Scalar speed limits and cosmology: Acceleration from D-cceleration

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(Received 21 July 2004; published 8 November 2004)

Causality on the gravity side of the AdS/CFT correspondence restricts motion on the moduli space of the $\mathcal{N} = 4$ super Yang-Mills theory by imposing a speed limit on how fast the scalar field may roll. This effect can be traced to higher-derivative operators arising from integrating out light degrees of freedom near the origin. In the strong coupling limit of the theory, the dynamics is well approximated by the Dirac-Born-Infeld Lagrangian for a probe D3-brane moving toward the horizon of the AdS Poincaré patch, combined with an estimate of the (ultimately suppressed) rate of particle and string production in the system. We analyze the motion of a rolling scalar field explicitly in the strong coupling regime of the field theory and extend the analysis to cosmological systems obtained by coupling this type of field theory to four-dimensional gravity. This leads to a mechanism for slow roll inflation for a massive scalar at sub-Planckian vacuum expectation value without need for a flat potential (realizing a version of k inflation in a microphysical framework). It also leads to a variety of novel Friedman-Roberston-Walker cosmologies, some of which are related to those obtained with tachyon matter.

DOI: 10.1103/PhysRevD.70.103505 PACS numbers: 98.80.Qc

I. INTRODUCTION

It is now almost 100 years since Einstein introduced the concept of a universal speed limit for all physical systems propagating in spacetime. However, in general, motion on the configuration space of a physical system is not constrained to obey a speed limit. For example, the vacuum expectation values (VEVs) of light scalar fields naturally move in an internal moduli space where motion is uninhibited; within classical, relativistic field theory there is no restriction on the rate of change of the VEV. The purpose of this paper is to show that this situation does not necessarily continue to hold in the quantum theory. We exhibit situations where the strong coupling dynamics do impose a speed limit on an internal moduli space and examine applications to cosmology, including a mechanism for slow roll inflation.

We focus on the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory. Although the (supersymmetry-protected) metric on the moduli space is flat, the quantum-induced speed limit ensures that a rolling scalar field slows down as it approaches the origin. This fact can be seen immediately from the gravity side of the AdS/CFT correspondence, where the process corresponds to a D3-brane domain wall in AdS_5 moving toward the horizon. The familiar causal speed limit in the bulk translates into a speed limit on moduli space, which becomes more pronounced as the brane approaches the origin. Indeed, from the perspective of a boundary observer, the probe brane takes an infinite time to cross the horizon (although, as we will discuss, the probe approximation breaks down due to backreaction at very late times). This speed limit on the moduli space, arising from causality in the bulk, was first stressed by Kabat and Lifschytz [1].

On the field theory side of the correspondence, this result reflects the breakdown of the moduli space σ-model approximation as a scalar field approaches a locus with new light degrees of freedom where higher-derivative terms become important. In AdS/CFT dual pairs, we can use the gravity side of the correspondence to determine the net effect of these higher-derivative corrections where they are summed into a Dirac-Born-Infeld (DBI) action. The resulting dynamics is dramatically different from the naive expectation based on the supersymmetric moduli space metric.

In this paper, we study the dynamics explicitly, both in quantum field theory in its own right and in the cosmological context arising from quantum field theory coupled to four-dimensional gravity. We use the dual picture of D3-branes and anti-D3-branes moving in an AdS-like throat and we exhibit late-time solutions describing the physics as the scalar field approaches the origin. Among our results, we find that the slowing down of the scalar field (relative to the behavior predicted by the two-derivative action) can lead to new regimes exhibiting inflationary behavior.

The idea of obtaining inflation from $\overline{\text{D3}}$- and D3-branes in a warped throat was studied recently in Ref. [2]. In particular, the authors observe that one can gain extra control from the warping, but some obstacles to obtaining explicit inflationary models in string compactifications were identified. Our results here concern the effects of the crucial higher-derivative terms in the effective action and allow us to probe a different regime from that studied in Ref. [2], which may help address the challenges of Ref. [2]. In particular, the slow motion of the scalar field, enforced by higher-derivative terms, leads to a new mechanism for slow roll inflation.

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A scalar field mass is required to obtain our simplest inflationary solution. The required mass scale is not finely tuned in ordinary four-dimensional effective field theory terms, which provides an interesting distinction from the usual inflationary models. In the case of the $\mathcal{N} = 4$ super Yang-Mills theory coupled to gravity, we propose some effective field theory couplings to other sectors generating such a mass while leaving intact the form of the crucial kinetic corrections to the moduli space approximation of the field theory. However, we should note that the couplings of Kaluza-Klein modes in the corresponding brane throat to other sectors may be important and may require tuned coefficients to avoid destabilization of the throat; this is a little understood aspect of current compactification technology. Because of the plethora of independent ingredients (and types of domain wall branes) available in string compactifications, we expect that the combination of approximate AdS-metric induced kinetic terms and scalar masses generated from other couplings is likely to be available in some set of examples. It remains the main weakness of our results, however, that we will not exhibit an explicit example here. The other field theoretic and cosmological phases we will exhibit are not tied to this subtlety.

Our results have further relations to previous work. The use of higher-derivative terms to change scalar field dynamics in ways interesting for cosmology has been dubbed "k inflation" [3] or "k essence" [4] . A particularly well-studied example occurs in the effective field theory describing the decay of branes and antibranes leading to "tachyon matter" [5–7] and the associated cosmology [8,9]. While our system shares many features with the rolling tachyon story, including the existence of a pressureless dust equation of state, it also differs in several ways important for cosmology. In particular, we argue that particle and string production is suppressed in our system and the dynamics is governed to good approximation by our equations of motion. For a few examples of previous works studying the application of rolling moduli to string cosmology, see Refs. [10–12] and the review [13]. The crucial physics of our model is ultimately extracted from the dynamics of D-branes; there are, of course, many interesting investigations in this area; for example, Refs. [14–16].

The manner in which higher-derivative terms can drastically affect the dynamics of a system raises many questions concerning potential applications to real models of cosmology as well as more theoretical issues. Among the latter set is the question of whether or not motion toward other finite distance singularities in field theory and string theory moduli spaces (such as the conifold singularity) also exhibits similar slowing down effects for some regime of the couplings. Noticeably, recent studies of the flop transition, which explore the effects of the states which become light at the singularity, suggest that the rolling scalar field does indeed become stuck in the region of the singularity [17,18].

The paper is organized as follows. We start in Sec. II by describing our basic setup, stressing the appearance of a speed limit on the moduli space. In Sec. III we study the consequences of this speed limit for the dynamics in the global conformal field theory. We then couple our system to gravity in Sec. IV. We take particular care to describe the possible deformations of the system arising from other sectors in a string compactification, including the generation of a potential on the moduli space and the effects this has on the AdS geometry seen by the probe brane. In Sec. V we study cosmologies arising from our low-energy effective actions. In Sec. VI we check that our solutions are not destabilized by perturbations or particle production.

II. THE BASIC SETUP

Typically, studies of scalar field dynamics consider an effective field theory Lagrangian containing a kinetic term (up to two derivatives) together with a potential energy on the space of scalar fields:

$$S_{\text{two deriv}} = \int d^4x\sqrt{g}[G_{ij}(\phi)g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j - V(\phi)]. \tag{2.1}$$

In supersymmetric situations, the moduli space metric G_{ij} and the form of the potential energy V are highly constrained, and much of the recent work on such quantum field theories has focused on determining these quantities and the Bogomoln'yi-Prasad-Sommerfield (BPS) spectrum of states exactly.

It is well known, however, that the moduli space approximation [in which the physics is governed by the action (2.1) along flat directions of the potential $V(\phi)$] can break down due to the presence of new light degrees of freedom arising on special loci of the space of scalar field VEVs.

In the U(N) $\mathcal{N} = 4$ SYM theory, one has scalar fields in the adjoint representation of the gauge group, which can be represented by $N \times N$ matrices Φ. The moduli space of the field theory is parametrized by diagonal (commuting) matrices, with eigenvalues $\phi_1, \ldots, \phi_N$ whose moduli space metric, including quantum corrections, is flat. Away from the origin $\phi_i = 0$ of this moduli space, the low-energy gauge symmetry is generically $U(1)^N$ and off diagonal matrix elements of the scalar fields, fermions, and gauge bosons obtain masses. We refer to these modes collectively as "W bosons." In particular, we consider a configuration in which $\langle\phi_1\rangle \equiv \langle\phi\rangle \neq 0$ but all the other ϕ_i, $i \neq 1$ have vanishing VEVs. This means that the theory has a low-energy unbroken gauge symmetry $U(N-1) \times U(1)$ with massive W bosons in the $(N-1, +) \oplus (\overline{N-1}, -)$ representation. The W bosons are BPS protected states with mass

$$m_W = \phi. \tag{2.2}$$

We will work expanding about the large N limit of the theory, in which the natural parameters in the field theory are the rank N and the 't Hooft coupling $\lambda = g_{\rm YM}^2 N$ in terms of the Yang-Mills coupling $g_{\rm YM}$.

A. The system at weak coupling

At weak coupling in the field theory, the effective action for ϕ gets contributions from virtual W bosons. For large $\langle\phi\rangle$, these contributions scale like powers of $\lambda\dot{\phi}^2/\phi^4$ in the planar limit (see [19] for a comprehensive discussion of these higher-derivative corrections). If we send the scalar field rolling toward the origin from a finite point on the Coulomb branch, the classical action (2.1) would predict a constant velocity $\dot{\phi} = v_0$, but the form of the corrections just noted shows that the action (2.1) becomes unreliable at the distance

$$\phi^2 = \sqrt{\lambda} v_0. \tag{2.3}$$

The question then arises about how to take into account these corrections. However, in the weak coupling regime, the point is moot as we have another issue to confront before we get this close to the origin. Since the W bosons become light, they may be produced on shell during the evolution. The time-dependent mass leads to particle production controlled by the parameter $\dot{m}_W/m_W^2$, which therefore becomes important at the distance

$$\phi^2 = v_0. \tag{2.4}$$

For $\lambda \ll 1$, we first reach the production point (2.4).

Naively extrapolating the above perturbative results to the strong coupling regime, we reach the point (2.3) first and therefore expect that the dynamics will be governed by the effective action for ϕ, taking into account the $\lambda\dot{\phi}^2/\phi^4$ corrections arising from virtual W bosons. In the following section, we confirm this expectation and show that the scale (2.3) is where the speed limit on the moduli space first becomes apparent.

