{\det(\eta_{ab} + \hat{e}_a^{-1m} \hat{e}_b^{-1n} \mathcal{F}_{mn})} \right. \\ &\quad \left. - \frac{1}{2} \Lambda^{mn} [F_{mn} - (\partial_m A_n - \partial_n A_m)] \right\} \\ &= -\sigma \int d^3 \xi \det \hat{e} \left\{ \sqrt{(1 + \mathcal{F}^a \mathcal{F}_a)} - F^a \Lambda_a \right. \\ &\quad \left. + \Lambda^{mn} \partial_m A_n \right\}, \end{aligned} \quad (48)$$

where for any antisymmetric 2-tensor,  $T_{mn}$ , the corresponding vector density,  $T^m$ , is given by  $T^m = \frac{1}{2} \epsilon^{mnr} T_{nr}$  and vice versa  $T_{mn} = \epsilon_{mnr} T^r$ . Likewise,  $T_m = \frac{1}{2} \epsilon_{mnr} T^{nr}$

and inversely  $T^{mn} = \epsilon^{mnr} T_r$ . It should be noted that the contravariant and covariant vector densities are related according to  $T^m = (\det \hat{\epsilon})^2 \hat{g}^{mn} T_n$ . It is useful to define tangent space tensors,  $T_{ab}$  and  $T^{ab}$ , and their associated tangent space vectors,  $T^a$  and  $T_a$ ,

$$\begin{aligned} T_{ab} &\equiv \hat{\epsilon}_a^{-1m} \hat{\epsilon}_b^{-1n} T_{mn}, & T^{ab} &\equiv \hat{\epsilon}_m^a \hat{\epsilon}_n^b T^{mn}, \\ T^a &= \frac{1}{2} \epsilon^{abc} T_{bc}, & T_a &= \frac{1}{2} \epsilon_{abc} T^{bc}. \end{aligned} \quad (49)$$

The contravariant and covariant tangent space vectors are related as  $T^a = \eta^{ab} T_b$ . Ordinary contravariant and covariant world volume vectors are related as usual by  $V^m = \hat{g}^{mn} V_n$ . Exploiting the relation

$$(\det \hat{\epsilon}) \epsilon_{mnr} = \epsilon_{abc} \hat{\epsilon}_m^a \hat{\epsilon}_n^b \hat{\epsilon}_r^c \quad (50)$$

and Eq. (49), world volume tensor expressions are readily converted to the corresponding tangent space tensor ones, for example,  $\frac{1}{2} T^{mn} T_{mn} = T^m T_m = T^a T_a$ .

The non-BPS  $p = 2$  brane action dual to this non-BPS D2-brane action can be obtained by eliminating the now independent field strength tensor. The field equations for the field strength,  $\delta\Gamma/\delta F^a = 0$ , yield an algebraic relation between it and the Lagrange multiplier and Goldstino fields

$$\Lambda_a = \frac{\mathcal{F}_a}{\sqrt{1 + \mathcal{F}^b \mathcal{F}_b}}. \quad (51)$$

Squaring this equation yields

$$\sqrt{1 + \mathcal{F}^2} = \frac{1}{\sqrt{1 - \Lambda^2}}, \quad (52)$$

and contracting the equation with  $\Lambda^a$  gives

$$\Lambda^a F_a = \Lambda^2 \sqrt{1 + \mathcal{F}^2} + \Lambda^a b_a. \quad (53)$$

Substituting these expressions into the action (48) results in

$$\begin{aligned} \Gamma &= -\sigma \int d^3 \xi \det \hat{\epsilon} \left\{ \sqrt{1 - \Lambda^2} - \Lambda^a b_a \right. \\ &\quad \left. + \frac{1}{2} \Lambda^{mn} (\partial_m A_n - \partial_n A_m) \right\}. \end{aligned} \quad (54)$$

Applying the gauge field equation of motion,  $\delta\Gamma/\delta A_n = 0 = +\sigma \partial_m [(\det \hat{\epsilon}) \Lambda^{mn}]$ , implies that the Lagrange multiplier is given by the curl of a scalar field  $(\det \hat{\epsilon}) \Lambda^{mn} = \epsilon^{mnr} \partial_r \phi$ , so that  $\Lambda_m = (1/\det \hat{\epsilon}) \partial_m \phi$ , or equivalently

$$\Lambda_a = \hat{\epsilon}_a^{-1m} \partial_m \phi = \hat{D}_a \phi. \quad (55)$$

Integrating the gauge field terms by parts and setting  $\partial_m [(\det \hat{\epsilon}) \Lambda^{mn}] = 0$  yields the dual action for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace,

