$D_s^{*+}(2317)$ and $D_s^{*+}(2460)$: Tetraquarks bound by the 't Hooft instanton-induced interaction?

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We calculate the influence of the 't Hooft interaction on the single-charm (C = 1) tetraquark ($cq\bar{q}^2$) mass spectrum and find that three SU(3)-flavor multiplets ($\mathbf{\bar{3}_A}$, **6**, $\mathbf{\overline{15}}$) are significantly lowered, while the $\mathbf{\bar{3}_S}$-plet is lifted, as compared with their unperturbed masses. States with the D_s^+ quantum numbers appear in two of the three lowered multiplets. The lower of the two states drops below both of the Okubo-Zweig-Iizuka (OZI)-allowed decay thresholds, thus being identified with $D_s^{*+}(2320)$, while the heavier state drops below the $\eta + D_s^+$ threshold, but stays above the $K + D$ one. A signature prediction of this theory is the lowering of the exotic tetraquark **6**-plet and $\mathbf{\overline{15}}$-plet: most of these states cannot consist of only one quark and one antiquark. As a test we suggest a search for two quasistable exotic tetraquarks with open-charm and strangeness: the (isotriplet) $cs\bar{q}\,\bar{q}$ and the (isodoublet) $cq\bar{s}\,\bar{s}$.

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I. INTRODUCTION

Recent discovery of positive parity states mesons with open-charm and strangeness $D_s^+(2317)$ [1] and $D_s^+(2460)$ [2,3], at least 200 MeV below their $c\bar{s}$ counterpart's expected masses and very close to the Silvestre-Brac and Semay's (10 yr old) [4] predictions of open-charm "tetraquark" ($q^2\bar{q}^2$) states at 2357 MeV (spin S = 0) and 2456 MeV (spin S = 1), respectively, has given a boost to their interpretation as tetraquarks. These predictions were made in a constituent quark model that leads to incorrect (too low) masses of η, η' mesons, which are, however, among the OZI-allowed decay products of these tetraquarks. Those predictions must therefore be taken with a grain of salt.

We are not the only ones to revive the notion of tetraquarks and suggest that the $D_s^+(2320)$ is one: that has been done in Refs. [1,5,6], and by others. Most authors do not discuss the discrepancy between the number of observed (one) and predicted (three) tetraquarks, however, nor do they try to explain their anomalously low mass(es). Although tetraquarks are by no means the only possible explanation of these states at present, indications do exist),[1] multiply observations of the $Z^+/\Theta^+(1540)$ exotic baryon, give additional credence to the multiquark interpretation of $D_s^+(2320)$.

In this Brief Report: (a) We include the (flavor-dependent instanton-induced) 't Hooft interaction in the constituent quark model. This leads not only to correct η, η' meson masses, but also to tetraquark mass splitting in apparent accord with experiment. States with quantum numbers of the D_s^+ appear in two of three lowered multiplets. After mixing, due to flavor symmetry breaking and the 't Hooft interaction, is taken into account, the lowest of the three states drops below all of the corresponding thresholds, thus being identified with $D_s^{*+}(2320)$, the second lowest state drops below the $\eta + D_s^+$ threshold, but stays above the $K + D$ one: it; perhaps it can be identified with $D_s^{*+}(2460)$. (b) We point out that if this theory is correct, then a large subset of the exotic tetraquarks" masses are also lowered by the 't Hooft interaction. Most significantly at least two exotic scalar tetraquark states with open-charm and strangeness ought to be quasistable with masses that are close to or even below their respective two-meson ("decay") thresholds: (1) the negative strangeness (S = −1) charmed (C = 1) isotriplet of states with the quark content $cs(\bar{q}\,\bar{q})_{I=1}$, where q denotes either u or d quark. This state can only decay into $D + \bar{K}$; and (2) the double-strangeness (S = 2) charmed (C = 1) isodoublet triplet, with the quark content $cq(\bar{s}\,\bar{s})$, whose mass is also lowered by the 't Hooft interaction below the $D_s + K$ threshold. As a test of the theory we suggest experimental searches for these two new tetraquark states.

This Brief Report is organized in four sections. In the next section we make some general remarks about the framework of our calculation. This is necessary as a prelude to the inclusion of the 't Hooft interaction that follows in Sect. III. Then we discuss our results, suggest new experiments and draw conclusion in Sect. IV.