B. The system at strong coupling and the speed limit

At strong coupling $\lambda \gg 1$, the effective description of the theory is via gravity and string theory using the AdS/CFT correspondence [20]. A point on the Coulomb branch is described on the gravity side in the Poincaré patch via a D3-brane at fixed radial position r in the metric

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx^2) + \frac{R^2}{r^2}dr^2 \tag{2.5}$$

and at a point on the S^5. The field theory coordinate ϕ on the Coulomb branch translates into r/α' in the gravity variables. We further have the relations $R = (g_s N)^{1/4}\sqrt{\alpha'}$ and $g_{\rm YM}^2 = 4\pi g_s$.

As in other applications of AdS/CFT [20] and Randall-Sundrum [21], the warp factor in (2.5) plays an important role in understanding the energy spectrum of the system. The string mass scale at position r is related to that at position r' by the ratio of warp factors r/r'. The open string oscillator modes on the brane at position $r = \langle\phi\rangle\alpha'$ have masses of order

$$m_s(\langle\phi\rangle) = \frac{\phi}{\lambda^{1/4}}. \tag{2.6}$$

As in the Randall-Sundrum scenario, the effective cutoff for modes on the brane is at this warped string energy scale (2.6) rather than the UV string scale $1/\sqrt{\alpha'}$.

The W bosons are strings stretched from the brane to the horizon at $r = 0$. Although this is an infinite proper distance for the string to stretch, the warp factor r/R reduces the effective tension of the string enough to produce a finite total mass $m_W = \phi$ in accord with the BPS formula (2.2).

We would now like to highlight the simple feature of our system which lies behind most of our detailed results to follow. It is immediately clear from (2.5) that, for a boundary observer using coordinates (t, x), a probe brane takes infinite time to reach the origin of the Coulomb branch at $r = 0$. (In our solutions it will turn out that the proper time for the probe to fall to the origin will be finite.) The radial velocity of the D3-brane in the AdS space is limited by the speed of light which, translated in field theory variables, becomes

$$\dot{\phi} \le \dot{\phi}_c = \frac{\phi^2}{\sqrt{\lambda}}. \tag{2.7}$$

This restriction was noted previously by Kabat and Lifschytz [1], who discussed some interesting aspects of the phenomenon. Note that the probe brane takes infinite time despite the fact that the moduli space metric in (2.1) for the $\mathcal{N} = 4$ SYM theory is uncorrected quantum mechanically; the distance to the origin in the field theory moduli space metric is finite for any value of the coupling. As discussed above, the fact that it takes an infinite time to reach the origin arises from crucial corrections to the moduli space approximation. These corrections apply to *any* physical process in which the scalar rolls toward the origin of the Coulomb branch.

Note that the $\lambda\dot{\phi}^2/\phi^4$ corrections, while they are higher-derivative corrections, are not suppressed by powers of the Planck mass; as we discussed above, they are suppressed only by the W-boson mass ϕ. Let us compare this situation to a brane in flat space: such an object has a Lagrangian proportional to $\sqrt{1 - v^2}$, where v is the proper velocity of the brane. Written in terms of canonically scaled brane fields ϕ, this becomes (for a D3-brane) $\sqrt{1 - \dot{\phi}^2\alpha'^2}$, which expands to a series of higher-derivative terms which are suppressed by powers of the string mass m_s. In the global limit $m_s \to \infty$, these corrections die and the motion on ϕ space is unconstrained. It is the warp factor in (2.5) that produces higher-derivative effects that are crucial at the field theory level, as we will see in detail in our analysis.

III. THE GLOBAL CFT

The corrections to the moduli space approximation, and the resulting dynamics, can be understood rather explicitly in a controlled analysis on the gravity side in which the rolling scalar VEV is modeled by a moving D3-brane probe in AdS_5. In this section we examine the resulting dynamics of the scalar field.

A. Effective action and approximation scheme

On the gravity side, the effective Lagrangian appropriate for a probe D-brane at arbitrary velocity (less than or equal to the speed of light) but low proper acceleration is the Dirac-Born-Infeld Lagrangian. This Lagrangian describes the effects of virtual open strings at the planar level; it includes the effects of the background geometry and field strengths but does not include production of on-shell closed strings or W bosons or loops of closed strings. We will analyze the motion of the 3-brane starting from this Lagrangian and check for self-consistency of the resulting solutions. This requires checking that the proper acceleration is small, and taking the string coupling to be small so that the DBI action is a good approximation to the effective action of the probe. It further requires determining the range of parameters and times for which the energy in the probe does not backreact significantly on the geometry, so that the probe approximation remains valid. Finally, we must also check that fluctuations about the solution, density perturbations, and particle and string production are not too large. In Sec. IV we consider generalizations of this system to include coupling to four-dimensional gravity and other sectors, including effects of a cutoff throat in the IR; we take into account new contributions to the action at the level of its most relevant terms at low energy.

In this section, we start by writing down the effective action applicable to the pure conformal field theory (CFT) without gravity (infinite AdS space). The DBI action for a probe D3-brane moving in $\mathrm{AdS}_5 \times \mathrm{S}^5$ can be found in Ref. [20] and we use their conventions. We concentrate only on radial fluctuations and set the field strength on the brane to vanish. Working in field theory variables $\phi \equiv r/\alpha'$, we have

$$S = -\frac{1}{g_{\mathrm{YM}}^2} \int dt d^3x f(\phi)^{-1}\{-\det[\eta_{\mu\nu} + f(\phi)\partial_\mu\phi\partial_\nu\phi]^{1/2} - 1\}, \tag{3.1}$$

where for now we take the background brane metric to be flat $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$, and $f(\phi)$ is the harmonic function

$$f(\phi) = \frac{\lambda}{\phi^4}. \tag{3.2}$$

Expanding the action out in derivatives leads to a canonical kinetic term for $\phi \equiv r/\alpha'$ and a series of higher order derivative interactions. The potential cancels out, reflecting the BPS nature of the D3-brane. In fact, for a single excited scalar field ϕ, this form of the higher-derivative action can be shown to hold even at weak coupling [22] without recourse to a dual gravitational description: it is fixed by the requirement of a nonlinearly realized conformal invariance, together with known nonrenormalization theorems.

So, in general, the dynamics of ϕ in our strongly coupled CFT differs in two important ways from a naive scalar field system governed by (2.1). First, the kinetic term is corrected to that in (3.1); as discussed above, we will find that the proper velocity approaches the speed of light so that the quantity in the square root approaches zero, resulting in a system far away from the regime where the two-derivative action suffices. Second, there is no potential $V(\phi)$. In later sections we shall remedy the latter issue by generalizing to systems in which potentials are generated, both by antibranes and by considering a field theory coupled to other sectors. The speed limit will remain a crucial ingredient in our generalizations.

B. Dynamics

Let us now study more explicitly the approach of the $\mathcal{N} = 4$ SYM theory to the origin of its Coulomb branch, using the action (3.1). Since we are interested only in the time dependence of the solution, we ignore the spatial derivatives, leaving us with the action

$$S = -\frac{N}{\lambda^2} \int d^4x \phi^4(\sqrt{1 - \lambda\dot{\phi}^2/\phi^4} - 1). \tag{3.3}$$

To determine the late-time behavior[1] of this system, we first compute the conserved energy density E, given by (up to a factor of $1/g_{\mathrm{YM}}^2$)

$$E = \frac{1}{\lambda}\phi^4\left(\frac{1}{\sqrt{1 - \lambda\dot{\phi}^2/\phi^4}} - 1\right) \approx \frac{1}{2}\dot{\phi}^2 + \frac{1}{8}\frac{\lambda\dot{\phi}^4}{\phi^4} + \ldots, \tag{3.4}$$

where, in the second line, we have expanded around large ϕ. The first term is the canonical kinetic energy for a rolling scalar field, with subsequent terms becoming important at small ϕ, capturing the effect of virtual W bosons.

[1] Readers who prefer $u = 1$ units may make the substitution in all expressions of this type.

As mentioned above, the Born-Infeld action is valid for arbitrarily high velocities $\dot{\phi}$ but only for small proper acceleration a,

$$a\sqrt{\alpha'} = \frac{\lambda^{1/4}}{\phi}\frac{d}{dt}\left(\frac{\sqrt{\lambda}\dot{\phi}}{\phi^2}\right).$$

So, in order for our analysis to be self-consistent, we must check that the proper acceleration is much smaller than string scale in our solution. Inverting (3.4) to solve for $\dot{\phi}$, and subsequently taking the time derivative, it is simple to see that we can trust the DBI analysis when we are in the strong coupling regime $\lambda \gg 1$.

Integrating once, the trajectory of the scalar ϕ is given by

$$t - t_0 = \frac{1}{\sqrt{E}}\int_{\phi_0}^{\phi} d\varphi \frac{1}{\varphi^2}\frac{\lambda E + \varphi^4}{\sqrt{\lambda E + 2\varphi^4}}.$$

We are interested in the dynamics of the scalar field after we hit the critical point on moduli space defined by (2.3). Using the classical expectation $E \sim v_0^2$, this means we are interested in the regime

$$\phi^4 \ll \lambda E. \tag{3.5}$$

Here we find the late-time behavior

$$\phi(t) \rightarrow \frac{\sqrt{\lambda}}{t} \tag{3.6}$$

saturating the speed limit (2.7). We therefore find that, from the perspective of the field theory observer, Xeno is vindicated and the scalar field takes an infinite time to reach the origin as advertised.

1. Background check

The above calculation treated the D3-brane as a probe and is valid only when the backreaction can be neglected. Since the brane travels at almost the speed of light at late times on the gravity side, it carries a lot of energy and we must determine the conditions under which its backreaction does not destabilize the AdS background.[2] The gravitational field surrounding a highly boosted object in locally flat space is given by the Aichelburg-Sexl metric [23], which in our case of codimension 6 is given by

$$h_{--} \sim \frac{l_s^8 g_s E_p}{r_\perp^3}\delta(x^-). \tag{3.7}$$

Here $E_p = E(R/r)^4$ is the proper energy density of the probe (related to the Poincaré observer's energy density E by the appropriate powers of the warp factor r/R), $r_\perp$ is the distance from the probe in the transverse dimensions, and x^- is the light cone coordinate with respect to which the brane trajectory is localized in locally flat coordinates. If we insist that the corresponding curvature $\mathcal{R}_{10} \sim h_{--}/r_\perp^2$ smeared over a string scale distance across the brane be smaller than the ambient AdS curvature $1/R^2$ at a distance R from the probe, this gives the condition

$$E < \frac{\phi^4}{\lambda^{1/4} g_s}. \tag{3.8}$$

If we impose this condition, we still obtain a window

$$\lambda^{1/4} g_s E < \phi^4 < \lambda E$$

in which our backreaction constraint (3.8) intersects with our regime (3.5) of limiting speed.

