PHYSICAL REVIEW D, VOLUME 70, 104029

Localizing gravity on exotic thick 3-branes

Oscar Castillo-Felisola,[1,2] Alejandra Melfo,[1] Nelson Pantoja,[1] and Alba Ramírez[1]
[1]Centro de Física Fundamental, Universidad de Los Andes, Mérida, Venezuela
[2]International Centre for Theoretical Physics, 34100 Trieste, Italy
(Received 15 April 2004; published 29 November 2004)

We consider localization of gravity on thick branes with a nontrivial structure. Double walls that generalize the thick Randall-Sundrum solution, and asymmetric walls that arise from a Z_2 symmetric scalar potential, are considered. We present a new asymmetric solution: a thick brane interpolating between two AdS_5 spacetimes with different cosmological constants, which can be derived from a "fake supergravity" superpotential, and show that it is possible to confine gravity on such branes.

DOI: 10.1103/PhysRevD.70.104029 PACS numbers: 04.50.+h, 11.27.+d

I. INTRODUCTION

As is well-known, gravity can be localized on an infinitely thin brane embedded in a five dimensional spacetime [1]. It is not surprising, although perhaps less known, that this result holds true when the infinitely thin brane is replaced by its regularized version [2], a thick wall whose distributional thin-wall limit reduces rigorously to the Randall-Sundrum (RS) spacetime [3]. Thick domain walls are solutions to the coupled Einstein scalar field equations with an appropriate discrete symmetry-breaking potential and boundary conditions that ensure topological stability.

Of course, domain wall solutions long predate the Randall-Sundrum result, and one can find a number of domain wall spacetimes with interesting features [4,5]. Afterwards, many other wall solutions have been found, and in some cases shown to also localize gravity. This, however, is not always the case.

In this paper, we address the issue of localizing gravity in some exotic types of domain walls. First, we present a new method for analyzing the spectrum of perturbations on the walls, which greatly simplifies the calculations when dealing with more involved spacetimes.

The regularized RS spacetime of Ref. [2] is a particular case of the more general thick wall found in [6]. In the general case, these are so-called "double" walls, that is, the energy density is not peaked around a certain value of the bulk coordinate, but has a double peak instead, representing two parallel walls, or, more exactly, a wall with some nontrivial internal structure. Similar double walls were later found in [7] and shown to localize gravity. We show how this holds true also for the double walls that are a generalization of the regularized RS walls.

An example of even more exotic walls are those lacking a Z_2 symmetry, i.e., walls that are not reflection symmetric although the scalar field potential is Z_2 invariant. A particular example of these walls was found in [5], and it is remarkable that this solution can be found for the same scalar field potential as the usual reflection symmetric one. This result, that the same scalar field configuration and potential can produce two different spacetimes, one reflection symmetric but dynamic, one asymmetric but static, was found to be true for a general class of walls in [3]. It is therefore interesting to investigate whether the two types of solutions can confine gravity in a similar way. We find below that these types of asymmetric walls cannot confine gravity, even when their dynamic counterparts do.

A perhaps more interesting type of asymmetric brane is the one that interpolates between spacetimes with different cosmological constants, or, equivalently, one where the scalar field interpolates between two nondegenerate minima of the potential. Thin brane world scenarios in which the reflection symmetry along the extra dimension was broken by gluing two AdS_5 spacetimes with different cosmological constants have been considered in [8–11].[1] Classically, these thin walls should arise as a well-defined distributional limit of smooth field configurations, i.e., thick walls, a fact that is far from being obvious due to the nonlinearities of general relativity [3]. To our knowledge, however, a thick wall with these features has not been reported. We find here one such solution, which in addition can be shown to be derivable from a superpotential [13–15] in the context of the so-called fake supergravity [16] (for domain wall solutions of the RS type in supergravity, see [17]). Localization of gravity in this wall is then shown to be possible.

