

Do fermions and bosons produce the same gravitational field?

John D. Barrow

DAMTP, Centre for Mathematical Sciences, Cambridge University, Cambridge CB3 0WA, United Kingdom

Robert J. Scherrer

Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235, USA

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We examine some cosmological consequences of gravity coupling with different strength to fermions and bosons. We show that this leads to a different perturbation of the standard picture of primordial nucleosynthesis than the addition of extra neutrino types or overall scaling of the value of G . Observed abundances of deuterium and ${}^4\text{He}$ place bounds on the ratio of the bosonic gravitational constant (G_B) to the fermionic gravitational constant (G_F) of $0.45 < G_B/G_F < 0.92$ at 1σ and $0.33 < G_B/G_F < 1.10$ at 2σ . A value of $G_B < G_F$ can reconcile the current “tension” between the abundances of deuterium and ${}^4\text{He}$ predicted by primordial nucleosynthesis. We comment briefly on other cosmological effects.

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We examine some cosmological consequences of assuming that gravity is not blind to statistics and that the gravitational coupling constant, G , is different for fermions and bosons. The gravitational constant is notoriously difficult to measure to high precision and is poorly known in comparison to other fundamental constants [1–3]. Hence there has been extensive experimental and theoretical exploration of possible deviations from the standard Newtonian and general relativistic theory of the gravitational interaction [4]. These investigations have recently been rejuvenated by the realization that “large” extra dimensions can be accommodated in string theories and can lead to anomalous gravitational interactions on scales as large as 10^{-5} m [5,6]. Supersymmetry assumes that there is a fundamental symmetry between fermions and bosons but this symmetry must have been broken at the TeV scale. Perhaps this breaking produces further asymmetries in fermion-boson properties or couplings at lower energies?

In allowing G to be different for bosons and fermions, the key question is the scale over which a particle is considered a boson or a fermion. We have chosen to set this scale equal to the nucleon scale, i.e., all nucleons are considered “fermions” in our discussion. There are certainly alternatives to this approach. For example, one could treat the constituents of the nucleons as the fundamental particles, with the quarks coupling to G as fermions and the gluons as bosons; this is certainly the correct approach prior to the quark-hadron phase transition. Alternatively, one could define bosons and fermions on a larger scale, so that a helium nucleus, for instance, would couple to gravity as a boson rather than as four fermions. We believe that our approach to this admittedly speculative topic is the most logical, but it is important to note that other definitions of bosons and fermions would yield other results.

Note that we assume that this differential coupling of bosons and fermions to gravity affects only the source

term in the Einstein equations. We assume that the Equivalence Principle still holds, so that a boson and a fermion in a given gravitational field will still follow exactly the same trajectories. Because of this distinction, the gravitational coupling to bosons is not probed experimentally. All direct terrestrial experimental measurements of G are made with a fermionic source (see Ref. [4] for a review), so that the measured G is really G_F , the coupling to fermions, and the bosonic G , denoted G_B , is undetermined. Observations of the gravitational deflection of light do not isolate G_B either, since they measure the behavior of bosons under a fermionic source, and under the assumptions noted above, a boson in the gravitational field generated by a fermion will follow the same trajectory as a fermion in this gravitational field.

The Shapiro time-delay effect has been considered as a constraint on Equivalence Principle violation under the assumption that gravity couples differently to neutrinos and photons in the PPN approximation. A comparison of the difference in arrival times for neutrinos and photons from SN1987A allows a bound to be placed on any PPN γ parameter difference between photons and neutrinos, with $|\gamma_{\text{photon}} - \gamma_{\text{neutrino}}| < O(10^{-3})$, [7,8], with small uncertainties due to the gravitational field of the Galaxy. Differences in gravitational coupling to ν_e and $\bar{\nu}_e$ have also been studied for SN1987A, by Pakvasa *et al.* [9], yielding $|\gamma_{\nu_e} - \gamma_{\bar{\nu}_e}| < 10^{-6}$. It is interesting to note that a difference only of order 10^{-14} in the coupling of gravity to ν_e and ν_μ would make the gravitational transformation of ν_e to ν_μ of similar strength to the conversion due to the MSW effect [4,10,11]. In what follows we will consider the simplest scenario of a difference in gravitational coupling constant for bosons and fermions, but assume that the coupling is the same for particles and antiparticles and for all fermionic species. These assumptions can be relaxed straightforwardly if required. (See, e.g., Ref. [12] for differential coupling to different families,

and Ref. [13] for different coupling to particles and antiparticles).