This constraint (3.8) may be too strong, since the novel behavior we found for ϕ based on the DBI action depends only on the AdS geometry. We can estimate the backreaction of our probe on the AdS geometry as follows. The probe forms a domain wall of energy density E_p in the five-dimensional gravity theory obtained by dimensional reduction on the S^5. Such a wall will jump the warp factor across it. By dimensional analysis, this gives the relation

$$\frac{1}{R'} - \frac{1}{R} \sim E_p l_5^3 \tag{3.9}$$

in terms of the five-dimensional Planck length l_5, where R' is the curvature radius on the IR side of the domain wall. Using $l_5^3 \sim l_{10}^8/R^5 = l_s^8 g_s^2/R^5$, the condition that the jump in warp factor is smaller than the original AdS warp factor is

$$E < \frac{\phi^4}{g_s}.$$

Again, this leads to a window at strong coupling in which the rolling scalar field saturates its speed limit (3.6) in the regime

$$g_s E < \phi^4 < \lambda E. \tag{3.10}$$

Similar backreaction criteria apply to all the cases discussed in the paper, and we will derive a related bound in our interesting inflationary phase. It would be interesting to explore what happens when this condition is violated. We will discuss the other consistency conditions for our background (including ruling out significant backreaction from particle production) in Sec. VI.

C. Antibranes

Here we introduce the first of our generalizations: anti-D3-branes moving in the AdS background (2.5). The action for the $\overline{\mathrm{D3}}$-brane probe in AdS_5 differs from the D3-brane case (3.3) merely by a change of sign in the final term,

$$S = -\frac{N}{\lambda^2}\int d^4x \phi^4(\sqrt{1 - \lambda\dot{\phi}^2/\phi^4} + 1). \tag{3.11}$$

Upon expanding the square root in powers of $\lambda\dot{\phi}^2/\phi^4$, this gives rise to a potential which is quartic in ϕ. There is

[2] This constraint was obtained via discussions with M. Fabinger.

no quadratic $m^2\phi^2$ term. This may be at first sight surprising since the system has broken supersymmetry in the presence of the antibrane, and loops of open strings which probe the supersymmetry (SUSY) breaking are included at the level of the action (3.11). However, this result is to be expected using simple Randall-Sundrum ideas. The local string scale at the position of the brane is $\phi/\lambda^{1/4}$ (2.6). Similarly, all hard masses are warped down by a factor of $r/R = \phi\sqrt{\alpha'}/\lambda^{1/4}$ at position ϕ in the throat. Applying this to the SUSY breaking scale $m_{\rm SUSY}$, a potential $V \sim m_{\rm SUSY}^2\phi^2$ really scales like ϕ^4. Said differently, there is no independent scale in the CFT on the Coulomb branch beyond the position ϕ.

The ϕ^4 potential implies that the $\overline{\rm D3}$-brane experiences a force towards the horizon at $r = 0$. Nonetheless, it is a simple matter to check that the late-time dynamics is dominated by the speed limit and is identical to that of the D3-brane (3.6).

In some sense, the $\overline{\rm D3}$-brane probe in AdS space can be thought of as a strong coupling limit of the usual tachyon matter system. Indeed, we see in Sec. V that the resulting dynamics bears some similarity to the results that have been obtained for the much-studied open string tachyon decay in the weakly coupled brane-antibrane system (see [24] for recent review of this system). However, there are some differences which we stress here. At weak coupling, the $\overline{\rm D3}$-brane system has tachyons from strings stretching between the antibrane and the $N - 1$ D3-branes described by the AdS throat. However, at strong coupling $\lambda \gg 1$, it is easy to see from (2.2) and (2.6) that this mode is not tachyonic. The negative contribution to the would-be tachyon mass squared, scaling like the (warped) string scale $m_s(\langle\phi\rangle)^2$, is dwarfed by the positive contribution m_W^2 to the mass squared from the stretching of the string from the antibrane to the AdS horizon. Thus, the brane-antibrane system at strong 't Hooft coupling (obtained by taking many branes and a single antibrane) has no tachyon. The system *is* unstable–as we will discuss in detail, the antibrane falls toward the horizon. However, this evolution is not condensation of a brane-antibrane tachyon since none exists; as in Ref. [25], the annihilation of the antibrane against the unit of D3-brane charge encoded in the Ramond-Ramond (RR) flux is a nonperturbative effect.

IV. COUPLING TO GRAVITY

In both the brane and antibrane cases, we will be interested in two further important generalizations. One is coupling to four-dimensional gravity, as well as to other sectors suppressed by higher-dimension operators that may arise in the corresponding string compactifications. The other is a generalization to a throat which is capped off in the region corresponding to the IR behavior of the field theory–i.e., the system with a mass gap. In a string theoretic setup, these effects can be achieved by gluing an AdS-throatlike solution onto a compactification geometry which acts as a UV-brane as in Refs. [26,27].

In this section, we discuss the effective actions which result from these generalizations, focusing on the relative strength of corrections to the potential energy and DBI kinetic energy terms in the effective action. Our motivation for considering these theories is the application of the speed-limit mechanism to cosmology. We postpone a full discussion of this until Sec. V. To avoid undue suspense, let us here summarize some of the important results.

Because our matter sector itself slows the scalar field ϕ in its progress toward the origin (in the field theory variables), the conditions for slow roll inflation are modified in a useful way. In a situation with a strong enough additional potential energy for ϕ (which may arise, for example, from a setup with a brane or antibrane in a cutoff AdS throat coupled to other sectors), we indeed find a phase of slow roll inflation. This can be obtained at sub-Planckian VEV for ϕ, with the slow roll provided by the DBI kinetic term corrections rather than from the usual gravitational damping or from an unnaturally flat potential. This allows us to evade some of the problems discussed in Ref. [2]. While we have not yet produced a complete model of real-world inflation from this mechanism, it seems a very promising ingredient.

More generally, we find some familiar behaviors for the Friedman-Roberston-Walker (FRW) scale factor $a(t)$ but arising in unusual ways from our matter sector. For example, as in tachyon matter we find a dust equation of state for motion of a D3-brane toward the origin of the Coulomb branch, though in our case production of massive matter is suppressed. With other potentials, we also find a novel steady-state late-time behavior for the universe that does not involve fine tuning of initial conditions.

The motion of the moduli themselves is an important aspect of string cosmology, and the basic result of the previous sections leads to interesting novelties in this area. In particular, if the moduli field ϕ corresponding to the motion of the 3-brane heads toward the origin of the Coulomb branch, it gets slowed down and, at least in the probe limit, takes an infinite time to reach the origin. This is radically different from the naive treatment based on the moduli space metric, in which it would shoot past the origin without pausing. Furthermore, if the scalar relaxes into a potential well surrounding the origin, it does not oscillate around the minimum in the same manner as moduli treated with the usual action (2.1). However, in the cases when the ϕ sector with all the DBI corrections leads to a dust equation of state, it can cause problems similar to the usual moduli problem.

A. CFT coupled to gravity

We first ask what becomes of the low-energy effective action (3.1) when the gauge theory is coupled to gravity by introducing a dynamical background metric $g_{\mu\nu}$. At the same time, we add a potential term $V(\phi)$ to the action that may arise when the system is coupled to four-dimensional gravity and other sectors involved in a full string compactification. A simple four-dimensional covariantization of (3.1) is

$$\mathcal{L}_0 = -\frac{1}{g_{\rm YM}^2}\sqrt{-g}[f(\phi)^{-1}\sqrt{1+f(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} + V(\phi) \mp f(\phi)^{-1}], \tag{4.1}$$

where $\pm$ refers to the D3-brane and $\overline{\rm D3}$-brane, respectively, while $f(\phi) = \lambda/\phi^4$ as before.

In the presence of four-dimensional gravity, there are in general further corrections to the action (4.1) coming from the following considerations. Dynamical $4d$ gravity leads generically to nontrivial four-dimensional curvature $\mathcal{R}$. In effective field theory, this leads to a series of contributions of the form

$$\mathcal{R}\,\phi^2(1 + c_1\mathcal{R}/\phi^2 + \ldots). \tag{4.2}$$

More generally, each contribution in the action (4.1) could be corrected by terms suppressed by powers of $\mathcal{R}/\phi^2$. In particular, there will be corrections depending on $\mathcal{R}$ and $\partial\phi$, which will be outside the scope of our analysis; our results will be contingent on these corrections being subdominant (which may require tuning in appropriate circumstances). In the weakly coupled $\mathcal{N} = 4$ SYM theory coupled to four-dimensional gravity, such corrections are evident from diagrams containing loops of W bosons. At strong coupling, such corrections were found in Refs. [28,29] starting from the DBI action for curved slices. From all these points of view, the coefficients c_i in (4.2) are expected to be of order 1 (i.e., not parametrically large or small as a function of λ).

In our solutions, we find that these curvature corrections are negligible. Our strategy will be to simply ignore them at this stage, analyze the dynamics, and then check that this is self-consistent (in Sec. V D below). In any case, we can collect these effects in an effective Lagrangian containing both gravity and matter:

$$S = \int[\tfrac{1}{2}\sqrt{-g}(M_p^2 + \phi^2)\mathcal{R} + \mathcal{L}_0 + \ldots], \tag{4.3}$$

where ... refers to corrections to the terms in (4.3), which are down by powers of $\mathcal{R}/\phi^2$ from the leading terms. The final action is a special case of those considered in k-inflation and k-essence scenarios [3,4]. In our case, the higher-derivative terms in $\mathcal{L}_0$ follow from the strong coupling dynamics of the field theory near the origin of its moduli space.

Let us now discuss the reliability and plausibility of this action including the coupling to four-dimensional gravity. In string theory, such a coupling is obtained by embedding the AdS geometry into a compactification as a throat emanating from a Calabi-Yau compactification of type IIB string theory [26,27]. The first question that arises is whether this geometrical combination exists or if, instead, the coupling of the AdS to the Calabi-Yau produces a large deformation of the AdS space. If such a geometry exists, we may move onto the second question: whether the low-energy effective field theory action is given by (4.1) to a good approximation for the system we wish to study, and what is the order of magnitude of the parameters in the potential V. Third, in the end we must check that the time-dependent solutions we find are stable against small fluctuations, particle production, and density perturbations. (This is in addition to the checks for small acceleration and small backreaction of the probe energy of the sort we completed in Sec. III for the global case.) This last question will be addressed in Sec. VI.