II. A FRAMEWORK FOR TETRAQUARKS: THE CONSTITUENT QUARK MODEL

Constituent quark models of tetraquarks have been based exclusively on the $F \cdot F$ Lorentz vector interaction induced two-quark potentials

$$V_{ij} = (\mathrm{F}_i \cdot \mathrm{F}_j)\mathcal{V}_{ij}, \tag{1}$$

where F_i^a is the SU(3) generator (Gell-Mann matrix divided by two) for the $i^{\rm th}$ quark. There are two possible color singlets for tetraquarks: one state is symmetric

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[1]For example, the 0^+ and 1^+ states might be anomalously light due to their chiral symmetry properties (Bardeen et al. [7]), or because of a particular mixture of tensor and LS forces (Chan and Jackson [8]).

($|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}\rangle$), and another antisymmetric ($|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle$) under the interchange of the color indices of the two quarks or antiquarks. As the Pauli principle applies only to identical particles, i.e., as it antisymmetrizes either quarks or (only) antiquarks, but not the $q\bar{q}$ pairs, the "unphysical" basis spanned by $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle$ and $|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}\rangle$ is better suited to the application of the Pauli principle than the ("physical") asymptotic state basis. As we know that the asymptotic two-meson state evolves into the $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle$ at short distances, we must apply the Pauli principle for that state and obtain the appropriate spin-flavor selection rules. The $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle$ state in isolation, i.e., when mixing with the $|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}\rangle$ state is ignored, is bound and the motion of the two $q\bar{q}$ clusters is confined in space, thus allowing easy application of the perturbative and variational approximations.

It has been shown in Ref. [9] that tetraquarks tend to be more stable (deeply bound) in the $F \cdot F$ quark model as the mass of one ("heavy") quark increases. This fact is related to both the increase of pseudoscalar meson[2] masses (due to the increased explicit chiral symmetry breaking) and to the quark mass difference effects in the kinetic energy, which lead to increased binding of mass/flavor asymmetric tetraquarks. Theoretical studies of tetraquark stability conducted in the late 1980s and early 1990s by Silvestre-Brac and Semay [4] indicated a number of potentially bound states.

Subsequent studies with the same "realistic" two-quark potential [10] and an improved variational method [11] have shown that only (doubly heavy) tetraquarks containing a bb pair can be reasonably expected to be bound [12]. A weak color dependent three-quark potential, of the kind introduced in Ref. [13], well within the constraints of three-quark spectroscopy, would bind even the (far lighter) $cc\bar{q}\,\bar{q}$ spin S $= 1$ state, Ref. [12], however. Recent discovery of light $c\bar{s}$ states indicates that perhaps even lighter tetraquark systems might be bound, but the precise origin of the necessary attraction, such as, e.g., the color dependent three-quark force [13], is not yet as well established as the 't Hooft force. One of the main results of this paper will be to provide evidence that the flavor-dependent 't Hooft interaction may yield sufficient attraction to bind both the $c\bar{s}q\bar{q}$ and the $cc\bar{q}\,\bar{q}$ states.

Unfortunately, the full problem is still beyond the reach of modern variational methods [14]: there are three flavor channels with three different thresholds and different flavor mixings in the asymptotic (two-meson) and "inner" (tetraquark) regions. For each of these flavor channels there are also two coupled color singlet channels. Thus one must solve the Schrödinger equation variationally in six channels, meaning that one must find three minimal energies at any given meson-meson separation and compare them with the asymptotic (two-meson) ones. The variational method allows, however, positive identification of only those states that lie below all three two-meson thresholds, i.e., only of truly bound tetraquark states. As only one tetraquark is likely to be bound, see below, the variational approach is facing serious difficulties and we must resort to perturbation theory.

One of the most acute problems of the constituent quark model is the fact that the standard two-body potentials, e.g., those used by Silvestre-Brac and Semay, are flavor independent, thus leaving the η degenerate with pions (this is just the ancient "$U_A(1)$ problem", although there is no chiral symmetry in the nonrelativistic quark model). Silvestre-Brac and Semay compare their results with the physical thresholds calculated with $m_\eta = 547$ MeV, however! This excess/discrepancy of 407 MeV in the height of the threshold is sufficient to artificially bind even the most underbound tetraquark state. Clearly, that is not good enough for comparison with experiment, so one must first cure the η, η' mass problem. We do so by introducing the 't Hooft interaction into tetraquarks.

