Skyrmions coupled to the electromagnetic field via the gauged Wess-Zumino term

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In soliton models expressed in terms of the nonlinear chiral field, the electric current has an anomalous gauge-field contribution as the baryon current does. We study the spin polarized Skyrmions coupled with the electromagnetic field via the gauged Wess-Zumino term and calculate configurations of the Skyrmion and the gauge field with boundary conditions to ensure the physical charge number for baryons. Although the electromagnetic field via the gauged Wess-Zumino term affects physical quantities in small amounts, we find that the magnetic field forms a dipole structure owing to a circular electric current around the spin-quantization axis of the soliton. This is understood on an analogy with the Meissner effect in the super conductor. The electric-charge distributions turn out to have characteristic structures depending on the total charge, which suggests the intrinsic deformation of baryons due to orbital motions of the constituents.

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I. INTRODUCTION

In recent years, the proton spin puzzle has attracted much attention. As reported in Ref. [1], the quark spin is found to be responsible for only a small amount of the proton spin. The gluon spin contribution is also estimated from the global analysis of the parton distribution function [2], but has yet large uncertainties. The experiment of polarized proton collision at RHIC in progress is expected to give some clue to the gluon spin content. Such being the situation, it is desirable to study the possibility that the orbital angular momentum may contribute to some extent to the proton spin and the orbital motion may induce intrinsic deformations of the baryons. Nonspherical components in baryons have been discussed in the context of the color magnetic interaction [3], the electromagnetic transition [4], and the generalized parton distribution [5].

Deformed baryon states are investigated also in soliton models [6–8]. Since the topological soliton is quantized to the fixed (iso)spin states by rotation, the intrinsic deformations directly correlate with orbital motions in this description. Considering that the shape of the charge density is probed via the photon, we study the Skyrme model coupled with the electromagnetic field. Note that there are two kinds of terms which are brought about by the coupling of the gauge field; nonanomalous terms through covariant derivatives and anomalous terms through the gauged Wess-Zumino (WZ) term.

The gauge fields are minimally incorporated through the covariant derivatives. Using variation of an action obtained by the minimal replacement, the authors of Ref. [9] computed configurations of the gauge field as well as the soliton and estimated the magnetic moment. They found that the dipole magnetic field is generated around the soliton and the magnetic moment has reasonable value. However, the topological current is not taken into account in their variational equations.

The topological baryon current is naturally incorporated in the Maxwell equation if one considers the gauged WZ term. The gauge fields affect the system not only through the covariant derivatives but also through the gauged WZ term, which is not obtained by the minimal replacement of derivatives in the WZ term. The gauged WZ term is designed to account for the nonconservation of the axial current by the anomaly and to describe correctly anomalous processes like $\pi^0 \rightarrow 2\gamma$ in the pion sector. This term possibly influences the system also in the soliton sector. Above all, the gauged WZ term is indispensable to assure the gauge invariance of conserved currents like the baryon current and the electric current as well. Actually, the gauged WZ term provides an anomalous contribution [10] of the magnetic field to the isospin charge of the soliton.

Although the electromagnetic field is usually treated perturbatively owing to the small coupling constant, it is worth estimating the anomalous contribution of the gauge field as the WZ term is essential for the anomalous baryon current. It is possible that the gauged WZ term exerts significant influences on the physical quantities of the isospin fixed solitons.

In this work, we adopt an action including the gauged WZ term and construct spin polarized Skyrmions with a proper electric charge imposing boundary conditions upon the chiral field and the gauge field. We calculate the configuration of these fields and study the properties of the spin polarized Skyrmions, particularly characteristic spatial structure of the electric charge, current, and the magnetic field. Through these results, we discuss the

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intrinsic deformations of the charge density as well as the significance of the effects of the electromagnetic fields from the gauged WZ term in the soliton model.

II. THE SKYRME MODEL WITH THE GAUGED WESS-ZUMINO TERM

We present a model written in terms of the chiral field, $U \in \mathrm{SU}(N_f)$, and the gauge fields. To make it manifest how the gauge fields are incorporated in the system, we consider a chirally gauged model first in a general context and then restrict ourselves to a model coupled with the electromagnetic field only. The action of the chiral field consists of the nonlinear sigma term, the Skyrme term with pion mass term, and the gauged Wess-Zumino term as

$$S = \int d^4x \left[\frac{F_\pi^2}{16} \mathrm{Tr}(D_\mu U D^\mu U^\dagger) + \frac{1}{32\epsilon^2} \mathrm{Tr}[D_\mu U U^\dagger, D_\nu U U^\dagger]^2 + \frac{1}{8} m_\pi^2 F_\pi^2 (\mathrm{Tr} U - 2) \right] + \Gamma_{\mathrm{WZ}}.$$