As far as the first question goes, as discussed in Refs. [26,27], the AdS throat is obtained from a collection of many D3-branes at a smooth point on the Calabi-Yau. At the level of the no scale potential obtained for Calabi-Yau compactifications of type IIB at large radius in the absence of α' corrections and nonperturbative effects [27], the scalars on the D3-branes have a potential identical to that in the global quantum field theory on their world volume. They are mutually BPS and, in this sense, are also mutually BPS with the rest of the compactification. This follows from the analysis in Ref. [27] and may be related to the fact that, in the $\mathcal{N} = 4$ SYM at strong coupling, the operator $tr\Phi^2$, which is relevant at weak coupling, is highly irrelevant at strong coupling (something exploited by, e.g., Ref. [30]). The quantum mechanical corrections to the Kahler potential and the superpotential of the model, which are important for fixing the Kahler moduli, will in general also determine the positions of the D3-branes. Since the configuration corresponding to their being on top of each other at a smooth point in the Calabi-Yau is an enhanced symmetry point, it is likely that an order 1 fraction of the models has a minimum containing to a good approximation an AdS throat arising from such a collection of D3-branes. We will momentarily estimate the size of corrections to this geometry as seen by our probe.

Now let us discuss the second question about the reliability of our effective action and the parameters in the potential. Attached to the Calabi-Yau, our AdS throat is coupled to four-dimensional gravity and other sectors. We start by discussing the effects of the coupling of these sectors to ϕ in effective field theory. Then we discuss the geometrical description of these effects. We argue that (consistently in both descriptions) significant corrections to the mass term in the potential can naturally arise from these couplings, while corrections to the kinetic terms arising from them are subleading to those in the original action. This ensures that we can preserve our slow roll effect for the scalar field coming from the kinetic terms while introducing potential energy sources for gravity which lead to interesting, and in one case, accelerating, cosmologies. As we also discuss, it is an interesting open question whether this setup requires tuning couplings of throat Kaluza-Klein modes to other sectors to avoid further effects on the kinetic terms for ϕ.

1. Effective field theory description of corrections

As well as curvature couplings such as $\mathcal{R}\phi^2$ arising from the introduction of gravity, we will generically have further couplings of ϕ to other sectors. These could be of the form

$$\phi^2\eta^2 \quad \text{or} \quad \frac{\phi^2(\partial\eta)^2}{M_*^2}, \tag{4.4}$$

where η is a field from another sector and M_* is a mass scale (such as the grand unified theory, string, or Planck scale) in the system above the energies we wish to consider. In the next subsection, we explain why we expect such couplings to be available naturally in string compactifications, but we have not constructed an explicit example and largely treat these couplings by effective field theory in our analysis of the cosmology.

We should emphasize that the effective field theory description we use in this subsection makes use of the weak coupling expansion available on the gravity side of AdS/CFT; it is just the couplings of the ϕ fields to the new sectors involved in the compactification which we treat here via effective field theory. A full treatment of the four-dimensional effective field theory on the gravity side also requires understanding the couplings and effects of the Kaluza-Klein modes. This involves understanding what tuning is required to preserve an (approximate) AdS throat in a Calabi-Yau compactification in the absence of a probe brane. This is not well understood. In any case, as we discuss in the next subsection and in Sec. V, the probe approximation for the brane in a large-radius approximate AdS throat is consistent with the presence of a large enough mass on the probe brane for interesting cosmological effects such as inflation.

More generally, we may view (4.4) as coupling the strongly coupled CFT to the η sector by hand in effective field theory. The action (4.1) contains no closed string (gravity) loops or loops of η from couplings such as (4.4). We must estimate contributions to our effective actions from these loop effects. We assume a weak enough string coupling for perturbative string theory to be valid on the gravity side.

Our first important task is to determine the scale of any relevant or marginal corrections to the action that may arise from the couplings involving ϕ. Let us start by discussing the potential $V(\phi)$, considering a power series expansion

$$V \equiv V_0 + V_2\phi^2 + V_4\phi^4 + \ldots. \tag{4.5}$$

The most relevant term in the Lagrangian at low energies is the hard cosmological constant V_0. This gets contributions, in principle, from all sectors in the system. In effective field theory (and approximately in string theory using the Bousso-Polchinski mechanism [31]) we may tune this (close) to the value of interest for a given application. We make use of this freedom in our analysis.

Now let us move on to the ϕ-dependent couplings, starting with the mass term $V_2\phi^2$. As we discussed in the global case, in an exact conformal field theory (which is either not coupled to gravity and other sectors or is not destabilized by their presence), a mass coupling $m_*^2\phi^2$ with m_* a constant mass parameter is ruled out by the conformal invariance encoded in the effects of the warping in the AdS throat. Thus, with an exact AdS-throat geometry, the potential is expected to receive corrections starting with the quartic $V_4\phi^4$ coupling. However, in general, the couplings to gravity and other sectors can generate corrections to the theory (4.3) which violate conformal invariance. For example, corrections to the scalar mass coming from loops containing virtual gravitons yield a contribution

$$m_*^2 \sim \Lambda_{\rm UV}^4/M_p^2, \tag{4.6}$$

where the fourth power of the UV loop momentum cutoff $\Lambda_{\rm UV}$ can be traced to the fact that the Feynman diagrams contain derivative couplings. Similar contributions arise from couplings to other sectors (4.4). A mass for ϕ can be generated either from a VEV for η or from loop corrections involving the derivative coupling, giving rise to respective contributions of the form

$$m_*^2 \sim \langle\eta\rangle^2 \quad \text{or} \quad m_*^2 \sim \Lambda_{\rm UV}^4/M_*^2. \tag{4.7}$$

We should also note that the quartic and higher couplings $V_4\phi^4 + \ldots$ in the effective potential will also get corrections from interactions such as (4.4). In our strategy for obtaining inflation below, we require the net quartic and higher couplings to be small enough relative to the quadratic coupling. This will not require significant tuning of parameters in our solutions.

Now let us consider the strength of corrections to the crucial generalized kinetic terms in the DBI action (i.e., the series in $v_p^2 = \lambda\dot\phi^2/\phi^4$) appearing in the action. Importantly, the corrections to these terms coming from (4.4) are suppressed relative to the existing terms that generate our slow roll effect [in contrast to the situation for the soft mass term just described, for which the (4.7) constitutes the leading effect]. For example, if we compute the effect of the coupling (4.4) on the $\dot\phi^4$ term in the effective action via a loop of ηs, we obtain a contribution scaling such as

$$\left(\frac{\dot\phi}{M_*}\right)^4 \log(\Lambda_{\rm UV}/M_*).$$

This is much smaller than the original contribution $N\dot\phi^4/\phi^4$ in the small ϕ regime of interest: $\phi^4 \ll NM_*^4$.

The fact that the DBI kinetic terms are robust against large corrections from (4.4) arises from the fact that they are marginal and get logarithmic corrections suppressed by the coupling constants in terms like (4.4). There are no soft contributions to the derivative terms in the action–they would have to be of the form $(m_*/\phi)^n(\dot{\phi}^2/\phi^4)$. Such corrections would be more infrared divergent as $\phi \to 0$ than the existing ones in (3.1). There are no perturbative diagrams producing such effects, and we find it implausible also in the strong coupling regime that coupling to gravity could worsen the IR behavior of the system.

Although these corrections to the DBI kinetic terms are small, they are nonzero in general. Since the kinetic terms arose from the DBI action given by the volume of the brane embedded in the ambient geometry, corrections to them correspond to corrections to the AdS geometry, at least as seen by our probe brane. Indeed, once we generate (or add by hand in effective field theory) a deformation of the theory by a mass term for ϕ, we expect deviations from exact AdS geometry in the IR. However, as above these deviations are small effects on our probe evolution relative to the effects of the original kinetic terms. In particular, if we deform the Lagrangian to introduce an $m^2\phi^2$ term, we can estimate its effects on the higher-derivative terms in the DBI action. These came from integrating out W bosons, with the importance of the higher-derivative terms arising from the fact that these modes become light at the origin $\phi \to 0$. The mass term for ϕ will at some order induce new contributions to the mass of the W bosons; up to logarithms these will at most scale like

$$m_W^2 \to \phi^2 + g_{\mathrm{YM}}^2 m^2. \tag{4.8}$$

In this manner, the contributions of virtual W bosons to the DBI Lagrangian for ϕ become a series in $\lambda\dot{\phi}^2/(\phi^2 + g_{\mathrm{YM}}^2 m^2)^2$. This means that, for $\phi \gg g_{\mathrm{YM}}^2 m$, the evolution is well approximated by that given by the original DBI Lagrangian. As we will see, this constraint preserves a window of interesting behavior for ϕ dictated by

$$\phi^2 > g_s m^2. \tag{4.9}$$

In Sec. V B, we see that the same constraint arises on the gravity side by requiring small backreaction so that the probe approximation holds.

In summary, the same couplings which give rise to a mass m for ϕ do not badly alter the generalized kinetic terms for $\phi \gg m$. In fact, the mass term itself is a small effect on the overall probe evolution, as we see from the ϕ equation of motion in which the DBI kinetic terms dominate. However, the mass term in the potential will be an important source for the four-dimensional spacetime geometry in our cosmological solutions. The corrections on the kinetic terms for $\phi < m$ do mean that ultimately the geometry is probably better approximated by a cutoff throat. We turn to this analysis in Sec. IV B.

As emphasized above, the four-dimensional effective field theory, however, contains more than just ϕ and the BPS W bosons, but also Kaluza-Klein modes in the AdS. It is important to check whether couplings similar to (4.4) and curvature couplings appear with ϕ replaced by Kaluza-Klein modes. If so, and if the induced masses m_{KK} of these Kaluza-Klein modes were as big as m, the corresponding deformation of the geometry in the IR region would remove our inflationary solution to be described in Sec. V. In the context of effective field theory, we may tune away these couplings if necessary, and, as discussed in the next section, we generically expect such tuning to be possible in string theory. It would be interesting to determine whether such tuning is necessary or if $m_{\mathrm{KK}} \ll m$ appears naturally.