II. GRAVITATIONAL PERTURBATIONS

We wish to obtain the equations for the perturbations to an arbitrary solution of the Einstein scalar field coupled system. In the usual approach one considers general fluctuations, with or without gauge fixing, around a background metric of a given form [15,18–20]. Instead, we present here a straightforward generalization of the well-known procedure to obtain the perturbation equations of an arbitrary solution to the Einstein equations in vacuum

[1]Within the domain wall context, nonreflection symmetric thin domain walls between spacetimes with negative and zero cosmological constant have been discussed previously in [12].

[21] to the case of an arbitrary solution of the Einstein scalar field coupled system.

Let g_{ab} and ϕ be exact solutions of

$$R_{ab} - \frac{1}{2} g_{ab} R = T_{ab}; \quad T_{ab} = \nabla_a \phi \nabla_b \phi - g_{ab}\left[\frac{1}{2}\nabla^c \phi \nabla_c \phi + V(\phi)\right]; \quad \nabla_a \nabla^a \phi = \frac{dV}{d\phi} \tag{1}$$

where ∇_a is the derivative in g_{ab}. Now, suppose that a one-parameter family of metrics $\tilde{g}_{ab}(\lambda)$ and a one-parameter family of scalar fields $\tilde{\phi}(\lambda)$ exist, such that they satisfy (1). Suppose also that

(i) $\tilde{g}_{ab}(\lambda)$ and $\tilde{\phi}(\lambda)$ depend differentiably on λ,

(ii) $\tilde{g}_{ab}(\lambda)|_{\lambda=0} = g_{ab}$ and $\tilde{\phi}(\lambda)|_{\lambda=0} = \phi$

We expect that small λ corresponds to small deviations from g_{ab} and ϕ, hence

$$\frac{d}{d\lambda}\tilde{g}_{ab}(\lambda)\bigg|_{\lambda=0} = h_{ab} \qquad \frac{d}{d\lambda}\tilde{\phi}(\lambda)\bigg|_{\lambda=0} = \varphi \tag{2}$$

where h_{ab} and φ are the metric and scalar field perturbations, respectively.

Now consider Eqs. (1) for $\tilde{g}_{ab}$ written in Ricci form, for a five dimensional spacetime

$$\tilde{R}_{ab} = \tilde{T}_{ab} - \frac{1}{3}\tilde{g}_{ab}\tilde{T}; \quad \tilde{T}_{ab} = \tilde{\nabla}_a\tilde{\phi}\tilde{\nabla}_b\tilde{\phi} - \tilde{g}_{ab}\left[\frac{1}{2}\tilde{\nabla}^c\tilde{\phi}\tilde{\nabla}_c\tilde{\phi} + \tilde{V}(\tilde{\phi})\right] \tag{3}$$

where $\tilde{T} = \tilde{g}^{ab}\tilde{T}_{ab}$. For the scalar field $\tilde{\phi}$ we have

$$\tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\tilde{\phi} = \frac{d\tilde{V}}{d\tilde{\phi}}. \tag{4}$$

Then, evaluating the λ derivatives in $\lambda = 0$ of the above, we have for the linearized Einstein scalar field equations for the metric perturbation h_{ab} and the scalar perturbation φ of the exact solutions g_{ab} and ϕ:

$$-\frac{1}{2}\nabla^d\nabla_d h_{ab} - \frac{1}{2}\nabla_a\nabla_b(g^{cd}h_{cd}) + \nabla_{(a}\nabla^c h_{b)c} + R^{c\ \ d}_{(ab)}h^{cd} + R^c_{(a}h_{b)c} = \frac{2}{3}h_{ab}V(\phi) + \frac{2}{3}g_{ab}\left\{g^{cd}h_{cd}\left[\frac{1}{2}\nabla^e\phi\nabla_e\phi + V(\phi)\right] + \frac{dV}{d\phi}\varphi\right\} + 2\nabla_{(a}\phi\nabla_{b)}\varphi \tag{5}$$

and

$$-h^{ab}\nabla_a\nabla_b\phi - \frac{1}{2}g^{ab}g^{cd}(\nabla_a h_{bd} + \nabla_b h_{ad} - \nabla_d h_{ab})\nabla_c\phi + g^{ab}\nabla_a\nabla_b\varphi - \frac{d^2V}{d\phi^2}\varphi = 0. \tag{6}$$