The gauged WZ term, Γ_{WZ}, is expressed [11] compactly by the use of differential forms as

$$\begin{aligned}\Gamma_{\mathrm{WZ}} = {}& \frac{iN_c}{240\pi^2} \int \mathrm{Tr}(U^\dagger dU)^5 + \frac{iN_c}{48\pi^2} \int \mathrm{Tr}[(U^\dagger dU)^3 A_L - (U^\dagger dU)(A_L dA_L + dA_L A_L + A_L^3) - \tfrac{1}{2}(U^\dagger dU A_L)^2 \\ & + (U^\dagger dU)^2 U^\dagger A_R U A_L - dA_L (U^\dagger dU) U^\dagger A_R U - (U^\dagger dU) A_L U^\dagger A_R U A_L - (U^\dagger A_R U)(A_L dA_L + dA_L A_L + A_L^3) \\ & - \tfrac{1}{4}(U^\dagger A_R U A_L)^2 - (\mathrm{p.c.})],\end{aligned}$$

with the abbreviation p.c. for parity conjugate: $U \leftrightarrow U^\dagger$, $A_L \leftrightarrow A_R$. The covariant derivative for the chiral field is defined as $dx^\mu D_\mu U = dU + A_R U - U A_L$. The gauge group of the one forms A_L and A_R is taken to be a subgroup of $\mathrm{SU_L}(N_f) \times \mathrm{SU_R}(N_f) \times \mathrm{U_V}(1)$.

This action except the Wess-Zumino term and the pion mass term has the gauge symmetry and is invariant under the gauge transformation:

$$\begin{aligned} U &\to g_R U g_L^{-1}, \\ A_L &\to g_L(A_L + d) g_L^{-1}, \\ A_R &\to g_R(A_R + d) g_R^{-1}, \end{aligned} \tag{1}$$

where g_L and g_R are elements of the gauge group concerned. The Wess-Zumino term is constructed so as to reproduce the chiral anomaly attended by this transformation and, hence, cannot be obtained by the minimal replacement of differential operators which makes an action gauge invariant. The Noether current [10] associated with the vector symmetry generated by Q is obtained by setting $g_L = g_R \in \mathrm{SU}(N_f) \times \mathrm{U}(1)$:

$$\begin{aligned} J^\mu = {}& -\frac{F_\pi^2}{8} \mathrm{Tr} iQ[U D^\mu U^\dagger + (\mathrm{p.c.})] \\ & + \frac{1}{8\epsilon^2} \mathrm{Tr}\{[iQ, U D_\nu U^\dagger][U D^\mu U^\dagger, U D^\nu U^\dagger] + (\mathrm{p.c.})\} \\ & - \frac{iN_c}{48\pi^2} \epsilon^{\mu\nu\lambda\rho} \mathrm{Tr} iQ \bigg[(UDU^\dagger)^3_{\nu\lambda\rho} \\ & - \left\{ U D_\nu U^\dagger, F_{R\lambda\rho} + \frac{1}{2} U F_{L\lambda\rho} U^\dagger \right\} - (\mathrm{p.c.}) \bigg], \end{aligned} \tag{2}$$

where F_R, F_L are the field strengths for the gauge fields A_R, A_L, respectively. If we set $Q = 1/N_c$ to get the baryon current, only the last term—the contribution from the WZ term—survives and we find the baryon number is equivalent to the winding number. Therefore we can identify a baryon with a topological soliton in this model.

Now we restrict ourselves to study of the SU(2) chiral field and the $\mathrm{U_{EM}}(1)$ gauge field, $A_L = A_R = ieQ_{\mathrm{EM}} A_\mu dx^\mu$. The electric-charge matrix satisfies the Gell-Mann–Nishijima relation: $Q_{\mathrm{EM}} = (\tau^3 + 1/N_c)/2$. A soliton of a unit baryon number is realized in the hedgehog form: $U = \exp[i\boldsymbol{\tau} \cdot \hat{\boldsymbol{r}} F(r)]$ with the boundary conditions $F(r=0) = \pi$, $F(r=\infty) = 0$. The electric potential is assumed to be spherical,

$$eA^0 = V(r),$$

for simplicity, and the angular dependence of the vector potential is fixed as

$$e\boldsymbol{A} = h(r) \sin^2\theta \boldsymbol{\nabla}\phi,$$

to match the Ampère law. Here, θ and ϕ are the polar and azimuthal angles, respectively. All these are assumed to be independent of time. We note that the time dependence of the chiral field by a collective rotation in ungauged models can be absorbed by the gauge transformation (1) in the chirally gauged model. One can fix the gauge so that the chiral field U is static, by regarding $g_R = g_L$ as the inverse of the rotation matrix. Even under this gauge fixing condition, the gauge field itself might have a time dependence in general because of the noncommutativity of elements of the gauge group. However, we restrict ourselves to gauging an Abelian subalgebra, and therefore the gauge potential is also taken to be a static, if one lets eA^0 take a constant value in proportion to the angular velocity. In Ref. [9], a more general ansatz for the fields is adopted with axial symmetry and the fields are dependent on the polar angle θ as well as the radial distance r. However, the polar-angle dependence used here is consistent with the perturbative configuration discussed in

Sec. IV of Ref. [9] and carries the advantage on the accuracy of numerical results and is practical to compare with other results of hedgehog ansatz.

