Black holes from colliding wavepackets

Steven B. Giddings*
Department of Physics† and Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA

Vyacheslav S. Rychkov‡
Institute for Theoretical Physics, University of Amsterdam, 1018XE Amsterdam, The Netherlands
(Received 21 September 2004; published 23 November 2004)

Arguments for black hole formation in collisions of high-energy particles have rested on the emergence of a closed trapped surface in the classical geometry of two colliding Aichelburg-Sexl solutions. Recent analysis has, however, shown that curvatures and quantum fluctuations are large on this apparent horizon, potentially invalidating a semiclassical analysis. We show that this problem is an artifact of the unphysical classical point-particle limit: for a particle described by a quantum wavepacket, or for a continuous matter distribution, trapped surfaces indeed form in a controlled regime.

DOI: 10.1103/PhysRevD.70.104026 PACS numbers: 04.70.–s, 04.60.–m

I. INTRODUCTION

Production of black holes in high-energy collisions has long been a topic of interest [1,2]. With the discovery of large extra dimension [3,4] or warped compactification [5,6] scenarios that lower the fundamental Planck scale to the TeV scale, it became clear that black holes might be experimentally accessible in accelerators; for early discussion of this possibility see [7,8]. Even more astounding, production rate estimates [9,10] showed that in the most optimistic version of these scenarios black holes could be produced copiously at LHC, at rates up to about 1 BH/s. These predictions were based on the so called geometric cross section estimate:

$$\sigma(2 \to BH) \sim R_h^2 \sim \frac{1}{M_p^2}\left[\frac{E}{M_p}\right]^{2/(D-3)}. \tag{1}$$

Here D is the total number of dimensions, $M_p \sim 1$ TeV is the fundamental D-dimensional Planck scale, E is the C.M. collision energy, which also provides the approximate black hole mass, and R_h is the horizon radius of the D-dimensional Schwarzschild BH of mass E.

The estimate (1) is based on the simple classical gravity intuition, encoded for example in the "hoop conjecture [11]," that gravitational collapse and BH formation will occur if the colliding particles pass within distance $\lesssim R_h$ from each other. This intuitive argument was made precise in [12] where it was argued that for impact parameters $\lesssim R_h$ a closed trapped surface (CTS) forms in the collision spacetime; for $D > 4$ numerical verification of this argument was supplied in [13]. Black hole formation then follows from this as a consequence of the singularity theorems and cosmic censorship conjecture of classical gravity.

Recently [14,15], one of us has found a loophole in this classical argument for black hole formation. Specifically, the analysis of [12] investigated the collision of two Aichelburg-Sexl [16] solutions, corresponding to the gravitational shock-waves of ultra-relativistic classical point particles. However, as [14] points out, and as we will review, in the intersection of the planes of the two shock-waves, there is a divergent curvature invariant.[1] This undermines the classical analysis, as do related arguments [15] about large fluctuations of the gravitational field.

This paper will address these issues, taking into account features that should be present in a more complete semiclassical analysis: specifically, the finite width of any wavepacket describing collision of quantum particles. As a consequence, we will argue that the geometric value of the black hole production cross section remains robust.

II. CLASSICAL GRAVITY DESCRIPTION

Let us briefly review the classical gravity description of the BH production process as presented in [12].

A classical BH may form in a collision of two particles with total C.M. energy $E \gg 1$. (Here and below the D-dimensional Planck units are used, with $8\pi G = 1$.) We will assume that the size of the created BH is much smaller than the size of the large extra dimensions (this assumption is satisfied in typical TeV-scale gravity scenarios). We will also neglect the brane tension. Under these assumptions, the BH production can be considered as happening in flat D-dimensional space.

BH production was described in [12] by considering two ultrarelativistic point particles in a grazing collision with an impact parameter b. The gravitational field of one such particle is given by the Aichelburg-Sexl metric

*Electronic address: giddings@physics.ucsb.edu
†Permanent address.
‡Electronic address: rychkov@science.uva.nl

[1] Concerns about high curvature were also previously expressed in [17].

[2,16,18,19],

$$ds^2 = -d\bar{u}d\bar{v} + d\bar{x}^{i2} + \Phi(\bar{x}^i)\delta(\bar{u})d\bar{u}^2. \tag{2}$$

Here $\bar{u} = t - z$, $\bar{v} = t + z$, and Φ depends only on the radius in the $D - 2$ transverse directions, $\bar{x}^i$, $\bar{\rho} = \sqrt{\bar{x}^i\bar{x}_i}$, and takes the form

$$\Phi = -\frac{E}{\pi}\log(\bar{\rho}), \qquad D = 4; \tag{3}$$

$$\Phi = \frac{2E}{(D-4)\Omega_{D-3}\bar{\rho}^{D-4}}, \qquad D > 4, \tag{4}$$

where Ω_{D-3} is the volume of the unit $(D - 3)$-sphere.