2. Gravity description of corrections

In the previous subsection, we gave arguments based on simple effective field theory couplings that the (soft) mass term for ϕ generated by (4.4) will generically be affected more strongly (relative to the original mass term for ϕ in the Lagrangian) than the generalized kinetic terms (relative to the original generalized kinetic terms in the Lagrangian). At first sight, this may appear surprising from the point of view of the geometrical picture of a brane probe. Such a probe has kinetic terms determined by the metric of the ambient spacetime and potential terms introduced via couplings to other background fields (such as the five form RR field strength in the $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution of type IIB supergravity). There is no general relation between these two effects, though in particular examples they are related in particular ways via the coupled equations of motion for the metric curvature and the other low-energy fields. In familiar examples, such as the $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution or the Klebanov-Strassler solution [32], the strength of the potential is smaller than the effects we found in the previous subsection.

These classical solutions, however, do not include the effects introduced by the other sectors (mocked up by η in the above analysis) located in the bulk of a Calabi-Yau to which the throat attaches. In general, the relation between the spacetime metric and the other background fields is different from that obtained in the familiar classical solutions. More general flux backgrounds (including AdS solutions) with different types of brane probes will have a different balance between fluxes felt by the probe and metric curvature due to the contribution of other ingredients. Our results in the above subsection reflect an aspect of this in the context of a coupling of ϕ in the familiar solutions to other sectors via, for example, an embedding in a Calabi-Yau.

To make this more concrete, let us use known scales of couplings between different sectors of a Calabi-Yau geometry to estimate the effective field theory parameters

M_* and $\Lambda_{\rm UV}$ in the effective field theory analysis of the last subsection. In Ref. [33] couplings between brane throats were computed. The results were consistent with couplings between a brane throat and another sector (which could be a much smaller throat) suppressed by powers of $M_* \sim 1/R$. Taking the UV cutoff $\Lambda_{\rm UV}$ of the second sector to also be of order $1/R$, one obtains an estimate for the mass m of order $m \sim 1/l_s\lambda^{1/4}$. As discussed in the previous section, the same coupling yields a suppressed correction to the kinetic terms if we lie within the window

$$\frac{1}{l_s\lambda^{1/4}} < \phi < \frac{N^{1/4}}{l_s\lambda^{1/4}}. \tag{4.10}$$

To summarize, the important corrections to the mass term for ϕ we derived via effective field theory couplings in the last subsection fit with known properties of geometrical embedding of brane probes, taking into account effects generated by sectors outside the brane throat corresponding to our original CFT. Therefore, we find it likely that such a mass term is available (and potentially generic given the presence of a large brane throat) in explicit string models. In any case, a mass for ϕ is physically consistent with a probe approximation for ϕ. This follows from the above considerations regarding the many independent ways of assembling ingredients such as the target space metric and other background fields, combined with the self-consistency against backreaction (to be checked in Sec. V) of the brane energy carried in the ϕ mass term.

However, we have not constructed an explicit example of this in a full compactification model. It is a subtle problem to determine the couplings among sectors in a full string compactification [2,33], and we plan to pursue it systematically in future work.

B. Quantum field theory with mass gap coupled to gravity

Motivated by the discussion above, we here consider geometries dual to theories with a mass gap, where the AdS throat is cut off at the IR end. The prototypical example of such a geometry is the Klebanov-Strassler solution of Ref. [32]. The effects of such an IR cutoff include the following: The warp factor $f(\phi)$ in (3.2) is changed, and further corrections to the action may arise if we start from a cutoff throat at the classical level. Here we consider only a toy model in which we focus on the effect on $f(\phi)$; similar comments to those in the previous subsection apply to the question of further corrections to the action.

For the purposes of our discussion, we replace the full Klebanov-Strassler solution with a simple toy model which reproduces the relevant features. We motivate this by considering a theory in which the formula for the W-boson mass (2.2) is deformed to

$$m_W^2 = \phi^2 + \mu^2, \tag{4.11}$$

where, in the effective field theory discussion above, we had $\mu^2 \sim g_{\rm YM}^2 m^2$. In general, μ will be related to m in a similar fashion, but for the purposes of this section we keep it arbitrary. To model the W mass (4.11) from the holographic perspective, we keep the action (4.1) but replace the warp factor f with the appropriate function. For simplicity, we consider

$$f(\phi) = \frac{\lambda}{(\phi^2 + \mu^2)^2}. \tag{4.12}$$

Such a geometry does not satisfy Einstein's equations by itself but, nonetheless, exhibits the important features of a capped off throat.

In the DBI Lagrangian (4.1) for the cutoff throat (4.12) in the antibrane case, there is a hard mass of order

$$m_{\rm DBI}^2 \sim \mu^2/\lambda. \tag{4.13}$$

Loop corrections arising from the ϕ^4 interactions drive this up to $m_{\rm DBI}^2 \sim \Lambda_\phi^2/\lambda$, where Λ_ϕ is the ultraviolet cutoff pertaining to the ϕ loop.

In our cutoff geometry we also have corrections to the masses and other couplings as discussed in the last subsection (where now there may be some new sectors associated with the physics at scale μ). Again, the effects on the kinetic terms are subleading but the mass corrections are important.

V. COSMOLOGICAL SOLUTIONS

In Sec. III, we saw that the scalar field motion in our system is radically different from the naive expectation based on the finite distance to the origin of the moduli space in the metric appearing in the two-derivative action (2.1). Here we ask what effects this novel matter sector has on cosmological solutions. We investigate this in detail by studying the possible FRW cosmologies solutions which follow from (4.1). We consider only spatially flat cosmologies,

$$ds^2 = -dt^2 + a(t)^2 dx^2.$$

Since spatially inhomogeneous terms are redshifted away during inflation, we consider the scalar field ansatz $\phi = \phi(t)$. With this ansatz, the equations of motion can be concisely expressed by first defining the analog of the Lorentz contraction factor in special relativity,

$$\gamma \equiv \frac{1}{\sqrt{1 - f(\phi)\dot{\phi}^2}}. \tag{5.1}$$

Our strategy is to analyze the solutions following from (4.1) by first ignoring the effects of the curvature coupling $\mathcal{R}\phi^2$ and later showing that its effects are self-consistently negligible on our solutions [unlike the situation in usual inflationary models based on (2.1)]. The energy density ρ and pressure p following from (4.1) (taking the $+$ sign, corresponding to a D3-brane, for definiteness) are given by

$$\rho = \frac{\gamma}{f} + (V - f^{-1}), \tag{5.2}$$

$$p = -\frac{1}{f\gamma} - (V - f^{-1}). \tag{5.3}$$

These definitions do not include the overall coefficient of $1/g_{\rm YM}^2$ in (4.1), which instead combines with M_p in the Einstein equations so that the scale $(M_p\sqrt{g_s}) = M_p g_{\rm YM}$ appears in all equations. The Friedmann equations read

$$3H^2 = \frac{1}{g_s M_p^2}\rho, \tag{5.4}$$

$$2\frac{\ddot{a}}{a} + H^2 = -\frac{1}{g_s M_p^2}p, \tag{5.5}$$

where $H = \dot{a}/a$ is the usual Hubble parameter. Finally, the equation of motion for ϕ reads

$$\ddot{\phi} + \frac{3f'}{2f}\dot{\phi}^2 - \frac{f'}{f^2} + \frac{3H}{\gamma^2}\dot{\phi} + \left(V' + \frac{f'}{f^2}\right)\frac{1}{\gamma^3} = 0, \tag{5.6}$$

where prime denotes a derivative with respect to ϕ. Note that, as we approach the speed limit on moduli space, so $\gamma \to \infty$, both the friction term and the potential term in the equation of motion for ϕ become subdominant. It is a simple matter to check that the second Friedmann equation (5.5) follows from the first (5.4) together with the equation of motion (5.6). The effective equation of state of our system, defined by $p = \omega\rho$, is therefore given by

$$\omega = \frac{-\gamma^{-2} - (Vf - 1)\gamma^{-1}}{1 + (Vf - 1)\gamma^{-1}}. \tag{5.7}$$

A. The Hamilton-Jacobi formalism

As occurred in the global case, we find solutions in which the brane asymptotes to the speed of light at late times in the gravity side background. This means that the quantity γ grows as t gets large, and the behavior of ρ will be substantially different from the usual case of $\rho = \frac{1}{2}\dot{\phi}^2 + V$ to which it reduces for small proper velocity.

First, we want to rewrite the Friedmann equations (5.4) and (5.6) in a more tractable form. In fact, they can be integrated once. In the inflation literature this is referred to as the Hamilton-Jacobi formalism [34]. Another perspective can be obtained by viewing the resulting cosmology as a Wick rotation of a BPS domain wall; the first order Friedmann equations are related to the Bogomoln'yi equations derived in Ref. [35].

To derive the Hamilton-Jacobi equations, the important step is to view the scalar field ϕ as the time variable. In practice, this means we consider $H = H(\phi)$ with $\phi = \phi(t)$. This immediately puts a limitation on the dynamics since it assumes ϕ is monotonic and H is a single-valued function of ϕ and therefore does not allow for oscillatory behavior in ϕ. We start by taking the time derivative of (5.4) and, using (5.6) along the way, we find

$$6HH'\dot{\phi} = -\frac{1}{g_s M_p^2}3H\gamma\dot{\phi}^2,$$

which can clearly be solved simply by

$$\dot{\phi} = -2(g_s M_p^2)\frac{H'}{\gamma}.$$

Since (γ^{-1}) depends on $\dot{\phi}$, it is useful to invert this to give us

$$\dot{\phi} = \frac{-2H'}{\sqrt{1/(g_s^2 M_p^4) + 4fH'^2}}. \tag{5.8}$$

Finally, we can substitute this expression for $\dot{\phi}$ into (5.4), using the full expression for ρ from (5.2) to get an expression for the potential $V(\phi)$ in terms of the Hubble parameter $H(\phi)$,

$$V = 3(g_s M_p^2)H^2 - \frac{(g_s M_p^2)}{f}\sqrt{1/(g_s^2 M_p^4) + 4fH'^2} + \frac{1}{f}. \tag{5.9}$$

The advantage of these equations is that they can be solved sequentially. Given a potential V, we solve (5.9) to find $H(\phi)$. This can then be plugged into (5.8) to find $\phi(t)$. Finally, this can be substituted back into $H(\phi)$ to find the dynamics of the universe. We now solve for the late-time behavior of these equations, both in the AdS throat where $f = \lambda/\phi^4$ and in our toy model for the cutoff geometry with $f = \lambda/(\phi^2 + \mu^2)^2$.