The only place where bosons act as a significant source term for gravity is in the overall expansion of the universe as a whole, in which photons contribute significantly (at least in the early universe) to the overall energy density. In seeking to constrain G_B , therefore, the best place to look is at early universe tests such as Big Bang Nucleosynthesis (BBN) and the cosmic microwave background (CMB). We will explore the former in detail and comment briefly on the latter.

In the standard flat Friedmann cosmological model, the overall Hubble expansion rate is given at early times by

$$H = \left(\frac{8}{3} \pi G \rho \right)^{1/2}, \quad (1)$$

where ρ is the total gravitating density. Now consider a model in which we have two separate gravitational constants: G_F for fermions and G_B for bosons. In our model, G_F is just set equal to the measured value of Newton's constant, $G = 6.6742 \pm 0.0010 \times 10^{-8} \text{ cm}^3 \text{ gm}^{-1} \text{ s}^{-2}$ [1], so all of the information in the model is contained in the ratio G_B/G_F , which we will denote by f_{BF} :

$$f_{BF} \equiv \frac{G_B}{G_F}. \quad (2)$$

Then Eq. (1) for the expansion rate becomes

$$H = \left[\frac{8}{3} \pi (G_B \rho_B + G_F \rho_F) \right]^{1/2}, \quad (3)$$

$$= \left[\frac{8}{3} \pi G (f_{BF} \rho_B + \rho_F) \right]^{1/2}, \quad (4)$$

and the problem is equivalent to changing the density of bosons by the factor of f_{BF} and leaving the fermion density unchanged.

This model is qualitatively similar to both adding (or subtracting) relativistic degrees of freedom, or to changing the overall value of G , both of which have been extensively explored in connection with BBN, but we will now show that it is different from either of these previously studied situations.

Consider first the effect on the expansion rate of adding additional relativistic degrees of freedom to ρ . (The resulting variation in the primordial element production can be used to constrain such a change, e.g., Refs. [14–19]). It is conventional to parametrize such a change in terms of the effective number of additional (i.e., beyond ν_e, ν_μ, ν_τ) relativistic two-component neutrino degrees of freedom, ΔN_ν . In this case the total relativistic energy density prior to e^+e^- annihilation is

$$\rho = [2 + (7/8)10 + (7/8)2\Delta N_\nu] \frac{\pi^2}{30} T^4, \quad (5)$$

where the first term in square brackets counts the boson degrees of the freedom (photons), the second counts the

fermionic degrees of freedom (e^+e^- and $\nu\bar{\nu}$), the third counts any hypothetical additional relativistic degrees of freedom, and T is the photon temperature. (We use units with $\hbar = c = k_B = 1$ throughout). After e^+e^- annihilation, when the photons are heated relative to the neutrinos, the corresponding energy density is

$$\rho = [2 + (7/8)(4/11)^{4/3}6 + (7/8)(4/11)^{4/3}2\Delta N_\nu] \frac{\pi^2}{30} T^4. \quad (6)$$

These expressions can be rewritten in terms of the photon density, ρ_γ , to give [18]

$$\rho_{\text{before}} = 5.375[1 + 0.1628\Delta N_\nu]\rho_\gamma, \quad (7)$$

$$\rho_{\text{after}} = 1.681[1 + 0.1351\Delta N_\nu]\rho_\gamma, \quad (8)$$

where the subscripts “before” and “after” refer to the density before and after e^+e^- annihilation, respectively.

Another way to parametrize the change in the expansion rate is through a change in the overall value of G , or, equivalently, multiplying Eq. (1) by a “speed-up” factor S . By incorporating such a change into BBN, the resulting changes in the element abundances can be used to constrain a time-shift in the value of G [20–23]. As emphasized by Kneller and Steigman [18], these two ways of modifying BBN (adding additional relativistic degrees of freedom, or multiplying G by a constant) while qualitatively similar, are inequivalent. This can be most easily seen by noting that changing G by some fixed factor f_G is completely equivalent to changing ρ by this same factor; the “effective” ρ which enters into Eq. (1) is then just

$$\rho_{\text{before}} = 5.375[f_G]\rho_\gamma, \quad (9)$$

$$\rho_{\text{after}} = 1.681[f_G]\rho_\gamma. \quad (10)$$

In order to make a change in G equivalent to a change in the number of effective neutrino degrees of freedom, we would need $f_G = 1 + 0.1628\Delta N_\nu$ before e^+e^- annihilation and $f_G = 1 + 0.1351\Delta N_\nu$ after e^+e^- annihilation; obviously, it is impossible to satisfy both equations simultaneously.