This solution has curvature concentrated on the plane transverse to the direction of motion. Indeed, the only nonzero components of the Riemann tensor for the right-moving particle are [14]

$$R_{uiuj} = -\frac{1}{2}\delta(\bar{u})\frac{\partial^2}{\partial\bar{x}_i\partial\bar{x}_j}\Phi. \tag{5}$$

This field should be superposed with the similar field of the left-moving particle, shifted by b in the transverse direction. The resulting field is valid outside the region $\bar{u}, \bar{v} > 0$, where the colliding shocks start influencing each other. The metric in this region should in principle be found by solving Einstein's equation, but it remains unknown even in the simplest $b = 0$ case. Thus, BH formation may be concluded only indirectly, looking for a CTS in the known part of the spacetime. The CTS constructed in [12] (with numerical solution provided for $D > 4$ in [13]) lies in the union of pre-collision parts of the shock planes $\bar{u} = 0$ and $\bar{v} = 0$. It looks like two roughly elliptically shaped surfaces, glued together across the shock waves at the collision plane $\bar{u} = \bar{v} = 0$. The size of the surface is comparable to the Schwarzschild radius,

$$R_h \sim E^{1/(D-3)}. \tag{6}$$

The maximal value of the impact parameter $b_{\max} \sim R_h$ for which a CTS is found to exist leads to the geometric cross section estimate (1).

The issue noticed in [14] follows directly from the curvature (5). For a single such shock wave, there is no divergent curvature invariant except at the precise location of the particle. But for the full solution corresponding to the combined shock waves, there is a divergent curvature invariant in the intersection of the two shocks at $\bar{u} = \bar{v} = 0$. Indeed, combining (5) with the oppositely-moving counterpart gives

$$(R_{\mu\nu\lambda\sigma})^2 \sim (E^2/\bar{\rho}^{2D-4})\delta(\bar{u})\delta(\bar{v}) \tag{7}$$

yielding a divergent result throughout the intersection $\bar{u} = \bar{v} = 0$. Since classical gravity must fail at such a curvature singularity, and since the closed trapped surface passes through the offending region, the argument for black hole formation, based on classical evolution of the trapped surface, is not on solid ground.

III. FROM CLASSICAL TO SEMICLASSICAL

Clearly the collision of two photons on Earth does not produce a gravitational singularity in the next galaxy. The effects that remove the singularity in (7) come from the fact that particles are intrinsically quantum, and thus have a quantum-mechanical width. We will argue that taking this into account suffices to reinstate the robustness of the argument for black hole formation.

A. Wavepackets

To go beyond the point-particle approximation, we must take into account the limitations imposed by quantum theory. In particular, for a highly relativistic particle with $p_z \sim E$, there is a typical position uncertainty $\Delta z \gtrsim 1/E$.

Of course, the position uncertainty may be even greater. For example, the fundamental quantum limits may be accounted for by considering minimal-uncertainty wavepackets of the form

$$\psi(z, t) \sim \exp\left\{-\frac{[z - z(t)]^2}{\Delta z^2} - ip_z[z - z(t)]\right\} \tag{8}$$

(with appropriate generalization for transverse coordinates). If we consider the collision of two such wavepackets, with widths Δz, the condition for us to be able to still use the above geometrical reasoning is that the wavepacket width be much less than the Schwarzschild radius,

$$\Delta z \ll R_h. \tag{9}$$

This is true because at distances large as compared to Δz the resulting solution will be a small perturbation of the point-particle spacetime. In particular, the δ-function in (5) will be smeared over an interval of length $\sim\Delta z$ (so that the shock acquires finite width), and the collision spacetime will still contain a CTS. Strictly speaking this relies on the argument, given in [12], that the CTS can be deformed out of the shock planes.

Note also that in a given experiment, for example at LHC, one may also be in practice working with wavepackets of a large, even macroscopic size. In this case we can think of decomposing these wavepackets into smaller wavepackets of size Δz satisfying the previous two limits:

$$E^{-1} \lesssim \Delta z \ll R_h. \tag{10}$$

This subdivision can be carried out in such a manner that different small wavepackets correspond to almost orthogonal states. Combining the contributions of the smaller wavepackets still results in the geometric cross section.

