Gravitational wave solutions in string and M-theory AdS backgrounds

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In this paper, we present several gravitational wave solutions in $\mathrm{AdS}_5 \times S^5$ string backgrounds, as well as in $\mathrm{AdS}_7 \times S^4$ and $\mathrm{AdS}_4 \times S^7$ backgrounds in M theory, generalizing the results of Phys. Lett. B **594**, 368 (2004).. In each case, we present the general form of such solutions and give explicit examples, preserving certain amount of supersymmetry, by taking limits on known Bogomol'nyi-Prasad-Sommerfield D3 and M2, M5-brane solutions in pp-wave backgrounds. A key feature of our examples is the possibility of a wider variety of wave profiles, than in pure gravity and string/M-theory examples known earlier, coming from the presence of various p-form field strengths appearing in the gravitational wave structure.

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I. INTRODUCTION

Gravitational waves have long been a subject of research [1,2]. Such solutions in general relativity, also known as pp waves, are mostly discussed in the context of asymptotically flat Minkowski spaces. On the other hand, exact solutions of Einstein equations representing gravitational waves in nonasymptotically flat backgrounds have also been analyzed [3–5] over a long period of time. Classical solutions representing gravitational waves provide a geometrical framework to understand gravitational radiation and may thus have astrophysical implications as well.

In this paper, gravitational wave solutions are obtained in various anti-de Sitter backgrounds in string and M theory [6–9]. In this context, we extend the results of a previous paper by one of the authors in [9], where gravitational waves in $\mathrm{AdS}_3 \times S^3 \times R^4$ were discussed, and write down several general solutions in $\mathrm{AdS}_5 \times S^5$ string backgrounds, as well as $\mathrm{AdS}_4 \times S^7$ and $\mathrm{AdS}_7 \times S^4$ backgrounds in M theory. In each of these cases, we also give explicit examples by applying scaling limits [10,11] (also identified as the near-horizon geometry) on known D3 and M2, M5-brane solutions [12–15] in pp-wave backgrounds. In particular, two explicit examples of gravitational waves in $\mathrm{AdS}_5 \times S^5$ are obtained by taking limits on known (supersymmetric) D3 branes given in [13,14]. In the first of these cases, in Sec. II A, the gravitational wave profile (H) turns out to depend only on AdS_5 coordinates, implying that only such graviton polarizations may exist for the gravitational wave, a fact normally seen by transforming to the "Rosen" coordinates. In the second $\mathrm{AdS}_5 \times S^5$ example in Sec. II B, however, we have H depending on S^5 coordinates as well. Examples for the $\mathrm{AdS}_7 \times S^4$ gravitational wave in Sec. III and $\mathrm{AdS}_4 \times S^7$ gravitational wave in Sec. IV are, respectively, obtained from M5-brane solutions in [13,15,16] and M2-brane solution in [15]. As will be noticed, guided by the general form of the AdS examples constructed from D branes and M branes, we are able to write a general class of solutions (with certain constraints) in each case. Supersymmetry is, however, discussed only for the special cases.

Earlier work on string theory in pp-wave backgrounds [17] are summarized in [18]. Recent developments [19–21], including "Penrose limits" [22] and applications to four dimensional gauge theories [21], are reviewed in [23]. Some other aspects of D branes in pp-wave backgrounds are discussed in [24]. A crucial feature of our solutions, compared to those in [6,7], is the presence of p-form field strengths affecting the structure of the gravitational waves. In other words, the gravitational wave equations in earlier examples [6–8] are identical to those appearing in a higher dimensional AdS $\times$ S pure gravity theory with a cosmological constant. The role of string or M theory in those solutions is only to provide a consistent background configuration without affecting the wave nature. In our examples, extending the results of [9], however, the p-form fields appear in the wave equations and thus provide room for a wider variety of solutions.

II. $\mathrm{AdS}_5 \times S^5$ GRAVITATIONAL WAVES

A. AdS_5 wave profile

We begin our discussion by considering first an example of gravitational wave in the $\mathrm{AdS}_5 \times S^5$ background in string theory. The general form of the metric used in this case has a form

$$ds^2 = q\left\{\frac{du^2}{u^2} + \frac{1}{u^2}\left[2dx^+dx^- + H(u, x^+, x^i, x^a)dx^{+2} + \sum_{i=1}^{2} dx^{i2}\right] + d\Omega_5^2\right\}, \tag{1}$$

where the parameter q gives the radius of curvature for

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the AdS_5 and S^5 spaces. In our examples, coordinates x^+, x^-, u, and x^i's run over AdS directions whereas x^a's denote the directions along S^n. The metric in Eq. (1) is a generalization of the one for the AdS_4 case in general relativity [4,5] to $AdS_5 \times S^5$. We will use a similar form of the metric for M theory AdS_7 and AdS_4 examples as well.

For the general solution above, we have assumed that the wave profile H in the above metric may depend on all the AdS as well as S^5 coordinates. However, specific coordinate dependence of H also gives the graviton polarizations that are turned on. In the present example, we also take an NS-NS 3-form flux of the form

$$H^{(3)} = dx^+ \wedge \bigg[\bigg(\frac{-2\mu q A_i(x^i, x^a, x^+)}{u^2} \bigg) du \wedge dx^i + \bigg(\frac{2\mu q A_{ia}(x^i, x^a, x^+)}{u} \bigg) dx^i \wedge dx^a \bigg], \quad (2)$$

and an R-R 3-form flux

$$F^{(3)} = dx^+ \wedge \bigg[\bigg(\frac{-2\mu q B_i(x^i, x^a, x^+)}{u^2} \bigg) du \wedge dx^i + \bigg(\frac{2\mu q B_{ia}(x^i, x^a, x^+)}{u} \bigg) dx^i \wedge dx^a \bigg], \quad (3)$$

with μ being a parameter characterizing the gravitational wave. By setting $\mu = 0$ one goes to the $AdS_5 \times S^5$ background solution.

One also has the usual R-R 5-form flux necessary for constructing the background $AdS_5 \times S^5$ solution in type IIB string theory,

$$F^{(5)} = -\frac{4q^2}{u^5} dx^+ \wedge dx^- \wedge du \wedge dx^1 \wedge dx^2 - 4q^2 \sqrt{g} d\theta^1 \wedge d\theta^2 \wedge d\theta^3 \wedge d\theta^4 \wedge d\theta^5, \quad (4)$$

where θ^i's are five angular coordinates on S^5 and g is the determinant of the metric (g_{ab}) on this space. Later on, while presenting the explicit example, we will also give expressions for the metric g_{ab} in terms of angles $\theta_1, \ldots, \theta_5$.