A domain wall spacetime is a solution of Eq. (1) with plane-parallel symmetry, with the scalar field depending only on the bulk coordinate, $\phi = \phi(\xi)$. Choosing the axial gauge

$$h_{\xi a} = 0, \tag{7}$$

the transverse traceless (TT) part of h_{ab} decouples from the scalar field fluctuations, as can be seen from (6). Therefore, working in this gauge, we can set the field fluctuations φ to zero. For the TT modes, Eq. (5) reduces to

$$-\frac{1}{2}\nabla^d\nabla_d h_{ab} + R^{c\ \ d}_{(ab)}h_{cd} + R^c_{(a}h_{b)c} = \frac{2}{3}h_{ab}V(\phi) \tag{8}$$

which describes linearized gravity in the transverse and traceless sector. We shall use this equation to find the TT modes for different domain wall spacetimes.

III. DOUBLE WALLS

In [6], a solution to Eq. (1) was found for the static spacetime of a domain wall with an internal structure. This so-called double wall solution is given by

$$ds^2 = e^{2A(\xi)}(\eta_{ab}dx^a dx^b + d\xi^2), \quad A(\xi) = -\frac{1}{2s}\ln[1 + (\alpha\xi)^{2s}], \tag{9}$$

where η_{ab} is the four dimensional Minkowski metric, and

$$\phi = \phi_0 \tan^{-1}(\alpha^s\xi^s), \qquad \phi_0 = \frac{\sqrt{3(2s-1)}}{s}, \tag{10}$$

with a potential

$$V(\phi) + \Lambda = 3\alpha^2 \sin(\phi/\phi_0)^{2-2/s}\left[\frac{2s+3}{2}\cos^2(\phi/\phi_0) - 2\right]. \tag{11}$$

For $s = 1$ it reduces to a well-known domain wall solution which is a regularized version of the RS thin brane [2,3], while for odd $s > 1$ the energy density is peaked around two values, as can be seen in Fig. 1. These walls interpolate between anti-de Sitter asymptotic vacua with $\Lambda = -6\alpha^2$ [6]. Similar solutions were also considered in

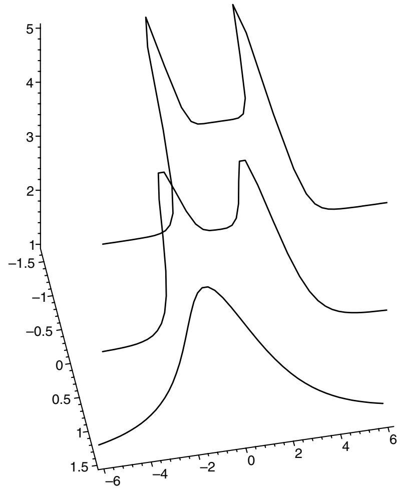

FIG. 1. Energy density as a function of ξ for double walls with $s = 1, 3, 5$.

[7]. We shall show that these double walls can confine gravity.

We write the metric fluctuations conveniently as

$$h_{\mu\nu} = e^{\imath p\cdot x} e^{A(\xi)/2} \psi_{\mu\nu}(\xi) \tag{12}$$

where $\mu, \nu = 0\ldots3$. From Eq. (8) we have

$$(-\partial_\xi^2 + V_{\rm QM})\psi_{\mu\nu} = m^2 \psi_{\mu\nu} \tag{13}$$

where

$$V_{\rm QM}(\xi) = \frac{3}{4\xi} \frac{5(\alpha\xi)^{4s} + 2(\alpha\xi)^{2s} - 4s(\alpha\xi)^{2s}}{[1 + (\alpha\xi)^{2s}]^2}. \tag{14}$$

The zero modes for each s are easily found to be (μ, ν indices omitted)

$$\psi_0 = N\left\{[1 + (\alpha\xi)^2] \sum_{m=0}^{s-1} (-\alpha^2\xi^2)^m\right\}^{-3/4s} \tag{15}$$

which are indeed the lowest mass ones. There is one discrete bound state at the threshold and one can show the existence of a continuum of states that asymptote to plane waves as $\xi \to \pm\infty$, as follows from the fact that $V_{\rm QM} \to 0$ asymptotically. The potential $V_{\rm QM}$ and the zero mode are plotted in Fig. 2 for s odd.