III. THE 'T HOOFT INTERACTION

A. Preliminaries

The 't Hooft interaction is induced by instantons in QCD [15]. it reads

$$\mathcal{L}_{\rm tH}^{(6)} = -K\{\det_f[\bar{\psi}(1+\gamma_5)\psi] + \det_f[\bar{\psi}(1-\gamma_5)\psi]\} \quad (2)$$

where, $\det_f[\bar{\psi}(1+\gamma_5)\psi]$ is a determinant in the flavor space only. Its primary purpose was to correct the η, η' masses, but in the meantime it has been shown that this interaction also affects the scalar meson- and the baryon spectra [16]. The effective coupling constant K has been determined [16] as

$$12K = -\langle\bar{q}q\rangle_0^{-3}[f_{\eta'}^2 m_{\eta'}^2 + f_\eta^2 m_\eta^2 - 2f_K^2 m_K^2]. \quad (3)$$

Inserting the experimental values of the pseudoscalar meson masses and decay constants into Eq. (3), as well as the quark condensate taken here as $\langle\bar{q}q\rangle = -(225\ \text{MeV})^3$, the 't Hooft coupling constant comes out at $K = 390\ \text{GeV}^{-5}$. Note, however, that the large uncertainty and the high exponent of the quark condensate $\langle\bar{q}q\rangle_0 = [-(225 \pm 25)\ \text{MeV}]^3$ lead to an even larger uncertainty in the value of K. This uncertainty shall translate into a wide margin of our tetraquark mass predictions.

The two-body 't Hooft interaction leads to the following effective two-quark potential

$$V_{12} = 4K\langle\bar{q}q\rangle_0 P_{12}^{\bar{3}}(1 + \gamma^5{}_1\gamma^5{}_2)\delta(\mathbf{r}_1 - \mathbf{r}_2)$$
$$P_{12}^{\bar{3}} = \left[\frac{1}{3} - \frac{1}{4}\lambda_1 \cdot \lambda_2\right]. \quad (4)$$

[2] which are the tetraquarks" OZI-allowed strong decay products.

TABLE I. Exotic and cryptoexotic $\overline{\mathbf{15}}$-plet tetraquark $cq\bar{q}^2$ states.

Exotics	Cryptoexotics
$T(cu\bar{d}\,\bar{s})_{I=1}$	$T(cd\bar{u}\,\bar{d})_{I=1/2}$
$T(cs\bar{u}\,\bar{d})_{I=0}$	$T(cu\bar{u}\,\bar{d})^{+}_{I=1/2}$
$T(cq\bar{s}\,\bar{s})_{I=1/2}$	$T(cq\bar{s}\,\bar{q})^{+}_{I=0}$
$T(cq\bar{q}\,\bar{q})_{I=3/2}$	

The flavor dependence of this potential is proportional to the $\bar{\mathbf{3}}$ projection operator $P_{12}^{\bar{3}}$, i.e., it only operates in the flavor antisymmetric qq state. Note, however, that in the $q\bar{q}$ channels the same flavor factor is *not* a flavor singlet projector.

The 't Hooft interaction also leads to the following three-quark potential

$$V_{123} = 12KP_{123}^{1}(1 + \sum_{i<j}^{3} \gamma^5{}_i\gamma^5{}_j)\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_3 - \mathbf{r}_2)$$

$$12P_{123}^{1} = [\frac{4}{9} - \frac{1}{3}\sum_{i<j}^{3} \lambda_i \cdot \lambda_j + d^{abc}\lambda_1^a\lambda_2^b\lambda_3^c]. \quad (5)$$

As can be seen from Eq. (5) the flavor-dependent part of the 't Hooft three-quark potential is just the three-quark flavor SU(3) singlet projection operator P_{123}^{1} [16]. Conversion of the above formulas to systems with one, or two antiquarks can be accomplished according to the rules and formulas spelled out in Ref. [13] (see "C-conjugation"). Depending on the number of charmed quarks in the tetraquark we may need only the two-body, or both the two- and three three-body potentials.

Perhaps the most important property of the $U_A(1)$ symmetry breaking 't Hooft interaction is its sensitivity to the flavor of the state. Of course, only the three lightest flavors enter the 't Hooft interaction, the charm and heavier flavors effectively decoupling. light (u, d, s) We must therefore make an $SU_F(3)$ flavor multiplet analysis of the open-charm light-heavy tetraquarks.