Ricci curvature tensors for the AdS_5 components for the above metric have the form

$$\begin{aligned} R_{++} &= -\frac{H_{,uu}}{2} + \frac{3}{2}\frac{H_{,u}}{u} - \frac{4H}{u^2} - \frac{1}{2}H^i_{,i} - \frac{1}{2u^2\sqrt{g}}\partial_a\Big(\sqrt{g}\partial^a H\Big), \\ R_{+-} &= R_{uu} = -\frac{4}{u^2}, \qquad R_{ij} = -\frac{4\delta_{ij}}{u^2}, \end{aligned} \quad (5)$$

and Ricci tensor for the S^5 components satisfy

$$R_{ab} = 4g_{ab}. \quad (6)$$

Now, before starting to solve the full type IIB equations of motion (see, for example, [25]), we make a few comments regarding the form of the stress energy tensor for the ansatz presented above. In particular, we note that although the 3-form flux in Eqs. (2) and (3) is inhomogeneous in the sphere direction due to the x^a dependence in A_i, B_i and A_{ia}, B_{ia}, they are *null* fluxes. Also, from our above metric, one easily deduces that $g^{+\mu} = 0$ unless $\mu = -$. Thus, when one contracts over the flux indices, for example, in a term such as $F_{\mu\nu\delta}F^{\mu\nu\delta}$, the result is zero. This is because none of these fluxes have a dx^- leg. Since the stress energy tensor coming from the various 3-form fluxes generically has a form

$$T_{ab} = \alpha F_{acd}F_b^{cd} + \beta F_{efg}F^{efg}g_{ab}, \quad (7)$$

where α and β are constants, one immediately reads off that the second term will always be zero, for all (a, b) for any flux that has a dx^+ leg. Further, by the exact same reasoning as above, the first term must vanish unless $(a, b) = (++)$. One deduces quickly that only the T_{++} components are modified by the introduction of our null fluxes, relative to the original ($H = 0$) solution.

Furthermore, note the introduction of the wave term only affects the Ricci tensor of the background $AdS_5 \times S^5$ by introducing a nonzero R_{++}, as we have written explicitly above in Eqs. (5) and (6). It is for this reason that we have chosen our null fluxes in the form (2) and (3); their contribution to T_{++} provides a source for R_{++} via the Einstein equations. For a thorough discussion of these points, see the discussion preceding Eq. (2.10) in [26].

We now proceed further and analyze the field equations and their solutions. The type IIB string theory equations of motion [25] reduce to the following conditions for our ansatz [in Eqs. (1)–(4)]:

$$-\frac{H_{,uu}}{2} + \frac{3}{2}\frac{H_{,u}}{u} - \frac{1}{2}H^i_{,i} - \frac{1}{2u^2\sqrt{g}}\partial_a\Big(\sqrt{g}\partial^a H\Big) = 2\mu^2\bigg(\sum_i [A_i^2 + B_i^2] + \sum_a [A_{ia}A_i^a + B_{ia}B_i^a]\bigg), \quad (8)$$

$$\partial_i A_i = \partial_i A_{ia} = \partial_i B_i = \partial_i B_{ia} = 0, \quad (9)$$

$$\begin{aligned} \nabla^a A_{ia} - A_i + 4\epsilon_{ij}B_j &= 0, \\ \nabla^a B_{ia} - B_i - 4\epsilon_{ij}A_j &= 0. \end{aligned} \quad (10)$$

The Bianchi identity on $H^{(3)}$ and $F^{(3)}$ imply the following conditions:

$$\partial_a A_i = -A_{ia}, \qquad \partial_a B_i = -B_{ia}. \quad (11)$$

Now, several explicit solutions for the above set of conditions can be obtained. First, we write down the type of solution already known in the literature. They correspond to the choice $A_i = B_i = A_{ia} = B_{ia} = 0$ and originate from certain "brane waves" [6–8]. Then, if one chooses H to be dependent only on coordinate "u" and x^+, we get a solution

$$H_0 = f_o(x^+)u^4 + f_1(x^+), \tag{12}$$

with f_0, f_1 being arbitrary functions of x^+. One can also add to H_0 harmonic functions H_1 and H_2 satisfying $H,i^i = 0$, and $\nabla^a \partial_a H = 0$, respectively. Several other solutions can be obtained by taking products of functions of the type H_0, H_1, and H_2.

We now discuss the new solutions which emerge due to the presence of nontrivial NS-NS and R-R 3-form field strengths in the gravitational wave equation. To write down a solution explicitly, we make a choice for the metric on S^5 as

$$d\Omega_5^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\alpha^2 + \cos^2\theta\cos^2\alpha d\beta^2 + \cos^2\theta\sin^2\alpha d\gamma^2. \tag{13}$$

These angular coordinates are related to the six dimensional Cartesian coordinates as

$$\begin{aligned} y_1 &= r\sin\theta\cos\phi, \qquad y_2 = r\sin\theta\sin\phi, \\ y_3 &= r\cos\theta\cos\alpha\cos\beta, \qquad y_4 = r\cos\theta\cos\alpha\sin\beta, \\ y_5 &= r\cos\theta\sin\alpha\cos\gamma, \qquad y_6 = r\cos\theta\sin\alpha\sin\gamma. \end{aligned} \tag{14}$$

The solutions for A_i, A_{ia}, B_i, B_{ia} in Eqs. (1)–(4) then are

$$\begin{aligned} A_1 &= \sin\theta\cos\phi, \qquad A_2 = \sin\theta\sin\phi, \\ A_{1\theta} &= -\cos\theta\cos\phi, \qquad A_{1\phi} = \sin\theta\sin\phi, \\ A_{2\theta} &= -\cos\theta\sin\phi, \qquad A_{2\phi} = -\sin\theta\cos\phi, \end{aligned} \tag{15}$$

and

$$\begin{aligned} B_1 &= \sin\theta\sin\phi, \qquad B_2 = -\sin\theta\cos\phi, \\ B_{1\theta} &= -\cos\theta\sin\phi, \qquad B_{1\phi} = -\sin\theta\cos\phi, \\ B_{2\theta} &= \cos\theta\cos\phi, \qquad B_{2\phi} = -\sin\theta\sin\phi. \end{aligned} \tag{16}$$

Also, the 5-form field strength can also be written in terms of angular variables θ, ϕ, α, β, and γ for the above parametrization by using $\sqrt{g} = \sin\theta\cos^3\theta\sin\alpha\cos\alpha$.