The total Lagrangian density including the kinetic term of the electromagnetic field is written in terms of $F(r)$, $V(r)$, and $h(r)$ as

$$\mathcal{L} = \frac{F_\pi^2}{8}\left[S_F^2 V^2 \sin^2\theta - F'^2 - S_F^2 \frac{1 + (1 + h\sin^2\theta)^2}{r^2}\right] + \frac{S_F^2}{2\epsilon^2}\left[\left(F'^2 + \frac{S_F^2}{r^2}\right)\left(V^2\sin^2\theta - \frac{(1 + h\sin^2\theta)^2}{r^2}\right) - \frac{F'^2}{r^2}\right] - \frac{1}{4}m_\pi^2 F_\pi^2 (1 - C_F) + \frac{1}{4\pi^2 r^2}\left[S_F^2 F'^2 V + \frac{1}{2}S_F C_F h V' \sin^2\theta - V\left(F' h\cos^2\theta + \frac{1}{2}C_F S_F h' \sin^2\theta\right)\right] + \frac{1}{2e^2}\left(V'^2 - 4\frac{h^2}{r^4}\cos^2\theta - \frac{h'^2}{r^2}\sin^2\theta\right),$$

where $S_F = \sin F(r)$ and $C_F = \cos F(r)$. After the angular integration and scaling r, V, m_π by ϵF_π to be dimensionless, we are led to the variational equations:

$$F'' + \frac{2}{r}F' + \frac{2}{r^2}(4S_F^2 F'' - S_F C_F + 4 S_F C_F F'^2)\left(1 + \frac{2}{3}h + \frac{4}{15}h^2\right) - \frac{8}{r^4}S_F^3 C_F\left(1 + \frac{4}{3}h + \frac{8}{15}h^2\right) + \frac{16 S_F^2}{3r^2}h' F'\left(1 + \frac{4}{5}h\right) - m_\pi^2 S_F + \frac{2}{3}S_F C_F V^2\left(1 - 4F'^2 + 8\frac{S_F^2}{r^2}\right) - \frac{8}{3}S_F^2 V\left(F'' V + 2F'V' + \frac{2}{r}F'V\right) - \frac{\epsilon^2}{\pi^2 r^2}\left(S_F^2 V' - \frac{2}{3}C_F^2 h V' - \frac{2}{3}S_F^2 h' V\right) = 0, \tag{3}$$

the Gauss law $\nabla \cdot \boldsymbol{E} = j^0_{\rm EM}$ with the angular-averaged charge density,

$$V'' + \frac{2}{r}V' = \alpha\left[\frac{1}{\pi r^2}S_F^2 F' - \frac{2}{3\pi r^2}(C_F^2 F' h + S_F C_F h') + \frac{2\pi}{3\epsilon^2}S_F^2 V\left(1 + 4F'^2 + 4\frac{S_F^2}{r^2}\right)\right], \tag{4}$$

and an equation deduced from the Ampère law $\nabla \times \boldsymbol{B} = \boldsymbol{j}_{\rm EM}$,

$$h'' - \frac{2}{r^2}h = \alpha\left[\frac{1}{\pi}(S_F^2 F' V - S_F C_F V') + \frac{\pi}{\epsilon^2}S_F^2\left(1 + 4F'^2 + 4\frac{S_F^2}{r^2}\right)\left(1 + \frac{4}{5}h\right)\right], \tag{5}$$

with the fine structure constant $\alpha = e^2/4\pi$.

Note that the magnetic field $\boldsymbol{B}$ [or h in the right-hand side (rhs) of Eq. (4)] is instrumental in partly producing the electric charge $j^0_{\rm EM}$ as a source of the electric field $\boldsymbol{E}$ and that $\boldsymbol{E}$ contributes the electric current $\boldsymbol{j}_{\rm EM}$ as a source of $\boldsymbol{B}$. This complementarity comes from the gauged WZ term in the action which provides the anomalous couplings between electromagnetic field and the chiral field like $\boldsymbol{E} \cdot \boldsymbol{B}\pi^0/F_\pi$. The term brings to $j^\mu_{\rm EM}$ an anomalous current of the dual field strength as $\epsilon^{\mu\nu\lambda\rho}\partial_\lambda A_\rho \partial_\nu \pi^0/F_\pi$ with the nonvanishing pion field $\partial_\nu \pi^0/F_\pi \sim i{\rm Tr}\tau^3(U\partial_\nu U^\dagger - U^\dagger \partial_\nu U)$ in the soliton sector. In addition to this, here we emphasize that the topological term of the baryon density is correctly incorporated in the charge density due to the gauged WZ term, as one can see in the first term in the rhs of Eq. (4). These contributions exhibit a contrast to the equations obtained in Ref. [9].

