Exact solutions of Lovelock-Born-Infeld black holes

Matías Aiello,[1,*] Rafael Ferraro,[1,2,†] and Gastón Giribet[1,3,‡]

[1]*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*

[2]*Instituto de Astronomía y Física del Espacio, Casilla de Correo 67, Sucursal 28, 1428 Buenos Aires, Argentina*

[3]*Institute for Advanced Study, Einstein Drive, Princeton, New Jersey 08540, USA*

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The exact five-dimensional charged black hole solution in Lovelock gravity coupled to Born-Infeld electrodynamics is presented. This solution interpolates between the Hoffmann black hole for the Einstein-Born-Infeld theory and other solutions in the Lovelock theory previously studied in the literature. It is shown how the conical singularity of the metric around the origin can be removed by a proper choice of the black hole parameters. The differences existing with the Reissner-Nordström black holes are discussed. In particular, we show the existence of charged black holes with a unique horizon.

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I. INTRODUCTION

The Einstein tensor is the only symmetric and conserved tensor depending on the metric and its derivatives up to the second order, which is linear in the second derivatives of the metric. Dropping the last condition, Lovelock [1] found the most general tensor satisfying the other ones. The obtained tensor is nonlinear in the Riemann tensor and differs from the Einstein tensor only if the space-time has more than four dimensions. Therefore the Lovelock theory is the most natural extension of general relativity in higher dimensional space-times. The Lovelock theory for a particular choice of the coefficients of the action could be thought as the gravitational analogue of Born-Infeld electrodynamics [2].

In the last decades a renewed interest in both Lovelock gravity and Born-Infeld electrodynamics has appeared because they emerge in the low energy limit of string theory [3–5]. Since the Lovelock tensor contains derivatives of the metric of order not higher than the second, the quantization of the linearized Lovelock theory is free of ghosts. For this reason the Lovelock Lagrangian appears in the low energy limit of string theory. In particular, the Gauss-Bonnet terms (quadratic in the Riemann tensor) were studied in Ref. [6] and the quartic terms in Refs. [7,8]. The Lovelock theory of gravity was also discussed in Refs. [9–11].

Hoffmann was the first one in relating general relativity and the Born-Infeld electromagnetic field [12]. He obtained a solution of the Einstein equations for a point-like Born-Infeld charge, which is devoid of the divergence of the metric at the origin that characterizes the Reissner-Nordström solution. However, a conical singularity remained there, as it was later objected by Einstein and Rosen. The Einstein-Born-Infeld black hole has been revisited in Refs. [13,14]

The aim of this paper is to study the charged black hole solutions in five-dimensional Lovelock gravity coupled to Born-Infeld electrodynamics.

The Lovelock Lagrangian density in dimensions $D = 5$ and $D = 6$ [1,15] is given by

$$\mathcal{L} = \sqrt{-g}[R - 2\Lambda + \alpha(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)], \tag{1}$$

where we recognize the usual Lagrangian for the cosmological term, the Einstein-Hilbert Lagrangian and the Lanczos Lagrangian [16,17], respectively.

The spaces of dimensions $D = 7$ and $D = 8$ includes the Lagrangian $\mathcal{L}_3$, which was first obtained by Müller-Hoissen [18].

Consequently, the analogous to the Einstein tensor is obtained by varying the Lanczos Lagrangian with respect to the metric, resulting in the five-dimensional Lovelock tensor

$$\mathcal{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} - \alpha\Big\{\frac{1}{2}g_{\mu\nu}(R_{\rho\delta\gamma\lambda}R^{\rho\delta\gamma\lambda} - 4R_{\rho\delta}R^{\rho\delta} + R^2) - 2RR_{\mu\nu} + 4R_{\mu\rho}R^{\rho}_{\nu} + 4R_{\rho\delta}R^{\rho\delta}_{\mu\nu} - 2R_{\mu\rho\delta\gamma}R^{\rho\delta\gamma}_{\nu}\Big\}. \tag{2}$$

The Gauss-Bonnet constant α will allow us to track the changes in the equations, when we compare with the respective ones of general relativity. The coupling constant α introduces a length scale $l_L \sim \sqrt{\alpha}$ in the theory which physically represents a short-distance range where the Einstein gravity turns out to be corrected.