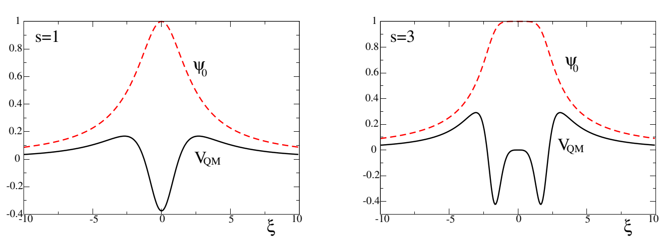

FIG. 2 (color online). $V_{\rm QM}$ and the zero mass mode for walls with $s = 1$ and $s = 3$.

IV. ASYMMETRIC VS. DYNAMIC WALLS

In [6], it was shown that for a general class of scalar field potentials, it is possible to obtain two different domain wall solutions: one representing a dynamic spacetime with a Z_2 symmetry on the wall's plane, and another lacking this symmetry, and with a static metric. Both solutions have essentially the same scalar field configuration. We wish to address in this section the issue of whether these configurations can localize gravity.

The most general five dimensional time-dependent metric representing the spacetime of a static domain wall can be written as

$$ds^2 = e^{2A(\xi)}[-dt^2 + e^{2\beta t} dx^i dx_i + d\xi^2], \tag{16}$$

where x^i ($i = 1, 2, 3$) are spatial coordinates on the wall. When

$$A(\xi) = -\delta \ln[\cosh(\beta\xi/\delta)], \tag{17}$$

Eq. (16) is a solution of (1) with

$$\begin{aligned} \phi(\xi) &= \phi_0 \tan^{-1}[\sinh(\beta\xi/\delta)], \\ \phi_0 &= \sqrt{\frac{3a^2\beta^2}{4a}} \frac{\sqrt{\delta(1-\delta)}}{\beta}, \end{aligned} \tag{18}$$

$$V(\phi) = \frac{[1 + \delta(a-1)]}{\delta} \frac{3a^2\beta^2}{8a} [\cos(\phi/\phi_0)]^{2(1-\delta)} \tag{19}$$

for $a = 4$. In [3], it was shown that this solution has a well-defined thin-wall limit when $\delta \to 0$, such that the metric and all the curvature tensor fields make sense as distributions. The analysis of gravitational fluctuations around this geometry, parametrized in a slightly different manner and for the case of a finite width, was studied in [22].

From Eq. (8) and (12) we find that the spectrum of perturbations consists of a zero mode

$$\psi_o = N[\cosh(\beta\xi/\delta)]^{-3\delta/2} \tag{20}$$

and a set of continuous modes separated by a mass gap given by $\frac{9}{4}\beta^2$, as in [22]. Interestingly enough, the thin-wall limit for the gravitational perturbations can also be obtained in the sense of distributions. We find

$$\lim_{\delta\to0} \psi_0 = \frac{2}{3\beta} \exp(-3\beta|\xi|/2). \tag{21}$$

On the other hand,

$$\lim_{\delta\to0} V_{\rm QM} = \frac{9}{4}\beta^2 - 3\beta\delta(\xi) \tag{22}$$

which, besides the δ term responsible for the unique bound state, clearly shows the mass gap whose existence is a generic property of the de Sitter branes [18–20,22].