For the choice of the metric in (13), very few nonzero components of the Christoffel connection exist, namely, $\Gamma^\theta_{\phi\phi}$, $\Gamma^\theta_{\alpha\alpha}$, $\Gamma^\theta_{\beta\beta}$, $\Gamma^\theta_{\gamma\gamma}$, $\Gamma^\alpha_{\beta\beta}$, $\Gamma^\alpha_{\gamma\gamma}$ and $\Gamma^\phi_{\theta\phi}$, $\Gamma^\alpha_{\theta\alpha}$, $\Gamma^\beta_{\theta\beta}$, $\Gamma^\gamma_{\theta\gamma}$, $\Gamma^\beta_{\alpha\beta}$, $\Gamma^\gamma_{\alpha\gamma}$. We skip their detailed form here. It can now be directly verified that A_i, A_{ia}, B_i, B_{ia} given in Eqs. (15) and (16) satisfy Eqs. (9)–(11). Moreover, we also have

$$\sum_i \left[A_i^2 + B_i^2 + \sum_a (A_{ia}A_i^a + B_{ia}B_i^a) \right] = 4, \tag{17}$$

leading to the following equation for the wave profile H [by using Eq. (8)]:

$$-\frac{H_{,uu}}{2} + \frac{3}{2}\frac{H_{,u}}{u} - \frac{1}{2}H^i_{,i} - \frac{1}{2u^2\sqrt{g}}\partial_a\left(\sqrt{g}\partial^a H\right) = 8\mu^2. \tag{18}$$

The condition (18) again has several solutions. When H is a function of u and x^+ only, one has

$$H_1 = 4\mu^2 u^2 + f_0(x^+), \tag{19}$$

with $f_0(x^+)$ being an arbitrary function of x^+ only.

One can, instead, take H to be a function of coordinates x^i and x^+ only, leading to

$$H_2 = -4\mu^2 \sum_{i=1}^{2} x^{i2} + \sum_{i=1}^{2} f_i(x^+)x^i + g(x^+), \tag{20}$$

with f_1, f_2, and g being functions of x^+ only. One can also take linear combinations of solutions of the types in Eqs. (19) and (20) with coefficients a_0 and a_1, such that $a_0 + a_1 = 1$.

We can further generalize the solutions by multiplying A_i, A_{ia}, B_i, B_{ia} in Eqs. (15) and (16) by an arbitrary function of x^+:$F(x^+)$, while simultaneously multiplying H by another function $G(x^+)$ such that $G = F^2$.

We now show that for a special case in Eq. (20), namely,

$$H \equiv \hat{H} = -4\mu^2((x^1)^2 + (x^2)^2) \tag{21}$$

(obtained by setting $f_{1,2}$ and g to zero in H_2), the gravitational wave solution follows from a (singular) scaling limit of a supersymmetric D3-brane solution in a pp-wave background [13]. Similar limits have been applied to obtain other examples of gravitational wave solutions in AdS backgrounds previously [6–9].

The (localized) D3-brane solution in a pp-wave background, giving the above gravitational wave solution, in a scaling limit $r \to 0$, with a wave profile $\hat{H}$, is written as [13]

$$\begin{aligned} ds^2 &= f^{-1/2}[2dy^+dy^- - 4\mu^2[\tilde{y}_2^2 + \tilde{y}_4^2](dy^+)^2 + d\tilde{y}_2^2 \\ &\quad + d\tilde{y}_4^2] + f^{1/2}(dr^2 + r^2 d\Omega_5^2), \\ F_{+32} &= F_{+41} = 2\mu, \qquad B_{+1} = 2\mu\tilde{y}_2, \\ B_{+3} &= 2\mu\tilde{y}_4, \qquad F_{mnlpq} = \epsilon_{mnlpqs}\partial_s f, \\ f &= 1 + \frac{q^2}{r^4}, \qquad q^2 = c_3 N g_s l_s^4. \end{aligned} \tag{22}$$

This is a D3-brane solution in a pp-wave background specified by the parameter μ. When one sets the D3-brane charge to zero, one gets the background metric of the pp wave, with NS-NS and R-R 3-form flux having components H_{+12}, H_{+34} and F_{+23}, F_{+14}, respectively. In the above D3-brane solution, directions $\tilde{y}^2$, $\tilde{y}^4$ are longitudinal coordinates of the brane, whereas $\tilde{y}^1, \tilde{y}^3, \tilde{y}^5, \ldots, \tilde{y}^8$ are transverse to the brane.

To obtain the gravitational wave metric in $AdS_5 \times S^5$ background as given in Eq. (1), from the D3-brane solution above, we take $r \to 0$ limit in the Green function (f) in Eq. (22). One then obtains the metric in Eq. (1) with the profile $H = \hat{H}$ given as in Eq. (21)], when one also identifies

$$\begin{aligned} &x^1 \equiv \tilde{y}^2, \qquad x^2 \equiv \tilde{y}^4, \qquad y^1 \equiv \tilde{y}^1, \\ &y^2 \equiv \tilde{y}^3, \qquad y^{3\cdots 6} \equiv \tilde{y}^{5\cdots 8}, \end{aligned} \tag{23}$$

and defines $r = \frac{q}{u}$. For the D3-brane solution in Eq. (22), the radial coordinate along transverse direction is $r^2 = \tilde{y}^{12} + \tilde{y}^{32} + \sum_{i=5}^{8} \tilde{y}^{i2}$. In the new variables that we are using, one gets $r^2 = \sum_{i=1}^{6} y^{i2}$. NS-NS and R-R 3-form field strengths in Eqs. (2) and (3) [with A's and B's as in Eqs. (15) and (16)] also arise from the ones in Eq. (22) by redefining the coordinates as in Eq. (23) and making use of the angular coordinates in Eq. (14). By such change of variables, one obtains the expressions in Eqs. (2) and (3) with solutions for A_i, A_{ia}, B_i, B_{ia} as in Eqs. (15) and (16).

We have therefore obtained a general class of $\mathrm{AdS}_5 \times S^5$ gravitational wave solution in type IIB string theory and also presented an explicit example characterized by functions A_i, A_{ia}, B_i, B_{ia} in Eqs. (15) and (16), and the wave profile $\hat{H}$ in Eq. (21). We have also shown how our gravitational wave arises from a D3 brane in a pp-wave background. This connection with D3 brane has been presented for the wave profile $\hat{H}$. It will be interesting to see if the wave-profile H_1 can also arise from a D brane in a similar way.