B. SU(3) multiplets

We find several new flavor SU(3) multiplets appearing in the charm C = 1 tetraquark system: $\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}} \oplus \bar{\mathbf{3}}' \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$. There are two distinct triplets in this Clebsch-Gordan series, that may be distinguished in two ways: either (i) by way of the $q\bar{q}$ pair's flavor multiplet: then we denote them with subscripts 1, 8, i.e., as $\mathbf{3}_{1,8}$; or (ii) by way of their (permutational) symmetry, or antisymmetry with respect to the interchange of the two quarks, in which case we denote the states with subscripts S, A, i.e., as $\mathbf{3}_{\mathrm{S,A}}$. Conversion from one basis to another is straightforward. The former basis may be considered "more physical", as it relates to the external, asymptotic two-meson states" flavor, but the latter basis has the advantage of simplifying the mixing matrix of the two antitriplets and the $\overline{\mathbf{15}}$-plet, due to either the s-u/d quark mass difference, or the 't Hooft interaction, as well as simplifying the application of the Pauli principle.

TABLE II. Quark contents of the $cq\bar{q}^2$ tetraquark $\bar{\mathbf{3}}$-plet, and $\overline{\mathbf{15}}$-plet states that mix, and their predicted masses (MeV). The states denoted by $(\bar{\mathbf{3}}_{\mathbf{A}} - \overline{\mathbf{15}})_{\pm}$ are the heavier and lighter admixtures, respectively.

	$\tilde{D}^0$ $T(cs\bar{u}\,\bar{s})$	$\tilde{D}^+$ $T(cs\bar{s}\,\bar{d})$	$\tilde{D}_s^+$ $T(cq\bar{s}\,\bar{q})_{I=0}$
$\bar{\mathbf{3}}_{\mathbf{A}}$	2317	2317	2317
$(\bar{\mathbf{3}}_{\mathbf{A}} - \overline{\mathbf{15}})_{-}$	2438	2443	2505
$(\bar{\mathbf{3}}_{\mathbf{A}} - \overline{\mathbf{15}})_{+}$	3101	3106	3309

Some of these multiplets have their counterparts in the $c\bar{q}$ meson sector, while others do not: the two antitriplets ($\bar{\mathbf{3}}$) are analogous to $c\bar{q}$ mesons, so they are called cryptoexotics; the sextet ($\mathbf{6}$) and (most of) the $\overline{\mathbf{15}}$-plet do not have $c\bar{q}$ analogons, so (most of) their members are exotics, with three significant exceptions, see Table I.

The sextet, the antitriplet and the $\overline{\mathbf{15}}$-plet states" quark contents are tabulated in Tables II, III, and IV, see below.

C. Matrix elements

We shall evaluate the 't Hooft interaction's effects in first order perturbation theory. We use the *chromo-harmonic* potential, i.e., $\mathcal{V}(\mathbf{r}_{ij})$ in Eq. (1) is the harmonic oscillator potential. We find the following 't Hooft interaction flavor-spin-spatial matrix elements for scalar tetraquarks in the color $|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle$ and spin $S = 0, 1$ states

$$\langle V\rangle_{\bar{3}_A} = 8K[\langle\bar{q}q\rangle_0\langle\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle - 2\langle\delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle] \quad (6)$$

$$\langle V\rangle_{\bar{3}_S} = -16K\langle\bar{q}q\rangle_0\langle\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle \quad (7)$$

$$\langle V\rangle_{\overline{15}} = 12K\langle\bar{q}q\rangle_0\langle\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle \quad (8)$$

$$\langle V\rangle_{6} = 8K[2\langle\bar{q}q\rangle_0\langle\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle + 2\langle\delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_1 - \mathbf{r}_2)\rangle] \quad (9)$$

TABLE III. Quark contents of the $cq\bar{q}^2$ tetraquark $\mathbf{6}$-plet and their predicted masses (MeV).

$\Sigma_c^0(s=1)$ $T(cd\bar{u}\,\bar{d})_{I=1/2}$	$T(cu\bar{u}\,\bar{d})_{I=1/2}$	$T(cs\bar{u}\,\bar{d})_{I=0}$	$T(cd\bar{u}\,\bar{s})_{I=1}$	$T(cq\bar{s}\,\bar{q})_{I=1}$	$T(cu\bar{d}\,\bar{s})_{I=1}$
2468	2468	2468	2468	2468	2468

TABLE IV. Quark contents of the $cq\bar{q}^2$ tetraquark $\overline{\mathbf{15}}$-plet and their predicted masses (MeV).