We solve the coupled Eqs. (3)–(5) imposing boundary conditions for the fields,

$$\begin{aligned} &F(0) = \pi, && F(r \to \infty) \propto \frac{1 + \mu_\pi r}{r^2}\exp(-\mu_\pi r), \\ &V'(0) = 0, && V(r \to \infty) = Z\frac{\alpha}{r} + V_\infty(\text{:const}), \\ &h(r \to 0) \propto r^2, && h(r \to \infty) \propto \frac{1}{r}, \end{aligned} \tag{6}$$

where Z is the charge number of the soliton (to be set at 1 or 0) and $\mu_\pi^2 = m_\pi^2 - 2V_\infty^2/3$ is a modified pion mass [7,12] caused by "rotation" of the soliton. The angular velocity is turned to an asymptotic constant V_∞ by a gauge transformation as mentioned previously and thus nonzero V_∞ corresponds to a rotation of the soliton. In practical computing, we impose a boundary condition on $V'(r)$ instead of $V(r)$ without any tuning of V_∞ and confirm finite values, actually.

However, the rotation matrix does not cover the whole SU(2) group because the generator $Q_{\rm EM}$ of $U_{\rm EM}(1)$ gauge group includes only the z component of the isospin matrix. Accordingly, Δ cannot be excluded from nucleon states without the Casimir projection as performed in Ref. [13]. In this sense, our treatment is not equivalent to the collective quantization of rotation fully, but to one-dimensional cranking. Nonetheless, the z component of isospin (as the electric charge minus one-half of the baryon number) is assured of a quantized value $\pm 1/2$ for the charged and neutral soliton ($Z = 1, 0$) only by the boundary conditions for F and V, though this does not mean a topological quantization of isospin in contrast to that of the baryon number. Since the asymptotic value V_∞ affects the boundary condition of $F(r)$, the self-consistent approach we took to solve the differential equations corresponds to the variation after projection. Furthermore, the z component of the spin takes $\mp 1/2$ for the charged and neutral solitons, since the spatial rotation means the inverse isorotation for the hedgehog configuration and the photon contribution to the spin is negligible which we confirm numerically. Therefore we regard the soliton as a spin polarized nucleon with possible admixture of the Δ component.

III. FIELD CONFIGURATIONS AND CHARGE DISTRIBUTION

We solve the differential equations with the boundary conditions for proton ($Z = 1$) and neutron ($Z = 0$) using the relaxation method. Three sets of parameters F_π and ϵ are chosen while m_π is fixed at the physical pion mass. Set I ($\epsilon = 5.37, F_\pi = 185$ MeV) is determined so that the charge radius of the proton is reproduced with physical pion decay constant. Set II ($\epsilon = 5.45, F_\pi = 129$ MeV) and Set III ($\epsilon = 4.84, F_\pi = 108$ MeV) are taken from Refs. [13,14], respectively. We find that, for Set I, r_0 such that $F(r_0) = \pi/2$ decreases by a few percent compared with that of Set II. While this means $F(r)$ for Set I is squeezed, $F(r)$ for Set III, on the contrary, swells compared to Set II. Although the boundary condition constrains the fields to have the different asymptotic behavior, profiles of $F(r)$ with Sets II and III are almost the same as that obtained in Refs. [13,14], respectively. The configurations of the electromagnetic field are also determined simultaneously, as shown in Fig. 1.

The electric potentials display correctly the Coulombic behavior for large r, and the smeared peak near the origin means finite structures of soliton's charge unlike a point particle. The function $h(r)$ gives the magnetic field as