We now discuss the supersymmetry property of the solutions described above. To show that the above gravitational wave, with H $(= \hat{H})$ as in Eq. (21), is supersymmetric, one also notes that the original D3 brane given in Eq. (22) preserves a certain amount of supersymmetry as well.[1] Therefore the limiting solution, appearing as gravitational wave above, is expected to be supersymmetric as well.

To elaborate more, the Killing spinors [13] for the D3-brane solution [13] are the ones which satisfy the following projections: (i) the D3-brane supersymmetry condition identical to the one in flat space, relating $\epsilon_\pm$ with $\epsilon_\mp$, (ii) the usual $\Gamma^{\hat{+}}$ projection of the gravitational wave solution implying either a left-moving or a right-moving wave, (iii) an additional condition due to the presence of NS-NS and R-R 3-form flux. For the relevant D3 brane discussed above, these conditions are mentioned in Eqs. (42), (38), and (43) of [13]. The important point to note is that these conditions are independent of the function f, namely, the Green function in the transverse space. As a result, the limiting procedure that we described for getting the $\mathrm{AdS}_5 \times S^5$ gravitational wave gives the identical Killing spinors as the D3 brane case. In the present case, however, one also expects the presence of additional Killing spinors since, as is known for flat D3 branes, the supersymmetry enhances in the near-horizon limit from one half to the maximal supersymmetry. Finding the exact amount of supersymmetry, by writing down all the Killing spinors explicitly, is important. We, however, do not go into them right now and simply end the section by saying that our gravitational wave solution is supersymmetric for the wave profile $\hat{H}$ [with nontrivial NS-NS and R-R 3-form flux as given in Eqs. (15) and (16)]. The case of other wave profile, namely H_1, is also expected to be supersymmetric, even though the D3 brane connection is not apparent for this example.

[1] In solution (22) above we have corrected a minus sign in one of the R-R 3-form component in [13].

B. Wave profile with S^5 dependence

We now give an example of the gravitational wave where the wave profile H depends on S^5 coordinates as well. An explicit example of this type originates from a D3-brane solution in [14]. The general form of the metric for this solution is same as in Eq. (1). The 5-form field is also identical to the one in Eq. (4). However, one now has only an NS-NS 3-form flux of the form:

$$\begin{aligned} H^{(3)} = dx^+ \wedge \bigg[&\bigg(\frac{-\mu q^2 A_a(x^i, x^a, x^+)}{u^3} \bigg) du \wedge dx^a \\ &+ \bigg(\frac{\mu q^2 A_{ab}(x^i, x^a, x^+)}{u^2} \bigg) dx^a \wedge dx^b \bigg]. \end{aligned} \tag{24}$$

One can also generate a combination of NS-NS and R-R 3-form field strengths by using S duality symmetry of the IIB string theory. We, however, do not go into this aspect here. One also notices that the general form of the NS-NS 3-form field strength above is identical to the one used in [9] for the $\mathrm{AdS}_3 \times S^3$ example.

In the present case, the gravitational wave is therefore characterized by the metric, R-R 5-form flux, and NS-NS 3-form flux as in Eqs. (1), (4), and (24), respectively. The equation for the wave profile H is now given as

$$\begin{aligned} -\frac{H_{,uu}}{2} + \frac{3}{2}\frac{H_{,u}}{u} - \frac{1}{2}H^{i}_{,i} - \frac{1}{2u^2\sqrt{g}}\partial_a\Big(\sqrt{g}\partial^a H\Big) \\ = \frac{\mu^2 q^2}{2u^4}\bigg[\sum_a A_a A^a + \frac{1}{2}\sum_{a,b}(A_{ab}A^{ab})\bigg]. \end{aligned} \tag{25}$$

One also has additional conditions on quantities A_a and A_{ab} [9]:

$$\nabla^a A_a = 0, \qquad \nabla^b A_{ab} = 4A_a, \tag{26}$$

coming from equations of motion. The Bianchi identity implies [9]

$$\partial_{[a}A_{b]} = A_{ab}. \tag{27}$$

Although the condition (27) coming from the Bianchi identity for this example is identical to the one for the $\mathrm{AdS}_3 \times S^3 \times R^4$ case in [9] due to the identical form of the NS-NS 3-form in the two cases, one of the equations of motion in (26) has a different factor than in [9] due to the presence of additional coordinates $x^{1,2}$ in AdS_5 with respect to the one in the AdS_3 example in [9]. An explicit solution for Eqs. (26) and (27) is given as

$$A_{\psi} = 2\cos^2\theta\cos^2\phi, \qquad A_{\omega} = 2\cos^2\theta\sin^2\phi, \tag{28}$$

and

$$\begin{aligned} A_{\theta\psi} &= -\sin 2\theta\cos^2\phi, \qquad A_{\theta\omega} = -\sin 2\theta\sin^2\phi, \\ -A_{\phi\psi} &= A_{\phi\omega} = \cos^2\theta\sin 2\phi, \end{aligned} \tag{29}$$

for the choice of S^5 metric:

$$\begin{aligned} d\Omega_5^2 = d\theta^2 &+ \cos^2\theta d\phi^2 + \cos^2\theta\cos^2\phi d\psi^2 \\ &+ \cos^2\theta\sin^2\phi d\omega^2 + \sin^2\theta d\gamma^2. \end{aligned} \tag{30}$$

Using the above expressions for A_a's and A_{ab}'s, the wave profile H in the present example can be shown to satisfy

$$-\frac{H_{,uu}}{2} + \frac{3}{2}\frac{H_{,u}}{u} - \frac{1}{2}H^i_{,i} - \frac{1}{2u^2\sqrt{g}}\partial_a\left(\sqrt{g}\partial^a H\right) = \frac{4\mu^2 q^2}{u^4}, \tag{31}$$

with a solution (independent of x^i's) given by

$$H = -\frac{\mu^2 q^2}{u^2} cos^2\theta. \tag{32}$$

Once again, the solution given in Eqs. (28), (29), and (32) can be generalized further by multiplying A's and H with functions $F(x^+)$ and $G(x^+)$, respectively, satisfying $G = F^2$. We have therefore again presented an explicit example of a gravitational wave in $\mathrm{AdS}_5 \times S^5$ background.