Now, as we argued above, the metric (16) is not the only solution for a scalar field potential of the form (19). There is a corresponding static solution with a static metric that is not reflection symmetric on the wall's plane, still given by Eq. (19), with $a = 1$, but with a static metric

$$ds^2 = e^{A(\xi)/2 - 3\beta\xi/2}(-dt^2 + e^{2\beta\xi} dx^i dx_i) + e^{2A(\xi)} d\xi^2. \tag{23}$$

It should be stressed that these asymmetric thick branes arise as solutions to the Einstein scalar field equations with a Z_2 symmetric potential for which $V[\phi(-\infty)] = V[\phi(\infty)] = 0$. The thin-wall limit of this spacetime can also be taken rigorously [6,23]. At one side of the wall, the spacetime is asymptotically Minkowski, while on the other side it tends to the Taub [24] spacetime.[2]

Most of the thin brane world scenarios assume that the extra dimension is Z_2 symmetric, although asymmetric brane world scenarios can also be considered. We now show that the asymmetric domain wall spacetime with metric (23) cannot confine gravity. Let us consider the spectrum of linearized gravity fluctuations around (23). For this background geometry, from (8) and in the axial gauge (7), it follows that the only nonzero TT modes are h_{ij} with $i, j = 1, 2, 3$. Next, for these modes, with the rescaling

$$h_{ij} = e^{[A(\xi)+\beta\xi]/2} \psi_{ij}(\xi) \tag{24}$$

we obtain

$$(\partial_\xi^2 + e^{2A}\Box^{(4)})\psi_{ij} = 0, \tag{25}$$

where $\Box^{(4)}$ is the $(3 + 1)$ dimensional covariant d'Alembertian

$$\Box^{(4)} = e^{-(A+\beta\xi)/2}(-e^{2\beta\xi}\partial_t^2 + \partial_x^2 + \partial_y^2 + \partial_z^2). \tag{26}$$

Now, we find that the linearized equation of motion for tensor fluctuations (25) cannot be rewritten as a Schrödinger equation, (26) depends explicitly on the additional coordinate ξ through $A(\xi)$, and all the modes propagate in the bulk. Hence, a normalizable zero mode cannot be found, and the usual method of demonstrating localization of gravity cannot be applied to this spacetime. Notice that (23) does not belong to the general class of metrics considered, for example, in [26]. One could try to add more ingredients to the theory (e.g., additional branes [27], mass terms for the matter fields [28], curvature terms on the brane [29], etc.); we do not pursue this direction here.

V. AN ASYMMETRIC THICK BRANE WORLD SCENARIO

Let us now consider an asymmetric thick domain wall spacetime where the C^∞ metric tensor produces a line element

$$ds^2 = e^{2A(\xi)}(-dt^2 + dx^i dx_i) + d\xi^2, \tag{27}$$

where

$$A(\xi) = -\frac{1}{12}\{-\alpha\xi + \delta \exp[-2\exp(-\beta\xi/\delta)] - \delta \mathrm{Ei}[-2\exp(-\beta\xi/\delta)]\}, \tag{28}$$

with Ei the exponential integral given by

$$\mathrm{E\,i}(u) \equiv -\int_{-u}^{\infty} d\tau \frac{e^{-\tau}}{\tau} \tag{29}$$

and where α, β, δ are real constants with $\delta > 0$. This represents a three-parameter family of plane symmetric static domain wall spacetimes without reflection symmetry along the direction perpendicular to the wall, being asymptotically (i.e., far away from the wall) AdS_5 with a cosmological constant $-\alpha^2/48$ for $\xi < 0$ and $-(\beta - \alpha)^2/48$ for $\xi > 0$.

The metric (27) is a solution to the coupled Einstein scalar field Eqs. (1) with

$$\phi(\xi) = \sqrt{\delta}\exp[-\exp(-\beta\xi/\delta)] \tag{30}$$

and

$$V(\phi) = \frac{1}{8}\left[\frac{\beta^2}{\delta^2}\phi^2 \ln^2\left(\frac{\phi^2}{\delta}\right) - \frac{4}{12}\left(\frac{\beta}{\delta}\phi^2\left[1 - \ln\left(\frac{\phi^2}{\delta}\right)\right] - \alpha\right)^2\right], \tag{31}$$