*Electronic address: aiello@iafe.uba.ar
†Electronic address: ferraro@iafe.uba.ar
‡Electronic address: gaston@ias.edu

Thus, the vacuum field equations are given by

$$\mathcal{G}_{\mu\nu} = 0 \leq \mu, \nu \leq 4 \tag{3}$$

and accept spherically symmetric solutions in five dimensions, which in terms of a suitable Schwarzschild-like *ansatz,* can be written as

$$ds^2 = -\Psi(r)dt^2 + \frac{1}{\Psi(r)}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\chi^2 + r^2\sin^2\theta\sin^2\chi d\varphi^2. \tag{4}$$

In this case, the solution of Eqs. (3) is

$$\Psi^{\pm}(r) = 1 + \frac{r^2}{4\alpha} \pm \sqrt{1 + \frac{M}{6\alpha} + \frac{r^4}{16\alpha^2} + \frac{\Lambda}{12\alpha}r^4}, \tag{5}$$

where M is an integration constant.

By requesting the proper Newtonian potential in the weak field region $r \to \infty$ ($\Lambda = 0$), it results that the Arnowitt-Deser-Misner mass is $m = \frac{\pi}{6}M + \pi\alpha$ [19] with $\alpha > 0$ for Ψ^- and $\alpha < 0$ for Ψ^+. Then

$$\Psi(r) = 1 + \frac{r^2}{4\alpha} - \frac{r^2}{4\alpha}\sqrt{1 + \frac{16m\alpha}{\pi r^4} + \frac{4\alpha\Lambda}{3}}. \tag{6}$$

Asymptotically, this solution goes to the general relativity solution in five dimensions when $\alpha \to 0$, as it is expected. Namely,

$$\Psi_{\rm GR}(r) = 1 - \frac{2m}{\pi r^2} - \frac{\Lambda}{6}r^2. \tag{7}$$

The purpose of this article is to present exact solutions of charged black holes in Lovelock theory coupled to Born-Infeld electrodynamics which will be shown to be generalizations of the solutions mentioned above. In Sec. II, we discuss the Born-Infeld electrodynamics which will provide us the necessary tools in order to eventually find, in Sec. III, the five-dimensional charged black hole solution in Lovelock-Born-Infeld field theory. We study the geometrical properties of the solution and discuss the similarities and distinctions existing with respect to the Reissner-Nordström black hole. The conclusions are contained in Sec. IV.

II. BORN-INFELD ELECTRODYNAMICS

In 1934 Born and Infeld [20,21] proposed a nonlinear electrodynamics with the aim of obtaining a finite value for the self-energy of a pointlike charge. The Born-Infeld Lagrangian leads to field equations whose spherically symmetric static solution gives a finite value b for the electrostatic field at the origin. The constant b appears in the Born-Infeld Lagrangian as a new universal constant. Following Einstein, Born and Infeld considered the metric tensor $g_{\mu\nu}$ and the electromagnetic field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as the symmetric and antisymmetric parts of a unique field $bg_{\mu\nu} + F_{\mu\nu}$. Then they postulated the Lagrangian density

$$\mathcal{L} = \sqrt{\det(bg_{\mu\nu} + F_{\mu\nu})} + \sqrt{-\det g_{\mu\nu}}, \tag{8}$$

where the second term is chosen so that the Born-Infeld Lagrangian tends to the Maxwell Lagrangian when $b \to \infty$. In four dimensions, this Lagrangian results to be

$$\mathcal{L} = \sqrt{-g}\frac{b^2}{4\pi}\left(1 - \sqrt{1 + \frac{2S}{b^2} - \frac{P^2}{b^4}}\right), \tag{9}$$

where S and P are the scalar and pseudoscalar field invariants

$$S = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(B^2 - E^2),$$
$$P = \frac{1}{8}\sqrt{-g}\epsilon_{\mu\nu\rho\gamma}F^{\mu\nu}F^{\rho\gamma} = E \cdot B.$$

The Born-Infeld Lagrangian is usually mentioned as an exceptional Lagrangian because its properties of being the unique structural function which: (1) Assures that the theory has a single characteristic surface equation; (2) Fulfills the positive energy density and the non-spacelike energy current character conditions; (3) Fulfills the strong correspondence principle. As a consequence of these conditions, the Lagrangian has timelike or null characteristic surfaces [22].