We now show the connection of our explicit solution given in Eqs. (28), (29), and (32) with a supersymmetric D3-brane solution [14]. To show this connection, we write down the relevant D3-brane solution in a pp-wave background (which is obtained by taking a Penrose limit [22] on an $\mathrm{AdS}_3 \times S^3 \times R^4$ background of string theory). The D-brane solution is

$$\begin{aligned} ds^2 = f^{-1/2}\Bigg[2dx^+dx^- &- \mu^2\sum_{i=1}^4 x_i^2(dx^+)^2 + (dx_5)^2 \\ &+ (dx_6)^2\Bigg] + f^{1/2}\sum_{a=1,\ldots,4,7,8}(dx_a)^2, \end{aligned} \tag{33}$$

$$H_{+12} = H_{+34} = 2\mu, \qquad F_{+-56a} = \partial_a f^{-1}, \qquad e^{2\phi} = 1,$$

with f being the Green function in six dimensional transverse space with coordinates $x^{1,\ldots,4}, x^7, x^8$. We write this Green function as $f = (1 + \frac{q^2}{r^4})$. We also mention that among the above coordinates, $x^{1\cdots 4}$ are also the pp-wave directions and x^+, x^-, x^5, x^6 are longitudinal directions of the brane.

To arrive at the metric and NS-NS 3-form of Eqs. (1) and (24) [with A_a's, A_{ab}'s, and H as in Eqs. (28), (29), and (32)] from the D3-brane solution in (33), we now make the following coordinate transformations from the Cartesian to radial and angular variables:

$$\begin{aligned} &x_1 = r\cos\theta\cos\phi\cos\psi, \qquad x_2 = r\cos\theta\cos\phi\sin\psi, \\ &x_3 = r\cos\theta\sin\phi\cos\omega, \qquad x_4 = r\cos\theta\sin\phi\sin\omega, \\ &x_7 = r\sin\theta\cos\gamma, \qquad x_8 = r\sin\theta\sin\gamma. \end{aligned} \tag{34}$$

Then, taking the scaling limit $r \to 0$, and defining $r = \frac{q}{u}$, one gets the result given in Eqs. (28), (29), and (32).

We have therefore shown the connection of our gravitational wave solution in this subsection with a D3 brane in a pp-wave background. The background pp wave itself follows from a Penrose limit on $\mathrm{AdS}_3 \times S^3 \times R^4$ geometry with an appropriate 3-form NS-NS or R-R flux. Just as in Sec. II A, we can also obtain other solutions for the wave profile H in Eq. (31). It will again be of interest to find out which D branes lead to a gravitational wave of this type in a singular limit $r \to 0$.

Because of arguments similar to the ones in Sec. II A, namely, that the supersymmetry projections are independent of the Green function f, we expect our gravitational wave solution to be supersymmetric. In particular, since the D3-brane solution of Eq. (33) preserves $1/8$ supersymmetry, as discussed in Sec. (3.1) of [14], we expect the gravitational wave solution to preserve at least this much supersymmetry as well. Possible enhancement of supersymmetry can be analyzed by solving the Killing spinor equations. However, at this point we move from type IIB supergravity to study the gravitational waves in eleven dimensional M theory.

III. $\mathrm{AdS}_7 \times S^4$ GRAVITATIONAL WAVE SOLUTION IN M THEORY

A. AdS_7 wave

We now write down the gravitational wave solution in $\mathrm{AdS}_7 \times S^4$ backgrounds. These background configurations appear in M theory in a near-horizon geometry of M5-brane solutions solving the 11-dimensional supergravity equations of motion. The metric ansatz for the gravitational wave solution that we obtain has the following form:

$$\begin{aligned} ds^2 = 4q\Bigg\{\frac{du^2}{u^2} + \frac{1}{u^2}\Bigg[2dx^+dx^- &+ H(u, x^+, x^i, x^a)dx^{+2} \\ &+ \sum_{i=1}^4 dx^{i2}\Bigg] + \frac{1}{4}d\Omega_4^2\Bigg\}. \end{aligned} \tag{35}$$

For $H = 0$ this metric reduces to that of $\mathrm{AdS}_7 \times S^4$. This theory, in addition, contains a 3-form field (or the corresponding 4-form flux). Note that we parametrize the space-time using coordinates analogous to the earlier section. In our case the 4-form flux has the following form:

$$F^{(4)} = \frac{16\mu q^{3/2}}{u^3} A_{ij}(u, x^+, x^i, x^a) dx^+ \wedge du \wedge dx^i \wedge dx^j + \frac{8\mu q^{3/2}}{u^2} A_{ija} dx^+ \wedge dx^i \wedge dx^j \wedge dx^a - 3q^{3/2}\sqrt{g} d\theta^1 \wedge d\theta^2 \wedge d\theta^3 \wedge d\theta^4. \quad (36)$$

Nonzero Ricci tensor components along AdS_7 directions, for the above metric, have a form:

$$R_{++} = -\frac{H_{,uu}}{2} + \frac{5}{2}\frac{H_{,u}}{u} - \frac{6H}{u^2} - \frac{1}{2}H^i_{,i} - \frac{2}{u^2\sqrt{g}}\partial_a\left(\sqrt{g}\partial^a H\right), \quad (37)$$

$$R_{+-} = R_{uu} = -\frac{6}{u^2}, \qquad R_{ij} = -\frac{6\delta_{ij}}{u^2}.$$

Ricci tensor components along S^4 directions are

$$R_{ab} = 3g_{ab}. \quad (38)$$

We now derive constraints on A_{ij}, A_{ija}, and H from the 11-dimensional supergravity equations of motion ($G_{\mu\nu}$ represents the 11-dimensional metric, as opposed to g_{ab} which denotes the metric on the S_4),

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = \frac{1}{12}\left(F_{\mu\alpha\beta\gamma}F_\nu^{\alpha\beta\gamma} - \frac{1}{8}G_{\mu\nu}F_{\alpha\beta\gamma\delta}F^{\alpha\beta\gamma\delta}\right), \quad (39)$$

and

$$\partial_\mu\left(\sqrt{-G}F^{\mu\nu\rho\sigma}\right) + \frac{1}{1152}\epsilon^{\nu\rho\sigma\alpha_1\alpha_2\alpha_3\alpha_4\beta_1\beta_2\beta_3\beta_4}F_{\alpha_1\alpha_2\alpha_3\alpha_4}F_{\beta_1\beta_2\beta_3\beta_4} = 0, \quad (40)$$

where $F_{\mu\nu\rho\sigma}$, as mentioned earlier, is a 4-form field strength in the 11-dimensional supergravity theory.