B. Curvature

In the context of collisions of such wavepackets, we can revisit the question of large curvature: namely, can we choose a wavepacket size such that

$$E^{-1} \lesssim \Delta z \ll R_h \tag{11}$$

that avoids the large curvature of (7)?

To answer this, note that for wavepackets of the form (8), the δ-function in (5) gets replaced by a Gaussian, with maximum strength $1/\Delta z$. This follows for example from the fact that the integral of the curvature across the shock should be independent of the width of the wavepacket. Thus (7) gets replaced by

$$(R_{\mu\nu\lambda\sigma})^2 \lesssim (E^2/\bar{\rho}^{2D-4})\Delta z^{-2}. \tag{12}$$

Evaluating this in the vicinity of the trapped surface, $\bar{\rho} \sim R_h$, gives

$$(R_{\mu\nu\lambda\sigma})^2 \lesssim R_h^{-2}\Delta z^{-2}. \tag{13}$$

From this we see that the curvature can be kept small for

$$\Delta z \gg 1/R_h, \tag{14}$$

which is compatible with the allowed range (11). Thus it is always possible to choose wavepackets small as compared to the size of the closed trapped surface, and such that the curvature remains small in the vicinity of the closed trapped surface. This avoids the singular curvatures found in [14,15].

C. Quantum fluctuations

Another test of the semiclassical description of scattering is to estimate the strength of quantum fluctuations in the gravitational field relative to the semiclassical solution. For the validity of the semiclassical description these fluctuations should be small compared to the field itself. This is also equivalent to requiring that the occupation numbers of the background graviton field be large—the condition proposed in [15].

We will focus on the part of the spacetime near the shock front $\bar{u} = 0$, since everywhere else spacetime is flat. The form of the shock-wave metric given in (2) is inconvenient to use, because of its divergent components. We will instead use the metric

$$ds^2 = -dudv + \left[1 + \frac{(D-3)E}{\Omega_{D-3}\rho^{D-2}}u\theta(u)\right]^2 d\rho^2 + \left[1 - \frac{E}{\Omega_{D-3}\rho^{D-2}}u\theta(u)\right]^2 \rho^2 d\Omega_{D-3}^2, \tag{15}$$

following from (2) by a coordinate transformation [2,12,14]. Near the shock front, this metric can be approximated as

$$ds^2 \approx dx_\mu^2 + \frac{2E}{\Omega_{D-3}\rho^{D-2}}u\theta(u)[(D-3)d\rho^2 - \rho^2 d\Omega_{D-3}^2]. \tag{16}$$

Since, as previously mentioned, the center portions of the CTS can be deformed away from the shock planes [12], the region where we have to check validity of the semiclassical approximation is the region where the two pieces cross the shocks and join. This is the region $\rho \sim R_h$ and at $|u| \ll R_h$. Here the deviation from the Minkowski metric is small:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \tag{17}$$

and we are in the linearized gravity situation.

To describe fluctuations, we will impose the transverse-traceless (TT) gauge, specified by conditions

$$h_{\mu 0} = 0, \quad h_{ab,b} = 0, \quad h_{aa} = 0, \tag{18}$$

where $a = 1, \ldots, D-1$. We will also take advantage of the fact that the transverse shock profile is slowly varying compared to the shock width Δz in the region of interest. We can thus work in the plane wave approximation [14], neglecting transverse derivatives of the metric:

$$h_{ab} \approx h_{ab}(u, v). \tag{19}$$

Our goal is to estimate fluctuations of the shock wave amplitude at $r \sim R_h$. The h_{ij} corresponding to (16) can be written as

$$h_{ij}^{cl} = \begin{pmatrix} (D-3)Cu\theta(u) & & & \\ & -Cu\theta(u) & & \\ & & \ddots & \\ & & & -Cu\theta(u) \end{pmatrix}. \tag{20}$$

In the plane wave approximation the difference between polar and Cartesian coordinates disappears. We can also neglect the transverse dependence of C, so that it becomes a constant $\sim R_h^{-1}$. After these simplifications, the classical field (20) becomes precisely of the form (18) and (19).

We will estimate the strength of fluctuations of the gravitational field compared to the semiclassical background in the region where the CTS intersects the shocks. These fluctuations are approximately controlled by the linearized Einstein-Hilbert action, which in the TT gauge takes the form

$$S = \frac{1}{8}\int d^D x h_{ij,\alpha} h_{ij}^{,\alpha}. \tag{21}$$

In the plane wave approximation this becomes

$$S = \frac{A}{8}\int dt dz[(\partial_t h_{ij})^2 - (\partial_z h_{ij})^2], \tag{22}$$

where A is the transverse area of the planar field configuration being considered, here $A \sim R_h^{D-2}$.