where ϕ interpolates between the two nondegenerate minima of $V(\phi)$, $\phi_0 = 0$ and $\phi_1 = \sqrt{\delta}$ [see Fig. 3(a)], and where δ plays the role of the wall's thickness. Following [3], the distributional $\delta \to 0$ thin-wall limit of this geometry can be obtained and we find

$$\lim_{\delta\to 0} e^{2A(\xi)} = e^{\alpha\xi/6}\Theta(-\xi) + e^{-(\beta-\alpha)\xi/6}\Theta(\xi), \tag{32}$$

where Θ is the Heaviside distribution. This clearly shows that this spacetime behaves asymptotically [23] as an AdS spacetime with different cosmological constants at

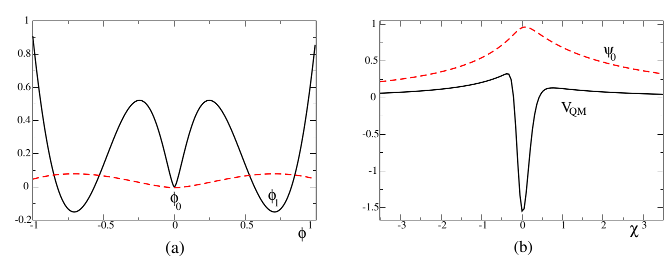

FIG. 3 (color online). Figure (a) shows the scalar field potential (continuous line) and superpotential (dashed line) for the asymmetric brane (27)–(31), with $\beta > \alpha > 0$. Figure (b) gives V_{QM} and the (arbitrarily normalized) zero mode for the same values of the parameters.

[2] Static thin domain walls without reflection symmetry embedded in spacetimes with vanishing cosmological constant have been considered previously in [25].

either side of the wall. Furthermore, it also shows that the emergence of two separate AdS_5 patches with different cosmological constants can arise (rigorously) from the thin-wall limit of a thick domain wall interpolating between nondegenerated minima of the scalar potential.

Remarkably, using the first order formalism of [13–15], (27) and (28) and (30) and (31) can be obtained from a single function, the superpotential $W(\phi)$ given by

$$W(\phi) = \frac{1}{24}\left\{\beta\frac{\phi^2}{\delta}\left[1 - \ln\left(\frac{\phi^2}{\delta}\right)\right] - \alpha\right\} \tag{33}$$

whose critical points are the asymptotic values of ϕ as given by (30) [see Fig. 3(a)]. Now, critical points of W are also critical points of V and in the context of supergravity theories the critical points of W yield stable AdS vacua [14]. Interestingly, the vacuum structure of the present model is described by a fake superpotential which resembles the true ones that appear in low energy global supersymmetric effective field theories.

Next, let us examine the stability of this domain wall spacetime as a thick brane world scenario. In the axial gauge (7), by making the change of variables $d\chi \equiv e^{-A}d\xi$ and writing the metric fluctuations as

$$h_{\mu\nu} = e^{ip\cdot x}e^{-3A(\chi)/2}\psi_{\mu\nu}(\chi) \tag{34}$$

where $\mu, \nu = 0\ldots 3$, we find from Eq. (8)

$$\left(-\frac{1}{2}\partial_\chi^2 + V_{\rm QM}\right)\psi_{\mu\nu}(\chi) = m^2\psi_{\mu\nu}(\chi), \tag{35}$$

where

$$V_{\rm QM} = \frac{1}{2}\left[\frac{9}{4}A'(\chi)^2 + \frac{3}{2}A''(\chi)\right]. \tag{36}$$

For the zero mass modes this is integrated as usual,

$$\psi_0(\chi) = N_0 e^{3A(\chi)/2}, \tag{37}$$

which is a normalizable mode if $\beta > \alpha > 0$. The zero mode and $V_{\rm QM}$ are given in Fig. 3(b).

