Is there maximal mixing in the lepton sector?

Stefan Antusch,[1] Patrick Huber,[2] Jörn Kersten,[2,*] Thomas Schwetz,[2] and Walter Winter[2]

[1]*Department of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, United Kingdom*

[2]*Physik-Department T30d, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany*

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We discuss the potential of long-baseline neutrino oscillation experiments to determine deviations from maximal ν_μ-ν_τ mixing. We compare the obtainable sensitivities to predictions from neutrino mass models and to the size of quantum corrections. We find that the theoretical expectations for deviations are typically well within experimental reach.

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One of the most interesting results in recent particle physics is the evidence for large generation mixing in the lepton sector, which has been established by neutrino oscillation experiments. Two of the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ commonly used to parametrize the lepton mixing matrix are large: The "solar" mixing angle θ_{12} is approximately 33°, where maximal mixing is excluded at more than 5σ [1]. The best-fit value of the "atmospheric" mixing angle θ_{23} is very close to maximal mixing [2], where current data are still consistent with rather large deviations from maximality: At 3σ the allowed range is $0.31 \le \sin^2\theta_{23} \le 0.72$ [1]. These results are in sharp contrast to the quark sector, where generation mixing is small. Maximal mixing is very interesting from the theoretical point of view, since it corresponds to a very particular flavor structure indicating an underlying symmetry. On the other hand, if significant deviations from maximality were established, the value of θ_{23} could just be a numerical coincidence.

A precise measurement for the leading atmospheric neutrino oscillation parameters will be mainly obtained from the ν_μ survival probability determined by future long-baseline experiments. In addition to this disappearance channel, we include all appearance channels available for a given experiment in the analysis, which in some cases slightly increases the sensitivity to θ_{23}. For quantitative evaluations, we discuss the next generation of conventional beam experiments, MINOS [3], ICARUS [4], and OPERA [5]. We show their combined results after five years of running time each. In addition, we investigate the potential of the first-generation superbeams JPARC-SK [6] and NuMI off-axis [7]. To estimate the potential ten years from now, we combine the conventional beams and first-generation superbeams [8]. Eventually, we consider also the JPARC-HK superbeam upgrade [6] and a representative setup for a neutrino factory (labeled NuFact-II). The analysis techniques and precise definitions for the discussed experiments can be found in [8–10]. The most important parameter values are also given in the caption of Fig. 1.

In this figure, we show the potential of the discussed experiments to exclude maximal θ_{23}. We simulate data for fixed "true values" of θ_{23} and Δm^2_{31} and test if they can be fitted by $\theta_{23} = \pi/4$. For a fixed set $(\theta_{23}, \Delta m^2_{31})$ realized by nature, one can exclude maximal mixing at a certain confidence level (C.L.) if these values are within the corresponding shaded region. Thus, one can easily read off how far θ_{23} has to be from $\pi/4$ in order to be distinguished from it. For the current best-fit value of Δm^2_{31}, these results are summarized in Table I. From Fig. 1 and Table I, one can read off that the best sensitivity to maximal mixing is obtained by JPARC-HK: Deviations as small as 4% of $\sin^2\theta_{23}$ from maximal mixing could be established at 90% C.L. The neutrino factory is not as good as one may expect, since it measures far away from the oscillation maximum. In fact, one can show that the sensitivity can be improved by about a factor of 2 for baselines much longer than 3000 km. For all experiments, the sensitivity strongly decreases for low values of $\Delta m^2_{31} \lesssim 2 \times 10^{-3}$ eV^2, which is well within the current 3σ range. In particular, because of the sharp energy spectrum the NuMI superbeam could provide excellent results only in a rather narrow region of Δm^2_{31} around 3×10^{-3} eV^2. Eventually, if Δm^2_{31} is not too low, the combination of conventional beams, JPARC-SK, and NuMI will provide a rather good measurement at the 10% level about ten years from now.

The results in Fig. 1 are calculated for the true value $\theta_{13} = 0$. For θ_{13} close to the current bound, none of the shown results change drastically. Only the neutrino factory potential is slightly improved, since the $\nu_e \to \nu_\mu$ channel contributes somewhat to the measurement of θ_{23}. For all experiments, the results are basically independent of the mass hierarchy. Note that although the sensitivity to $|0.5 - \sin^2\theta_{23}|$ is rather good, in general it is very difficult to determine the sign ("θ_{23} degeneracy" [11,12]). Finally, we remark that, irrespective of the true value of $\sin^2\theta_{23}$, the achievable accuracy is very similar to the sensitivities shown in Fig. 1 and Table I.