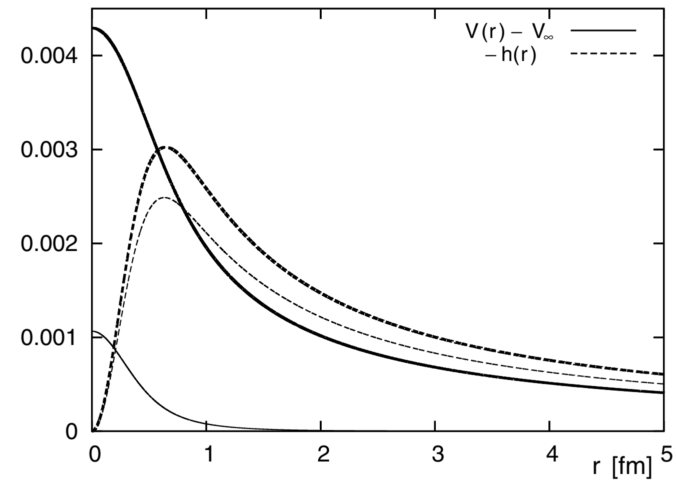

FIG. 1. Electromagnetic fields with the parameter Set II are plotted as a function of radial distance r. Bold lines are for the charged soliton and thin lines are for the neutral one. The electric potentials (normalized by ϵF_π) with the asymptotic constants subtracted are shown by solid lines and $-h(r)$'s which determine the magnetic field are by dashed lines.

$$e\boldsymbol{B} = \frac{h}{r^2} 2\cos\theta \hat{\boldsymbol{r}} - \frac{h'}{r} \sin\theta \mathbf{e}_\theta, \tag{7}$$

and the radial distance of their peak determines the size of the dipole magnetic field. The spatial distribution of the fields discussed in detail later. Physical quantities with these parameter sets are shown in Table I.

For Sets I and III, the mass of solitons deviates from the experimental values, but these receive corrections of meson loop and higher chiral-order terms [15]. Mass differences between proton and neutron are of proper magnitude for all sets of parameters but show the opposite sign. This is because the spherical ansatz for the electric field is oversimple to give the correct sign, though we take the electric field into account even for the neu-

TABLE I. Static properties of the nucleon described by the soliton with several sets of parameters. $\langle r^2\rangle_{\rm ch}$, $\langle r^2\rangle_{\rm b}$, and $\langle r^2\rangle_{\rm M}$ are the mean square charge, baryon number, and magnetic radii.

		Mass [MeV]	$\langle r^2\rangle_{\rm ch}$ [fm^2]	$\langle r^2\rangle_{\rm b}^{1/2}$ [fm]	$\langle r^2\rangle_{\rm M}^{1/2}$ [fm]	μ [μ_N]	$g_{\rm A}$	$g_{\pi NN}$	σ [MeV]
Set I	p	1326	0.757	0.416	1.128	3.27	0.616	6.08	31.4
	n	1325	−0.556	0.415	1.164	−2.84	0.616	6.25	31.3
Set II	p	933	0.773	0.563	1.118	3.95	0.572	8.20	34.1
	n	931	−0.446	0.563	1.136	−3.28	0.571	8.35	34.0
Set III	p	889	0.890	0.708	1.086	5.53	0.672	11.9	42.8
	n	887	−0.385	0.707	1.124	−4.79	0.671	12.1	42.7
exp	p	938	0.757	0.801	0.81	2.79	1.26	13.5	~30
	n	940	−0.116			−1.91			

tron. In fact, the energy of the electric field contributes about 0.2% to the proton mass which is reasonable for the mass difference, while it almost vanishes for the neutron case.

This setting is also reflected in the charge radius defined as $\langle r^2\rangle_{\rm ch} = \int d^3x r^2 j^0_{\rm EM}/e$ with the charge density,

$$j^0_{\rm EM} = \frac{e}{4\pi^2 r^2}(-S_F^2 F' - S_F^2 F' h\sin^2\theta + S_F C_F h'\sin^2\theta + 2F'h\cos^2\theta) - \frac{e}{4\epsilon^2}\left(1 + 4F'^2 + 4\frac{S_F^2}{r^2}\right)S_F^2 V\sin^2\theta, \tag{8}$$

which is obtained with assignment $Q = Q_{\rm EM}$ in Eq. (2) and scaling to the dimensionless variables. For the proton, the positive charge density is piled up and the charge radius is well reproduced comparatively. For the neutron, however, the positive and negative charge densities compete with each other and the charge radius of neutron is overestimated several times as much.

The main contribution to the charge density, (8), comes from the first term of the baryon-number density near the origin and from the last term of the isospin charge for the medium range. Since the latter has a remarkable polar-angle dependence, the spatial distribution of the electric-charge density presents a deformed or a toric structure with a core according to the isospin, as shown in Fig. 2.