We were not able to find exact solutions for $m^2 > 0$. Nevertheless, in the thin-wall limit we can find the exact massive modes. From (36) we find

$$\lim_{\delta\to 0} V_{\rm QM} = -\frac{3}{48}\beta\delta(\chi) + \frac{15}{8}\frac{\alpha^2}{(12-\alpha\chi)^2}\Theta(-\chi) + \frac{15}{8}\frac{(\beta-\alpha)^2}{[12+(\beta-\alpha)\chi]^2}\Theta(\chi). \tag{38}$$

Hence, we expect a single normalizable bound state mode at the threshold and a continuum of Kaluza-Klein states for all possible $m^2 > 0$. In fact, in this limit, the zero mass solution of (35) is given by

$$\lim_{\delta\to 0}\psi_0(\chi) \sim (1 - k_-\chi)^{-3/2}\Theta(-\chi) + (1 + k_+\chi)^{-3/2}\Theta(\chi), \tag{39}$$

where $k_- \equiv \alpha/12$ and $k_+ \equiv (\beta - \alpha)/12$, while the massive ones are given by

$$\lim_{\delta\to 0}\psi_m(\chi) \sim (k_-^{-1} - \chi)^{1/2}\left\{Y_2[m(k_-^{-1} - \chi)] + \frac{8k_-^2}{\pi m^2}C_- J_2[m(k_-^{-1} - \chi)]\right\}\Theta(-\chi) + (k_+^{-1} + \chi)^{1/2}\left\{Y_2[m(k_+^{-1} + \chi)] + \frac{8k_+^2}{\pi m^2}C_+ J_2[m(k_+^{-1} + \chi)]\right\}\Theta(\chi), \tag{40}$$

where Y_2 and J_2 are the Bessel functions of order 2. In (40), C_- and C_+ are constants such that, as follows from (35) and (38), ψ_m is continuous and its derivative discontinuous at $\chi = 0$. We find, for small $m/k_\pm$

$$C_+ = \left(1 + \frac{k_+}{k_-}\right)^{-1}\left\{1 + \frac{1}{2}\left(\frac{k_+}{k_-} - \sqrt{\frac{k_+}{k_-}}\right) + 4\frac{k_+^2}{m^2}\left[1 - \left(\frac{k_-}{k_+}\right)^{3/2}\right]\right\} \tag{41}$$

and C_- is given by interchanging k_+ and k_- above. As expected, the massive modes asymptote to plane waves. For $\beta = 2\alpha$ we have $k_+ = k_-$ and we recover the original Z_2 symmetric RS scenario.

Now we can calculate the gravitational potential between two particles (m_1, m_2) induced by the massless and the massive modes on the brane in this approximation. In order to compare with the Z_2 case, consider the nearly symmetric scenario $k_- \sim k_+$. To first order in $(k_-/k_+ - 1)$, we obtain from (39) and (40)

$$V(r) \sim G_N\frac{m_1 m_2}{r} + \int_0^\infty dm\frac{G_N}{k_e}\frac{m_1 m_2 e^{-mr}}{r}\frac{m}{k_e}\left[1 + 3\left(\frac{k_-}{k_+} - 1\right)\right] \tag{42}$$

where we have defined

$$k_e^{-1} = \frac{k_+^{-1} + k_-^{-1}}{2} \tag{43}$$

and then as usual

$$G_N = G_5 k_e \tag{44}$$

with G_5 the gravitational constant in five dimensions.

VI. SUMMARY AND OUTLOOK

We have explored the possibility of localizing gravity in some exotic thick brane spacetimes, using a technique that allows straightforward calculation of the perturbation equations.

First, we studied the double wall spacetime which is a generalization of the regularized (thick) RS spacetime, in the sense that it reproduces the latter for a particular value of a discrete parameter. We have shown that there is always a normalizable zero mode, and that the quantum-mechanical potential for the modes is such that there is no gap between the massless and massive modes for any value of the parameter.

Next, we consider asymmetric walls arising from a Z_2 symmetric scalar field potential, where the asymmetry is induced by the boundary conditions at spatial infinity, i.e., it is manifest on the spacetime metric only. These walls share the scalar field configuration and potential with the more familiar symmetric and dynamic walls, which have been shown elsewhere to confine gravity. However, this feature is shown to be lost in the asymmetric ones.