Now consider the equivalent expressions in our model, when $G_B \neq G_F$. In Eqs. (5) and (6) the first term in brackets gives the contribution to the energy density from the photons, which are the only bosonic degrees of freedom present during BBN. Hence, when $G_B \neq G_F$, the effective density becomes

$$\rho_{\text{before}} = [2f_{BF} + (7/8)10] \frac{\pi^2}{30} T^4, \quad (11)$$

$$\rho_{\text{after}} = [2f_{BF} + (7/8)(4/11)^{4/3}6] \frac{\pi^2}{30} T^4. \quad (12)$$

These expressions can be rewritten in terms of the photon density as

$$\rho_{\text{before}} = [f_{BF} + 4.375]\rho_{\gamma}, \quad (13)$$

$$\rho_{\text{after}} = [f_{BF} + 0.681]\rho_{\gamma}. \quad (14)$$

A comparison of Eqs. (13) and (14) with Eqs. (7) and (8) and with Eqs. (9) and (10) shows that changing G_B/G_F is inequivalent (in terms of its effect on the expansion rate) to changing either the overall value of G , or adding additional relativistic degrees of freedom. This can be seen most easily by fixing ρ_{before} to the same multiple of ρ_{γ} in all three cases; it is easy to see that the resulting values of ρ_{after} are all different.

We now consider the effect of taking $f_{BF} \neq 1$ on the primordial element abundances. The primordial production of ${}^4\text{He}$ is controlled by the competition between the expansion rate and the rates for the weak interactions which govern the interconversion of neutrons and protons:

$$\begin{aligned} n + \nu_e &\leftrightarrow p + e^-, \\ n + e^+ &\leftrightarrow p + \bar{\nu}_e, \\ n &\leftrightarrow p + e^- + \bar{\nu}_e. \end{aligned} \quad (15)$$

At high temperatures, $T \gtrsim 1$ MeV, the weak-interaction rates are faster than the expansion rate, H , and the neutron-to-proton ratio (n/p) tracks its equilibrium value $\exp[-\Delta m/T]$, where Δm is the neutron-proton mass difference. As the universe expands and cools, the expansion rate becomes too fast for the kinetic equilibrium to be maintained by weak interactions and n/p freezes out. Nearly all the neutrons which survive this freeze-out are converted into ${}^4\text{He}$ as soon as deuterium becomes stable against photodisintegration, but trace amounts of other elements are produced, including deuterium (see, e.g., Ref. [24] for a review). Therefore, the primordial production of ${}^4\text{He}$ is very sensitive to the expansion rate of the Universe at temperatures ~ 1 MeV, so BBN has been used many times to constrain any change in this expansion rate.

As is the case for other models which change the expansion rate, the primordial deuterium abundance is most sensitive to changing the baryon-photon ratio η , and it essentially provides the upper and lower bounds on η . The predicted abundance of ${}^4\text{He}$ within this range for η can then be calculated as f_{BF} varies, allowing bounds to be placed on f_{BF} . (This is something of an oversimplification, as the deuterium abundance also depends weakly on f_{BF} ; our calculation correctly incorporates this dependence).

The primordial abundance of deuterium has been inferred from QSO absorption systems. We use the abundance estimated by Kirkman *et al.* [25]:

$$\log(D/H) = -4.556 \pm 0.064, \quad (16)$$

where all errors are quoted at the $1 - \sigma$ level.

The abundance of ${}^4\text{He}$ can be inferred from low-metallicity HII regions, but there are significant discrepan-

cies between different estimates (see Ref. [19] for a recent analysis, and references therein). We will follow Ref. [19] and take the primordial ${}^4\text{He}$ mass fraction, Y_P , to be

$$Y_P = 0.238 \pm 0.005, \quad (17)$$

an estimate which is consistent, for example, with that found in the review of Olive *et al.* [24].