$\Sigma_c^0(s=1)$ $T(cd\bar{s}\,\bar{s})$	$T(cu\bar{s}\,\bar{s})$	$T(cq\bar{q}\,\bar{q})$	$T(cd\bar{u}\,\bar{s})$	$T(cq\bar{s}\,\bar{q})_{I=1}$	$T(cu\bar{d}\,\bar{s})$
2529	2524	2255 $\pm$10	2392	2392	2392
$\Sigma_c^+(s=-1)$					
$T(cq\bar{u}\,\bar{q})$	$T(cq\bar{d}\,\bar{q})$	$T(cq\bar{s}\,\bar{q})$	$T(cs\bar{u}\,\bar{d})$	$T(cs\bar{q}\,\bar{q})_{I=1}$	$T(cs\bar{d}\,\bar{u})$
			2392	2392	2392

In the lowest nonrelativistic approximation and ignoring the $m_s - m_{u/d}$ mass difference we find

$$\langle\Psi|\delta(\mathbf{r}_1-\mathbf{r}_2)|\Psi\rangle = \left(\frac{m_q\omega}{2\pi}\right)^{3/2} \equiv I \tag{10}$$

$$\langle\Psi|\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)|\Psi\rangle = \left(\frac{m_q\omega}{\sqrt{3}\pi}\right)^3 = \left(\frac{2}{\sqrt{3}}\right)^3 I^2 \simeq 1.54 I^2, \tag{11}$$

where ω= 500 MeV is the (harmonic) oscillator frequency in the model, and the constituent quark mass, $m_q =$ 313 MeV, has been taken as one third of the nucleon's. Note that here again the uncertainty in $\langle\bar{q}q\rangle_0$ enters the picture: therefore the ratio of the two- and three-body contributions is a variable. Thus we have the following tetraquark energy/mass shifts

$$\delta E(\bar{\mathbf{3}}_{\mathbf{A}}) = 8KI[\langle\bar{q}q\rangle_0 - 3.08I] \simeq -3.7A < 0 \tag{12}$$

$$\delta E(\bar{\mathbf{3}}_{\mathbf{S}}) = -16K\langle\bar{q}q\rangle_0 I \equiv 4A > 0 \tag{13}$$

$$\delta E(\overline{\mathbf{15}}) = 12K\langle\bar{q}q\rangle_0 I \equiv -3A < 0 \tag{14}$$

$$\delta E(\mathbf{6}) = 8KI[2\langle\bar{q}q\rangle_0 + 3.08I] \simeq -2.3A < 0, \tag{15}$$

where the numerical estimate is based on the value of the quark condensate $\langle\bar{q}q\rangle_0 = -(225\ \text{MeV})^3$, which leads to $K = 390\ \text{GeV}^{-5}$ and $A = 70$MeV. Because of the aforementioned uncertainty in the value of the quark condensate $\langle\bar{q}q\rangle_0$, we have an uncertainty in A of at least a factor of 2, i.e., A may certainly vary between 70 MeV and 150 MeV, and the ratio of strengths of the two- and three-body contributions varies with it.[3] We shall turn this procedure around and use the experimental mass of $D_s(2320)$ to fix A.

D. Mass and state mixing

Flavor state mixing determines the flavor content of the physical tetraquarks, which in turn determines their decay channels. Flavor SU(3) symmetry breaking by quark mass differences, leads to splitting within flavor multiplets and to mixing of members of different multiplets. The 't Hooft interaction, although SU(3) symmetric, also adds to the mixing.