The baryon-number density depends also on the polar angle through the gauge field,

$$b^0 = \frac{1}{2\pi^2 r^2}\left(-S_F^2 F' - S_F^2 F' h\sin^2\theta + \frac{S_F C_F}{2} h'\sin^2\theta + F'h\cos^2\theta\right),$$

but it is almost spherical because the small fine structure constant suppresses the magnitude of field h. On the other hand, the isospin charge is mainly contributed from the nonlinear sigma term proportional to $S_F^2 V\sin^2\theta$ as is seen in Eq. (8). This term affects the electric charge with the opposite sign for charged and neutral solitons, especially in the region such that $F(r) \sim \pi/2$ and $\sin\theta \sim 1$. As seen in Fig. 3, the solid curves of the charged soliton change according to the polar angle and smoothly merge into the dashed lines of the neutral one at the polar region. Although the angular-averaged density of the electric charge agrees with that obtained in Ref. [13], the θ dependence makes an oblate shape for the charged soliton and a prolate positive charge surrounded by a toric negative charge for the neutral soliton. The deformed charge density is interpreted as a consequence of the centrifugal force accompanying the rotation of soliton. The radius of the negative-charge torus is about 0.7 fm with the parameter Set II, which is governed by the distance r_0 such that $F(r_0) \sim \pi/2$. Since such a deformed structure cannot be realized for the nucleon with spin 1/2, the density distribution is interpreted to stand for an intrinsic deformation of the nucleon and also to reflect the admixed Δ component.

The magnetic moment μ is evaluated from the electric current as $\frac{1}{2}\int d^3x\boldsymbol{r} \times \boldsymbol{j}_{\rm EM}$ with

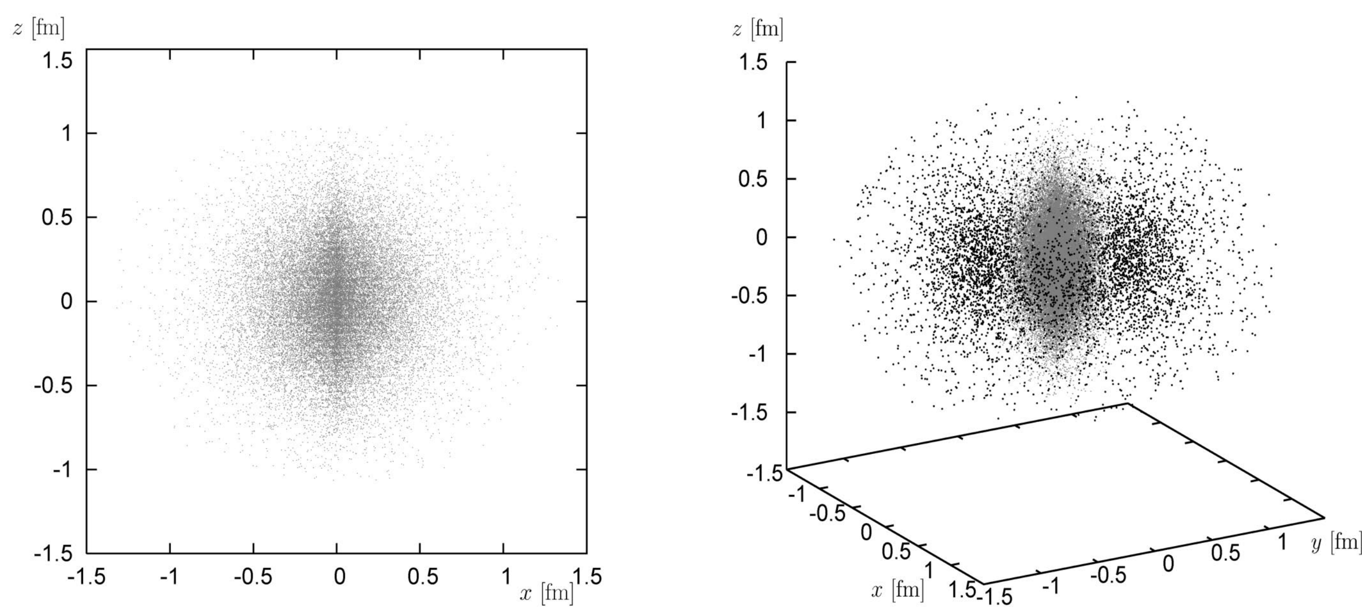

FIG. 2. The charge distribution of the charged soliton in the xz plane (left) and the neutral soliton in the xyz space (right) calculated with the parameter Set II. Note that the soliton spin is polarized along the z axis. Gray and black dots are positive and negative charges. The charge density of the charged soliton is deformed to an oblate shape and that of the neutral soliton has a Saturnian structure.