In order to obtain the static spherically symmetric solution in five dimensions, we will replace $F = E(r)dt \wedge dr$ and the metric (4) in the Born-Infeld Lagragian (8); then we will vary the action (this procedure is valid due to the high symmetry of the solution for which we are looking). Therefore

$$\mathcal{L}_{\rm BI} = \frac{b^2\sqrt{-g}}{4\pi}\left(1 - \sqrt{1 - \frac{E^2(r)}{b^2}}\right). \tag{10}$$

The field equation derived from this Lagrangian (10) is

$$\frac{\partial}{\partial r}\left[\frac{\sqrt{-g}E(r)}{\sqrt{1 - \frac{E^2(r)}{b^2}}}\right] = 0,$$

where $\sqrt{-g} = r^3\sin^2\theta\sin\chi$. So the Born-Infeld point charge field in five dimensions is

$$E(r) = \frac{Q}{\sqrt{r^6 + L^6}}; \qquad L = \left(\frac{Q}{b}\right)^{1/3}. \tag{11}$$

The energy-momentum tensor is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_{\rm BI}}{\partial g^{\mu\nu}}.$$

In the static isotropic case it results to be diagonal:

$$T_0^0 = T_r^r = \frac{b^2}{4\pi}\left[1 - \frac{1}{\sqrt{1-\frac{E^2(r)}{b^2}}}\right],$$
$$T_\chi^\chi = T_\theta^\theta = T_\varphi^\varphi = \frac{b^2}{4\pi}\left[1 - \sqrt{1-\frac{E^2(r)}{b^2}}\right]. \qquad (12)$$

The energy of this field is finite in contrast to the energy of the Maxwell field:

$$U = 2\pi^2 \int_0^\infty T_0^0 r^3 dr = \frac{-b^2 L^4\sqrt{\pi}}{12}\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{1}{4}\right). \qquad (13)$$

III. LOVELOCK-BORN-INFELD SOLUTIONS

We will study the exact solutions of Lovelock gravity for a Born-Infeld isotropic electrostatic source. The field equations to be solved are $\mathcal{G}_{\mu\nu} = 8\pi T_{\mu\nu}$, where $\mathcal{G}_{\mu\nu}$ is the Lovelock tensor (2) and $T_{\mu\nu}$ is the Born-Infeld energy-momentum tensor corresponding to a point charge located in the origin (12). Because of the symmetry of the source we repit the *ansatz* (4) for the metric. In this case only the diagonal components of the Lovelock tensor survive: the components $\mu = \nu = 0, 1$ are equal, and they are integrals of the components $\mu = \nu = 2, 3, 4$. Therefore, it is enough to solve $\mathcal{G}_{00} = 8\pi T_{00}$, which amounts to the equation

$$-\frac{\Psi(r)}{2r^3}\left[3r^2\frac{d\Psi(r)}{dr} + 12\alpha\frac{d\Psi(r)}{dr} - 12\alpha\Psi(r)\frac{d\Psi(r)}{dr} + 6\Psi(r)r - 6r + 2r^3\Lambda\right] = \frac{2b^2\Psi(r)}{r^3}\left(\sqrt{r^6+L^6} - r^3\right).$$

The left-hand side can be written as a total radial derivative, to be easily integrated. The solution is

$$\Psi^\pm[r] = 1 + \frac{r^2}{4\alpha}\left[1 + \frac{M}{6\alpha} + \left(\frac{1}{4\alpha} - \frac{4b^2-2\Lambda}{6}\right)\frac{r^4}{4\alpha} + \frac{2b^2}{3\alpha}\int_0^r dr\sqrt{r^6+L^6}\right)^{1/2}, \qquad (14)$$

where M is an integration constant. The integral inside the square involves an incomplete elliptic integral of the first kind $F(a, b)$ [23], namely,

$$\int_0^r dr\sqrt{r^6+L^6} = \frac{1}{4}r\sqrt{r^6+l^6} + \frac{3}{4}L^4\int_0^{\frac{r}{L}}\frac{dt}{\sqrt{1+t^6}},$$

where

$$\int_0^{\frac{r}{L}}\frac{dt}{\sqrt{1+t^6}} = \frac{1}{23^{1/4}}F\left(\arccos\left[\frac{L^2+(1-\sqrt{3})r^2}{L^2+(1+\sqrt{3})r^2}\right], \frac{2+\sqrt{3}}{4}\right).$$

Thus, we obtain two solutions for the metric, but the sign of α is determined requiring that in the limit $r \to \infty$ we must recover the Newtonian potential in five dimensions $\Phi(r) = \frac{m}{\pi r^2}$, so we obtain $\alpha > 0$ for Ψ^- and $\alpha < 0$ for Ψ^+. In that limit the solution is

$$\Psi(r) = 1 - \frac{2m}{\pi r^2},$$

with

$$m = \frac{\pi}{6}M + \pi\alpha + \frac{\pi}{2}L^4 b^2\gamma,$$

where

$$\gamma = \int_0^\infty \frac{dt}{\sqrt{1+t^6}} = \frac{\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{7}{6}\right)}{\sqrt{\pi}} \approx 1.40218.$$

In terms of the Arnowitt-Deser-Misner mass m, $\Psi(r)$ becomes

$$\Psi(r) = 1 + \frac{r^2}{4\alpha} - \frac{r^2}{4\alpha}\left[1 + \frac{16m\alpha}{\pi r^4} - \frac{2}{3}\alpha(4b^2 - 2\Lambda) + \frac{8b^2\alpha}{3r^3}\sqrt{r^6+L^6} - \frac{8b^2L^6\alpha}{r^4}\int_r^\infty \frac{dr}{\sqrt{r^6+L^6}}\right]^{1/2}. \qquad (15)$$

This class of solutions was also studied in reference [24].