Eq. (4),

$$\Gamma = -\sigma \int d^3 \xi \left[ \det \hat{\epsilon} \sqrt{1 - \hat{D}_a \phi \hat{D}^a \phi} - \frac{1}{2} \epsilon^{mnr} b_{mn} \partial_r \phi \right]. \quad (56)$$

#### IV. FIELD REDEFINITIONS AND EQUIVALENT ACTIONS

As stated in the introduction, the non-BPS  $p = 2$  brane action obtained by the Green-Schwarz and coset methods, Eq. (1), differs from the above action, Eq. (56) [(4)], obtained by duality from the direct construction of the non-BPS D2-brane action, Eq. (47) [(3)]. Or, mutatis mutandis, the Akulov-Volkov-Nambu-Goto action in Eq. (27) [(1)] can itself be obtained as the dual to the non-BPS D2-brane Born-Infeld action in Eq. (41) [(2)]. Proceeding analogously to the above, the action (41) with Lagrange multiplier is introduced

$$\begin{aligned} \Gamma &= -\sigma \int d^3 \xi \left\{ \sqrt{\det(\hat{g}_{mn} + F_{mn})} - F^m (\bar{\theta} \vec{\partial}_m \lambda) \right. \\ &\quad \left. - \frac{1}{2} (\det \hat{\epsilon}) \Lambda^{mn} [F_{mn} - (\partial_m A_n - \partial_n A_m)] \right\} \\ &= -\sigma \int d^3 \xi \det \hat{\epsilon} \left\{ \sqrt{(1 + F^a F_a)} - F^a \Lambda_a \right. \\ &\quad \left. - F^a (\bar{\theta} \vec{\partial}_a \lambda) + \frac{1}{2} \Lambda^{mn} (\partial_m A_n - \partial_n A_m) \right\}. \end{aligned} \quad (57)$$

Applying the equation of motion for the field strength,  $\delta\Gamma/\delta F_a = 0$ , it is found that

$$\Lambda_a + (\bar{\theta} \vec{\mathcal{D}}_a \lambda) = \frac{F_a}{\sqrt{1 + F^b F_b}}. \quad (58)$$

Once again, squaring the equation yields

$$\sqrt{1 + F^2} = \frac{1}{\sqrt{1 - [\Lambda + (\bar{\theta} \vec{\mathcal{D}} \lambda)]^2}}, \quad (59)$$

while contracting with  $[\Lambda_a + (\bar{\theta} \vec{\mathcal{D}}_a \lambda)]$  yields

$$F^a \Lambda_a + F^a (\bar{\theta} \vec{\mathcal{D}}_a \lambda) = [\Lambda + (\bar{\theta} \vec{\mathcal{D}} \lambda)]^2 \sqrt{1 + F^2}. \quad (60)$$

Substituting these into Eq. (57), the action becomes

$$\begin{aligned} \Gamma &= -\sigma \int d^3 \xi \det \hat{\epsilon} \left\{ \sqrt{1 - [\Lambda + (\bar{\theta} \vec{\mathcal{D}} \lambda)]^2} \right. \\ &\quad \left. + \frac{1}{2} \Lambda^{mn} (\partial_m A_n - \partial_n A_m) \right\}. \end{aligned} \quad (61)$$

Applying the  $A_n$  field equation results in  $\Lambda_a = \hat{D}_a \phi$  and hence the Nambu-Goto-Akulov-Volkov action of Eq. (27) for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$

superspace is obtained

$$\Gamma = -\sigma \int d^3\xi \det \hat{e} \sqrt{1 - \hat{V}^a \phi \hat{V}_a \phi}. \quad (62)$$

Since these actions describe the dynamics of the same extended objects, either a non-BPS  $p = 2$  brane or a non-BPS D2-brane, there is an equivalence relation between them. Returning to the direct Green-Schwarz form of the D2-brane action, Eq. (47), a field redefinition for the field strength tensor in Eq. (48), now denoted with a prime,  $F'_{mn} = F_{mn} + y_{mn}$ , is required in order to express the action in the alternate form of Eq. (57). The field redefinition  $y_{mn}$  is determined by requiring the Lagrangian of Eq. (48) to be equal to that in Eq. (57)

$$\begin{aligned} & \sqrt{\det(\hat{g}_{mn} + \mathcal{F}'_{mn})} - \frac{1}{2}(\det \hat{e}) \Lambda^{mn} F'_{mn} \\ &= \sqrt{\det(\hat{g}_{mn} + F_{mn})} - F^m (\bar{\theta} \leftarrow \hat{\partial}_m \lambda) - \frac{1}{2}(\det \hat{e}) \Lambda^{mn} F_{mn}. \end{aligned} \quad (63)$$