Let us now analyze theoretical expectations for the deviation from maximal atmospheric mixing. It could either be a feature of a mass model itself, or it could stem from quantum corrections due to the running of θ_{23}

*Electronic address: jkersten@ph.tum.de

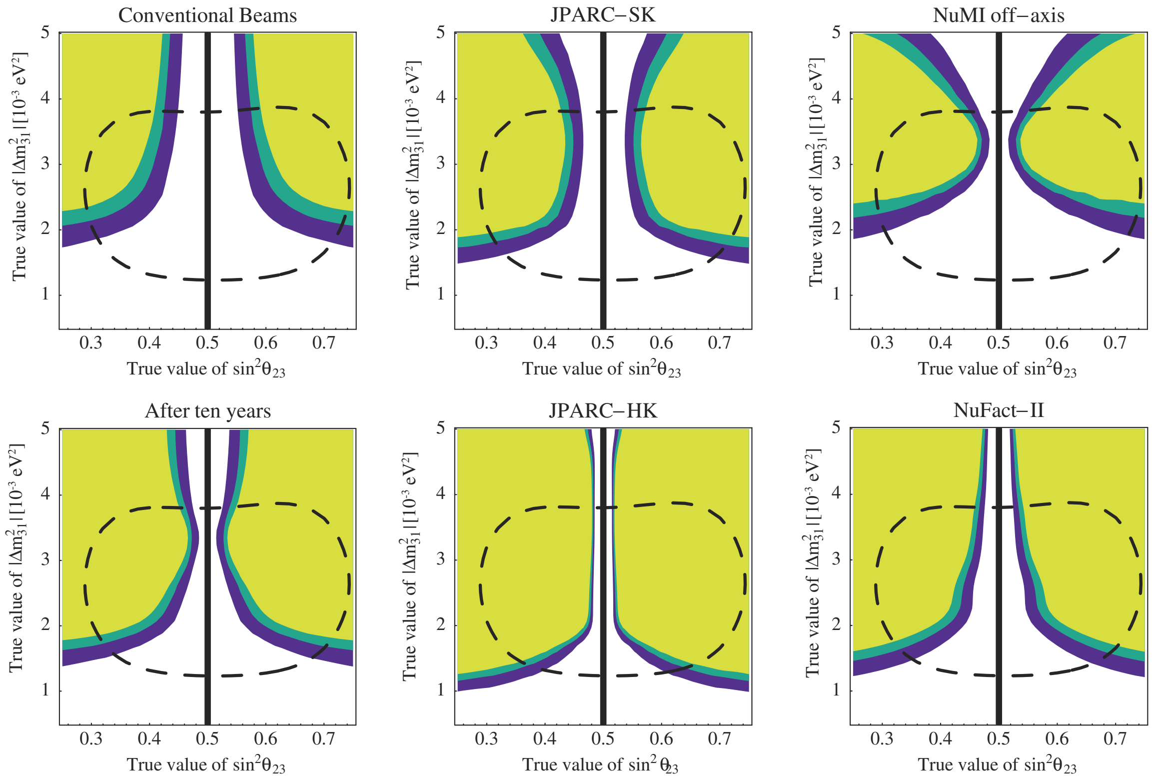

FIG. 1 (color online). The regions of the true values of $\sin^2\theta_{23}$ and $|\Delta m^2_{31}|$ where maximal mixing can be rejected by the considered experiment(s) at 1σ, 2σ, and 3σ (from dark to light shading). The currently allowed region is shown as the dashed curve at the 3σ C.L. [1]. The "conventional beams" refer to the combined MINOS, ICARUS, and OPERA experiments after five years of running time each [8]. The JPARC-SK experimental parameters are used as given in the LOI [6] with five years of neutrino running, and for the JPARC-HK upgrade a target power of 4 MW, a fiducial detector mass of 1 Mt, and two years of neutrino running followed by six years of antineutrino running are assumed [9,10]. For the NuMI superbeam, we use a target power of 0.43 MW, a detector mass of 50 kt, and five years of neutrino running at a baseline of 812 km and an off-axis angle of 0.72° [8,10]. The label "after ten years" refers to the combined potential of MINOS, ICARUS, OPERA, JPARC-SK, and NuMI [8]. For the neutrino factory NuFact-II we assume 5.3×10^{20} useful muon decays per year, a detector mass of 50 kt, a baseline of 3000 km, and operation of four years with a neutrino beam and four years with an antineutrino beam [9]. For the oscillation parameters not displayed, we choose $\theta_{13} = 0$, $\Delta m^2_{21} = 7 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{12} = 0.8$, and a normal mass hierarchy. In addition, we assume external precisions of 10% on each of Δm^2_{21} and $\sin 2\theta_{12}$, as expected from the KamLAND experiment, as well as a matter density uncertainty of 5% on the average matter density [53].