In the limit $\alpha \to 0$ the solution tends to

$$\Psi(r) \approx 1 - \frac{\frac{6}{\pi}m - 2L^4b^2\gamma}{3r^2} - \frac{\Lambda}{6}r^2 - \frac{4b^2}{3r^2}\times\int_0^r dr\left(\sqrt{r^6+L^6} - r^3\right). \qquad (16)$$

This limit agrees with the quoted four-dimensional Hoffmann solution [12] with a conical singularity in the origin of the black hole. Consequently, by performing the limit $b \to \infty$ in Eq. (16) we recover the Reissner-Nordström solution in five dimensions with cosmological constant,

$$\Psi_{\mathrm{RN}}(r) = 1 - \frac{2m}{\pi r^2} + \frac{Q^2}{3r^4} - \frac{\Lambda}{6}r^2.$$

On the other hand, by taking the limit $b \to \infty$ in (15), we also recover the Lovelock-Maxwell solution found by Wiltshire in [25]; namely,

$$\Psi_{\mathrm{W}}(r) = 1 + \frac{r^2}{4\alpha} - \frac{r^2}{4\alpha}\left[1 + \frac{16m\alpha}{\pi r^4} - \frac{8Q^2\alpha}{3r^6} + \frac{4\Lambda\alpha}{3}\right]^{1/2}. \qquad (17)$$

Notice that, because to the existence of a Birkhoff-like theorem (see Appendix A in Ref. [19] and references therein), this limit turns out to be more than a simple heuristical argument to check the solution (15), representing a necessary condition which must be proved.

We also observe that the uncharged solution (6) is recovered in the limit $b \to 0$. In this case, a fact which deserves to be emphasized is that for nonvanishing Λ, besides the leading term in the expansion (7), we find finite-α corrections to the black hole parameters. Namely,

$$\Psi(r) = 1 - \frac{2m_d}{\pi r^2} - \frac{\Lambda_d}{6} r^2 + \mathcal{O}(\alpha r^{-6}), \tag{18}$$

where the *dressed* parameters m_d and Λ_d are given by

$$\Lambda_d = \Lambda\left(1 + \sum_{n=2}^{\infty} c_n \mu^{n-1}\right), \tag{19}$$

$$m_d = m\left(1 + \sum_{n=2}^{\infty} n c_n \mu^{n-1}\right), \tag{20}$$

being

$$c_n = \frac{(2n-3)!!}{2^{n-1} n!}, \qquad \mu = -\frac{4}{3}\Lambda\alpha.$$

Furthermore, in the case of the charged solution we are discussing in this section, the (charge) parameter Q receives similar corrections due to these finite-α effects, resulting

$$Q_d^2 = Q^2\left(1 + \sum_{n=2}^{\infty} n\ c_n \mu^{n-1}\right). \tag{21}$$

Notice that the parameter μ controls the *dressing* of the whole set of black hole parameters. The above power expansion converges for values such that $\mu < 1$. Besides, for the case $\mu > 1$ we find a different expansion, leading to the following *dressed* parameters in the large r regime

$$m_d = \frac{m}{\sqrt{|\mu|}}\left(1 + \sum_{n=2}^{\infty} n\ c_n \mu^{1-n}\right). \tag{22}$$

Thus, we note that the Newtonian term $\sim m_d r^{-2}$ vanishes in the limit $|\Lambda\alpha| \to \infty$. The particular case $\mu = 1$ is discussed below. Moreover, it is possible to see that, if one considers the case $\alpha\Lambda > 0$, the effective cosmological constant in the large μ limit turns out to be

$$\Lambda_d = \sqrt{\frac{3\Lambda}{\alpha}} - \frac{3}{2\alpha} + \mathcal{O}(1/\sqrt{|\mu|}). \tag{23}$$

On the other hand, for the case of vanishing cosmological constant ($\Lambda = 0$), the solution (6) displays an event horizon located at $r_h = \sqrt{\frac{2m}{\pi} - 2\alpha}$ when $m \geq \pi\alpha$. Then, the horizon can reach the point $r = 0$ for a massive object with $m = \pi\alpha$; in this case $r = 0$ is a naked singularity. If $m < \pi\alpha$ there is no horizon.

