Landau-gauge condensates from the quark propagator on the lattice

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We compute the dimension-2 condensate $\langle A^2 \rangle$ and dimension-4 mixed condensate $\langle \bar{q} \not{A} q \rangle$ from the recent quenched lattice results for the quark propagator in the Landau gauge. We fit the lattice data to the Operator Product Expansion in the fiducial region 1.2 GeV $\leq Q \leq$ 3 GeV. Our result for the dynamical gluon mass at the scale of 10 GeV^2 is $m_A = 600$–650 MeV, in agreement with independent determinations. For the dimension-4 mixed Landau-gauge condensate we get $\alpha_s \langle \bar{q} g \not{A} q \rangle = (-0.11 \pm 0.03)$ GeV^4. This value is an order of magnitude larger than the $\langle G^2 \rangle$ gluon condensate.

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I. INTRODUCTION

The question of how constituent quarks arise dynamically has always been one of the most intriguing problems of QCD. The issue has prompted perturbative and nonperturbative approaches both in the continuum as well as on the lattice. Politzer [1] was the first one to compute the quark mass function using the Operator Product Expansion (OPE) in the high momentum regime in terms of the quark condensate $\langle \bar{q} q \rangle$ in the Landau gauge. This calculation was corrected and extended to a general Lorentz gauge by Pascual and de Rafael [2]. The gauge-independent gluon condensate $\langle G^2 \rangle$ was included by Lavelle and Schaden [3], where it was also foreseen that a dimension-2 condensate $\langle A^2 \rangle$ should be present. Originally it was interpreted as a signature of spontaneous gauge symmetry breaking. The dimension-4 mixed quark-gluon condensate $\langle \bar{q} \not{A} q \rangle$ was included in the analysis of Lavelle and Oleszczuk [4].

More recently, Schwinger-Dyson approaches (for reviews see, e.g., Refs. [5–8] and references therein) have been intensely applied in an attempt to understand the nonperturbative physics in the infrared domain. The phenomenological success of this approach has triggered a lot of activity on the lattice where the quark propagator has recently been computed after gauge fixing [9–12]. However, the discussion of Ref. [12] regarding the matching to the OPE is limited to the mass function. Remarkably, the dimension-2 $\langle A^2 \rangle$ condensate is related to the quark wave function renormalization [3].

Early implications of a nonvanishing dimension-2 condensate have been explored by Celenza and Shakin [13,14]. More recently Chetyrkin, Narison, and Zakharov [15,16] found that the inclusion of a tachyonic gluon mass parameter $m_A \sim 700$ MeV improves the phenomenology of the QCD sum rules in mesonic channels. For heavy quarks m_A^2 is proportional to the string tension of a short string, so it provides the short-range behavior of confining forces. Other phenomenological determinations of a nonvanishing gluon mass can be traced from the review [17].

Although the dimension-2 $\langle A^2 \rangle$ condensate naïvely breaks gauge invariance in the perturbative sense, a detailed analysis reveals that this is not so. As suggested in Refs. [18–20], there exists a nonlocal gauge-invariant condensate,

$$\langle A^2_{\min} \rangle = \frac{1}{VT} \min_g \int d^4x \langle (gA_\mu g^\dagger + g\partial_\mu g^\dagger)^2 \rangle, \quad (1)$$

which reduces to the $\langle A^2 \rangle$ condensate in the Landau gauge. Here g denotes the group element. A physical meaning has also been attached to this condensate by a perturbative gauge-covariant redefinition of the gluon field [21]. Further mounting evidence for the existence and physical relevance of the dimension-2 condensate in QCD has been also gathered from the lattice calculations [22], analytic estimates [23], purely theoretical considerations [24], and microscopic approaches [25]. Anomalous dimensions for the A^2 condensate were calculated in Refs. [26–31].

The comparison of numerical lattice QCD calculations with analytic continuum approaches, such as the perturbation theory, the OPE, or the Dyson-Schwinger approaches, requires a local gauge fixing condition on the lattice. Thereafter it is possible to give a meaning of quark and gluon correlation functions. However, it is well-known that there is no known local gauge fixing condition free of the Gribov copies (see, e.g., Ref. [32]

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and references therein). Therefore, one should keep in mind that when fixing the gauge there may still be differences in physical observables which become nonanalytic functions of the coupling constant due to the influence of the Gribov copies. If one restricts, however, to the fundamental modular region by a partial local gauge fixing, there may still be gauge-invariant operators under the residual subgroup and the BRST transformation [24]. In the Landau gauge the only dimension-2 operator satisfying the above condition is precisely A_μ^2.