By using our ansatz for the metric and $F_{\mu\nu\rho\sigma}$ in Eqs. (35) and (36) we have, for example,

$$R_{+-} - \frac{1}{2}g_{+-}R = -\frac{1}{96}g_{+-}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} = -\frac{9}{u^2}. \quad (41)$$

For all other components of $R_{\mu\nu}$, except R_{++}, one gets identical field equations. For the $(++)$ component, on the other hand, we get

$$-\frac{H_{,uu}}{2} + \frac{5}{2}\frac{H_{,u}}{u} - \frac{1}{2}H^i_{,i} - \frac{2}{u^2\sqrt{g}}\partial_a\left(\sqrt{g}\partial^a H\right) = \mu^2\left[\sum_a A_{ij}^2 + \sum_{ija}(A_{ija}A^a_{ij})\right]. \quad (42)$$

The 4-form field equation (40) implies the following conditions for quantities A_{ij} and A_{ija}:

$$\partial_j A_{ij} = 0, \qquad \partial_j A_{ija} = 0, \quad (43)$$

$$-\frac{1}{\sqrt{g}}\partial_a\left(\sqrt{g}A^a_{ij}\right) + A_{ij} + \frac{3}{2}\epsilon^{ijkl}A_{kl} = 0. \quad (44)$$

From the Bianchi identity, one also has

$$A_{ija} = -\partial_a A_{ij}. \quad (45)$$

We now find that for the S^4 metric

$$d\Omega_4^2 = d\theta^2 + \sin^2\theta[d\phi^2 + \cos^2\phi d\psi^2 + \sin^2\phi d\omega^2], \quad (46)$$

A_{ij} and A_{ija},

$$A_{23} = A_{14} = -\cos\theta, \qquad A_{23\theta} = A_{14\theta} = -\sin\theta, \quad (47)$$

and

$$H = -\mu^2\left(\sum_{i=1}^4 x_i^2\right) \quad (48)$$

provide an explicit solution for Eqs. (42)–(45). Other examples can also be obtained, as in previous sections, by multiplying A's and H by functions of x^+, as well as by solving Eq. (42) directly.

We now show that the gravitational wave solution, with its explicit form as in Eqs. (47) and (48), is obtained from a supersymmetric M5-brane solution in a pp-wave background [13,16] by applying a limit, taking $r \to 0$. The M5-brane solution is given as

$$ds^2 = f^{-1/3}\left[2dx^+dx^- - \mu^2\sum_{i=2,3,4,11}^{4}\tilde{x}_i^2(dx^+)^2 + \sum_{i=2,3,4,11}(dx_i)^2\right] + f^{2/3}(dr^2 + r^2 d\Omega_4^2), \quad (49)$$

$$F^{(4)} = 2\mu dx^+ \wedge (d\tilde{x}_1 \wedge d\tilde{x}_3 \wedge d\tilde{x}_4 + d\tilde{x}_1 \wedge d\tilde{x}_2 \wedge d\tilde{x}_{11}) + \epsilon_{mnlpq}\partial_q f d\tilde{x}^m \wedge d\tilde{x}^n \wedge d\tilde{x}^l \wedge d\tilde{x}^p,$$

$$f = \left(1 + \frac{q^{3/2}}{r^3}\right),$$

where we have kept the longitudinal coordinate indices in a manner such that its origin as a dimensionally "oxidized" D4 brane [13] becomes more clear. In our notation here, the five transverse coordinates, denoted by $\tilde{x}^m$, representing r and $d\Omega_4$, are given as $\tilde{x}^1, y^{1,\ldots,4}$. Now, to obtain the gravitational wave solution in Eqs. (35), (36), (47), and (48), we define coordinates as

$$\tilde{x}_1 = r\cos\theta, \qquad y_1 = r\sin\theta\cos\phi\cos\psi,$$
$$y_2 = r\sin\theta\cos\phi\sin\psi, \qquad y_3 = r\sin\theta\sin\phi\cos\omega, \quad (50)$$
$$y_4 = r\sin\theta\sin\phi\sin\omega,$$

and apply the limit on the transverse radius $r \to 0$, while redefining $r = 4\frac{q^{3/2}}{u^2}$. Identifications for the longitudinal coordinates used in (47) and (48) are

$$x_1 \equiv \tilde{x}_2, \qquad x_2 \equiv \tilde{x}_3, \qquad x_3 \equiv \tilde{x}_4, \qquad x_4 \equiv \tilde{x}_{11}. \quad (51)$$

Supersymmetry of the gravitational wave is once again expected, due to the above connection with a 3/16 supersymmetric M5 brane. The supersymmetry of both M5 [16] and the related D4 [13] branes have been obtained explicitly. The projection conditions again turn out to be independent of f, implying once more that the final solution is supersymmetric as well.

B. Wave profiles dependent on the S^4

We now present another class of $\mathrm{AdS}_7 \times S^4$ gravitational wave solution. The new solutions are obtained by considering variations of the flux ansatz (36), allowing for the wave profile to be dependent on the transverse S^4. To this end, consider the following ansatz:

$$\begin{aligned} F^{(4)} = {}& \frac{32\mu q^3}{u^5} B_{ai}(u, x^+, x^i, x^a) dx^+ \wedge du \wedge dx^a \wedge dx^i \\ & + \frac{8\mu q^3}{u^4} B_{abi} dx^+ \wedge dx^a \wedge dx^b \wedge dx^i \\ & - 3q^{3/2}\sqrt{g} d\theta^1 \wedge d\theta^2 \wedge d\theta^3 \wedge d\theta^4, \end{aligned} \quad (52)$$

with i running over $1, \ldots, 4$ and x^a's are the four angular coordinates. As before, we can derive constraints on the undetermined functions. From the Bianchi identity

$$B_{[ai,b]} = B_{abi}, \quad (53)$$

while the equation of motion $d * F = -F \wedge F$ yields

$$\partial_j B_{ja} = 0, \qquad \partial_j B_{abj} = 0, \quad (54)$$

$$B_i^a = \frac{1}{6\sqrt{g}} \partial_b \Big(\sqrt{g} B_i^{ba} \Big). \quad (55)$$