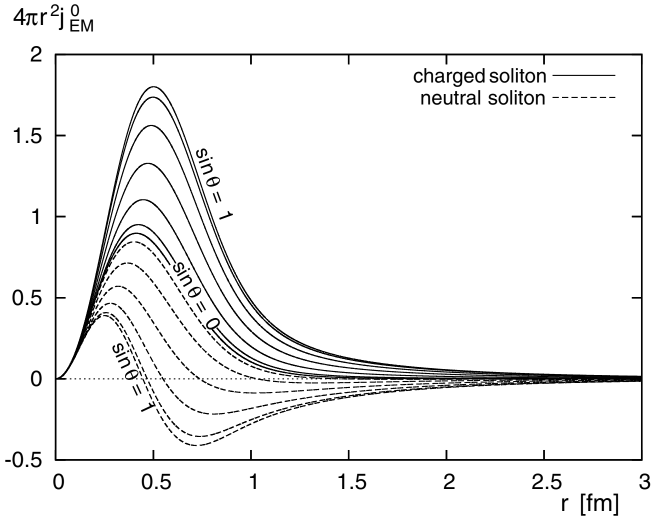

FIG. 3. The charge densities as functions of the radial distance r with several polar angles θ.

$$\boldsymbol{j}_{\rm EM} = \frac{e\sin\theta\mathbf{e}_\phi}{r}\bigg[-\frac{1}{4\pi^2}S_F^2F'V + \frac{1}{4\pi^2}S_FC_FV' - \frac{S_F^2}{4\epsilon^2}\bigg(1 + 4F'^2 + 4\frac{S_F^2}{r^2}\bigg)(1 + h\sin^2\theta)\bigg], \qquad (9)$$

which is equivalent to the estimation from asymptotic behavior of the magnetic field, $\lim_{r\to\infty} rh(r)/\alpha$, owing to the Ampère law. Our results in units of the nuclear magneton are comparable to the experimental values including their sign.

The reasonable estimation of the magnetic moment results from the dipole magnetic field generated by the current of soliton. Actually, the magnetic field expressed by Eq. (7) has a dipole structure induced by the circular current Eq. (9), as shown in Fig. 4. These field configurations are the same for both of the charged and neutral cases, while the spin of the neutral (charged) soliton is polarized (anti)parallel to the z axis. The radius of the circular current is also controlled by the scale r_0 ($\sim$0.7 fm) satisfying $F(r_0) \sim \pi/2$. The spatial distribution of the magnetic field is also understood in an analogy with the Meissner effect in the superconductor. The factor

$$\bigg(\frac{F_\pi^2}{4} - \frac{1}{\epsilon^2}\{(\partial_\mu F)^2 + \sin^2F(\partial_\mu\theta)^2\}\bigg)\sin^2F\sin^2\theta, \qquad (10)$$

which can be read as the coefficient of $(eA_\mu)^2$ in the action, plays the role of a square "mass" of the gauge field. In case of the hedgehog configuration, the large mass region forms a torus whose core, as one can see from the factor $\sin^2F\sin^2\theta$ in (10), is located at the circle around the z axis such that $F(r) = \pi/2$ and $\theta = \pi/2$. The configuration of the magnetic field is determined so as to avoid the large "Meissner mass" region to save the total energy. As a consequence of this fact, the magnetic field coils around a genus of the torus.

The axial coupling g_A defined through the axial current matrix element tends to be one-half of experiments, which agrees with Refs. [13,14]. The pion-nucleon sigma term σ evaluated from the symmetry breaking term results in the adequate value as shown in Table I.

In any case, contributions of the electromagnetic field via the gauged Wess-Zumino term are controlled by the fine structure constant and, hence, amount to a few percent of the physical quantities, at most. Therefore the complementary dynamics of the electromagnetic field is not conspicuous in the present situation. However, this anomalous contribution might be influential in high dense matter.

The gauge field, generated by the chiral field, produces a back reaction to the chiral field, as is seen in its asymptotic behavior, Eq. (6). This effect on the chiral field brings about the main difference between the physical quantities in Refs. [13,14] and ours. Taking account of the gauge field is effective also from the viewpoint of the variation after projection explained in Sec. II, though we must attend to the admixed Δ.

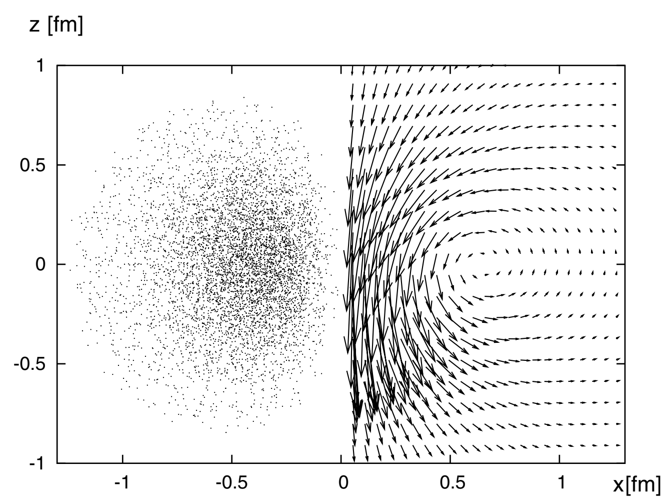

FIG. 4. The arrows in the xz plane show the magnetic field, which is axial symmetric. Dots in the negative x region represent the current density running around the z axis clockwise.