B. Cosmology in the AdS throat

We start by considering cosmology in the CFT, corresponding to a brane moving in the full AdS throat. In this case, the first order Friedmann equations (5.8) and (5.9) read

$$\dot{\phi} = \frac{-2H'}{\sqrt{1/(g_s^2 M_p^4) + 4\lambda H'^2/\phi^4}} \tag{5.10}$$

and

$$\lambda V = 3\lambda(g_s M_p^2)H^2 - (g_s M_p^2)\phi^4\sqrt{1/(g_s^2 M_p^4) + 4\lambda H'^2/\phi^4} + \phi^4. \tag{5.11}$$

We consider potentials of the form

$$V = V_0 + V_2\phi^2 + V_4\phi^4 + \ldots \tag{5.12}$$

and determine the behavior of the scale factor as $\phi \to 0$.

For now, we simply introduce these parameters by hand to analyze the equations but, as mentioned above, there are constraints on these coefficients in the $\mathcal{N} = 4$ SYM and in this theory coupled to gravity which we should include in the final analysis. For now, however, we work with the *ad hoc* potential above. There are three possible cases, depending on which of V_0, V_2, or V_4 is the first non-vanishing coefficient. (We in fact see that if the first two vanish, and the potential is positive, then the result is universal.)

1. Case A: $V_0 \neq 0$

If V is simply a constant, then Eq. (5.11) is solved by $H = \sqrt{V_0/3(g_s M_p^2)}$, in which case Eq. (5.10) shows that $\phi = \text{const}$, and we get a standard de Sitter phase. Of course, our kinetic terms are not playing a role here since ϕ is not moving. Note that Eq. (5.10) requires $V_0 > 0$ here.

Suppose instead that we take $V = V_0 + V_4\phi^4$, which suffices to get ϕ moving. Then for small ϕ, we may choose the simple ansatz $H = h_0 + h_4\phi^4 + \ldots$, where, from (5.11), we must choose the coefficients $h_0 = \sqrt{V_0/3(g_s M_p^2)}$ and $h_4 = V_4/6(g_s M_p^2)h_0$. Substituting this into the first of the Friedmann equations, we have the late-time behavior for ϕ,

$$\phi \to \frac{1}{4(M_p\sqrt{g_s})\sqrt{h_4}}\frac{1}{\sqrt{t}}.$$

We see that, as in the case of the global CFT considered in Sec. III, the scalar field takes infinite time to reach the origin. However, in this case the friction from the expanding universe means that ϕ does not saturate the speed limit (2.7). From the late-time behavior of ϕ, we may immediately determine the late-time behavior of the scale factor. We have

$$H = \sqrt{\frac{V_0}{3(g_s M_p^2)}}\left[1 + \frac{3}{2^7 V_4(g_s M_p^2)}\frac{1}{t^2}\right].$$

This result highlights the peculiar cosmologies that can arise due to strong coupling effects of the gauge theory. Here we find a standard de Sitter phase, superposed with a cosmology in which the scale factor has the form $a(t) \sim \exp(-c/tM_p)$. We see more cosmologies with this characteristic exponential behavior in case D.

2. Case B: $V_0 = 0$

Let us now consider the interesting case that will give rise to inflation. We suppose that $V_2 \neq 0$ so that this quadratic mass term dominates at late times when ϕ is small.

We work with the simple ansatz

$$H = h_1\phi + \ldots. \tag{5.13}$$

Substituting this into (5.11), we get contributions to V_2 from both the H^2 term as well as the $\sqrt{\cdots}$ term. We find the potential takes the form

$$V = \left(3h_1^2 - \frac{2h_1}{\sqrt{\lambda}}\right)(g_s M_p^2)\phi^2 + \mathcal{O}(\phi^4) \equiv V_2\phi^2 + \mathcal{O}(\phi^4), \tag{5.14}$$

which, solving for h_1 in terms of V_2, gives

$$h_1 = \frac{1}{3\sqrt{\lambda}}[1 + \sqrt{1 + 3V_2\lambda/(g_s M_p^2)}]. \tag{5.15}$$

Substituting our ansatz for H into Eq. (5.10), we find

$$\dot{\phi} = \frac{-2h_1\phi^2}{\sqrt{\phi^4/(g_s^2 M_p^4) + 4\lambda h_1^2}}.$$

As $\phi \to 0$, we simply throw away the ϕ^4 term in the denominator to find the late-time behavior

$$\phi \to \frac{\sqrt{\lambda}}{t} + \ldots.$$

This coincides with the result (3.6) from the global analysis of Sec. III; in this theory the slowing down of the scalar field is driven by the speed limit rather than friction from the expanding spacetime. We now substitute this back into our ansatz (5.13) to find the late-time behavior of the scale factor,

$$a(t) \to a_0 t^{h_1\sqrt{\lambda}}. \tag{5.16}$$

Using (5.15), the exponent is

$$h_1\sqrt{\lambda} = \frac{1}{3}\left[1 + \sqrt{1 + \frac{3V_2\lambda}{(g_s M_p^2)}}\right] \approx \sqrt{\frac{V_2\lambda}{3(g_s M_p^2)}}, \tag{5.17}$$

where the approximation is true only for $V_2\lambda \gg g_s M_p^2$. We see that we can orchestrate an inflationary universe if the potential V_2 is sufficiently large. Specifically,

$$\frac{V_2\lambda}{g_s M_p^2} > 1. \tag{5.18}$$

It is amusing to recall that usual slow roll inflation occurs only if the potential is suitably flat. Here we find the opposite result: with a speed limit on the scalar field, inflation occurs only if V_2 corresponds to a high enough mass scale [we see from (5.18) that this mass scale can be much lower than M_p, but the acceleration gets stronger for larger V_2].

Interestingly, we have obtained a form of power-law inflation, since the scale factor behaves as a power of t. However, it arises in the case of a polynomial (quadratic) potential, which under the usual circumstances (2.1) leads to exponential inflation $a \sim e^{Ht}$. [The potential required to obtain power-law inflation from (2.1) is exponential in ϕ.]

Naively, it appears that this result suggests an alternative to the usual quintessence scenario where a scalar field rolls towards an asymptotic regime of moduli space. Here the scalar field could roll a finite distance in the field theory moduli space metric in the interior of the moduli space and still lead to asymptotic acceleration. However, we are limited to a finite period of controlled speed of light evolution because of the backreaction of the probe on the geometry. In the global case we determined this backreaction in (3.10). It is a simple matter to repeat the calculation here where the energy of the brane is dominated by the potential $V_2\phi^2$. Once again, translating from the proper energy E_p to the observer's energy density $E = E_p(r/R)^4$, we have the constraint

$$E_p l_5^3 \sim \frac{V_2\phi^2}{g_{YM}^2}\left(\frac{R}{r}\right)^4 \frac{\alpha'^4 g_s^2}{R^5} < \frac{1}{R},$$

which gives us the constraint that our solution is only to be trusted for ϕ greater than

$$\phi^2 > g_s V_2.$$

Notice that this is identical to the lower bound on ϕ arising derived in Sec. IV by calculating one-loop corrections to the W-boson mass using effective field theory methods (4.9). Despite this infrared cutoff to our inflationary phase, we shall shortly argue that we still retain a viable window of inflation in this scenario.

3. Case C: $V_0 = 0$ *and* $V_2 = 0$

Let us now suppose that the potential has only $V_4 \neq 0$. As we have seen, such a potential is generated by a $\overline{\mathrm{D3}}$-brane moving in the AdS throat. The late-time analysis of the Hamilton-Jacobi equations (5.10) and (5.11) is actually just a special case of the above discussion, where we put $h_1 = 2/3\sqrt{\lambda}$. We can read of the final result from (5.16): we have

$$a \to a_0 t^{2/3},$$

which looks like dust. In fact, for any potential for which $V_0 = V_2 = 0$ with higher order terms nonvanishing, the speed limit on the moduli space ensures that the rolling scalar field looks like equation of state $\omega \approx 0$: a kinetic dust.

4. Case D: $V < 0$

To find equations of state with $\omega > 0$, we require negative V_2. From (5.15), we see that for $0 < h_1 < 2/3\sqrt{\lambda}$ we have

$$-\lambda(g_s M^2)^2 \frac{\lambda}{3} < V_2 < 0.$$

The late-time behavior of the scale factor is once again given by (5.16). During this expansion, the scalar field is rolling up the inverted harmonic oscillator potential. Without the higher kinetic terms, the scalar would overshoot the top and roll down the other side. However, the speed limit does not allow this to happen and slows the field to prevent it reaching the top at $\phi = 0$ in finite time.

Let us now consider the case $V_2 = 0$ with $V_4 < 0$. We can generate such a potential by considering, for example, $H = h_3\phi^3$, which gives rise to the potential $V = V_4\phi^4 + V_6\phi^6$, where

$$\lambda V_4 = -\sqrt{1 + 36\lambda h_3^2(g_s^2 M_p^4)} + 1,$$
$$\lambda V_6 = 3\lambda h_3^2(g_s M_p^2).$$

At late times the scalar field rolls like $\phi \sim 1/t$, and the equation of state parameter diverges: $\omega \to \infty$. Meanwhile, the scale factor a has the peculiar behavior

$$a = a_0 \exp\left\{-\frac{[1/(g_s M_p^2) + 36\lambda h_3^2]^{3/2}}{2^4 3^3 h_3^2}\frac{1}{t^2}\right\},$$

which undergoes a period of accelerated expansion before settling down to a steady-state cosmology at late times. The period of acceleration coincides with the period when the $V_6\phi^6$ term dominates over the $V_4\phi^4$ term in the potential; in the solution this also correlates with a period in which the potential dominates in the gravitational dynamics determined by ω (5.7). As in case B, the effect becomes stronger with a stronger potential (so a flat potential is not required for slow roll since this is provided by the DBI kinetic terms).

Scale factors with the characteristic $\exp(-1/t^2)$ behavior also occur in standard cosmology but only for very finely tuned initial conditions. The novelty here is that such dynamics is generic for a large enough initial velocity. A similar steady-state dynamics also arises from $H = h_4\phi^4$, this time with $a \sim \exp(1/M_p t)$. Each of these scenarios finds Xeno exonerated and Hoyle happy.