The 3×3 mass matrix for the D_s states is nondiagonal in general, but in the aforementioned basis ($\bar{\mathbf{3}}_{\mathbf{A}}, \bar{\mathbf{3}}_{\mathbf{S}}, \overline{\mathbf{15}}$) there is no mixing between the $\bar{\mathbf{3}}_{\mathbf{A}}$, whose mass equals $[m_c + 3\bar{m} + \delta E(\bar{\mathbf{3}}_{\mathbf{A}})]$, and the other two multiplets ($\bar{\mathbf{3}}_{\mathbf{S}}, \overline{\mathbf{15}}$), i.e., the mass matrix reduces to:

$$M_{\mathrm{D_s}} = \begin{pmatrix} [m_c + \frac{3}{2}(\bar{m}+m_s) + \delta E(\bar{\mathbf{3}}_{\mathbf{S}})] & \frac{3}{2}(\bar{m}-m_s) \\ \frac{3}{2}(\bar{m}-m_s) & [m_c + \frac{3}{2}(\bar{m}+m_s) + \delta E(\overline{\mathbf{15}})] \end{pmatrix} \tag{16}$$

where

$$\bar{m} = \frac{1}{3}(m_u + m_d + m_s). \tag{17}$$

Complete diagonalization is accomplished by way of mixing of the $\bar{\mathbf{3}}_{\mathbf{S}}$, and the $\overline{\mathbf{15}}$ states; the mixing angle is

$$\tan 2\theta_{\mathrm{D_s}} = \frac{3(\bar{m}-m_s)}{\delta E(\bar{\mathbf{3}}_{\mathbf{S}}) - \delta E(\overline{\mathbf{15}})}. \tag{18}$$

Similar mass matrices and mixing angles can be written for the nonstrange tetraquark members of the two triplets and the $\overline{\mathbf{15}}$-plet.

E. Without 't Hooft interaction

We set $A = 0$ in the equations above. Diagonalization of the mass matrix leads to two degenerate lighter tetraquark triplets with opposite exchange symmetries at $m_c + m_u + m_d + m_s$, and a third one at $m_c + 3m_s$, i.e., the heavier and the two lighter states are separated by exactly two strange u,d quark mass differences $m_u + m_d - 2m_s = -300$ MeV . This is equivalent to the ideal tetraquark mixing angle $\theta_{\mathrm{D_s}} = -45^o$.

F. With 't Hooft interaction

In Fig. 1 we show the results of the diagonalization: we see a clear splitting of two diagonalized states that grows with the 't Hooft coupling strength $A = -4K\langle\bar{q}q\rangle_0 I > 0$, see Eq. (15). At $A = 108$ MeV the lowest energy solution ($E(\bar{\mathbf{3}}_{\mathbf{A}})$) crosses 2317 MeV, i.e., this tetraquark may be identified with $\tilde{D}_s^+(2317)$. The mixing, in general, is not ideal: at the fitted value of $A = 108$ MeV (see above)

[3] Reproducing the η, η' masses in the nonrelativistic quark model requires $A \simeq 85$ MeV, which is roughly in line with the above arguments.

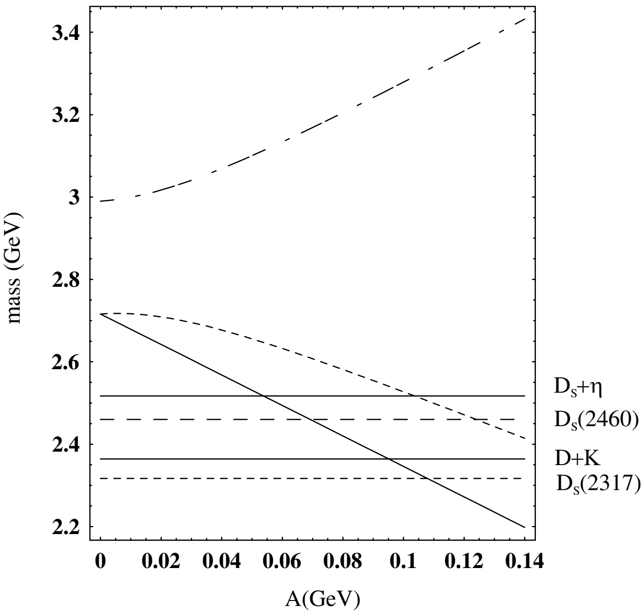

FIG. 1. Masses of the mixed states as a function of the matrix element A, i.e., of the 't Hooft coupling constant K and the quark condensate (see text): $\bar{\mathbf{3}}_{\mathbf{A}}$ (solid line), and the two solutions to the $\bar{\mathbf{3}}_{\mathbf{S}}$-$\overline{\mathbf{15}}$ mixing (dot-dashed line and dashed line). Horizontal lines denote the corresponding thresholds (solid line) and the observed states'' masses (dashed line): $D_s^*(2317)$ and $D_s^*(2460)$.