Using a modified version of the Kawano nucleosynthesis code [26] we scan over the (η, f_{BF}) plane, calculating the likelihood for a given pair of values. (In doing so, we take the distribution of $\log(D/H)$ to be Gaussian as in, e.g., Refs. [23,25], rather than taking the distribution of D/H to be Gaussian as in, e.g., Ref. [19]. In practice, this should have only a small effect). The $1 - \sigma$ and $2 - \sigma$ contours are shown in Fig. 1 (where we take $\eta = \eta_{10} \times 10^{-10}$). The limits on f_{BF} , at the $1 - \sigma$ and $2 - \sigma$ levels, are:

$$0.45 < f_{BF} < 0.92, (1 - \sigma), \quad (18)$$

$$0.33 < f_{BF} < 1.10, (2 - \sigma). \quad (19)$$

These limits are the main result of our paper.

Note that the standard model is excluded at the $1 - \sigma$ level. This is not surprising; it is due to the tension which currently exists between the observed deuterium abundance and the observed ${}^4\text{He}$ abundance; the former prefers a higher value of η than does the latter. This sort of result, then, also shows up in discussions of changing ΔN_{ν} [19] and in analyses of other allowed deviations from standard physics (e.g., Ref. [27]). Note that a value of G_B less than G_F provides yet another mechanism for resolving this

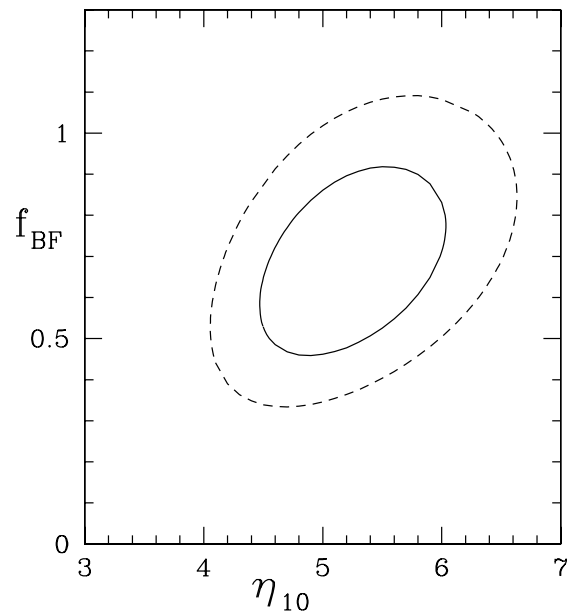


FIG. 1. The $1 - \sigma$ (solid) and $2 - \sigma$ (dashed) contours in the η_{10}, f_{BF} plane, where $\eta = \eta_{10} \times 10^{-10}$ and $f_{BF} = G_B/G_F$.

tension. Clearly, $G_B \gg G_F$ is excluded by BBN, but G_B could be much smaller than G_F .

Of course, these conclusions are strongly dependent on the parameters that go into the calculation. A higher primordial ${}^4\text{He}$ abundance, such as that claimed, for instance, in Ref. [28], would eliminate the tension between the deuterium and ${}^4\text{He}$ abundances, and shift our allowed region upward in Fig. 1. A similar effect would occur if the neutron lifetime were measured to be shorter than the currently accepted value [29].

Including CMB data in our analysis will improve these limits, but not by much. Although, as we have emphasized, our model differs both from changing ΔN_ν , or from changing G , it is qualitatively similar to both of these. For the case of changing ΔN_ν , the addition of CMB data constrains η more tightly than BBN alone, but it has only a small effect on the overall limits on ΔN_ν , compared to using BBN alone [19]. Similarly, BBN provides a stronger constraint on a change in G than does the CMB (compare, for example, the results in Ref. [2] for BBN with those in Ref. [30] for the CMB).

It is difficult to imagine other environments in which the value of G_B could manifest itself. If a scenario of

baryon number generation by out of equilibrium decay of superheavy vector or scalar bosons [31,32] were precisely established as the source of the value of η , then there would be a dependence of η on f_{BF} via the ratio of boson decay rate to the total expansion rate, but this is not the case at present. If the dark-matter were bosonic, then the overall expansion rate would also be altered in the matter-dominated era. This could only be detected, however, if the number density and mass of the dark-matter particle were independently determined, rather than being inferred from the Hubble expansion itself. The effect on large-scale structure in this case is equivalent to a model with one gravitational interaction for dark-matter and a different one for luminous matter [33]. It remains to be seen whether there are further consequences of $G_B \neq G_F$ at very early times (when $T \gg 1$ MeV) which could lead to observable consequences at low energy.

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