One of the relevant differences existing between the black hole solutions in Einstein and Lovelock theories is the fact that $\Psi(r)$ is not singular at the origin. Instead, the metric is regular everywhere, as it can be directly seen by setting $r = 0$ in (6). In fact, we find a similar aspect for the case of the Lovelock-Born-Infeld charged black hole [see (25) below].

Besides, if the object has no mass ($m = 0$), one gets de Sitter ($a < 0 \Rightarrow \Lambda > 0$) and anti-de Sitter (AdS) solutions ($a > 0 \Rightarrow \Lambda < 0$) as particular cases, namely,

$$\Psi_{\mathrm{(A)dS}}(r) = 1 + ar^2$$

where $a = \frac{1}{4\alpha}(1 - \sqrt{1 + \frac{4}{3}\Lambda\alpha})$. On the other hand, another interesting geometry is found in the particular case $\alpha\Lambda = -\frac{3}{4}$. At this point of the space of parameters, the solution (5) becomes

$$\Psi_{\mathrm{BTZ}}(r) = \frac{r^2}{4\alpha} - \mathcal{M}, \tag{24}$$

where we have considered $\Lambda < 0$ and introduced the notation $\mathcal{M} + 1 = \sqrt{\frac{m}{\pi\alpha}}$. Certainly, we could refer to this particular black hole solution as the *Bañados, Teiltelboim, and Zanelli (BTZ) branch*, due to its reminiscence of BTZ black hole [26–28]. Indeed, the parameter $\mathcal{M}$ in Eq. (24) plays the role of the mass M_{BTZ} in the BTZ solution. For instance, as well as AdS_3 space-time is obtained as a particular case of the BTZ geometry by setting the negative mass $M_{\mathrm{BTZ}} = -1$, also the five-dimensional anti-de sitter space corresponds to setting $\mathcal{M} = -1$. Let us notice that, in a consistent way, if the large α limit is taken while fixing the condition $\Lambda\alpha = -\frac{3}{4}$ one finds that the solution becomes the metric which represents the near boundary limit of AdS_5, like it happens with the *massless BTZ* ($M_{\mathrm{BTZ}} = 0$) which is obtained by making the three-dimensional black hole to disappear. Then, the parallelism with the solutions in $D = 3$ turns out to be exact since the five-dimensional metric obtained by keeping only the leading terms in the near boundary limit of AdS_5 corresponds to $\mathcal{M} = 0$ in (24) as well, which is precisely the Lovelock black hole solution (5) with *minimal* mass $m = \pi\alpha$. Besides, a conical singularity is found in the range $0 < m < \pi\alpha$ (corresponding to $-1 < \mathcal{M} < 0$) in a complete analogy.

Coming back to the general solution (15), we can see that, differing from Reissner-Nordström solution, the metric has no singularities (Fig. 1)

$$\Psi(r = 0) = 1 - \left(\frac{m}{\pi\alpha} - \frac{b^2 L^4 \gamma}{2\alpha}\right)^{1/2}. \tag{25}$$

Depending on the values of the parameters of the black hole (charge Q, mass m, Born-Infeld constant b and Gauss-Bonnet constant α) the square root could be imaginary. If $\alpha > 0$ and $m \geq \frac{\pi}{2} b^2 L^4 \gamma = m_c$ the solution is valid for all $r \geq 0$.

If $m > m_c$ then $\Psi(0) \neq 1$, so the metric has a conical singularity at the origin: whereas the equator measures $2\pi r$, the radius at the equator is $\int_o^r \frac{dr}{\sqrt{\Psi(r)}}$. In the critical

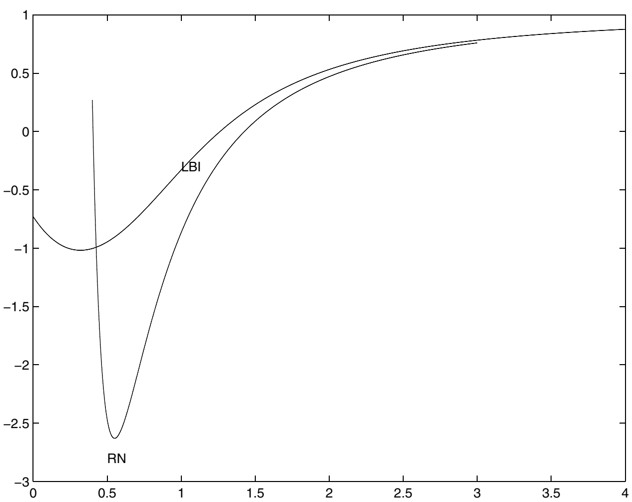

FIG. 1. Comparison between Reissner-Nordström (RN) and $\Psi(r)$ Lovelock-Born-Infeld (LBI) [$\alpha = 0.1$, $Q = 1$, $\Lambda = 0$, $b = 1$, $m = \pi$].