Finally, we note that the R_{++} equation of motion implies that

$$\begin{aligned} & -\frac{H_{,uu}}{2} + \frac{5}{2}\frac{H_{,u}}{u} - \frac{1}{2} H^i_{,i} - \frac{2}{u^2\sqrt{g}} \partial_a \Big(\sqrt{g} \partial^a H \Big) \\ & \quad = \frac{\mu^2 q^3}{u^6} \left[32 \sum_{ai} B_{ai} B_i^a + 4 \sum_{abi} (B_{abi} B_i^{ab}) \right]. \end{aligned} \quad (56)$$

Writing the S^4 in coordinates such that the metric is once again of the form (46), one has the solution

$$H = -\frac{4q^3\mu^2}{u^4} \sin^2\theta, \quad (57)$$

with flux determined by

$$B_{\psi 1} = -\sin^2\theta \cos^2\phi, \qquad B_{\omega 1} = -\sin^2\theta \sin^2\phi, \quad (58)$$

and

$$\begin{aligned} B_{\theta\psi 1} &= \sin 2\theta \cos^2\phi, \qquad & B_{\phi\psi 1} &= -\sin^2\theta \sin 2\phi, \\ B_{\theta\omega 1} &= \sin 2\theta \sin^2\phi, \qquad & B_{\phi\omega 1} &= \sin^2\theta \sin 2\phi. \end{aligned} \quad (59)$$

This gravitational wave can also be found as the near-horizon limit of a supergravity solution describing a stack of M5 branes in the plane wave background with 20 supersymmetries [15]. The resulting configuration has eight supersymmetries and is given by

$$\begin{aligned} ds^2 &= f^{-1/3} \left[2dx^+ dx^- - \frac{\mu^2}{4} \sum_{m=5}^{8} x_m^2 (dx^+)^2 + \sum_{i=1}^{4} (dx_i)^2 \right] \\ & \quad + f^{2/3} \sum_{m=5}^{9} (dx^m)^2, \\ F^{(4)} &= \mu dx^+ \wedge (dx^5 \wedge dx^6 \wedge dx^1 + dx^7 \wedge dx^8 \wedge dx^1) \\ & \quad + \frac{\epsilon_{mnlpq}}{4!} \partial_q f dx^m \wedge dx^n \wedge dx^l \wedge dx^p, \\ f &= \left(1 + \frac{q^{3/2}}{r^3} \right). \end{aligned} \quad (60)$$

Upon parametrizing the transverse coordinates $(x^5, \ldots, x^8, x^9 \equiv y^1, \ldots, y^4, \tilde{x}_1)$ exactly as above in Eq. (50), defining the coordinate u as before, and taking the appropriate near-horizon limit, one recovers the gravitational wave solution outlined above.

Supersymmetry of the solution can be discussed along the lines of other examples in Sec. II. We now go over to the gravitational wave solution in $\mathrm{AdS}_4 \times S^7$ background.

IV. $\mathrm{AdS}_4 \times S^7$ SOLUTION

Gravitational waves in AdS_4 backgrounds are of particular interest, due to their connection with the physics in four dimensions. In pure gravity theory, the gravitational waves in such backgrounds require the presence of cosmological constant term. In 11 dimensional M theory that we are considering, one does not have any such cosmological constant term and the background AdS_4 is accompanied by an S^7 in order to compensate for the opposite Ricci curvature terms. Phenomenological consequences of such a gravitational wave in $\mathrm{AdS}_4 \times S^7$ background will also be of interest to examine along the lines of [3].

We now give an example of a gravitational wave in $\mathrm{AdS}_4 \times S^7$ background. Later on in this section, we also show the connection of our solution with certain supersymmetric "localized" M2 branes of [15] in the same way as was done above for other branes. The metric is now written as

$$\begin{aligned} ds^2 = \frac{q}{4} \bigg\{ & \frac{du^2}{u^2} + \frac{1}{u^2} \Big[2dx^+ dx^- + H(u, x^+, x, x^a) dx^{+2} \\ & + dx^2 \Big] + 4d\Omega_7^2 \bigg\}. \end{aligned} \quad (61)$$

The 4-form flux is of the form

$$F^{(4)} = \frac{3q^{3/2}}{8u^4} dx^+ \wedge dx^- \wedge dx \wedge du - \frac{\mu q^{9/4}}{4\sqrt{2}} \frac{A_{ab}}{u^{5/2}} dx^+ \wedge du \wedge dx^a \wedge dx^b + \frac{\mu q^{9/4}}{2\sqrt{2}} \frac{A_{abc}}{u^{3/2}} dx^+ \wedge dx^a \wedge dx^b \wedge dx^c, \quad (62)$$

where powers of q are chosen appropriately to have q-independent solution for A_{ab}'s, etc., below. For the Ricci curvature components we now have

$$R_{++} = -\frac{H_{,uu}}{2} + \frac{H_{,u}}{u} - \frac{3H}{u^2} - \frac{1}{2} H^i_{,i} - \frac{1}{8u^2\sqrt{g}} \partial_a\Big(\sqrt{g}\partial^a H\Big), \quad (63)$$

$$R_{+-} = R_{uu} = R_{xx} = -\frac{3}{u^2}, \qquad R_{ab} = 6\delta_{ab},$$

with a, b denoting the S^7 coordinates. Equations of motion then simplify to

$$\frac{1}{\sqrt{g}} \partial_c\Big(\sqrt{g}A^{abc}\Big) + 5A^{ab} = 0, \qquad \frac{1}{\sqrt{g}} \partial_b\Big(\sqrt{g}A^{ab}\Big) = 0, \quad (64)$$

and

$$-\frac{H_{,uu}}{2} + \frac{H_{,u}}{u} - \frac{1}{2} H^i_{,i} - \frac{1}{8u^2\sqrt{g}} \partial_a\Big(\sqrt{g}\partial^a H\Big) = \frac{1}{32} \frac{\mu^2 q^{3/2}}{u^3} \Bigg[\sum_{a,b} A_{ab}A^{ab} + \frac{1}{3} \sum_{a,b,c} (A_{abc}A^{abc})\Bigg]. \quad (65)$$

The Bianchi identity gives

$$A_{abc} = \partial_{[a}A_{bc]}. \quad (66)$$

One can also write an explicit solution for all the conditions, namely, Eqs. (64)–(66). For this we write down a metric on S^7 as

$$ds^2 = d\theta^2 + \cos^2\theta[d\phi^2 + \cos^2\phi d\psi^2 + \sin^2\phi d\omega^2] + \sin^2\theta[d\gamma^2 + \sin^2\gamma d\eta^2 + \sin^2\gamma \sin^2\eta d\beta^2]. \quad (67)$$