C. Cosmology in the cutoff throat

The interesting case B of the previous section has $V_2 \neq 0$, which is not compatible with the conformal symmetry implied by an exact AdS throat. Instead, one ultimately expects a geometry in which the throat is cut off in the infrared. We described such geometries in Sec. IV. Here we work with the toy model in which the warp factor is taken to be

$$f(\phi) = \frac{\lambda}{(\phi^2 + \mu^2)^2}.$$

For $\phi \gg \mu$, the geometry is essentially that of the AdS throat and the analysis of the previous sections holds. However, for $\phi \leq \mu$, the speed limit becomes simply

$$\dot{\phi} \leq \frac{\mu^2}{\sqrt{\lambda}}.$$

So the scalar field can now reach the origin in finite time and happily sail right through at finite speed. This has the advantage that at late times the field oscillates around the minimum of the potential as usual and reheating may occur (depending on the couplings to the standard model). However, we must check that we can still arrange for a period of acceleration. The condition for acceleration requires large enough V_2 as we saw in (5.18). Generally, we have

$$V_2 = m^2,$$

where masses m^2 which could be generated in various ways were listed in Sec. IV and include contributions from graviton loop (4.6), couplings to other sectors (4.7), and a hard mass from the deformed geometry (4.13).

Let us start by examining the upper limit for ϕ. In fact, we are quite entitled to take ϕ as large as we wish and, for $\phi \gg M_p$, one finds the standard slow roll expansion of chaotic inflation. However, as we discuss in Sec. V D, models with super-Planckian VEVs suffer from destabilization from a slew of quantum corrections involving, for example, gravitational curvature couplings $\mathcal{R}\phi^2$ [2]. To avoid this, we instead restrict ourselves to *sub-Planckian* VEVs

$$\phi \ll M_p, \tag{5.19}$$

where, as the speed limit is saturated, we may still obtain inflationary behavior.[3]

In our cutoff geometry we also need ϕ to be suitably large so that the probe brane is experiencing the AdS-throatlike background before the cutoff is reached. It can be checked that the solution described in case B of Sec. V B continues to hold in the cutoff geometry provided $\phi \gg \mu$. We therefore find our window of inflation when the scalar field lives in the regime,

$$\mu \ll \phi \ll M_p, \tag{5.20}$$

during which time we have the cosmological solution $a(t) = a_0 t^{h_1\sqrt{\lambda}}$ while the scalar field slows down as $\phi = \sqrt{\lambda}/t$. Let us assume that this solution actually holds all the way to the end points (5.20). This may be overly optimistic, but it gives us a quick and ready way to calculate the number of e-foldings. We require that at the initial time $t = t_0$ we have $\phi(t_0 = 0) = M_p$, while at the final time $t = t_f$ we have $\phi(t = t_f) = \mu$. Then

$$t_0 = \frac{\sqrt{\lambda}}{M_p} \quad \text{and} \quad t_f = \frac{\sqrt{\lambda}}{\mu}.$$

The number of e-foldings is therefore given by

$$n = \log\left[\frac{a(t = t_f)}{a(t = 0)}\right] = h_1\sqrt{\lambda}\log(M_p/\mu).$$

The argument of the logarithm is roughly the distance travelled by the scalar field. Any large number of e-foldings comes from the prefactor. Using (5.17) we have

$$n \sim h_1\sqrt{\lambda} \sim \frac{m\sqrt{\lambda}}{\sqrt{g_s}M_p} \geq 100. \tag{5.21}$$

This criterion is the same as the criterion that the potential V dominate in the equation of state (in particular in ρ).

This inflationary phase is novel in several ways. Most importantly, the inflation can occur on a steeper potential hill than works for ordinary slow roll inflation. Moreover, at least before considering observational constraints from density perturbations, we find inflation to occur at sub-Planckian field strength for ϕ. We show in Sec. V D that, when combined with the novel form of the equation of motion for ϕ, this allows us to avoid destabilizing effects from a curvature coupling $\mathcal{R}\phi^2$, circumventing some of the difficulties involved in placing inflation within a string compactification [2]. Moreover, as noted in Sec. V B, we have obtained *power-law* inflation from a quadratic potential.

At later times, the brane reaches the end of the cutoff throat and the novelties of the DBI action wear off. At this stage, we expect to reduce to an ordinary matter or radiation dominated phase, though we have not yet controlled the details of the exit from our inflationary stage.

The number of e-foldings (5.21) can be made large if we can generate a large mass term $m^2\phi^2$ for our scalar field. In Sec. IV, we discussed possible mechanisms for generating such a term and argued that corrections to our crucial higher-derivative terms were subleading. The simplest of the mass corrections was the hard contributions (4.13) from the DBI action itself: $m_{\rm DBI} \sim \mu/\sqrt{\lambda}$. Comparing with (5.21) we immediately see that in our strong coupling regime $\lambda \gg 1$ there is no inflationary window with such a mass: we need another mechanism. As discussed at length in Sec. IV, such a mechanism may arise from coupling to other sectors which (depending on the strength of the couplings and cutoff scales for the other sectors in the system) may produce a large enough mass. More simply, as discussed above we may consider deforming the theory by a mass term for ϕ and note that this induces only subdominant corrections to the DBI kinetic terms. Again, it would be very interesting to determine the magnitude of mass m arising in specific string compactifications.

A comment on V_4 terms.—In the above discussion, we concentrated on the possible mass terms that could be generated and the associated window of applicability of the inflationary solution described in case B of Sec. V B. However, the solution itself is valid only if the quadratic

[3]In our subsequent analysis of density perturbations [36], we discovered that, to satisfy all the observational constraints including those on non-Gaussianity, we require starting inflation at Planck scale ϕ, which may require fine tuning.

term in the potential dominates over the quartic term; otherwise, we obtain the dustlike evolution of case C. We therefore require

$$V_4\phi^2 < m^2 \Rightarrow \phi^2 < m^2/V_4. \tag{5.22}$$

If V_4 is too big, then this constraint on ϕ would be much stronger than (5.19) and potentially ruin our window of inflation. Let us therefore examine the possible V_4 couplings.

In the DBI action, V_4 scales like $1/\lambda$. However, quantum corrections change this. Let us focus on the virtual effects of other sectors, since these were successful in giving us an inflationary window. The derivative coupling (4.4) leads to a contribution of order

$$\Delta V_4 \sim \Lambda_{\rm UV}^4/M_*^4 \tag{5.23}$$

while at the same time inducing a mass $m_* \sim \Lambda_{\rm UV}^2/M_*$ (4.7) as we discussed earlier. Plugging this into (5.22) leads to the constraint $\phi < M_*$, which is stronger than (5.19). At the level of our effective field theory analysis (in terms of an unknown UV cutoff parameter $\Lambda_{\rm UV}$ and a coupling scale $1/M_*$), this is consistent with our inflationary window for appropriate values of these parameters.

D. Curvature corrections

Let us now restore into our analysis the curvature coupling $\mathcal{R}\phi^2$ and the other curvature corrections scaling as powers of $\mathcal{R}/\phi^2$. We consider the regime of our solutions where $\phi \ll M_p$ and show that the solutions are not destabilized by the addition of these curvature couplings (in other regimes more fine tuning would be required [36]). We should note here that the *conformal* coupling required to render the stress-energy tensor traceless in the case of large γ will be more complicated than a $\mathcal{R}\phi^2$ coupling.

Let us first consider the $\mathcal{R}/\phi^2$ corrections to the terms in the Lagrangian (4.3) which scale like H^2/ϕ^2. In our inflating solution, these terms are of the order m^2/M_p^2. As long as this quantity is sufficiently smaller than 1, then the correction terms are negligible compared to the terms in (4.3) that we did include in the initial analysis. In the noninflating solutions, $H \sim 1/t$ and these corrections are suppressed by powers of λ, which makes them very safe from affecting the results above.

Now let us turn to the curvature coupling $\mathcal{R}\phi^2$. First, let us consider the Einstein equations for the metric. The effect of the conformal coupling is to replace M_p^2 with $M_p^2 + \phi^2$ in these equations, changing the effective Planck mass. Since $\phi \ll M_p$, this does not change the solutions for $a(t)$ significantly. Similar comments apply to the $H^2\phi^2$ terms which plague F-term inflation in supergravity models.

Second, we must check whether the scalar field equation of motion is self-consistently solved by the original solution in the presence of a curvature coupling. The coupling $\xi\mathcal{R}\phi^2$ contributes to the equation of motion for ϕ, (5.6), as

$$\frac{\lambda\ddot{\phi} - 6\lambda\dot{\phi}^2/\phi + 4\phi^3}{(1-\lambda\dot{\phi}^2/\phi^4)^{3/2}} + \frac{3\lambda H\dot{\phi}}{(1-\lambda\dot{\phi}^2/\phi^4)^{1/2}} + (V' + 2\xi\mathcal{R}\phi - 4\phi^3) = 0. \tag{5.24}$$

Because $\gamma(t)^{-1} = \sqrt{1-\lambda\dot{\phi}^2/\phi^4}$ is going to zero at late times, the terms involving V and $\mathcal{R}$ are subdominant in this regime. This is reflected in the fact that, for cases B and C of Sec. V B, the late-time behavior of the scalar field saturates the speed limit and is independent of the potential.

In summary, the curvature couplings we have considered here do not change our results. This is in contrast to the usual situation (2.1), where, for example, the slow roll conditions for inflation in a polynomial potential are impossible to satisfy in the presence of the coupling $\mathcal{R}\phi^2$. This point was emphasized for the case of 3-branes in warped throats in Ref. [2]. Here we see that near the origin, where the DBI velocity corrections are crucial, the structure of the scalar equation of motion is dramatically different due to the factors of $\gamma(t)$ in the denominator in (5.24).

E. Relation to other works

The behavior of our system, especially in case C in Sec. V B, bears a strong resemblance to the tachyon matter system studied in recent years [5,8]. Indeed, the antibrane moving in the AdS throat is, in some sense, a strong coupling limit of the tachyon matter system. There are important differences between our case and the weakly coupled tachyon matter system, most notably the fact that as we discussed above in our situation the tachyon has been lifted and, as we shall see, particle production is suppressed. Moreover, the spectra of particles and strings in the two backgrounds differ.