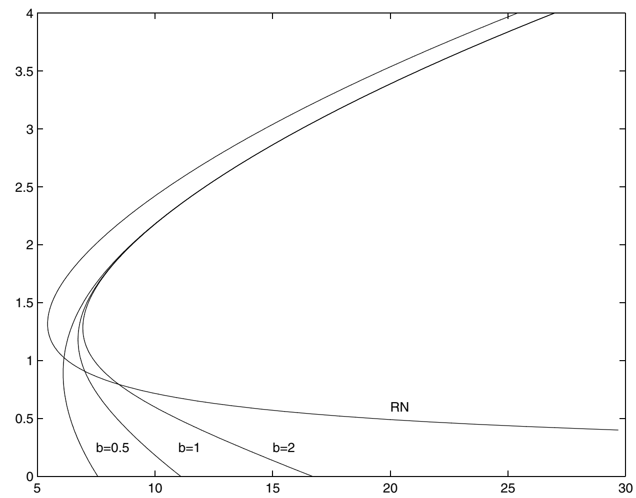

FIG. 2. Event horizon position (vertical axis) as a function of the black hole mass (horizontal axis) for different values of b (solid curves) and in the Reissner-Nordström case (RN) [$\alpha = 0.5$, $Q = 3$, $\Lambda = 0$].

case $m = m_c$ one finds $\Psi(0) = 1$ and the conical singularity disappears from the metric since in this case $\int_o^r \frac{dr}{\sqrt{\Psi(r)}} \approx r$ when $r \approx 0$.

Conversely, we can think in the critical value m_c as follows: we can define a critical value $Q_c^2 = b^{-1}(\frac{2m}{\pi\gamma})^{3/2}$ which represents an upper bound on Q for the metric (24) to be well defined in the whole space-time. Thus, a critical value for the black hole charge appears in this context as a direct consequence of the finite-b effects. The role played by b is setting the critical value $Q_c \rightarrow 0$ in the Maxwellian regime $b \rightarrow \infty$, which is consistent with the fact that the Lovelock-Maxwell black hole geometry is not regular if $m \neq 0$.

Besides, the finite-b effects act in (15) as a kind of effective cosmological constant $\Lambda_{\text{eff}}(r) = \Lambda + 2b^2(1 - e^{u_b(r)})$, with $u_0 = 0$ and $\lim_{r\rightarrow\infty} u_b(r) = 0$. Thus, this fact could lead one to infer that the *dressing* of the black hole parameters (19)–(21) discussed above can also receive contribution due to the presence of b. However, this is not the case, as it can be verified by noting that no b-dependent quadratic terms in r arise when expanding the right-hand side of (15).

In this geometry, the position of the horizon r_h is defined by $\Psi(r_h) = 0$, then

$$r_h^2 = \frac{2m}{\pi} - 2\alpha - \frac{r_h^4}{12}(4b^2 - 2\Lambda) + \frac{b^2}{3} r_h \sqrt{r_h^6 + L^6} - b^2 L^4 \int_{r_h/L}^{\infty} \frac{dt}{\sqrt{1 + t^6}},$$

and thus, for $\Lambda = 0$ and $m > m_c + \pi\alpha$ there is only one horizon. If $m \leq m_c + \pi\alpha$ then the solution is similar to Reissner-Nordström in the sense that there could be two horizons. When the equality holds one of the horizons is at the origin [see Eq. (25)]. Figure 2 shows the position of the horizon (r_h) as a function of the mass m for the Lovelock-Born-Infeld black hole and the Reissner-Nordström case ($b \rightarrow \infty$). These graphics exhibit the crucial difference existing between the Lovelock-Born-Infeld black hole and the Reissner-Nordström black hole, namely, the fact that for a given charge Q there exists a finite value for the black hole mass m such that the black hole geometry presents only one horizon, because the *internal* one reached the origin. This enhancement of the region bounded by both internal and external horizons is also b-dependent and represents, by itself, one of the principal distinctions between the black hole geometries in both theories. Moreover, Fig. 2 shows how the extremal configuration $r_+ = r_-$, which is translated into a complicated expression in terms of the parameters m, Q, α and b, experiments a displacement for finite values of b with respect to the Reissner-Nordström configuration $m^2 = \frac{\pi^2}{3} Q^2$. Figure 3 shows different behaviors of the solution $\Psi(r)$ for different values of b. This parameter controls the qualitative behavior close to the origin. The metric for the subcritical case ($m < m_c$) is not well defined in the whole space-time.