This yields the equation for  $y_{mn}$

$$\begin{aligned} & \sqrt{(1 + F^2)} \left[ \sqrt{1 + \frac{(y - b)^2 + 2F(y - b)}{1 + F^2}} - 1 \right] \\ &= y^a \Lambda_a - F^a (\bar{\theta} \leftarrow \hat{\partial}_a \lambda). \end{aligned} \quad (64)$$

Isolating the square root containing  $y^a$  and squaring the equation leads to the vanishing of a quadratic form in  $y^a$

$$y^a A_{ab} y^b + 2B_a y^a + C = 0, \quad (65)$$

with the coefficients given by

$$\begin{aligned} A_{ab} &= \eta_{ab} - \Lambda_a \Lambda_b, \\ B_a &= (F_a - b_a) + [(F \cdot J) - \mathcal{L}] \Lambda_a, \\ C &= b^2 - 2(F \cdot b) + 2\mathcal{L}(F \cdot J) - (F \cdot J)^2, \end{aligned} \quad (66)$$

where  $J_a = (\bar{\theta} \leftarrow \hat{\partial}_a \lambda)$  and  $\mathcal{L} = \sqrt{1 + F^2}$ .

The solution to this equation is a surface of the general form

$$y^a = \sqrt{(BA^{-1}B - C)A_b^{-1/2a} u^b - A_b^{-1a} B^b}, \quad (67)$$

where  $u^a$  is an arbitrary timelike unit vector,  $u^a u_a = 1$ , and

$$\begin{aligned} A^{-1ab} &= \eta^{ab} + \frac{1}{[1 - \Lambda^2]} \Lambda^a \Lambda^b, \\ A^{-(1/2)ab} &= \eta^{ab} + \frac{1}{\Lambda^2} \left[ 1 - \frac{1}{\sqrt{1 - \Lambda^2}} \right] \Lambda^a \Lambda^b, \end{aligned} \quad (68)$$

so that

$$\begin{aligned} (BA^{-1}B - C) &= \frac{1}{[1 - \Lambda^2]} (\Lambda \cdot B)^2 + \Lambda^2 [(F \cdot J) - \mathcal{L}]^2 \\ &\quad + F^2 - 2\mathcal{L}(F \cdot J) + (F \cdot J)^2 \\ &\quad + 2(\Lambda \cdot F - \Lambda \cdot b) [(F \cdot J) - \mathcal{L}]. \end{aligned} \quad (69)$$

In the purely bosonic case, when the Goldstinos are absent, the action reduces to the pure Born-Infeld action in both cases. Hence the unit vector  $u^a$  is fixed by requiring no field redefinition in the case of  $\theta_i = 0 = \lambda_i$ . Thus  $y_{mn}|_{\theta=0=\lambda} = 0$  is an equation for  $u^a$

$$u = \frac{1}{\sqrt{BA^{-1}B}} A^{-1/2} B|_{\theta=0=\lambda}, \quad (70)$$

where  $C|_{\theta=0=\lambda} = 0$ . With zero subscripts denoting the value of the quantities at  $\theta_i = 0 = \lambda_i$ , the solution for the required field redefinition is found

$$y = \sqrt{1 - \frac{C}{BA^{-1}B}} \frac{\sqrt{BA^{-1}B}}{\sqrt{B_0 A_0^{-1} B_0}} A^{-1/2} A_0^{-1/2} B_0 - A^{-1} B, \quad (71)$$

with

$$\begin{aligned} B_0 A_0^{-1} B_0 &= \left[ \frac{1}{[1 - \Lambda^2]} [(F \cdot \Lambda) - \mathcal{L} \Lambda^2] + \Lambda^2 \mathcal{L}^2 \right. \\ &\quad \left. + F^2 - 2\mathcal{L}(\Lambda \cdot F) \right] \Big|_{\theta=0=\lambda}. \end{aligned} \quad (72)$$

The non-BPS  $p = 2$  brane action, Eq. (56), obtained by duality from the non-BPS D2-brane action, Eq. (47), also can be shown to be equivalent to the Green-Schwarz method and coset method action, Eq. (27), by a field redefinition directly. Consider the  $p = 2$  brane action of Eq. (56), including a Lagrange multiplier vector density  $L^m$  to define the vector field  $v'_m = \partial_m \phi$ ,

$$\begin{aligned} \Gamma &= -\sigma \int d^3\xi \left[ \det \hat{e} \sqrt{1 - \hat{e}_a^{-1m} v'_m \eta^{ab} \hat{e}_b^{-1n} v'_n} \right. \\ &\quad \left. - \frac{1}{2} \epsilon^{mnr} b_{mn} v'_r - L^m (v'_m - \partial_m \phi) \right]. \end{aligned} \quad (73)$$