between high energy, where the model is defined, and low energy, where the experiments are performed. As to the first possibility, there exists a large variety of models aiming to explain the observed neutrino properties, utilizing various approaches such as Grand Unification (GUTs), flavor symmetries, sequential right-handed (RH) neutrino dominance, textures, or combinations of these. Many of them are based on a version of the seesaw mechanism. There are models where the predicted θ_{23} lies in a range that does not include maximal mixing at all [13–15]. In many other cases a large atmospheric angle can be explained, while almost maximal mixing would require some tuning (see, e.g., [16–23]). Other works, for instance [24–27], predict a value of θ_{23} rather close to $\pi/4$ at leading order, but various sources cause deviations that are typically still within the reach of future experiments.

In many cases, these deviations are related to small parameters, such as mass ratios. For example, even if we assume that maximal θ_{23} is predicted from properties of the neutrino mass matrix, corrections can stem from the charged lepton sector, with a typical order of magnitude of $|0.5 - \sin^2\theta_{23}| = \mathcal{O}(m_\mu/m_\tau) \sim 0.06$. Analogously, assuming that maximal θ_{23} is predicted from the charged lepton mass matrix, a hierarchical neutrino mass matrix might induce $|0.5 - \sin^2\theta_{23}| = \mathcal{O}(m_2/m_3) \sim 0.17$ [28]. Deviations of this order of magnitude are also typical in models based on sequential right-handed neutrino dominance, where maximal θ_{23} in leading order can originate from the dominant right-handed neutrino and the subdominant contribution leads to corrections (see, e.g., [14,29–32]).

Thus, the described classes of models, summarized in Table II, favor deviations from maximal atmospheric mixing that will be measurable unless Δm^2_{31} is very small. If they are not found experimentally, some new ingredients will be necessary. A value of θ_{23} very close to $\pi/4$ corresponds to a rather particular configuration of lepton

TABLE I. Minimal values of $|0.5 - \sin^2\theta_{23}|$ required to exclude maximal mixing at 90% C.L. and 3σ (absolute and relative values). For the oscillation parameters, we use the same values as in Fig. 1 and $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2.

Experiment(s)	$\|0.5 - \sin^2\theta_{23}\|$ 90% C.L.		3σ	
Conventional beams	0.100	20%	0.148	30%
JPARC-SK	0.057	11%	0.078	16%
NuMI	0.079	16%	0.126	25%
After ten years	0.050	10%	0.069	14%
JPARC-HK	0.020	4%	0.024	5%
NuFact-II	0.055	11%	0.075	15%

mixing parameters, which is clearly not compatible with the assumption of a neutrino mass matrix without any structure [47] and would require some theoretical reason. One option is employing flavor symmetries that enforce virtually maximal atmospheric mixing (see, e.g., [33–39]). On the other hand, if maximal mixing is excluded experimentally by a broad margin, this will favor either a numerical coincidence without an underlying symmetry or models which can accommodate or even predict significant deviations. Either way, precise measurements of θ_{23} will provide crucial information on the flavor structure of lepton mass models.

Models employing flavor symmetries, GUT relations, or textures typically operate at a very high energy scale. Consequently, their predictions are modified by radiative corrections, i.e., the renormalization group (RG) running to low energy, where experiments take place. This means that, even for a model predicting exactly maximal atmospheric mixing, one expects to measure deviations of the order of magnitude of the running effects [48]. Of course, the combination of deviations from $\pi/4$ at high energy and quantum corrections could, in principle, produce nearly maximal mixing at low energy. However, this

TABLE II. Selection of theoretical expectations for $|0.5 - \sin^2\theta_{23}|$ at tree level. The numbers should be considered as order of magnitude statements.