IV. CONCLUSIONS

In this paper we studied a solution (15) representing five-dimensional charged black holes in Lovelock gravity coupled to Born-Infeld electrodynamics. The corrections induced by the quadratic terms in the Lagrangian (Gauss-Bonnet terms) correspond to short-distance modifications to general relativity and, therefore, the relevant differences between both theories appear for small radius.

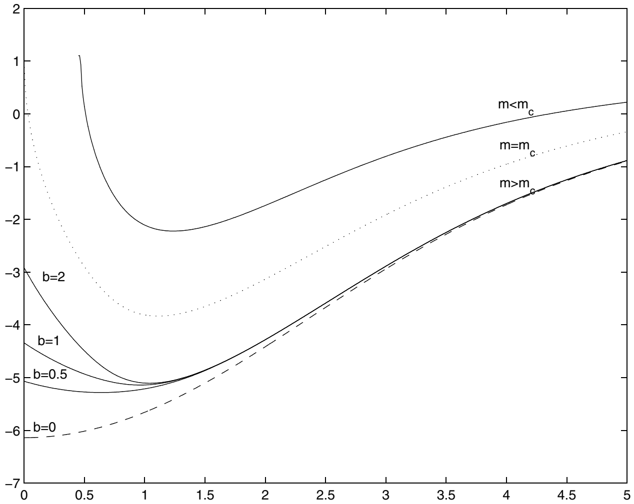

FIG. 3. Graphics of $\Psi(r)$ for different values of b, for $m < m_c$: [$m = 2$, $\alpha = 0.5$, $Q = 1$, $b = 2$, $\Lambda = 0$], for $m = m_c$ [$m = 3.49$, $\alpha = 0.5$, $Q = 1$, $b = 2$, $\Lambda = 0$] and for $m > m_c$ [$m = 5$, $\alpha = 0.5$, $Q = 1$, $\Lambda = 0$].

The Lovelock-Born-Infeld black holes are characterized by the mass (m), the Gauss-Bonnet constant (α), the charge (Q), and the Born-Infeld constant (b). The constant α must be positive in order to have a well-behaved solution for all value of r. The metric does not diverge at $r = 0$; for the critical mass $m = m_c$ the conical singularity, which is characteristic of the Hoffmann-Born-Infeld solution, is removed (nevertheless, the origin is a curvature singularity).

We commented on the differences existing with respect to the Reissner-Nordström black holes and, from this analysis, we observed that, unlike the general relativity, the Lovelock-Born-Infeld theory admits charged black hole solutions with only one horizon. This is due to the fact that for a given charge Q, there exist values of mass that force the internal black hole radius to reach the origin.

Besides, there is another important distinction between the solutions of both theories, which is regarding to the thermodynamical properties: also in contrast to the Schwarzschild solution, the temperature of the black hole remains finite, in particular, it is feasible to show the temperature goes to zero when the horizon radius approximates to the origin, and there is not Hawking radiation. This fact leads to an infinite lifetime for Lovelock solutions because the short-distance effects render the small black holes stable. Moreover, it can be shown that black hole solutions in Lovelock-Born-Infeld theory do not satisfy the Bekenstein-Hawking area formula, presenting an additional contribution which is linear in the horizon radius. Indeed, the usual black hole thermodynamics is recovered in the $\alpha \to 0$ limit, consistent with the results of general relativity. The thermodynamical properties were discussed in detail in Ref. [29].

We discussed different limits of the solutions in terms of the coupling constant of Lanczos Lagrangian α and Born-Infeld Lagrangian b, and we proved that these limits correspond to the expected geometries. Hence, the solution we present here represents a geometry interpolating between the quoted Hoffmann metric for Einstein-Born-Infeld theory and the solution found by Wiltshire for the case of Lovelock-Maxwell field theory. Furthermore, we showed how other solutions studied in the literature are included as particular cases, representing a *BTZ phase* which arises on the curve $\Lambda\alpha = -\frac{3}{4}$ in the space of parameters. We discussed the similar features of this phase and the anti-de sitter black holes. This parallelism led us to show a bridge connecting the charged solution we studied with the black hole geometries discussed in the literature within the context of Lovelock-Chern-Simon gravity.