In the present work we extract the dimension-2 $\langle A^2 \rangle$ condensate by comparing the lattice results for the quark propagator in the Landau gauge, presented in Ref. [10,12], to the OPE results of Refs. [3,4]. Our determination yields a novel estimate of the gluon mass m_A, as well as provides the first determination of the mixed dimension-4 condensate $\langle \bar{q} g \not{A} q \rangle$ (hereafter g denotes the strong coupling constant).

II. LATTICE DATA FOR THE QUARK PROPAGATOR IN THE LANDAU GAUGE

The inverse quark propagator can be parametrized as $S^{-1}(p) = \not{p} A(p) - B(p)$, where A and B, dependent on the quark momentum, have the meaning of the vector and scalar quark self-energies. An equivalent parametrization is via the wave function renormalization Z and the mass function M, defined as

$$S(p) = \frac{Z(p)}{\not{p} - M(p)}, \tag{2}$$

$$Z(p) = A^{-1}(p), \qquad M(p) = B(p)/A(p). \tag{3}$$

The quark propagator was calculated in Landau gauge using the "Asqtad" improved staggered action. The gauge ensemble is made of 100 quenched, $16^3 \times 32$ lattices with a nominal lattice spacing $a = 0.124$ fm, set from the static quark potential. This data was first published in Ref. [10].

The results for M and Z as functions of the Euclidean momentum Q are shown in Fig. 1 at various values of the current quark mass m. The data for M asymptote at large Q to the value of m, indicated by the horizontal lines. We note that the data at highest values of Q are not perfect, with some visible wiggles and a tendency of falling off at the end, which may be attributed to the finite-size effect. Yet, up to $Q \sim 3$ GeV the tails in M and Z look very reasonable, reaching plateaus before "hitting the wall."

As a matter of fact, the tail in M in the "fiducial" region of 1.9 GeV $\leq Q \leq$ 2.9 GeV was used successfully in Ref. [12] to verify the expression

$$M(Q) = -\frac{4\pi^2 d_M \langle \bar{q} q \rangle_\mu [\log(Q^2/\Lambda_{\mathrm{QCD}}^2)]^{d_M - 1}}{3Q^2 [\log(\mu^2/\Lambda_{\mathrm{QCD}}^2)]^{d_M}} + \frac{m(\mu^2)[\log(\mu^2/\Lambda_{\mathrm{QCD}}^2)]^{d_M}}{[\log(Q^2/\Lambda_{\mathrm{QCD}}^2)]^{d_M}}, \tag{4}$$

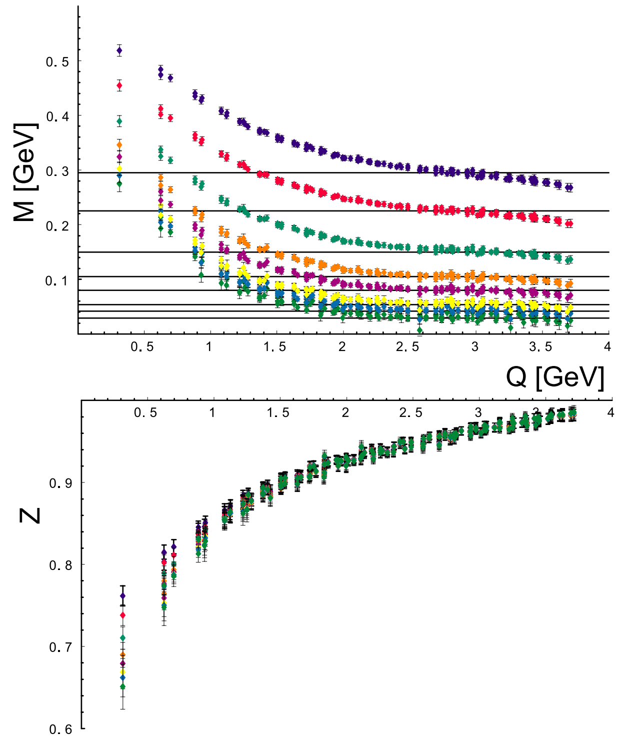

FIG. 1 (color online). The quark mass function M (top) and the wave function renormalization Z (bottom), plotted as functions of the Euclidean momentum Q. The data comes from quenched lattice calculations in the Landau gauge of Ref. [9]. Various sets of points correspond to the current quark masses $m = 29$, 42, 54, 80, 105, 150, 225, and 295 MeV, indicated by horizontal lines in the top panel. In both panels the highest sets of points correspond to the highest values of m.

where $d_M = 12/(33 - 2N_f)$ with $N_f = 0$ flavors, $\langle \bar{q} q \rangle_\mu$ and $m(\mu)$ are the quark condensate and the current quark mass at the scale μ, respectively, and $\Lambda_{\mathrm{QCD}} = 691$ MeV in the MOM scheme. This shows that the data is accurate enough to be verified against the perturbative QCD predictions.