Model(s)	References	$\|0.5 - \sin^2\theta_{23}\|$
Minimal SO(10)	[13]	>0.16
SO(10) + flavor symmetry	[24–26]	$\lesssim 0.05$
SO(10) + texture	[16]	$\lesssim 0.11$
Flavor symmetries	[33–39]	0
	[40]	0.02
	[41]	0.04
Sequential RH neutrino dominance	[29,30]	0.1
+ Flavor symmetries	[14,31,32]	0.1
+ Type II seesaw upgrade	[42]	0.01–0.1
Texture zeros	[43]	0.07
	[15]	>0.1
Perturbations of textures	[44]	$\lesssim 0.16$
	[45,46]	0.005–0.1

possibility appears unnatural, since it requires a conspiracy between two effects that are not related in general.

One can easily estimate the size of the RG effects using the differential equation for θ_{23} derived in [48]. It immediately follows that the effects are negligible in the standard model due to the smallness of the charged lepton Yukawa couplings. In the minimal supersymmetric standard model (MSSM), these are enhanced by $\tan\beta$, the ratio of the two Higgs vacuum expectation values, so that the situation can change. In addition to the oscillation parameters, the running depends on the mass of the lightest neutrino, the value of the Majorana CP phases in the lepton mixing matrix, and $\tan\beta$. The MSSM results are shown in Fig. 2. For a considerable parameter range, one finds corrections to θ_{23} comparable to the precision of future experiments. Note that this is a conservative estimate, as we have neglected additional contributions coming from neutrino Yukawa couplings above the seesaw scale [49–51], which can cause sizable effects even in the standard model. Physics above the GUT scale could also contribute [52]. This provides a further argument why precision experiments have a good chance of measuring deviations from maximal atmospheric mixing.

In summary, we have discussed the potential of future long-baseline experiments to test maximal atmospheric mixing. The comparison with fermion mass models has shown that the deviations from maximal mixing predicted by many of them are large enough to be experimentally accessible. We have furthermore discussed the effects of renormalization group running, which connects the models built at very high energy scales with the measurements at low energies. These effects are also likely to cause deviations from maximality accessible by planned experiments. We conclude that, if no deviation

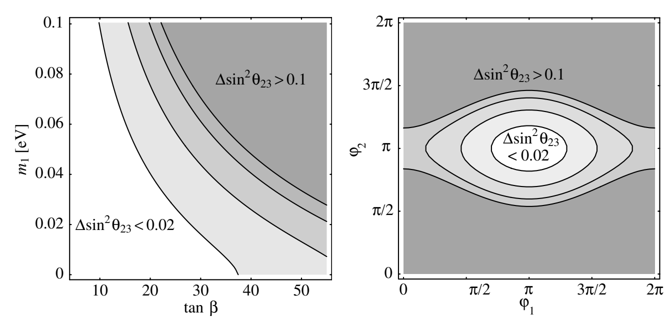

FIG. 2. Deviations of $\sin^2\theta_{23}$ from 0.5 due to the running in the MSSM between high energy $M_U \approx 2 \times 10^{16}$ GeV, where maximal θ_{23} has been taken as initial condition, and low energy $M_{EW} \approx 10^2$ GeV. The contour lines correspond to deviations roughly equal to the 90% C.L. sensitivities listed in Table I, $\Delta\sin^2\theta_{23} = 0.02, 0.05, 0.08$, and 0.1, respectively. In the left figure, the corrections are shown as a function of $\tan\beta$ and m_1, the mass of the lightest neutrino for a normal mass scheme. The right plot illustrates the dependence on the Majorana CP phases φ_1 and φ_2 (as defined in [48]) for $m_1 = 0.075$ eV. We have used $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2 and the same values as in Fig. 1 for the other parameters as further boundary conditions.

from maximal mixing can be established, the models will be severely constrained. This result will point towards a symmetry for maximal θ_{23} and indicate small quantum corrections. Finally, compared to experiments involving quarks, measurements in the leptonic sector do not suffer from the limitation by hadronic uncertainties. Thus, in the long term, the combination of precision measurements of the atmospheric angle and other neutrino parameters, such as θ_{13}, has the potential to play an important role for exploring GUT-scale physics.

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