Our solutions for A_{ab} and A_{abc} are then

$$\begin{aligned} &A_{\psi\theta} = \cos\theta\cos^2\phi\cos\gamma, \quad A_{\omega\theta} = \cos\theta\sin^2\phi\cos\gamma, \quad A_{\psi\gamma} = -\cos^2\theta\cos^2\phi\sin\theta\sin\gamma, \\ &A_{\omega\gamma} = -\cos^2\theta\sin^2\phi\sin\theta\sin\gamma, \quad A_{\phi\psi} = -\cos^2\theta\sin\theta\sin\phi\cos\phi\cos\gamma, \quad A_{\phi\omega} = \cos^2\theta\sin\theta\sin\phi\cos\phi\cos\gamma, \end{aligned} \quad (68)$$

and

$$\begin{aligned} &A_{\theta\psi\gamma} = \sin^2\theta\cos\theta\cos^2\phi\sin\gamma, \quad A_{\theta\omega\gamma} = \sin^2\theta\cos\theta\sin^2\phi\sin\gamma, \quad A_{\phi\psi\theta} = -\cos^3\theta\sin\phi\cos\phi\cos\gamma, \\ &A_{\phi\psi\gamma} = \cos^2\theta\sin\theta\sin\phi\cos\phi\sin\gamma, \quad A_{\phi\omega\theta} = \cos^3\theta\sin\phi\cos\phi\cos\gamma, \quad A_{\phi\omega\gamma} = -\cos^2\theta\sin\theta\sin\phi\cos\phi\sin\gamma. \end{aligned} \quad (69)$$

For the wave profile (H) we have

$$H = -\frac{\mu^2 q^{3/2}}{8u}\cos^2\theta. \quad (70)$$

We now show that the $\mathrm{AdS}_4 \times S^7$ gravitational wave solution, characterized by functions A_{ab}, A_{abc}, and H in Eqs. (68)–(70), is obtained from an M2-brane solution [15] in a pp-wave background in a near-horizon geometry. The M2-brane solution is given as

$$\begin{aligned} ds^2 &= f^{-2/3}[2dx^+dx^- - H(dx^+)^2 + (dx)^2] + f^{1/3}\Bigg(\sum_{i=1}^{8} dx_i^2\Bigg), \\ F^{(4)} &= dx^+ \wedge (\mu_1 dx_1 \wedge dx_2 + \mu_2 dx_3 \wedge dx_4 + \mu_3 dx_5 \wedge dx_6) \wedge dx_8, \\ f &= \Bigg(1 + \frac{q^3}{r^6}\Bigg), \end{aligned} \quad (71)$$

with H in Eqs. (71) being

$$H = -\frac{\mu_1^2}{4}(x_1^2 + x_2^2) - \frac{\mu_2^2}{4}(x_3^2 + x_4^2) - \frac{\mu_3^2}{4}(x_5^2 + x_6^2). \quad (72)$$

The general solution above, for the M2 brane, has three independent parameters μ_1, μ_2, and μ_3. However, to obtain the gravitational wave solution above, in Eqs. (68)–(70) we have set the parameter μ_3 to zero. Moreover, we have also set $\mu_1 = \mu_2$. The other supersymmetric solution $\mu_1 = -\mu_2$ in [15] is similar to the one we have written in (68)–(70), the only change being that ω components in A_{ab} and A_{abc} are changed by a minus sign.

Now, to obtain the gravitational wave solution obtained above, we make the following coordinate transformations:

$$\begin{aligned} x_1 &= r\cos\theta\cos\phi\cos\psi, & x_2 &= r\cos\theta\cos\phi\sin\psi,\\ x_3 &= r\cos\theta\sin\phi\cos\omega, & x_4 &= r\cos\theta\sin\phi\sin\omega,\\ x_5 &= r\sin\theta\sin\gamma\cos\eta, & x_6 &= r\sin\theta\sin\gamma\sin\eta\cos\beta,\\ x_7 &= r\sin\theta\sin\gamma\sin\eta\sin\beta, & x_8 &= r\sin\theta\cos\gamma, \end{aligned} \tag{73}$$

and take the limit $r \to 0$ while also defining $r = \frac{q^{4/3}}{\sqrt{2}} u^{-1/2}$. We then obtain the solution in Eqs. (68)–(70). The supersymmetry of the gravitational wave solution is once again expected, following similar arguments as in previous sections.

V. CONCLUSION

In this paper we have obtained several examples of gravitational wave solutions in string theory and M theory. A new feature of our solution is the presence of new p-form fluxes that are present. These fluxes also dictate the form of the wave profiles that one obtains by solving the wave equations. We have also presented many examples. The general structure of these examples has been dictated by certain D branes and M2, M5 branes in pp-wave backgrounds. It should certainly be possible to extend these solutions further and obtain the gravitational waves in AdS backgrounds in a large number of other possible cases, coming from various other brane solutions. In this context, D branes of maximally supersymmetric pp-wave backgrounds will be particularly interesting to study, as the corresponding gravitational wave may provide interpolation between $\mathrm{AdS}_5 \times S^5$ geometry and a pp wave with maximal supersymmetry, by using procedures outlined in previous sections. Such an interpolating solution may have an interesting interpretation in $N = 4$ supersymmetric gauge theories as well. Further, it would also be interesting to compute the Penrose limit along null geodesics on the spheres of the space-times given above. Such an analysis was carried out earlier in the case of Kaigorodov space-times [8], which can be derived from our ansatz above for the $\mathrm{AdS}_4 \times S^7$ wave by setting $\mu = 0$. One should in principle find a supersymmetric plane wave, but the form of the resulting wave profile would be highly nontrivial. It should also be noted that a Penrose limit along the perturbed AdS part of the metrics above should also be nontrivial, since the space-times are not conformally flat.

Our results can also possibly be of use for discussing holography in a more general context than pure $\mathrm{AdS}_m \times S^n$ type solutions [7]. An interesting example from this point of view may be the one obtained from a D3 brane, with wave profile given in Eq. (21). We notice that the boundary geometry in this case is a four dimensional pp wave. What implications this observation may have on the conformal field theory (CFT) structure is worth examining. In particular, it may be interesting to find out the meaning of an additional parameter (μ), in the examples discussed in this paper, in the CFT side, same way as the radius of curvature for $\mathrm{AdS}_5 \times S^5$ is related to the rank of the gauge group N.

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Note added.—After the submission of this paper to the archive, we have also come across another paper [27] where gravitational wave solutions have been obtained in AdS backgrounds.

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