Finally, we turn to what we consider the central result of this paper. We have shown that there exist asymmetric thick brane solutions interpolating between two nondegenerate minima of a Z_2 symmetric potential. These branes tend asymptotically to AdS_5 spacetimes with different cosmological constants at each side, and can be derived from a fake supergravity superpotential. We have found the zero modes for the metric fluctuations and shown that gravity can be localized on this brane. Furthermore, their thin-wall limit is well-defined, and we find the massive modes on this limit. When the two cosmological constants at either side of the wall coincide, the usual RS result is recovered.

ACKNOWLEDGMENTS

We wish to thank Daniel Morales for fruitful discussions. This work was supported by FONACIT (Project No. S1-2000000820) and CDCHT-ULA (Project No. C-1267-04-05-A). O. C-F. wishes to thank ICTP for hospitality during the completion of this work.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999); M. Gogberashvili, Int. J. Mod. Phys. D **11**, 1635 (2002).

[2] M. Gremm, Phys. Lett. B **478**, 434 (2000).

[3] R. Guerrero, A. Melfo, and N. Pantoja, Phys. Rev. D **65**, 125010 (2002).

[4] G. Goetz, J. Math. Phys. (N.Y.) **31**, 2683 (1990).

[5] R. Gass and M. Mukherjee, Phys. Rev. D **60**, 065011 (1999).

[6] A. Melfo, N. Pantoja, and A. Skirzewski, Phys. Rev. D **67**, 105003 (2003).

[7] D. Bazeia, C. Furtado, and A. R. Gomes, J. Cosmol. Astropart. Phys. 02 (2004) 002; D. Bazeia, J. Menezes, and R. Menezes, Phys. Rev. Lett. **91**, 241601 (2003).

[8] D. Ida, J. High Energy Phys. 09 (2000) 014.

[9] N. Deruelle and T. Dolezel, Phys. Rev. D **62**, 103502 (2000).

[10] H. Stoica, S. H. H. Tye, and I. Wasserman, Phys. Lett. B **482**, 205 (2000).

[11] W. B. Perkins, Phys. Lett. B **504**, 28 (2001).

[12] B. Jensen and H. H. Soleng, Classical Quantum Gravity **14**, 1821 (1997).

[13] K. Behrndt and M. Cvetic, Phys. Lett. B **475**, 253 (2000).

[14] K. Skenderis and P. K. Townsend, Phys. Lett. B **468**, 46 (1999).

[15] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).

[16] D. Z. Freedman, C. Nunez, M. Schnabl, and K. Skenderis, Phys. Rev. D **69**, 104027 (2004).

[17] K. Behrndt and G. Dall'Agata, Nucl. Phys. **B627**, 357 (2002).

[18] J. Garriga and M. Sasaki, Phys. Rev. D **62**, 043523 (2000).

[19] N. Alonso-Alberca, P. Meessen, and T. Ortin, Phys. Lett. B **482**, 400 (2000).

[20] A. Karch and L. Randall, J. High Energy Phys. 05 (2001) 008.

[21] R. Wald, *General Relativity* (University of Chicago, Chicago, 1984).

[22] A. z. Wang, Phys. Rev. D **66**, 024024 (2002).

[23] N. Pantoja and A. Sanoja, gr-qc/0312032.

[24] A. H. Taub, Ann. Math. **53**, 472 (1951).

[25] K. Tomita, Phys. Lett. **162B**, 287 (1985).

[26] C. Csaki, J. Erlich, T. J. Hollowood, and Y. Shirman, Nucl. Phys. **B581**, 309 (2000).

[27] R. Gregory, V. A. Rubakov, and S. M. Sibiryakov, Phys. Rev. Lett. **84**, 5928 (2000).

[28] S. L. Dubovsky, V. A. Rubakov, and P. G. Tinyakov, Phys. Rev. D **62**, 105011 (2000).

[29] G. R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **485**, 208 (2000).