we have $\theta_{D_s} = -5.1^o$ vs ideal mixing angle of -45^o (at $A = 0$). Similarly, in the nonstrange sector, the mixing angle is $\theta_D = -5.4^o$ vs. ideal mixing angle of -69.5^o (at $A = 0$) in this channel. Note that in this simple calculation we have ignored the color-spin hyperfine interaction. This may lead to inadequate hyperfine mass splittings between the S = 0 and S = 1 tetraquarks, but it should not affect the ordering of the different flavor states, however.[4]

IV. RESULTS, EXPERIMENTAL SUGGESTIONS AND CONCLUSIONS

Our results show that the 't Hooft interaction may be the cause of $\tilde{D}_s^+(2317)$'s anomalously low mass. Note that for the same value of 't Hooft couplings K and A, another scalar $\tilde{D}_s^+$ tetraquark state ought to exist around 2505 MeV, i.e., between the two thresholds (this state would be a degenerate partner of $D_s(2317)$ in the absence of 't Hooft interaction). A manifest candidate for this state appears to be the $D_s^+(2460)$, even though its spin is likely unity [3], as that conclusion depends on the inclusion of the color-spin hyperfine interaction, as explained above. A doubling of *scalar* states with the D_s^+ quantum numbers would be a definite "smoking gun" evidence of tetraquarks.

With the value of A fixed at 108 MeV in the aforementioned manner in our simple model, we find the open-charm (C = 1) tetraquark spectrum shown in Fig. 2 and tabulated in Tables II, III, and IV,.

Most dramatically, several exotics from the $\overline{\mathbf{15}}$-plet are close to being bound;, e.g., the single-strangeness exotic isotriplet $T(cq\bar{s}\bar{q})_{I=1}(2392) \subset \overline{\mathbf{15}}$ is very close to being bound. For example, the double-charge $T(cq\bar{s}\bar{q})_{I=1}^{++}(2392)$ is forbidden from decaying into $\eta + D_s^+$ by charge and isospin conservation, so only the $K + D$ threshold at 2365 MeV is relevant. But in that regard this state does not differ from the $D_s^*(2460)$ which is also above the threshold and yet it has been observed. In other words, the $T(cq\bar{s}\bar{q})_{I=1}^{++} \times (2392) \subset \overline{\mathbf{15}}$ might be observable. By the same token, the $T(cq\bar{s}\bar{q})_{I=1}^{+}(2392)$ state, even though only singly charged, could be detectable by way of its unique decay signature of an antikaon in the final state. The same remarks, though to a lesser extent due to the higher mass, also hold for their isosinglet partner $T(cs\bar{q}\bar{q})_{I=0}^{0}(2468) \subset \mathbf{6}$.

Moreover, the doubly strange isodoublet of states $T(cq\bar{s}\bar{s})_{I=1/2}^{+,++}(2529) \subset \overline{\mathbf{15}}$ are also relatively close to their

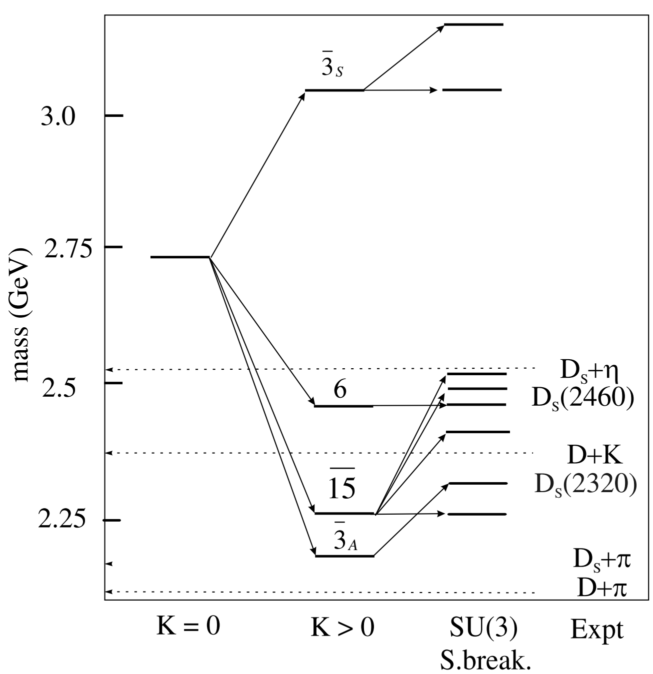

FIG. 2. Open-charm (C = 1) tetraquark mass spectrum as a function of the 't Hooft coupling constant K and $SU_F(3)$ symmetry breaking. Dashed horizontal lines with arrows indicate the corresponding thresholds.