IV. CONCLUDING REMARKS

We have studied the effects of the electromagnetic field coupling to the soliton configuration via the gauged Wess-Zumino term. We pointed out that the gauged Wess-Zumino term provides the Maxwell equation with anomalous terms in the presence of the soliton, which contribute largely to the charge radii of the soliton. Unless this term is considered, the topological baryon current cannot correctly be incorporated in the charge density of variational equations. Furthermore nonvanishing pion fields make the anomalous coupling $\pi^0\boldsymbol{E}\cdot\boldsymbol{B}$ effective in the soliton sector, and the electric and magnetic fields contribute complementarily to the source current of each other. Such a current from the dual field strength is interesting theoretically, but in reality these effects are suppressed by the small coupling constant.

Because the anomalous contribution of the electric field is small, the spatial distribution of the electric current is mainly determined by the nonanomalous term of the isospin current. From this, it is concluded that the electric current has the toric configuration, and accordingly that the magnetic field has a poloidal structure coiling around the circular current according to the Ampère law. This dipole magnetic field is understood as a consequence of the Meissner-like effect. Actually, the Meissner mass of the gauge field is estimated from the chiral angle F as $\sin^2 F \sin^2\theta$ corresponding to the condensate of the Cooper pairs. In contrast to the superconductor, the large mass region does not extend uniformly but forms a finite toric structure, and, hence, the magnetic flux quantization cannot be achieved in our case.

As discussed above, the symmetric axis of the torus of the field configuration is to be interpreted physically as a spin-quantization axis of the soliton. This leads us to a reasonable consequence that the magnetic moments evaluated from the asymptotic form of the poloidal magnetic fields are (anti)aligned with the spin of the charged (neutral) soliton. The theoretical value of the magnetic moment derived from the variational solution is of the same order as the experimental value of the nucleon.

We have found that the electric charge distributions are distinctive for their polar angle dependence. The equatorial swelling of the isospin charge makes the difference while the spherical core of baryon number density is common to charged and neutral solitons. Our results show that the charged soliton has the oblate shape of the charge density and the neutral soliton has the toric negative charge surrounding the prolate shape of positive charge. These distributions are consistent with the pion cloud rounding the nucleon or may imply a u-d diquark core accompanied by the other valence quark. In any case, the characteristic distributions of the charge density suggest the intrinsic deformation of baryons and the isospin-dependent quadrupole moment, which might be observed in the $N\Delta$ transition.

As in the dynamo theory for the planetary magnetic field, a poloidal magnetic field may induce a toroidal magnetic field through the rotation of the soliton, and vice versa, even though the physical scale is quite different. It is our future work to take account of this effect by extending the variational space of the magnetic field. The nonradial electric field is also important to estimate the mass of the proton and the neutron and to discuss a correlation between the orbital motion and the intrinsic deformation.

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[1] European Muon Collaboration, J. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988); Nucl. Phys. **328**, 1 (1989).
[2] M. Hirai, S. Kumano, and N. Saito, Phys. Rev. D **69**, 054021 (2004).
[3] N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. D **25**, 2394 (1982).
[4] A. Wirzba and W. Weise, Phys. Lett. B **188**, 6 (1987); A. Hosaka, M. Takayama, and H. Toki, Nucl. Phys. **A678**, 147 (2000).
[5] A. V. Belitsky, X. Ji, and F. Yuan, Phys. Rev. D **69**, 074014 (2004).
[6] C. Hajduk and B. Schwesinger, Phys. Lett. B **145**, 171 (1984); J. Wambach, H. W. Wyld, and H. M. Sommerman, Phys. Lett. B **186**, 272 (1987).
[7] J. P. Blaizot and G. Ripka, Phys. Rev. D **38**, 1556 (1988).
[8] F. Leblond and L. Marleau, Phys. Rev. D **58**, 054002 (1998).
[9] B. M. A. G. Piette and D. H. Tchrakian, Phys. Rev. D **62**, 025020 (2000).
[10] Y. Brihaye, N. K. Pak, and P. Rossi, Phys. Lett. B **149**, 191 (1984).
[11] C. M. Hull and B. Spence, Mod. Phys. Lett. A **6**, 969 (1991).
[12] N. Dorey, J. Hughes, and M. P. Mattis, Phys. Rev. D **50**, 5816 (1994); M. Schleif, R. Wunsch, and T. Meissner, Int. J. Mod. Phys. E **7**, 121 (1998).
[13] G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).
[14] G. Adkins and C. Nappi, Nucl. Phys. **B233**, 109 (1984).
[15] B. Moussallam, Ann. Phys. (N.Y.) **225**, 264 (1993); G. Holzwarth and H. Walliser, Nucl. Phys. **A587**, 721 (1995).