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[1] D. Lovelock, J. Math. Phys. (N.Y.) **12**, 498 (1971).
[2] M. Bañados, C. Teitelboim, and J. Zanelli, *Lovelock-Born-Infeld Theory of Gravity* in J. J. Giambiagi Festschrift, La Plata (edited by H. Falomir, R. Gamboa, P. Leal, and F. Schaposnik, (World Scientific, Singapore 1990).
[3] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B **163**, 12 (1985).
[4] E. Bergeshoeff, E. Sezgin, C. N. Pope, and P. K. Townsend, Phys. Lett. B **188**, 70 (1987).
[5] R. R. Metsaev, M. A. Rahmanov, and A. A. Tseytlin,Phys. Lett. B **193**, 207 (1987).
[6] B. Zwiebach, Phys. Lett. B **156**, 315 (1985).
[7] Y. Cai and C. Nuñez, Nucl. Phys. **B287**, 279 (1987).
[8] D. J. Gross and J. H. Sloan, Nucl. Phys. **B291**, 41 (1987).

[9] J. Crisostomo, S. del Campo, and J. Saavedra, hep-th/0311259.
[10] Rong-Gen Cai, Phys. Lett. B **582**, 237, (2004); Rong-Gen Cai,Phys. Rev. D **63**, 124018 (2001).
[11] F. Izaurrieta, E. Rodríguez, and P. Salgado, Phys. Lett. B **586**, 397 (2004); I. Navarro and J. Santiago, J. High Energy Phys. **04**, (2004) 62; Rong-Gen Cai and Kwang-Sup Soh, Phys. Rev. D **59**, 044013 (1999); J. P. Lemos, *A Profusion of Black Holes from Two to Ten Dimensions, Proceedings of the XVIIth Brazilian Meeting of Particle Physics and Fields*, edited by Adilson J. da Silva *et al.* (Sociedade Brasileira de Fisica, Serra Negra, Brazil, 1996); G. Allemandi, M. Francaviglia, and M. Raiteri, Classical Quantum Gravity **20**, 5130 (2003); A. Ilha, A. Kleber, and J. P. Lemos, J. Math. Phys. (N.Y.) **40**, 3509 (1999).
[12] B. Hoffmann, Phys. Rev. **47**, 877 (1935).
[13] G.W. Gibbons and D. A. Rasheed, Nucl. Phys. **B454**, 185 (1995).
[14] H. D'Olivera, Classical Quantum Gravity **11**, 1469 (1994).
[15] N. Deruelle, Phys. Rev. D **41**, 3696 (1990).
[16] C. Lanczos, Z. Phys. **73**, 147 (1932).
[17] C. Lanczos, Ann. Math. **39**, 842 (1938).
[18] F. Müller-Hoissen, Phys. Lett. **163B**, 106 (1985).
[19] J. Louko, J. Z. Simon, and S. N. Winters-Hilt, Phys. Rev. D **55**, 3525 (1997).
[20] M. Born and L. Infeld, Nature (London) **132**, 1004 (1933).
[21] M. Born and L. Infeld, Proc. R. Soc. London A **144**, 425 (1934).
[22] J. Plebanski, *Lectures On Non-Linear Electrodynamics* (Nordita Lecture Notes, Copenhagen, 1968).
[23] P. Franklin, *Methods of Advanced Calculus* (Mc.Graw-Hill, London, 1944).
[24] D. Wiltshire, Phys. Rev. D **38**, 2445 (1988).
[25] D. L. Wiltshire, Phys. Lett. **169B**, 36 (1986).
[26] M. Bañados, C. Teiltelboim, and J. Zanelli, Phys. Rev. Lett. **69**, 13 (1992).
[27] M. Bañados, M. Henneaux, C. Teiltelboim, and J. Zanelli, Phys. Rev. D **48**, 1506 (1993); G. Horowitz and D. Welch, Phys. Rev. Lett. **71**, 328 (1993); O. Coussaert, M. Henneauxm, and P. van Driel, Classical Quantum Gravity **12**, 2961 (1995); O. Coussaert and M. Henneaux, Phys. Rev. Lett. **72**, 183 (1994).
[28] M. Bañados, Mod. Phys. Lett. A **8**, 3673 (1993).M. Bañados, C. Teitelboim, and J. Zanelli, Phys. Rev. D **49**, 975 (1994); J. Crisostomo, R. Troncoso, and J. Zanelli, Phys. Rev. D **62**, 084013 (2000).
[29] R.C. Myers and J.Z. Simon, Phys. Rev. D **38**, 2434 (1988).