The data for $Z(Q)$ from Ref. [9] are shown in the bottom panel of Fig. 1. A very weak dependence on m has been noted, except perhaps at low Q. Asymptotically, $Z(Q) \to 1$, as requested by the canonical normalization of the quark fields. At lower values of Q the departure of Z from unity is sizeable, with a long-range tail clearly visible.

III. MATCHING OPE TO LATTICE DATA

In our further analysis we will work with the function $A(Q)$. The data for the vector quark self-energy $A(Q)$ may be parametrized at sufficiently large values of Q as

$$A(Q) = 1 + \frac{c_2}{Q^2} + \frac{c_4}{Q^4}. \tag{5}$$

In the fitting procedure we must decide on the matching region in Q. Certainly, this choice will affect the results,

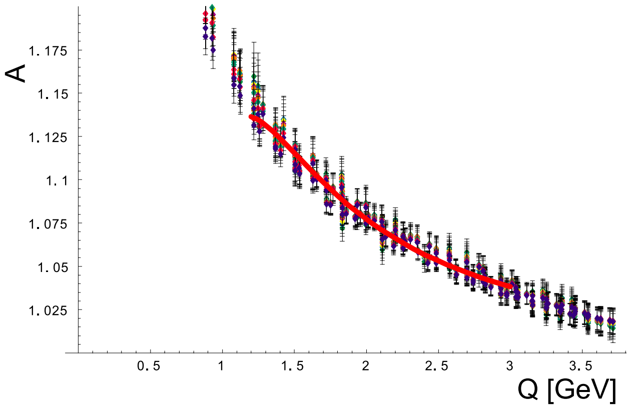

FIG. 2 (color online). The fit to the tail of the vector quark self-energy $A(Q)$. The solid line corresponds to the asymptotic formula (5) with the optimum parameters (6), while the data (including all values of the current quark masses m) are taken from the quenched lattice calculation in the Landau gauge of Ref. [9]. The asymptotic curve is drawn in the fiducial region of 1.2 GeV $\leq Q \leq 3 <$ GeV.

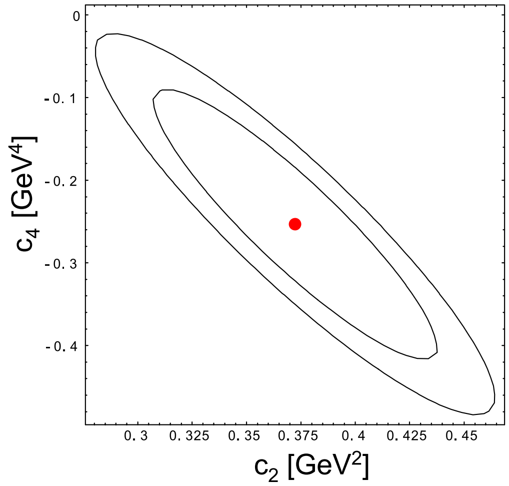

FIG. 3 (color online). The χ^2 contours corresponding to the fit of the data of Ref. [9] for the vector self-energy $A(Q)$ in the Landau gauge to formula (5). The dot indicates the optimum values of Eq. (6). The curves correspond to 68% and 95% confidence levels. Note a large correlation between c_2 and c_4.

yielding a systematic error. The values of Q cannot be too large due to finite-size effects, nor too small, where the expansion (5) is no longer accurate. We perform the χ^2 fit in the range 1.2 GeV $\leq Q \leq$ 3 GeV, which yields the optimum values

$$\begin{aligned} c_2 &= (0.37 \pm 0.04)\ \mathrm{GeV}^2, \\ c_4 &= (-0.25 \pm 0.06)\ \mathrm{GeV}^4. \end{aligned} \tag{6}$$

The errors have been calculated by jackknife, and the fit is shown in Fig. 2. The value of χ^2/DOF is 0.51, but one can see from the plot of Fig. 3 that there is a sizeable correlation between c_2 and c_4.

We have also performed the fit with three terms, appending formula (5) with the term c_6/Q^6. The result is $c_2 = 0.39\ \mathrm{GeV}^2$, $c_4 = -0.37\ \mathrm{GeV}^4$, and $c_6 = 0.15\ \mathrm{GeV}^6$ with errors that overlap with the central values for c_2 and c_4 of Eq. (6). However, due to large correlations between c_4 and c_6, no reliable information may be extracted from this three-parameter fit. More accurate data and a larger range of momenta will allow for a better determination of the $1/Q^2$expansion.