Up to a constant factor this is the action of a 2-dimensional massless scalar field. An estimate of the size of the quantum fluctuations follows straightforwardly from this:

$$(\delta h_{ij})^2 \sim A^{-1} \int_{k \sim \delta z^{-1}} \frac{dk}{k} \sim A^{-1}. \tag{23}$$

These fluctuations should be much smaller than the typical size of h_{ij}^{cl} at a distance $\sim\delta z$ from the shock front, which is $\sim C\delta z$ [see (20)]. This condition becomes most restrictive when applied at the smallest scale $\delta z \sim \Delta z$ existing in the classical solution (shock wave width). Thus we get the final condition for the smallness of quantum fluctuations:

$$A^{-1/2} \ll C\Delta z \quad \Leftrightarrow \quad \Delta z \gg R_h^{2-(D/2)}, \tag{24}$$

which is also compatible with (11).

IV. CONCLUSIONS

In this paper we have argued that it is possible to carry out the analysis of BH production in transplanckian elementary particle collisions in a semiclassical approximation, taking into account quantum spreading of the wavepacket for a particle. The crucial difference with [15] (which led to the opposite conclusion) was that instead of using the minimally allowed uncertainty E^{-1} as the particle wavepacket size Δz, we noticed that it is unnecessary to insist on such a choice, and kept Δz as a free parameter. It was found that both criteria of semiclassicality—low curvatures and small quantum fluctuations—can be reconciled with the classical gravity analysis of BH formation, provided that Δz is in the range [see (11), (14), and (24)]

$$\max(R_h^{-1}, R_h^{2-(D/2)}) \ll \Delta z \ll R_h. \tag{25}$$

Note that, in practice, to produce a black hole with $R_h \gg 1$, all this requires is a shock width larger than the Planck size. It is nonetheless interesting that a careful treatment of black hole creation requires quantum wavepackets, or classical continuous matter distributions. This analysis then puts the geometric cross section estimate on a more solid ground.

ACKNOWLEDGMENTS

We would like to thank T. Banks, D. Eardley, and G. Horowitz for important conversations. The work of S. B. G. was supported in part by the Department of Energy under Contract No. DE-FG02-91ER40618, and part of it was carried out during the Workshop on QCD and String Theory at the Kavli Institute for Theoretical Physics, whose support is gratefully acknowledged. The work of V. S. R. was supported by Stichting FOM.

[1] R. Penrose (unpublished).
[2] P. D. D'Eath, Phys. Rev. D **18**, 990 (1978); P. D. D'Eath and P. N. Payne, Phys. Rev. D **46**, 658 (1992); P. D. D'Eath and P. N. Payne, Phys. Rev. D **46**, 675 (1992); P. D. D'Eath and P. N. Payne, Phys. Rev. D **46**, 694 (1992).
[3] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [hep-ph/9803315].
[4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436**, 257 (1998) [hep-ph/9804398].
[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].
[6] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002) [hep-th/0105097].
[7] T. Banks and W. Fischler, hep-th/9906038.
[8] S. B. Giddings and E. Katz, J. Math. Phys. (N.Y.) **42**, 3082 (2001) [hep-th/0009176].
[9] S. B. Giddings and S. Thomas, Phys. Rev. D **65**, 056010 (2002) [hep-ph/0106219].
[10] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001) [hep-ph/0106295].
[11] K. S. Thorne, *Magic Without Magic*, edited by J. R. Klauder (Freeman, San Francisco, 1972), pp. 231–258.
[12] D. M. Eardley and S. B. Giddings, Phys. Rev. D **66**, 044011 (2002) [gr-qc/0201034].
[13] H. Yoshino and Y. Nambu, Phys. Rev. D **67**, 024009 (2003) [gr-qc/0209003].
[14] V. S. Rychkov, Phys. Rev. D **70**, 044003 (2004) [hep-ph/0401116].
[15] V. S. Rychkov, hep-ph/0405104.
[16] P. C. Aichelburg and R. U. Sexl, Gen. Relativ. Gravit. **2**, 303 (1971).
[17] S. D. H. Hsu, Phys. Lett. B **555**, 92 (2003) [hep-ph/0203154];O. V. Kancheli, hep-ph/0208021.
[18] F. A. E. Pirani, Proc. R. Soc. London A **252**, 96 (1959).
[19] T. Dray and G. 't Hooft, Nucl. Phys. **B253**, 173 (1985).