[4]One may, if one wishes, replace in all the mass formulas the pseudoscalar meson masses with the weighted average masses of corresponding vector and pseudoscalar mesons, but that would merely lift the two-meson thresholds without otherwise affecting our results. In this sense our results may be viewed as a conservative limit on the actual values.

(only) decay threshold $K + D_s^+$ at 2465 MeV. They might be observable due to their double-strangeness signature.

Next we consider two cases with charm different from one: (1) Note that in the hidden charm ($c\bar{c}$) nonet, the masses of tetraquarks are not changed by the 't Hooft interaction as compared with the two asymptotic mesons. In other words, whatever mass shifts occur in the asymptotic two-meson states due to the 't Hooft interaction, they also happen in the tetraquarks, so we do not expect to find any extra attraction in this channel; therefore no new (bound) states in this channel.

(2) Finally we are left with double-charm (C = 2) tetraquark states as the last option; they come in two varieties: antisextet and triplet ($\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \bar{\mathbf{6}} \oplus \mathbf{3}$). Only the latter is susceptible to the 't Hooft interaction (see comments below Eq. (4)): the spin singlet mass is shifted by

$$\delta E(\mathbf{3}_{cc}) = 16K\langle\bar{q}q\rangle_0 I \equiv -4A \quad (19)$$

$$\delta E(\bar{\mathbf{6}}_{cc}) = 0. \quad (20)$$

The two heavier (strange) states in the triplet, $cc\bar{u}\,\bar{s}$ and $cc\bar{d}\,\bar{s}$, are more likely to be bound, as there is only one rather high decay threshold into $D + D_s$ at 3840 MeV, but even the nonstrange state $cc\bar{u}\,\bar{d}$ is likely to be seen below its DD threshold at 3740 MeV. Thus the primary experiment we suggest is the search for double-charm tetraquark states that may also have strangeness.

The double-charm tetraquarks have been candidates for bound states even before including the 't Hooft interaction, see [4,12,17,18]. In particular Pepin et al. [17], have predicted a stable $cc\bar{q}\,\bar{q}$ tetraquark in the presence of a flavor-dependent two-body hyperfine interaction, which however does not reproduce the observed $q\bar{q}$ light meson level ordering. Moreover, Janc has recently shown [12] that when one adds a weak (about 10 MeV) three-body force, that does not change baryon spectra significantly, to the conventional Bhaduri Hamiltonian, this tetraquark becomes bound.

In conclusion, we have looked into the question of tetraquarks with open and hidden charm in the presence of 't Hooft interaction and found that:

(1) The lowest open-charm and strangeness scalar cryptoexotic tetraquark belonging to the mixture of $\bar{\mathbf{3}}_{\mathrm{A}}$, $\bar{\mathbf{3}}_{\mathrm{S}}$, $\overline{\mathbf{15}}$-plets is moved by 't Hooft interaction below both of its strong decay thresholds; the second lowest state is lowered between the two thresholds. The two tetraquarks might be identifiable with the recently observed states $D_s^+(2317)$ and $D_s^+(2460)$.

(2) Four sets of exotica are very close to their decay thresholds and thus might be detectable: the $T(cq\bar{s}\,\bar{q})^{+,+}_{I=1}(2392) \subset \overline{\mathbf{15}}$, the $T(cs\bar{q}\,\bar{q})^{+}_{I=1} \times (2392) \subset \overline{\mathbf{15}}$, the $T(cs\bar{q}\,\bar{q})^{0}_{I=0}(2468) \subset \mathbf{6}$ and the doubly strange isodoublet of states $T(cq\bar{s}\,\bar{s})^{+,++}_{I=1/2}(2529) \subset \overline{\mathbf{15}}$. Each one of them lies above the lowest corresponding threshold, however.

(3) If our interpretation of $D_s^+(2317)$ and $D_s^+(2460)$ as scalar tetraquarks is correct, then the isodoublet of strangeness 1, double-charm members of the flavor triplet tetraquark states ought to be bound by the 't Hooft interaction, with masses (just) below the $D_s + D$ threshold at 3840 MeV, as well as the isosinglet nonstrange double-charm state below 3740 MeV.

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