A simple field redefinition can be used to relate the effective field theories in the two cases as follows. In tachyon cosmology, the action for the tachyon is of the form [37]

$$\mathcal{L} = -F(T)\sqrt{1-\dot{T}^2}, \tag{5.25}$$

which, upon expanding the square root, has a potential term $F(T)$. We can simply generalize this action to include a further potential term $G(T)$, which we could use either to cancel F or simply to make the potential different from the coefficient of the kinetic terms. Such a potential may be generated in situations where the brane-antibrane system is embedded in a compactification. We therefore consider

$$\mathcal{L} = -F(T)\sqrt{1-\dot{T}^2} - G(T).$$

The stress-energy tensor and pressure can be easily calculated to yield energy density

$$\rho = \frac{F}{\sqrt{1 - \dot{T}^2}} + G, \qquad p = -F\sqrt{1 - \dot{T}^2} - G.$$

Tachyon cosmologists work with this Lagrangian imposing $G = 0$. Of particular relevance for the present work is the observation [9] that the power-law inflationary phase of case B can be obtained by the choice $F(T) \sim 1/T^2$ in the Lagrangian (5.25). It was further shown that higher order (inverse) potentials give rise to dust behavior as we saw in case C. To return to our favorite Lagrangian and compare these results with those above, we employ the field redefinitions

$$T = \frac{\sqrt{\lambda}}{\phi}, \qquad F(T) = \frac{\lambda^2}{T^4}, \qquad G(T) = V(\phi) - \phi^4.$$

It is very interesting to ask how generally motion on internal scalar field configuration spaces can be imbued with a geometric interpretation (and therefore a speed limit induced by causality). It is intriguing that the tachyon Lagrangian (5.25) has a structure reminiscent of that of a relativistic particle moving in spacetime.

The motion of the probe brane in the cutoff geometry is also somewhat similar to the discussion of Ref. [38]. These authors consider the induced mirage cosmology [39] on a probe brane as it moves in the Klebanov-Strassler geometry [32]. This differs from our analysis in two important respects: first, in Ref. [38] the probe brane was taken to move slowly through the background geometry, so that the DBI action could be approximated by two-derivative terms. Second, the cosmology was viewed from an observer on the probe brane rather than from the perspective of a boundary observer as is relevant for our discussion.

VI. PERTURBATIONS AND PARTICLE PRODUCTION

We now come to the promised self-consistency checks of our time-dependent evolution. In order to trust our results in the previous sections, we must check that particle and string production and fluctuations about the solution do not destabilize it. We have already checked that acceleration is small enough to avoid closed string production and that backreaction from the probe's large kinetic energy is small in the window (3.10).

A. W-boson and string production

First, we consider the production of massive strings on the brane and W bosons. Our effective DBI action is blind to the possible on-shell creation of these modes and we must check by hand that the production is suppressed in the $\lambda \gg 1$ regime. This is rather simple. The time-dependent solution we found entails time-dependent masses for both W bosons (2.2) and massive string modes on the brane (2.6) since these masses are proportional to $\phi \sim \sqrt{\lambda}/t$.

The strength of particle production for a time-dependent frequency $\omega(t)$ is controlled by $\dot{\omega}/\omega^2$. This is a rough estimate which can be obtained by starting from the requirement for adiabatic evolution for a frequency which changes over a time period δt:

$$\delta t \gg 1/\omega_{\min}, \tag{6.1}$$

where $\omega_{\min}$ is the smallest value of ω obtained in the evolution. Dividing by the change $\delta\omega$ arising in the process yields

$$\delta t/\delta\omega \gg 1/(\omega_{\min}\delta\omega). \tag{6.2}$$

Consider a long enough evolution so that $\delta\omega$ is of order $\omega_{\max}$. Replacing $\delta\omega$ on the right hand side by $\omega_{\max}$ and replacing $\omega_{\max}\omega_{\min}$ by ω^2 yields a condition under which particle production may be consistently neglected:

$$\dot{\omega} \ll \omega^2. \tag{6.3}$$

This can be thought of as expressing the fact that (on average) the energy from the time dependence of the frequency is smaller than the jump in energy needed to produce an on-shell particle. We find in our situation that the inequality (6.3) is satisfied parametrically in λ for large λ. In particular, for the W boson zero modes we find

$$\frac{\dot{m}_W}{m_W^2} \sim \frac{1}{\sqrt{\lambda}}, \tag{6.4}$$

which is suppressed in our large λ regime. Nonzero modes are more suppressed. We also need to check the production of open string oscillator modes on the brane. The masses for these modes are given by the warped value $m_s(\phi) \sim \phi/\lambda^{1/4}$. If they were created at rest, a similar calculation to the one above would yield again parametric suppression of $\dot{m}_s/m_s^2 \sim 1/\lambda^{1/4}$. Since the brane is moving with a large velocity $\sqrt{\lambda}\dot{\phi}/\phi^2$, the strings created on the brane have energies boosted to

$$\omega \sim \gamma\sqrt{N_{\rm osc} m_s(\phi)^2 + \vec{p}^2}, \tag{6.5}$$

where $N_{\rm osc}$ is an integer coming from the oscillator level of the string. These energies are large, and we find that again the time-dependent background does not inject enough energy to create these states. Therefore, our solution is stable against string and W-boson production in the strong coupling regime.

B. Perturbations of ϕ

In the remainder of this section, we analyze the fluctuations of ϕ itself about the solutions we have found. Our main goal will be to determine in what regime the perturbations are not dangerous for our solutions above. These results also pertain to the spectrum of density perturbations produced in our inflationary phase. We defer a detailed discussion of any predictions of that model for the cosmic microwave background radiation to a later investigation. We perturb our solutions as

$$\phi(t) \rightarrow \phi(t) + \alpha(x, t).$$

Expanding in Fourier modes, $\alpha(x, t) = \alpha_k(t)e^{ikx}$, the equation of motion for α obtained by expanding the action (4.1) is

$$\ddot{\alpha}_k + \left(\frac{6}{t} + 3H\right)\dot{\alpha}_k + \left(\frac{6}{t^2} + \frac{6H}{t} + \frac{k^2}{\gamma_0^2 a^2 t^4}\right)\alpha_k = 0,$$

where we have used the late-time behavior of the function γ defined in (5.1) which, for most of our solutions, is $\gamma \rightarrow t^2/\gamma_0$ for constant γ_0. In particular, for the inflationary solution of case B we have $\gamma_0 \sim mM_p/\sqrt{\lambda}$.

If we analyze this equation of motion in the global case, where $H = 0$, we obtain the results $\delta\phi \propto t^{-2}$ and $\delta\phi \propto t^{-3}$ for the $k = 0$ mode. For the $k \neq 0$ modes, the perturbations will be further suppressed. So we see that the perturbations do not grow relative to the background solution $\phi \sim \sqrt{\lambda}/t$; i.e., these perturbations are nontachyonic.

Let us now analyze the equation with gravity turned on, focusing on the inflationary solution. We would like to estimate the density perturbations that result from the fluctuation $\delta\phi$ in the field. In our inflationary phase, we have

$$H = \frac{1}{\epsilon_0 t}, \tag{6.6}$$

where, from (5.16), we have $\epsilon_0 = (h_1\sqrt{\lambda})^{-1}$. Since we have $\epsilon_0 \ll 1$, in the inflationary phase the terms involving H and H/t dominate over the terms involving $1/t$ and $1/t^2$, respectively. The relative importance of the k terms depends on the wavelength under consideration. In general, we therefore have

$$\ddot{\alpha}_k + \frac{3}{\epsilon_0 t}\dot{\alpha}_k + \left(\frac{6}{\epsilon_0 t^2} + \frac{ck^2\lambda}{M_p^2 m^2}\frac{1}{a^2 t^4}\right)\alpha_k = 0, \tag{6.7}$$

where c is a constant of order 1, parametrizing our ignorance of γ_0.

For superhorizon fluctuations, defined by $k^2 \ll a^2H^2$, where aH is the comoving horizon size, we may neglect the spatial kinetic terms in (6.7). This leads to a simple pair of solutions to the classical equation (6.7) for the modes

$$\alpha^{(0)} \sim \frac{\alpha_0(k)}{t^2} \tag{6.8}$$

and

$$\alpha^{(1)} \sim \frac{\alpha_1(k)}{t^{3/\epsilon_0}}. \tag{6.9}$$

These modes decay faster at large t than the perturbations of the scalar field in ordinary slow roll inflation based on (2.1), which go to a constant as the modes cross the horizon. There are reasonable regimes of parameters for which all the sub-Planckian k/a modes have the gradient term in (6.7) suppressed relative to the effective mass squared term there.

To obtain a prediction for the observed density perturbations, we must be careful to follow them as they reenter the horizon at later times. For now, let us simply check that the perturbations do not destabilize our inflationary phase.

During inflation, the density perturbations are simply

$$\frac{\delta\rho}{\rho}\bigg|_{k,\text{during inflation}} \sim \frac{V'\delta\phi_k}{V}. \tag{6.10}$$

For our polynomial potentials, this yields $\delta\phi/\phi$. As we have seen, our perturbations die faster than in ordinary inflation due to the stronger role for the mass term in (6.7), and they certainly do not compete with ϕ in magnitude. Thus, our density perturbations during inflation are small enough to avoid backreaction. The question of whether one obtains eternal inflation depends on the magnitude of $H\delta\phi/\dot{\phi}$, which can be small in our speed of light phase. We plan to pursue in future work a detailed study of the predictions of our inflationary models as well as an analysis of reheating and other required features.

ACKNOWLEDGMENTS

At the risk of sounding like overly grateful Oscar recipients, we thank B. Acharya, T. Banks, S. Cherkis, N. Constable, E. Copeland, M. Fabinger, D. Freedman, A. Guth, M. Headrick, S. Kachru, B. Kors, N. Lambert, A. Linde, H. Liu, J. Maldacena, E. Martinec, J. Minahan, C. Nunez, H. Ooguri, M. Peskin, J. Polchinski, M. Schnabl, S. Shenker, W. Taylor, J. Troost, and D. Wands for useful and interesting discussions. We are grateful to the organizers and participants of the 2003 Benasque string theory workshop for providing a stimulating environment where this project was initiated. Thanks also to the organizers and participants of the KITP string cosmology workshop for hospitality and useful advice on some of the issues in this paper and to those of the 2003 International String School and Workshop in Iran, where these results were first unveiled. D.T. thanks SLAC for hospitality while this work was completed. E.S. is supported in part by the DOE under Contract No. DE-AC03-76SF00515 and by the NSF under Contract No. 9870115. This research was supported in part by the National Science Foundation under Grant No. PHY99-07949. D.T. is grateful to Neil Pappalardo for his generosity. This work was also supported in part by funds provided by the U.S. Department of Energy (DOE) under cooperative research agreement No. DF-FC02-94ER40818.

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