Next, we will compare the obtained values of Eq. (6) to theoretical predictions and extract estimates for the Landau-gauge condensates. At $D = 4$ the vector self-energy read out from the propagator of Ref. [3,4] is

$$A(Q) = 1 + \frac{\pi\alpha_s(\mu^2)\langle A^2\rangle_\mu}{N_c Q^2} - \frac{\pi\alpha_s(\mu^2)\langle G^2\rangle_\mu}{3N_c Q^4} + \frac{3\pi\alpha_s(\mu^2)\langle \bar{q}g\not{A}q\rangle_\mu}{4Q^4}, \tag{7}$$

where μ denotes the renormalization scale. Comparing to Eq. (6) we find for three colors

$$\alpha_s(\mu^2)\langle A^2\rangle_\mu = (0.36 \pm 0.04)\ \mathrm{GeV}^2, \tag{8}$$

or

$$g^2\langle A^2\rangle = (2.1 \pm 0.1\ \mathrm{GeV})^2, \tag{9}$$

and

$$\alpha_s(\mu^2)\langle \bar{q}g\not{A}q\rangle_\mu - \frac{4\pi}{27}\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle = (-0.11 \pm 0.03)\ \mathrm{GeV}^4. \tag{10}$$

Since $\langle \frac{\alpha_s}{\pi} G^2\rangle \simeq 0.009\ \mathrm{GeV}^4$ [33], the contribution of the $\langle G^2\rangle$ condensate to Eq. (10) is negligible compared to the mixed-condensate term. Thus $\alpha_s(\mu^2)\langle \bar{q}g\not{A}q\rangle_\mu = (-0.11 \pm 0.03)\ \mathrm{GeV}^4$.

The errors quoted throughout the paper are statistical. In addition, there are certain systematic errors originating from the choice of the fitted function $A(Q)$ of Eq. (5) and from the choice of the fiducial region in Q. Quantities quoted in physical units are also subject to the uncertainty in scale that goes with quenched lattice simulations.

IV. COMPARISON OF $\langle A^2\rangle$ TO OTHER DETERMINATIONS

The Landau-gauge condensates considered in this paper are not renormalization-invariant quantities, thus their values evolve perturbatively with the scale. The QCD evolution for $\langle A^2\rangle$ has been worked out in Ref. [27–31], with the leading-order result

$$\alpha_s(\mu^2)\langle A^2\rangle_\mu \sim \alpha_s(\mu^2)^{1-\gamma_{A^2}/\beta_0}, \tag{11}$$

where $\gamma_{A^2} = 35/4$ and $\beta_0 = 11$ correspond to evolution with no flavor. We use $\alpha_s(\mu^2) = 4\pi/(9\log[\mu^2/\Lambda^2])$, with $\Lambda = 226$ MeV for the evolution at the leading order. The exponent in Eq. (11) is equal to 9/44, hence the evolution is very slow. For instance, the change of μ^2 from 1 GeV^2 up to 10 GeV^2 results in a reduction of $\alpha_s\langle A^2\rangle$ by 10% only.

Most estimates in the literature refer to the gluon mass, related to the $\langle A^2\rangle$ by the formula $m_A^2 = \frac{3}{32}g^2\langle A^2\rangle$. Our estimate (8), when evolved with Eq. (11) from 2 to 10 GeV^2, yields

$$m_A = (625 \pm 33)\ \text{MeV}. \tag{12}$$

Evolution from 1 to 10 GeV^2 gives $m_A = (611 \pm 32)$ MeV, while evolution from 4 to 10 GeV^2 produces $m_A = (635 \pm 34)$ MeV. These values are close to many estimates made in other approaches. In particular, most of the numbers listed in Table 15 of Ref. [17] and obtained by very different techniques are in the range 0.5–1.5 GeV.

V. CONCLUSIONS

We have attempted to match the OPE to the quenched lattice data for the vector quark energy in the Landau gauge. The obtained value of the dimension-2 Landau-gauge gluon condensate $\langle A^2\rangle$ of Eq. (8) and the corresponding estimate for the gluon mass of Eq. (12) are consistent with other estimates in the literature. Thus the lattice provides an independent way of determining this condensate. The estimate for the dimension-4 mixed quark-gluon condensate of Eq. (10), made to our knowledge for the first time, is an order of magnitude larger compared to the $\langle G^2\rangle$ condensate.

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