The vector field can be redefined  $v'_m = v_m + y_m$  so that this action becomes that of Eq. (27), including a Lagrange multiplier,

$$\begin{aligned} \Gamma &= -\sigma \int d^3\xi \left[ \det \hat{e} \sqrt{1 - \hat{e}_a^{-1m} v_m \eta^{ab} \hat{e}_b^{-1n} v_n} \right. \\ &\quad \left. - L^m \{v_m - [\partial_m \phi + (\bar{\theta} \leftarrow \hat{\partial}_m \lambda)]\} \right]. \end{aligned} \quad (74)$$

Setting the Lagrangians equal implies the equation for the required field redefinition  $y_m$

$$\begin{aligned} & \det \hat{e} \sqrt{1 - (v + y)^2} - \frac{1}{2} \epsilon^{mnr} b_{mn} (v_r + y_r) - L^m (y_m + J_m) \\ &= \det \hat{e} \sqrt{1 - v^2}, \end{aligned} \quad (75)$$



where  $J_m \equiv (\bar{\theta} \leftrightarrow^{\partial} \lambda)$ . As earlier, isolating the square root involving the vector  $y_m$  and squaring yields an equation for the quadratic form in  $y$

$$y_m A^{mn} y_n + 2B^m y_m + C = 0, \quad (76)$$

with

$$\begin{aligned} A^{mn} &= (\det \hat{e})^2 \hat{g}^{mn} + (b + L)^m (b + L)^n, \\ B^m &= (\det \hat{e})^2 \hat{g}^{mn} v_n + [(b^n v_n) + (L^n J_n) + \mathcal{L}] \\ &\quad \times (b + L)^m, \\ C &= [2\mathcal{L} + (b^m v_m) + (L^m J_m)][(b^n v_n) + (L^n J_n)], \end{aligned} \quad (77)$$

where now  $\mathcal{L} = \det \hat{e} \sqrt{1 - v^2}$ ,  $b^m = \frac{1}{2} \epsilon^{mnr} b_{nr}$ , and recall  $\hat{g}_{mn} = \hat{e}_m^a \eta_{ab} \hat{e}_n^b$  so that  $\hat{g}^{mn} = \hat{e}_a^{-1m} \eta^{ab} \hat{e}_b^{-1n}$  with  $\hat{g}^{mn} \hat{g}_{nr} = \delta_r^m$ . Analogous to the previous case, the solution is given by the surface

$$y_m = \sqrt{BA^{-1}B - CA_{mn}^{-1/2} u^n - A_{mn}^{-1} B^n}. \quad (78)$$

The actions are already identical in the bosonic case, hence the field redefinition must vanish for  $\theta = 0 = \lambda$ . This defines the timelike unit vector  $u^m$  to be

$$u = \frac{A^{-1/2} B}{\sqrt{BA^{-1}B}} \Big|_{\theta=0=\lambda} = \frac{A_0^{-1/2} B_0}{\sqrt{B_0 A_0^{-1} B_0}}. \quad (79)$$

The final form of the field redefinition is as in Eq. (71) along with Eq. (77) and

$$\begin{aligned} A_0^{mn} &= A^{mn} \Big|_{\theta=0=\lambda} = (\det \hat{e}_0)^2 \hat{g}_0^{mn} + L^m L^n, \\ B_0^m &= B^m \Big|_{\theta=0=\lambda} = (\det \hat{e}_0)^2 \hat{g}_0^{mn} v_n + \mathcal{L}_0 L^m, \\ C_0 &= C \Big|_{\theta=0=\lambda} = 0. \end{aligned} \quad (80)$$

Also additional field redefinitions are possible which include changes in the Lagrange multiplier field as well.

Once again these lead to a quadratic form in  $y_m$  equal to zero with modified coefficients.

The Akulov-Volkov-Nambu-Goto action in Eq. (27) [(1)] for a non-BPS  $p = 2$  brane embedded in  $N = 1$ ,  $D = 4$  superspace was the same whether obtained by means of the non-BPS Green-Schwarz method or the coset method. This action was shown to be dual to the supersymmetric Born-Infeld action, Eq. (41) [(2)], for a space-filling non-BPS D2-brane in  $N = 2$ ,  $D = 3$  superspace, which was first obtained by the coset method in [17]. Applying the Green-Schwarz methods of Sen [7] to the case at hand, another form of the action for the non-BPS D2-brane was obtained, Eq. (47) [(3)]. The dual non-BPS  $p = 2$  brane action to this, Eq. (56) [(4)], had a form different from the non-BPS Green-Schwarz method and coset method action of Eq. (27) [(1)], as it should. In each case the required field redefinition was found to show that each non-BPS  $p = 2$  brane action was equivalent and that each non-BPS D2-brane action was equivalent. Finally, coset construction techniques can be readily applied to the case of a non-BPS brane in arbitrary space-time dimensions, in particular, for 10 or 11 dimensions relevant for string theory.

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